MOMENTS OF RUNOFF COEFFICIENT AND PEAK DISCHARGE ESTIMATION IN URBAN CATCHMENTS

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Abstract: A study of hydrologic losses in urban catchments is presented, on the basis of rainfall-runoff data of events recorded in 21 urban experimental catchments. Interest was mainly focused on runoff coefficient, usually used in simple conceptual models for discharge estimation in the design of drainage networks. This coefficient shows characteristics that are typical of random variates and that are usually neglected. From analysis of experimental data its probability distribution function was found to be approximately normal and simple relationships for estimation of main moments were developed. These relationships can be used in problems concerning reliability analysis and risk design of drainage networks. A modification of the rational method, taking into account the randomness of runoff coefficient to achieve a correct estimation of risk level of design discharges, is then proposed for urban catchments.

1. Introduction

Estimation of hydrologic losses is a key factor in the design of urban storm drainage networks. In many cases, when small and simple networks are concerned, the design is carried out by means of the so-called *rational formula* $Q = C\ i\ A$, proposed more than a hundred years ago by Kuichling [1889]. The runoff coefficient C, ratio between the peak discharge Q and the inflow rate $i\ A$ averaged on the time of concentration of the catchment, takes simultaneously into account the effects of hydrologic losses and of attenuation of peak flow rates due to the rainfall-runoff transformation. It is often useful to adopt a conceptual scheme in which the two effects are separated, introducing a coefficient φ, ratio between the net and gross rainfall depths evaluated at the end of a storm event, to account only for hydrologic losses, and a "model" coefficient ε, to account for the dynamics of hydrologic response. Following this scheme, the term *runoff coefficient* is here used for the coefficient φ, according to one of its common definitions [Chow et al, 1988]. If the hydrologic losses are considered as a constant percentage of rainfall intensities during the event, as usual in urban catchments where the losses are often related to an invariant rate of pervious area, the rational formula can then be written in the notation:

$$Q = \varphi\ i(t_d)\ \varepsilon A \qquad (1)$$

where Q = peak discharge [m^3/s]
A = catchment area [m^2]
φ = runoff coefficient [-]
ε = coefficient ($\varepsilon \le 1$) depending on the net rainfall-runoff model characteristics[2] [-]
$i(t_d)$ = average rainfall intensity [m/s]
t_d = rainfall intensity averaging time, depending on the net rainfall-runoff model characteristics [s]

In the traditional use of the rational formula it is assumed that, among all the parameters on which Q depends, only the average rainfall intensity $i(t_d)$, supposed uniform on the catchment, is a random variable and, consequently, that Q and $i(t_d)$ have the same probability distribution [McPherson, 1969]. In engineering practice, in fact, the runoff coefficient φ is usually evaluated in a deterministic way on the basis of catchment characteristics and, sometimes, also of rainfall event. Analysis of real events showed, however, that the return periods of rainfall and runoff for individual storm events may be quite different, as pointed out by several researchers in the last decades [see e.g Hiemstra and Reich, 1967]. For this reason, in design problems, relatively high values of φ are often chosen, with the intention of taking into account, in a simplified way, a sort of statistical trend of this coefficient to increase with the importance of the rainfall event [ASCE-WEF, 1992]. The more sophisticated distributed models, on the other hand, usually employ procedures based on infiltration (Horton, SCS-CN, Green-Ampt, Philips, etc.) and depression storage modeling. In this

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[2] If the catchment is supposed to perform as a linear hydrologic system and a constant hyetograph of duration t_p is considered, the coefficient ε is the maximum of the function:

$$\varepsilon(t_p) = \max_{t \ge t_p} \left\{ \int_{t-t_p}^{t} u(\tau) d\tau \right\}$$

where $u(\tau)$ is the IUH (Instantaneous Unit Hydrograph). Under the same hypotheses the design peak discharge $Q_{max}(T)$ for a chosen return period T is defined as the maximum value of Q, corresponding to the duration t_p for which the product $i(t_p)\cdot\varepsilon(t_p)$ is maximum.

way, although being estimated in a deterministic way, the hydrologic losses are made to decrease in percentage with the importance of the rainfall event, while the corresponding values of runoff are made to increase.

The numerous data available nowadays from experimental urban catchments show, however, that a sure correspondence of high values of φ with high values of rainfall depth cannot be identified in real events. This result is surely affected by measurement errors, both of rainfall depths, especially due to the uncertainty of its spatial distribution, and of flow measurements in sewers. However, the scatter of data seems too high to be explained only in this way (see Fig. 1). It seems, then, that the values of the runoff coefficient are significantly influenced by factors which are independent from rainfall characteristics. Some of these factors are known, even though of difficult determination, as the antecedent moisture conditions of catchment surfaces, some are probably unknown, as, for instance, the effective infiltration rate in urban surfaces usually defined as "impervious". It is then clear that the great variability of runoff coefficient values can't be satisfactorily explained inside a deterministic framework and that is more convenient to consider this coefficient as a random variable.

Even though this hypothesis has been suggested by many authors in the last forty years [Chow,1957 and 1959; Schaake et al., 1967; Becciu and Paoletti, 1994 and 1997], being the natural consequence of recognizing the random nature of the conditions of the catchment, to which the hydrologic losses are related, the randomness of the runoff coefficient is usually ignored or empirically taken into account in engineering practice. This approach is not justified both because it can result in significant errors in estimating the risk related to design discharges and because the increase of complication is really small, being sufficient to estimate the main moments of φ. In this paper the rainfall-runoff data of 319 events recorded in 21 experimental urban catchments set up in 10 countries of Europe and America are analyzed to get simple relationships for the estimation of moments μ_φ and σ_φ of runoff coefficient. A new procedure for the application of the rational method, using these relationships, is then proposed.

2. Hydrologic losses in urban catchments.

In Table 1 the main characteristics of the catchments and events used in this study are reported. For 12 urban catchments data were taken from Maksimovic and Radojkovic [1986], for other 8 (Italian) from Calomino and Paoletti [1994]. The data of the Baggio catchment in the city of Milano, Italy, which was recently set up by the Authors, are yet unpublished.

As can be seen the experimental values of runoff coefficient φ vary greatly for the same catchment and are almost always less than its impermeability ratio *Imp*, defined as the ratio between the so-called impermeable areas connected to the drainage system and the overall area (see Fig. 1). Moreover, the runoff coefficient φ is often well below *Imp* even when the rainfall event is characterised by significant periods of high intensity and by a considerable total rainfall depth, meaning that losses in "impermeable" areas are not negligible as usually assumed. It has to be noted that the estimation of *Imp* may be very uncertain, being related to the accuracy of survey of surfaces characteristics, including their effective connection to the drainage network. The levels of this accuracy are probably different for the 21 catchments considered in this study, even if they should be high in any case, due to the fact that these catchments are experimental sites.

It has to be noted that while the values of runoff coefficient are so variable and seem independent from rainfall characteristics, the observed values of the hydrologic loss *L*, defined as the part of rainfall depth *h* that does not produce runoff in the drainage system, show a certain relation to the characteristics of both the catchment and the rainfall (see Fig. 2). It seems more convenient, then, to search firstly for an estimation model of *L*. The relevant number of factors affecting the hydrologic losses makes it very difficult to achieve an estimation by a detailed analysis of each of them, also taking into account the great difficulty to get, in the practical applications, all the data that are needed. The estimation problem has, then, to be afforded at the catchment scale, starting from general considerations on the nature of the phenomenon.

Firstly, it can be noted that hydrologic losses show a different behavior during rainfall, so that often the initial losses are distinguished from those happening afterwards. In the initial phase of a meteoric event, rainfall is almost completely lost to evaporation, filling of depression storages and wetting of surfaces; consequently the runoff amounts are almost to zero. Afterwards, a second phase begins, in which infiltration into the ground becomes prevalent. In this phase, the losses go on generally increasing, but with decreasing increments, for the infiltration velocity decreases tending to an asymptotic limit.

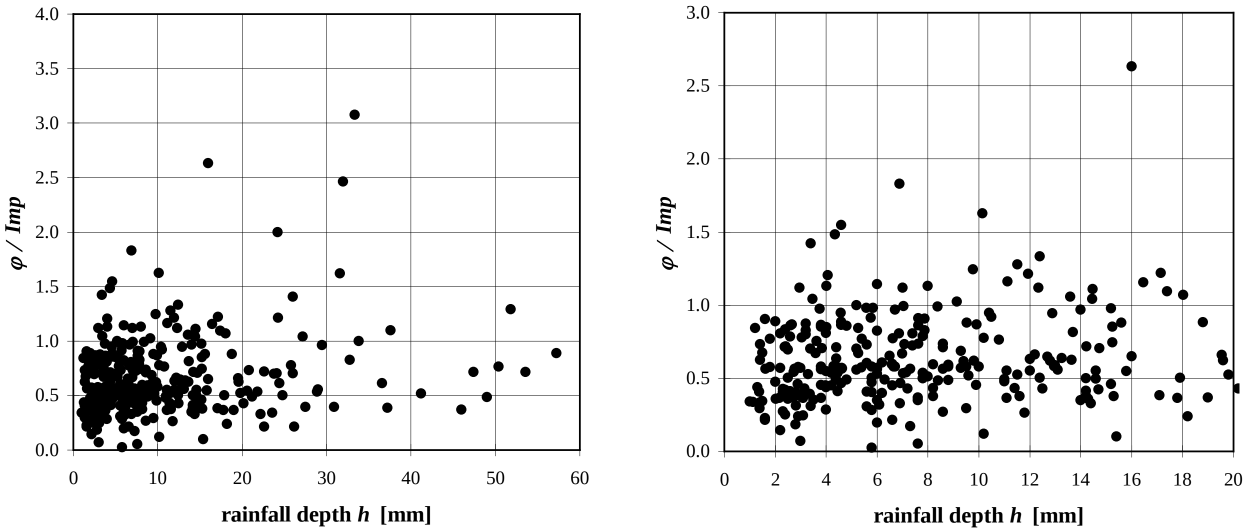
According to this scheme and also to the general trend of the experimental data shown in Fig. 2, the hydrologic balance of a rainfall event seems to be expressible, in terms of loss depth *L*, by equations of the following form:

$$L = h \qquad \text{if } h \le L_o$$
$$L = L_o + \alpha\, h^{\beta} \qquad \text{if } h > L_o \qquad (2)$$

where L_o is the maximum value of initial losses, *h* is the total rainfall depth and the term αh^{β} corresponds to the losses in the second phase (the dimensions of all terms are [L]). The observed trend of experimental data and also known considerations on the progressive soil saturation, suggest that the exponent β should have a value lower than one.

Catchment name	*Catchment area A [ha]*	*Impermeability ratio Imp*	*n° events*	*Rainfall depth range [mm]*	*φ range*
Luzzi (1) (2)	1.89	0.913	25	1.4 – 22.6	0.46 – 0.81
Parco d'Orleans (2)	14.29	0.700	14	2.4 – 30.9	0.18 – 0.34
Malvaccaro (2)	8.10	0.850	23	3.1 – 23.8	0.31 – 0.69
Cascina Scala (2)	11.35	0.650	32	4.6 – 53.6	0.17 – 0.73
Mulinu Becciu (2)	13.34	0.444	6	4.0 – 8.6	0.24 – 0.36
Fossolo (2)	40.71	0.748	9	2.8 – 87.4	0.04 – 0.39
Merate (2)	21.90	0.420	31	1.4 – 62.8	0.01 – 0.84
Casal Palocco (2)	28.21	0.380	10	3.4 – 50.4	0.17 – 0.54
Baggio (3)	199.44	0.291	7	11.0 – 49.0	0.10 – 0.14
Malvern (1)	23.33	0.338	24	3.0 – 37.6	0.22 – 0.41
East York (1)	155.84	0.393	13	1.5 – 24.3	0.21 – 0.48
Pompano Beach (1)	16.49	0.059	6	7.0 – 33.3	0.04 – 0.18
Sample Road (1)	22.96	0.186	6	6.5 – 57.2	0.12 – 0.24
Munkerisparken (1)	6.42	0.318	8	2.6 – 14.5	0.28 – 0.36
Livry Gargan (1)	253.50	0.326	38	1.5 – 28.9	0.06 – 0.28
Clifton Grove (1)	10.60	0.403	19	1.0 – 6.7	0.13 – 0.23
St. Marks Road (1)	7.32	0.456	14	2.4 – 13.6	0.21 – 0.35
Miskolc (1)	25.24	0.158	6	4.3 – 26.0	0.15 – 0.41
Vika (1)	9.90	0.965	13	1.2 – 14.2	0.54 – 0.87
Porsoberg (1)	13.00	0.397	7	1.4 – 11.0	0.12 – 0.21
Miljakovac (1)	25.50	0.349	8	2.6 – 19.5	0.13 – 0.25

Table 1 - Main characteristics of experimental catchments and events used in the study: (1) from Maksimovic and Radojkovic [1986]; (2) from Calomino and Paoletti [1994]; (3) catchment recently set up in the city of Milano.



a) b)

Fig. 1 - Observed values of ratio between runoff coefficient φ and percentage of impermeable areas *Imp* in the experimental catchments of Table 1: a) all events; b) blow up of graph for events with $h \leq 20$ mm.

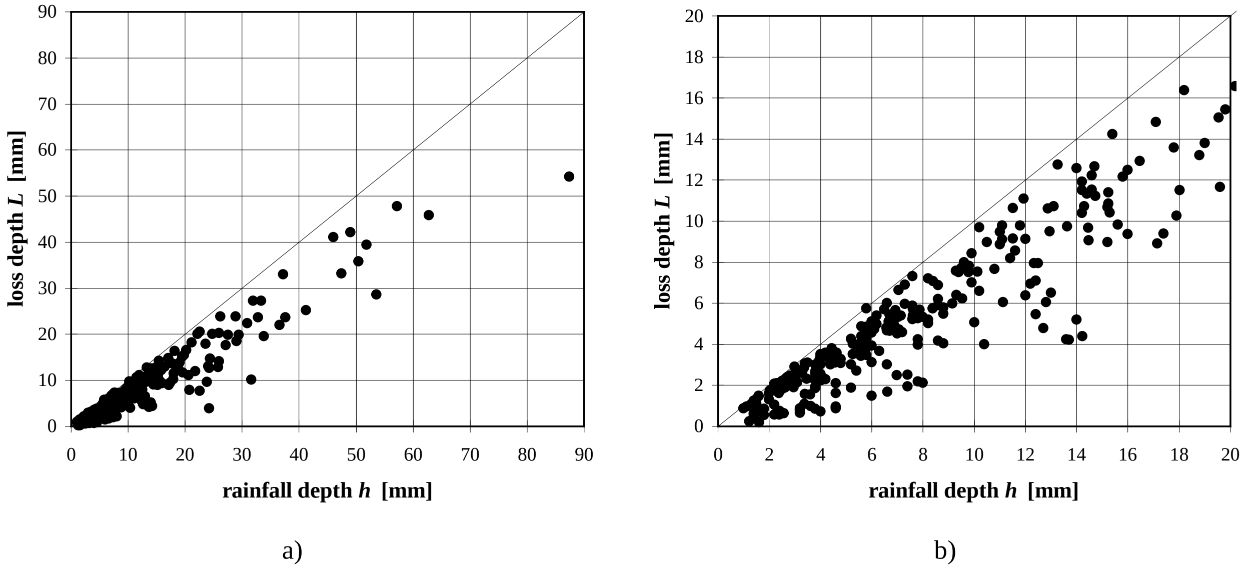


Fig. 2 - Observed values of loss depth in the experimental catchments of Table 1: a) all events; b) blow up of graph for events with $h \leq 20$ mm.

Before going on with the estimation of parameters L_o, α and β, it has to be highlighted that the term αh^{β}, even if it has been stated empirically, approximates well the total infiltration losses estimated by any Hortonian model, provided that the rainfall intensity is always greater than the infiltrability. In fact, from a well-known approximated solution of Richards equation on monodimensional vertical percolation flow in unsaturated porous media, the infiltrability f can be expressed as [Philip, 1969]:

$$f(t) = \frac{1}{2}\, s\, t^{-1/2} + C \qquad (3)$$

where s is the so-called *sorptivity*, having dimensions [$L \cdot T^{-1/2}$], and C is a factor that, but for very small times t, is almost equal to the hydraulic conductivity K of the soil and has dimensions [$L \cdot T^{-1}$]. The integral of equation (3), that is

$$L(t_p) = s\, t_p^{\,1/2} + K\, t_p\,, \qquad (4)$$

expresses the depth L globally infiltrated at the end of a rainfall event of duration t_p and intensity always greater than infiltrability. If the well known relationship $h = a\, t_p^{\,n}$, between the duration t_p and the total rainfall depth h, is assumed, the equation (4) can be written in the form:

$$L(h) = a^{-1/(2n)}\, S h^{1/(2n)} + a^{-1/n}\, K h^{1/n} \qquad (5)$$

which can be very well fitted, for whatever set of values of the parameters a, n, S and K, by the equation $L = \alpha h^{\beta}$.

Analysing the experimental data, a very clear functional relationship between loss depth L and rainfall depth h, of the type of equation (2) can be observed in all catchments (see Fig. 3). In some cases a certain scatter of data around this theoretical trend was observed, so equation (2) really explains an average behavior, that is expresses the relationship between the mean loss depth and the rainfall depth. Then, it can be written that:

$$\mu_L = L_o + \alpha h^{\beta} \qquad (6)$$

The optimal values, in terms of minimization of square errors, for the parameters L_o, α and β, are reported in Table 2, together with the values of correlation coefficient r^2 that are in all cases significantly high ($0.839 \leq r^2 \leq 0.999$). In Fig. 3 the best and worst correlation cases are shown.

These first results prove that, at least in the range of experimental data, the model expressed by equation (6) can be regarded as sound. Nevertheless, the found values of parameter β, even if always not much different from unity, are in some cases greater than one, in contrast with the above made considerations. The reason of this inconsistency has to be searched in the set of data available for the parameter calibration: many of them are extracted from medium-small events and only few from important ones for which the concavity of the L–h relationship is expected to show itself. To increase the weight of greater events, the analysis was then repeated on the subset of 111 events with at least 10 mm of total rainfall depth, obtaining the results reported in the last four columns of Table 2. In this analysis were excluded the data of the catchments in which had been recorded less than three events with the chosen characteristics (at least 10 mm of total rainfall depth). Results of this second analysis show, in most cases, a reduction of correlation coefficient r^2, due to the greater scatter of the observed points (L , h) when events with greater h are considered (see, for example, the case of Malvaccaro catchment in Fig. 3). This fact can be explained with the reduction of the number of events considered in the analysis, in some cases significant (e.g., for East York catchment, the number of events changes from 13 to 4). In fact, it is known that the average deviation of a random variable from its mean shows an increment when the dimension of the sample is reduced [Kottegoda & Rosso, 1997]. Also the strange value of $\beta = 1.40$ for St. Marks Road catchment is to ascribe to the small number of available data: in this case only three events, two of which with very similar characteristics, satisfy the condition of $h \geq 10$ mm and so were used for the fitting of equation (6).

	All events				*Events with h ≥ 10 mm*			
Name of the catchment	L_O *[mm]*	α	β	r^2	L_O *[mm]*	α	β	r^2
Luzzi	0.00	0.30	1.07	0.871				
Parco d'Orleans	0.00	0.81	0.96	0.998	0.00	0.78	0.98	0.998
Malvaccaro	0.00	0.70	0.88	0.839	0.00	0.75	0.85	0.548
Cascina Scala	0.00	0.72	0.93	0.920	0.00	0.57	1.00	0.738
Mulinu Becciu	0.00	0.71	0.99	0.979				
Fossolo	0.00	1.05	0.88	0.993	4.90	0.52	1.00	0.978
Merate	0.15	0.79	0.92	0.858	0.00	0.93	0.87	0.457
Casal Palocco	0.00	0.60	1.08	0.992	0.00	0.82	0.98	0.989
Baggio	0.00	0.95	0.98	0.999	0.00	0.95	0.98	0.999
Malvern	0.00	0.71	0.99	0.987	0.00	0.90	0.91	0.953
East York	0.15	0.63	0.96	0.946	0.15	1.31	0.70	0.690
Pompano Beach	0.00	1.16	0.91	0.996	0.00	1.25	0.89	0.997
Sample Road	0.50	0.74	1.01	0.995	0.50	0.74	1.02	0.997
Munkerisparken	0.00	0.70	0.98	0.997				
Livry Gargan	0.10	0.82	1.00	0.997	0.10	0.87	0.98	0.993
Clifton Grove	0.00	0.87	0.94	0.994				
St. Marks Road	0.00	0.75	0.97	0.986	0.00	0.25	1.40	0.988
Miskolc	0.50	0.62	1.04	0.951	0.50	0.76	0.97	0.871
Vika	0.00	0.21	1.19	0.941				
Porsoberg	0.20	0.74	1.03	0.999				
Miljakovac	0.35	0.64	1.07	0.992				

Table 2 - Optimal values of parameters of equation (6).

Taking into account that equation (6) is intended to express an average behaviour of the phenomenon, it seems acceptable to assume, in the range of the experimental data, a linear approximation of it (β =1). Moreover, observing that the values of initial losses L_o shown in Table 2 are always very small and in most of cases null, it is also assumed that, within the above mentioned intent and limits, $L_o = 0$. Then, the mean total depth of losses can be expressed by the very simple linear relationship:

$$\mu_L = \alpha h \tag{7}$$

According to this very simplified model, the new values of parameter α were estimated again. These values are shown in Table 3, together with the mean deviation between the model and the experimental data. In Fig. 3 are shown, for comparison, the functions (6) and (7) for the two catchments having the best and worst fitting for eq. (6) with all events. As it can be seen the results are almost as good as those obtained with equation (6) and are very similar both when all events and events with h≥10 mm are taken into account. The values of α are variable, but are always less than unity, as it was expected because it must be $\mu_L \leq h$.

	All events		*Events with h ≥ 10 mm*	
Name of catchment	α	*mean square error [mm]*	α	*mean square error [mm]*
Luzzi	0.36	0.92		
Parco d'Orleans	0.73	2.37	0.73	1.63
Malvaccaro	0.51	1.32	0.50	2.22
Cascina Scala	0.58	1.78	0.58	4.26
Mulinu Becciu	0.70	0.17		
Fossolo	0.63	3.59	0.63	5.13
Merate	0.67	3.86	0.67	1.14
Casal Palocco	0.74	0.92	0.74	3.89
Baggio	0.89	3.45	0.89	1.81
Malvern	0.68	1.35	0.68	3.09
East York	0.57	1.79	0.55	1.02
Pompano Beach	0.85	1.15	0.85	1.49
Sample Road	0.80	1.43	0.80	1.51
Munkerisparken	0.66	0.80		
Livry Gargan	0.83	0.74	0.83	0.46
Clifton Grove	0.79	0.29		
St. Marks Road	0.71	0.33	0.71	1.67
Miskolc	0.74	1.85	0.74	1.64
Vika	0.33	0.79		
Porsoberg	0.81	0.51		
Miljakovac	0.81	0.80		

Table 3 - Optimal values of parameters α of equation (7) and mean square error of fitting.

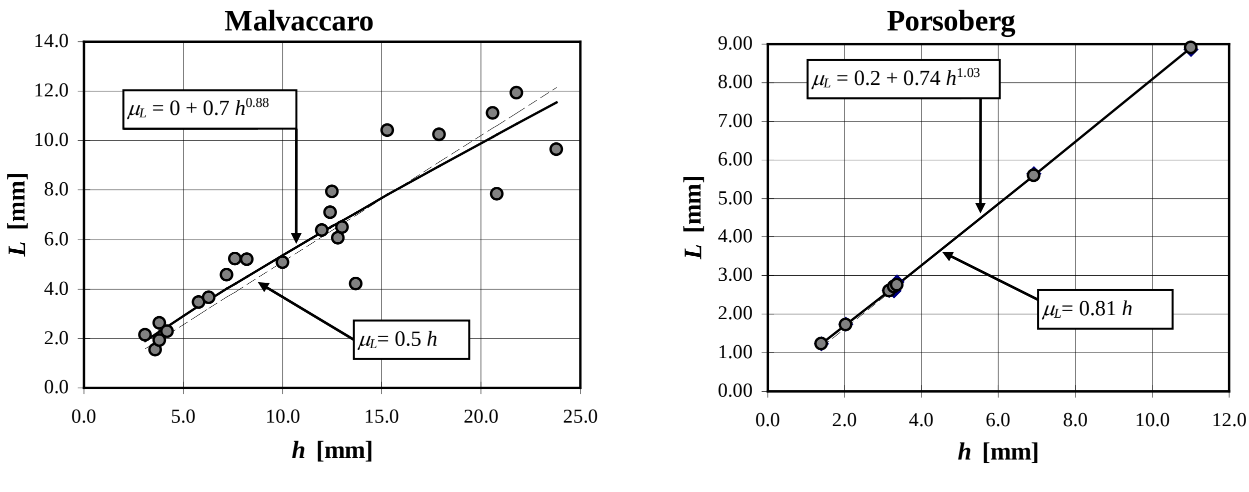


Fig. 3 - Relationships between *L* and *h* (equations (6) and (7)) for the catchments of Malvaccaro (worst case) and Porsoberg (best case).

A relationship between α and the characteristics of the catchments was searched, finding a clear correlation with the impermeability ratio *Imp* (Fig. 4). From regression analysis, the relationships were found:

$$\alpha = 0.92 - 0.49 Imp \qquad r^2 = 0.673 \qquad \text{(all events)} \qquad (8a)$$

$$\alpha = 0.87 - 0.36 Imp \qquad r^2 = 0.505 \qquad \text{(events with } h \geq 10 \text{ mm)} \qquad (8b)$$

In conclusion, equations (7) and (8) can be combined to give the relationships

$\mu_L = (0.92 - 0.49Imp)h$ [mm] (all events) (9a)

$\mu_L = (0.87 - 0.36Imp)h$ [mm] (events with $h \geq 10$ mm) (9b)

From these equations it can be deduced that the hydrologic losses in percentage of the rainfall depth, range, on average, in the two cases, between 43% and 51% in the paved areas (*Imp*=1) and between 87% and 92% in the permeable areas (*Imp*=0). The average value of the standard errors of rainfall losses from the mean expressed by equations (9), evaluated in each catchment, is 1.44 mm in the case of all events and 2.23 mm in the case of events with $h \geq 10$ mm. In Fig. 5 the comparison, for the data of all catchments, between the observed values of L and μ_L, estimated with equations (9), is reported.

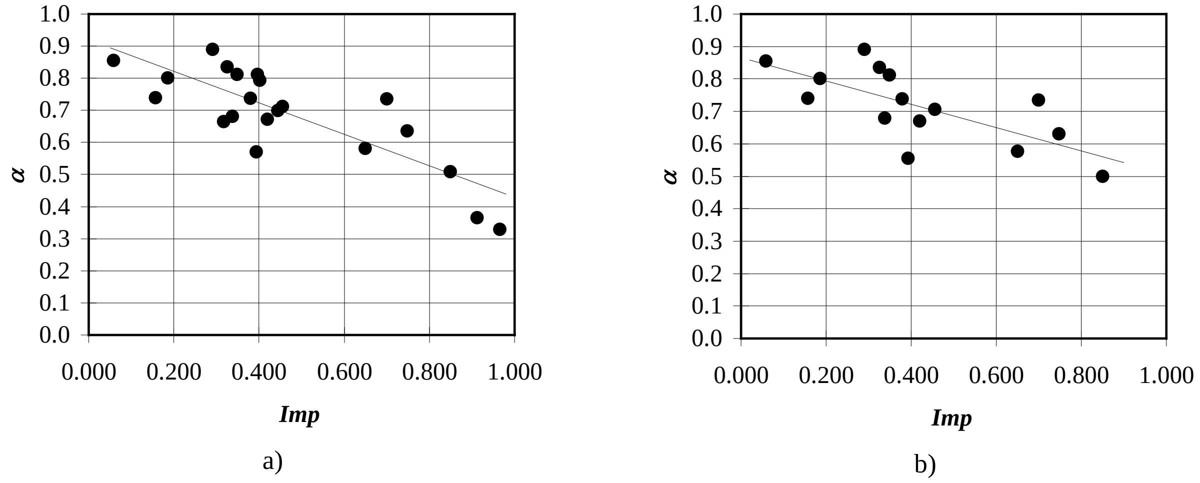


Fig. 4 - Regression between observed values of α and *Imp*: a) all events; b) events with $h \geq 10$ mm.

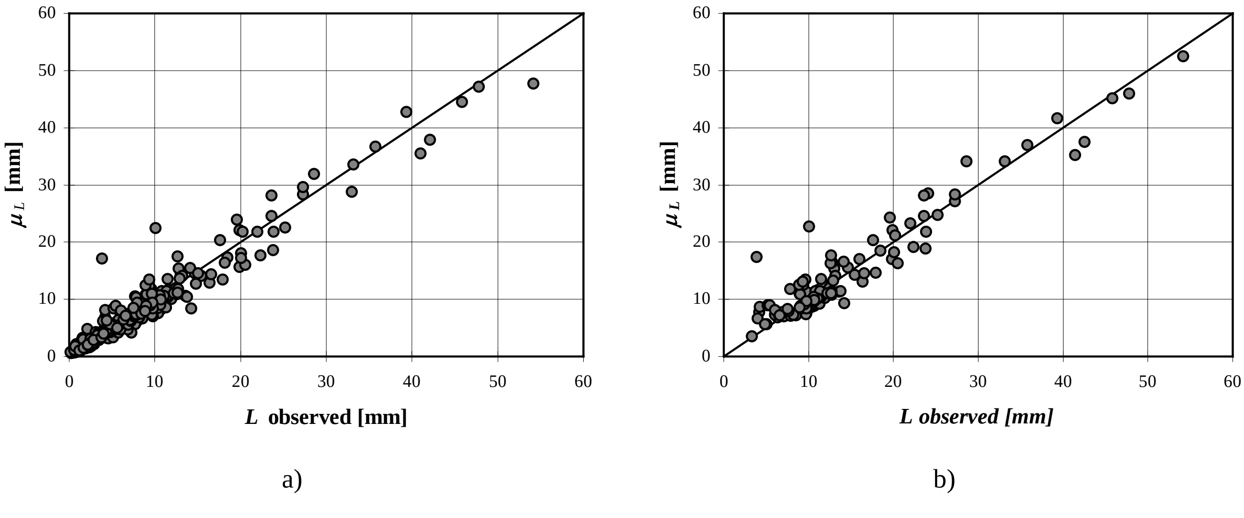


Fig. 5 - Comparison between the observed values of L and μ_L, estimated with equations (9): a) all events; b) events with $h \geq 10$ mm.

In conclusion, it can be noted that these results show that, on average, the hydrologic losses in the so-called impermeable areas are significant and that the permeable areas give their contribution to the discharges in the drainage network. Moreover, it can be noted that the differences between the results obtained in the case with all the events and in that with events $h \geq 10$ mm are not so significant.

3. *Probability distribution and moments of runoff coefficient*

Empirical probability histograms of runoff coefficient are approximately bell-shaped. The hypothesis of normal probability distribution was tested with the Pearson's goodness-of-fit test. Five classes were chosen for catchments with at least 10 events and four classes for the others, obtaining that the hypothesis was not rejected at the 0.05 level of significance for all catchments. In Table 4 the main sample statistics of runoff coefficient and the non-exceedance probability of the statistic X^2 are reported[3]. In Fig. 6 the theoretical and observed probability density functions of runoff coefficient for the cathments of East York and Vika, the two cases of lower and higher value of the function $F(X^2)$, are reported.

Catchment	*Mean obs. φ*	*Standard deviation of obs. φ.*	*Skewness obs. φ.*	*F(X²)*
Luzzi	0.655	0.113	-0.313	0.95
Parco d'Orleans	0.240	0.048	0.199	0.63
Malvaccaro	0.450	0.104	0.562	0.29
Cascina Scala	0.378	0.143	0.877	0.64
Mulinu Becciu	0.297	0.041	0.317	0.59
Fossolo	0.206	0.123	0.362	0.62
Merate	0.276	0.211	0.948	0.42
Casal Palocco	0.262	0.107	2.246	0.78
Baggio	0.120	0.020	0.124	0.90
Malvern	0.305	0.047	0.319	0.51
East York	0.366	0.085	-0.299	0.96
Pompano Beach	0.095	0.056	0.894	0.93
Sample Road	0.191	0.046	-0.542	0.68
Munkerisparken	0.328	0.029	-0.819	0.78
Livry Gargan	0.152	0.042	0.305	0.78
Clifton Grove	0.184	0.037	0.012	0.54
St. Marks Road	0.281	0.044	0.223	0.81
Miskolc	0.260	0.090	0.893	0.84
Vika	0.704	0.100	-0.099	0.18
Porsoberg	0.168	0.032	-0.374	0.60
Miljakovac	0.190	0.045	-0.075	0.92
Average values	*0.291*	*0.074*	*0.274*	*0.684*

Table 4 – Mean sample statistics of runoff coefficient and non-exceedance probability of X^2 statistic (Pearson's goodness-of-fit test).

As known, the normal distribution is completely defined by the two main moments μ and σ^2, that is the mean and the variance. Taking into account that between the mean of runoff coefficient and the mean of loss depth holds the relationship

$$\mu_\varphi = 1 - \frac{\mu_L}{h} = 1 - \alpha \quad , \tag{10}$$

it can be obtained, from equations (9), that

$$\mu_\varphi = 0.08 + 0.49 Imp = 0.57 Imp + 0.08\,(1 - Imp) \qquad \text{(all events)} \tag{11a}$$

$$\mu_\varphi = 0.13 + 0.36 Imp = 0.49 Imp + 0.13\,(1 - Imp) \qquad \text{(events with } h \geq 10 \text{ mm)} \tag{11b}$$

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[3] As known, the variate

$$X^2 = \sum_{i=1}^{k} \frac{(o_i - e_i)^2}{e_i}$$

where k is the number of classes and o_i and e_i are the observed and expected frequency for the ith class, has a Chi-square distribution with k – s – 1 degrees of freedom, where s is the number of parameters of the distribution estimated from the sample.

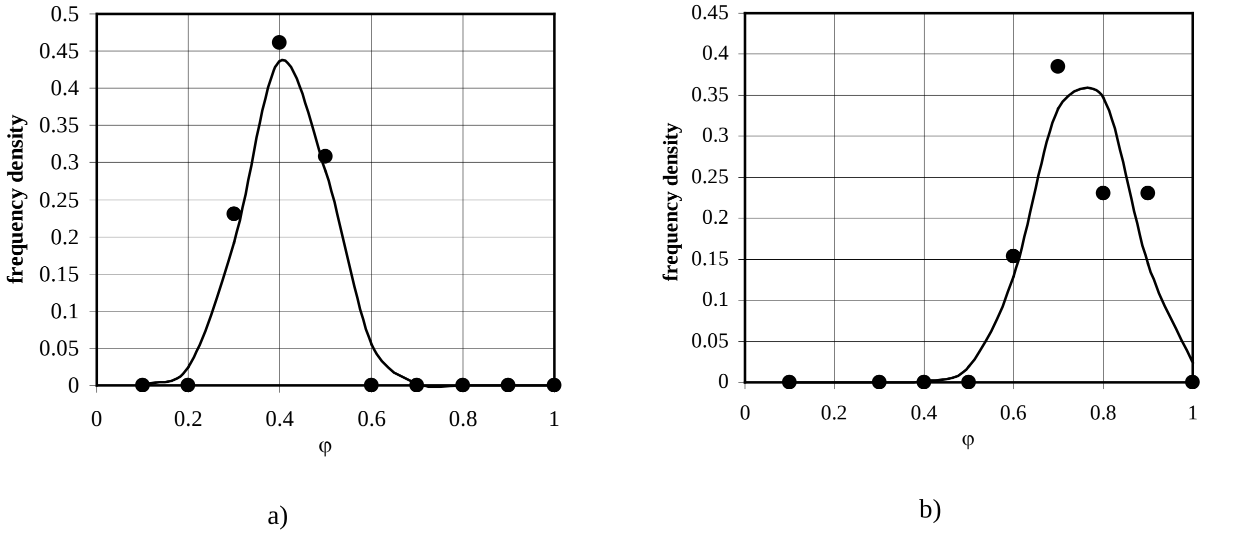

Fig. 6 - Theoretical and observed probability density functions of runoff coefficients for the catchments of East York (a) and Vika (b).

In other words, the mean runoff coefficient of paved areas φ_{imp} ranges between 0.49 and 0.57, while the same coefficient φ_{per} for the pervious areas ranges between 0.08 and 0.13. From the analysis of the deviations between observed values of the runoff coefficient and their mean estimated by these relationships, average values of 0.118 (for all 319 data) and 0.111 (for event with $h \geq 10$ mm) were obtained. It can be noted that these deviations increase with the impermeability ratio, as can be seen in Fig. 7. The dispersion of the values is, however, considerable. In Table 5 the values of the coefficient of variation for each catchment, that is the ratio between the mean standard error of estimation (evaluated as the mean square difference between the observed values of the runoff coefficient and their estimated mean μ_φ) and the estimated μ_φ, are reported: as can be seen in both cases the mean value of this coefficient is about CV_φ = 0.40. Taking into account the average CV_φ values from Table 5, the following relationships can be deduced (Fig. 7):

$$\sigma_\varphi = 0.39(0.08 + 0.49Imp) = 0.03 + 0.20Imp = 0.23Imp + 0.03\,(1 - Imp) \qquad \text{(all events)} \qquad (12a)$$

$$\sigma_\varphi = 0.40(0.13 + 0.36Imp) = 0.05 + 0.14Imp = 0.19Imp + 0.05\,(1 - Imp) \qquad \text{(events with } h \geq 10 \text{ mm)} \qquad (12b)$$

In Fig. 8 the comparison, for the data of all events, between the observed values of φ and μ_φ, estimated with equations (11), together with the confidence limits of 95%, that is the lines of equation $\mu_\varphi \pm 1.96\sigma_\varphi$, is reported.

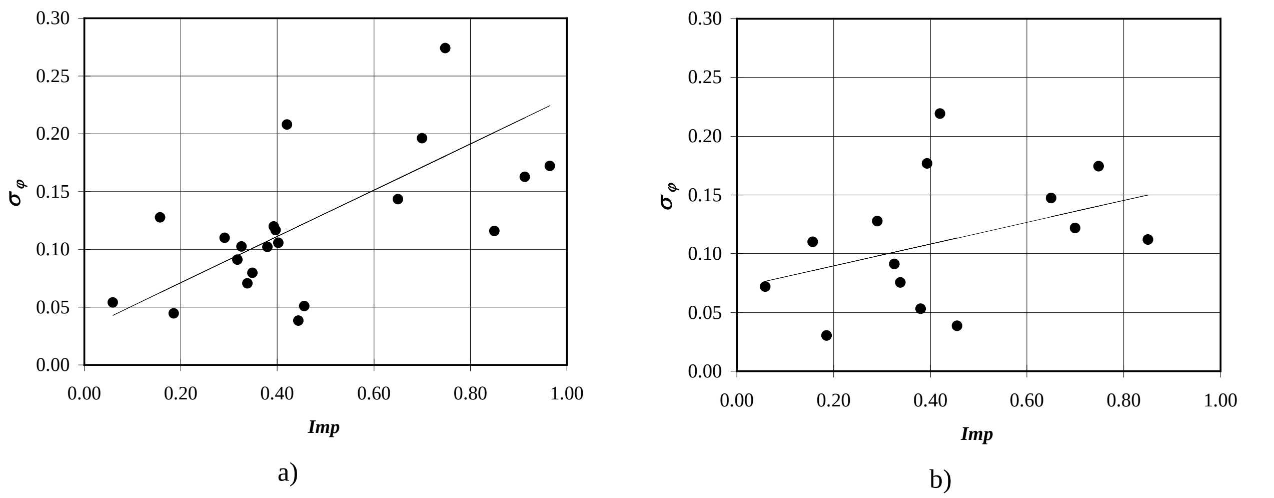

Fig. 7 - Estimated and observed standard deviations of runoff coefficient: a) all events; b) events with $h \geq 10$ mm.

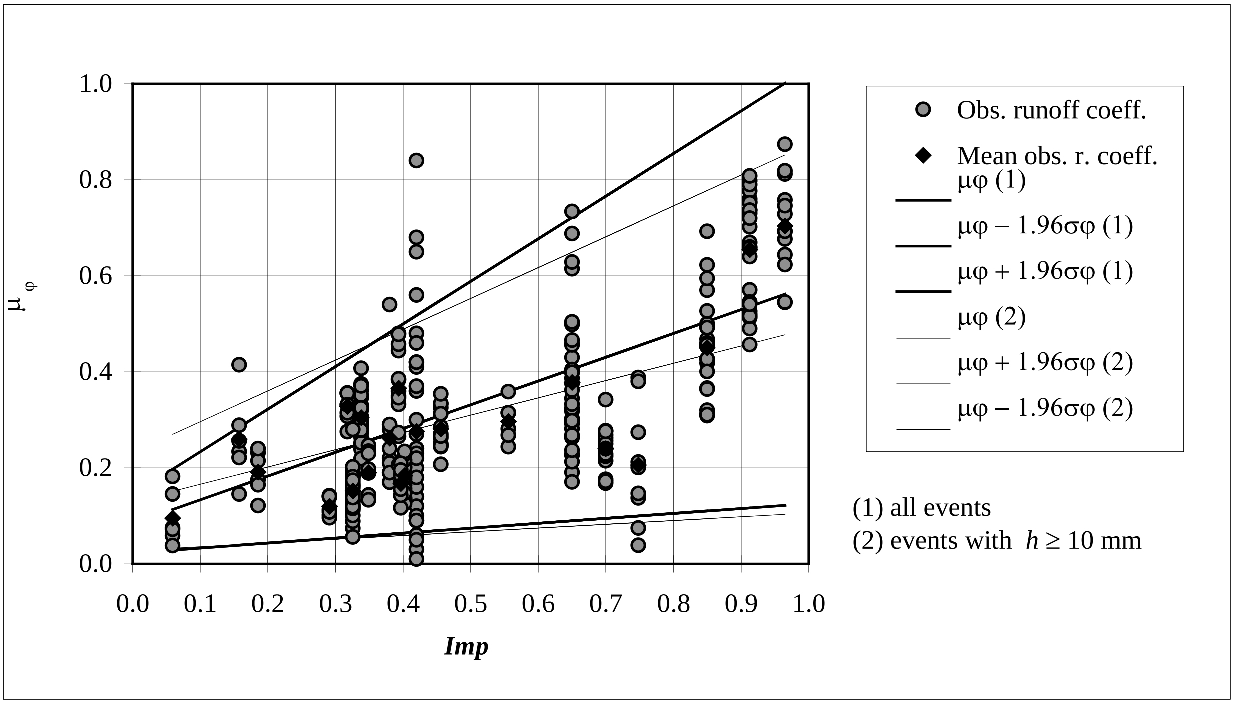

Fig. 8 - Observed values of runoff coefficient, with their means for each catchment, and relationships between estimated μ_φ, with their confidence limits (95%).

	All events	*Events with h ≥ 10 mm*
Name of catchment	CV_φ	CV_φ
Luzzi	0.303	
Parco d'Orleans	0.456	0.318
Malvaccaro	0.229	0.256
Cascina Scala	0.353	0.405
Mulinu Becciu	0.125	
Fossolo	0.604	0.436
Merate	0.712	0.779
Casal Palocco	0.375	0.198
Baggio	0.481	0.544
Malvern	0.281	0.299
East York	0.430	0.650
Pompano Beach	0.478	0.476
Sample Road	0.252	0.154
Munkerisparken	0.377	
Livry Gargan	0.417	0.368
Clifton Grove	0.373	
St. Marks Road	0.164	0.131
Miskolc	0.788	0.588
Vika	0.306	
Porsoberg	0.416	
Miljakovac	0.309	
Average	0.392	0.400

Table 5 - Values of coefficients of variation.

4. Estimation of design discharges

Considering both rainfall intensity and runoff coefficient φ as random variables, equation (1) should be written in the following more proper form:

$$Q(T) = A\, \varepsilon\, q(T) \tag{1b}$$

where $q = \varphi \cdot i(t_d)$ is a product of two random variables and T is the return period of peak discharge. According to the general expression of moments of a function of random variables [Kottegoda and Rosso, 1997], the main moments of the peak discharge can be expressed as functions of the corresponding moments of $i(\theta)$ and φ, obtaining the following equations:

$$\mu_Q = \varepsilon A \left(\mu_i \mu_\varphi + \rho_{i\varphi} \sigma_i \sigma_\varphi \right) \tag{13a}$$

$$\begin{aligned} \sigma_Q^2 = \varepsilon^2 A^2 \Big\{ & \mu_i^2 \sigma_\varphi^2 + \mu_\varphi^2 \sigma_i^2 + 2\rho_{i\varphi} \mu_i \mu_\varphi \sigma_i \sigma_\varphi - \rho_{i\varphi}^2 \sigma_i^2 \sigma_\varphi^2 + E\left[(i-\mu_i)^2 (\varphi-\mu_\varphi)^2 \right] + \\ & + 2\mu_i E\left[(i-\mu_i)(\varphi-\mu_\varphi)^2 \right] + 2\mu_\varphi E\left[(i-\mu_i)^2 (\varphi-\mu_\varphi) \right] \Big\} \end{aligned} \tag{13b}$$

where μ_Q, μ_i, μ_φ and σ_Q, σ_i, σ_φ are the means and the standard deviations of peak discharge, rainfall intensity and runoff coefficient, $\rho_{i\varphi}$ is the correlation coefficient between $i(t_d)$ and φ and E[..] is an operator denoting the average value. If φ and $i(t_d)$ are supposed to be independent, that is $\rho_{i\varphi} = 0$, equations (13) can be written in the following simpler form:

$$\mu_Q = \varepsilon\, A\, \mu_i\, \mu_\varphi \tag{14a}$$

$$\sigma_Q^2 = \varepsilon^2 A^2 \left(\mu_i^2 \sigma_\varphi^2 + \mu_\varphi^2 \sigma_i^2 + \sigma_i^2 \sigma_\varphi^2 \right) \tag{14b}$$

With regards to the correlation coefficient $\rho_{i\varphi}$ in equations (13), it can be pointed out that the analysis of real events in urban catchments shows a very low value of this coefficient [Chow, 1957; Shaake et al., 1967, Merkle, 1968]. In Table 6 the correlation coefficients $\rho_{i\varphi}$ for the averaging times of 10, 15, 20, 25 minutes and for the total rainfall duration, calculated for the 32 events recorded in the catchment of Cascina Scala[4], having the larger set of available and complete rainfall-runoff data among the 21 considered catchments, are reported as an example. As can be seen, the $\rho_{i\varphi}$ values are in any case so small that it seems sound to assume no significant correlation. This can probably be explained by the fact that, in urban catchments, the variability of runoff coefficient from one event to another is mainly due to factors, like antecedent conditions of catchment surfaces, which are independent from rainfall characteristics. Similar results are obtained also in natural catchments when rainfall-runoff single events are analyzed. However, if, in these catchments, rainfall-runoff data are analyzed on annual basis, a certain correlation between average values of runoff coefficient and average annual rainfall is sometimes observed, even if also in this case the only significant factor on which runoff coefficient seems to depend is the type of soils [Titmarsh et al., 1995]. If, then, the correlation between runoff coefficient and rainfall intensity is neglected ($\rho_{i\varphi}$=0), equations (14) result, which are more convenient to be used for estimating the moments of peak discharges.

	$\rho_{i\varphi}$
Mean intensities of events	0.101
Maximum intensities of 10 min duration	0.012
Maximum intensities of 15 min duration	0.009
Maximum intensities of 20 min duration	0.009
Maximum intensities of 25 min duration	0.001

Table 6 - Values of coefficients of correlation $\rho_{i\varphi}$ between runoff coefficients and the mean rainfall intensity of the events and between runoff coefficients and the maximum values of rainfall intensity corresponding to the averaging times of 10, 15, 20 and 25 minutes, for the 32 events recorded in the catchment of Cascina Scala.

[4] The concentration time of the catchment is about 19 minutes.

In design problems usually the level of risk is expressed in terms of return period T and annual maxima of peak discharge Q_{max} are considered. In this case the moments of rainfall intensity and runoff coefficient in the right parts of equations (14) have to be referred to the events for which these annual maxima occur. As a detailed knowledge of rainfall intensity and runoff coefficient for a convenient number of events with the annual maxima of peak discharge is usually not available, these moments are normally unknown. It is then of great interest to transform these equations in order to express them in terms of moments μ_{imax} and σ_{imax} of the annual maxima of rainfall intensity, which are usually available, and of moments μ_φ and σ_φ of runoff coefficient relating to all events in the available observation period, expressed in equations (11) and (12).

If the following symbols are introduced

$$K_{\mu_Q} = \frac{\mu_{Qmax}}{\mu_Q} \qquad K_{\sigma_Q} = \frac{\sigma_{Qmax}}{\sigma_Q}$$

$$K_{\mu_i} = \frac{\mu_{imax}}{\mu_i} \qquad K_{\sigma_i} = \frac{\sigma_{imax}}{\sigma_i}$$

$$K_1 = \frac{K_{\mu_Q}}{K_{\mu_i}} \qquad K_2 = \frac{K_{\sigma_Q}}{K_{\sigma_i}} \qquad K_3 = \frac{K_{\sigma_Q}}{K_{\mu_i}}$$

the moments of the annual maxima of peak discharge μ_{Qmax} and σ_{Qmax} can be derived from equations (14) as:

$$\mu_{Qmax} = K_{\mu_Q}\, \mu_Q = K_{\mu_Q}\, \varepsilon\, A\, \mu_i\, \mu_\varphi = K_1\, \varepsilon\, A \mu_{imax}\, \mu_\varphi \tag{15a}$$

$$\sigma^2_{Qmax} = K^2_{\sigma_Q}\, \sigma^2_Q = \varepsilon^2 A^2 \left(K_3^2\, \mu^2_{i\max}\, \sigma^2_\varphi + K_2^2\, \mu^2_\varphi\, \sigma^2_{i\max} + K_2^2\, \sigma^2_{i\max}\, \sigma^2_\varphi \right) \tag{15b}$$

The values of parameters K_1, K_2, K_3 depend on the probability distributions of discharges and rainfall intensities. These distributions are different, but usually are supposed to be of the same type [Eagleson, 1978; Salas, 1993]; in a first approximation, then, it can be assumed a value equal to one for the parameters K_1 and K_2. The parameter K_3 depends also on the characteristics of the stochastic processes of discharges and rainfall intensities. In the hypothesis that both these processes are Poissonian and homogeneous with the same mean number λ of events per year and that the maxima of both peak discharge and rainfall intensity have an Extreme Value of the first type probability distribution function, this parameter can be expressed as :

$$K_3 = \frac{CV_{i\max}}{CV_i} = \frac{CV_{Q\max}}{CV_Q} = \frac{\sqrt{1.645}}{\ln(\lambda) + 0.577} \tag{16}$$

where the $CV = \sigma/\mu$ are the coefficients of variation of the variables. Even if λ can be in theory any real number greater than zero, it seems reasonable for this kind of processes to consider $\lambda \geq 2$. As K_3 decreases with λ from K_3=1 for $\lambda \cong 2$, a conservative hypothesis is to adopt K_3=1 when not enough information is available to perform the estimation of this parameter with the (16). In fact, from equation (15b) it results that σ_{Qmax} increases with K_3. Assuming K_1 and K_2 equal to one, equations (16) can be simplified in the approximated form:

$$\mu_{Q\max} = \varepsilon\, A\, \mu_{i\max}\, \mu_\varphi \tag{17a}$$

$$\sigma^2_{Q\max} = \varepsilon^2 A^2 \left(K_3^2\, \mu^2_{i\max}\, \sigma^2_\varphi + \mu^2_\varphi\, \sigma^2_{i\max} + \sigma^2_{i\max}\, \sigma^2_\varphi \right) \tag{17b}$$

Equations (17) can be used to define the distribution of the annual maxima of peak discharge and then to estimate the design discharge $Q_{max}(T)$. A different formulation of the rational formula is then possible to achieve, according to the following general relationship:

$$Q_{max}(T) = \mu_{Qmax}\; f(T, \sigma_{Qmax}) = \varepsilon\; A\; \mu_{imax}\, \mu_\varphi\; f(T, \sigma_{imax}, \sigma_\varphi) \tag{18}$$

The form of the function f depends on that of the probability distribution the design discharge is supposed to follow. If this form is such that the quantile can be expressed as $\xi(T) = \mu + \sigma K_T$, where K_T is an increasing function of the return period T, and equations (17) are taken into account, equation (18) becomes:

$$Q_{max}(T) = \mu_{Q\max} + \sigma_{Q\max}\, K_T = \varepsilon\, A\, \mu_{i\max}\, \mu_\varphi \left(1 + K_T \sqrt{K_3^2 CV_\varphi^2 + CV_{i\max}^2 + CV_{i\max}^2 CV_\varphi^2}\right) \quad (19)$$

where CV_φ and CV_{imax} are the coefficients of variation of φ and of annual maxima of rainfall intensity. Equation (19) can also be written in the following form:

$$Q_{max}(T) = \varepsilon\, A\, \mu_\varphi\, K_\varphi(T)\; i_{\max}(T) \quad (20)$$

where the coefficient K_φ is given by the equation:

$$K_\varphi = \frac{1 + K_T \sqrt{K_3^2\, CV_\varphi^2 + CV_{i\max}^2 + CV_{i\max}^2 CV_\varphi^2}}{1 + K_T CV_{i\max}} \quad (21)$$

Equations (20) and (21) can be used to estimate the design discharge for a chosen return period once the function K_T is defined, that is once a certain probability distribution is assumed for Q_{max}. For example, if this distribution is the Normal or the Log-Normal, K_T is the standard normal deviate. If the probability distribution of Q_{max} is supposed to be, accordingly to a common practice in hydrology, EV1, K_T becomes the so-called frequency factor and is defined by the relationship [Chow, 1951]:

$$K_T = -0.45 - 0.779 \cdot \ln\left[-\ln\left(1 - \frac{1}{T}\right)\right] \quad (22)$$

In Fig. 9 the relationship (21) between K_φ and CV_{imax} is reported for some return periods T, considering the expression (22) for K_T and assuming, according to the results reported in the previous paragraph, $CV_\varphi = 0.4$ and $K_3 = 1$. As can be seen, equation (20) corresponds to the rational formula (1) if the runoff coefficient is expressed as the product $\varphi = \mu_\varphi K_\varphi$. Equation (21) shows that the coefficient K_φ increases with T and is greater than or equal to one when $K_T \geq 0$, that is when T is greater than about 2 years.

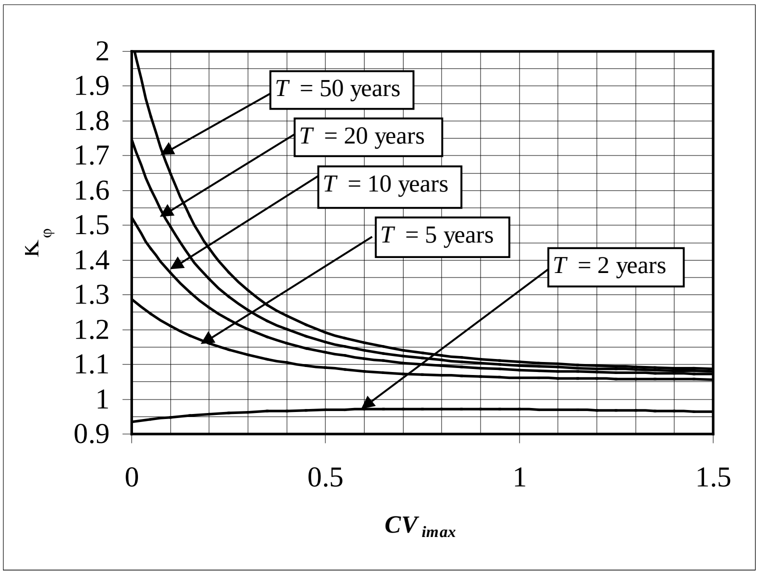

Fig. 9 - Relationship K_φ - CV_{imax} for some return periods T, assuming $CV_\varphi = 0.4$ and $K_3 = 1$.

As can be seen from equation (17b), the random variability of the runoff coefficient makes the variance of design discharge to be greater than that of rainfall intensity and consequently makes $Q_{max}(T)$ increase, other parameters being equal. This means that neglecting the randomness of runoff coefficient (CV_φ=0), assuming the same return period for design discharges and rainfall intensities, may lead to incorrect estimates of design discharge or, that is the same, of its return period. In particular, a constant value of φ greater than μ_φ causes an overestimation of design discharges for low return periods (about 2 years). This may happen, for example, when $\varphi = Imp$ is chosen, according to a common engineering practice, as relationship (11a) gives estimates of μ_φ lower than *Imp*, except for catchments with small values of this parameter ($Imp \leq 0.16$). Analogously, considering a constant value $\varphi = \mu_\varphi$ results in an underestimation of design discharge for return periods higher than 2 years.

More correct estimates of design discharge $Q_{max}(T)$ can be obtained using the proposed form (20) of the rational formula, in which the runoff coefficient $\varphi = \mu_\varphi K_\varphi$ is expressed as a function of the return period of Q_{max}. It has to be noted that in the engineering practice, values of runoff coefficient increasing with the design return period are sometimes used, trying to account empirically for its variability. In these cases, however, the real return period associated to the design discharge Q_{max} remains uncertain.

To give an example, the design discharges Q_{max} were calculated, for different return periods, for the catchment of Baggio, whose IDF curves were available (t_d = 15 min, μ_{imax} = 19.4 mm, CV_{imax} = 0.32), using equation (1) with $\varphi = \mu_\varphi$, and equations (20) and (21) with K_3=1. Results, reported in Table 7, show differences in the estimations of Q_{max} varying from about – 4% for T=2 years to 25% for T= 100 years.

***T* [years]**	**K_T**	**$K\varphi$**	**$Q_{max}(T)$ eq. (1) [m^3/s]**	**$Q_{max}(T)$ eq. (20) [m^3/s]**	***Difference* [%]**
2	-0.164	0.964	5.846	5.635	-3.7
5	0.718	1.122	7.589	8.511	10.8
10	1.303	1.191	8.744	10.416	16.1
50	2.590	1.295	11.284	14.608	22.8
100	3.134	1.325	12.358	16.380	24.6

Table 7 - Comparison between estimates of Q_{max} performed with equation (1) and (20) for the catchment of Baggio (CV_φ=0.4, μ_{imax} = 19.4 mm, CV_{imax} = 0.32, K_3=1).

5. Conclusions

The analysis of rainfall-runoff data of 319 events recorded in 21 urban experimental catchments throughout the world highlighted the need to consider the runoff coefficient as a random variable. The probability distribution of this parameter has proved to be normal and the two simple relationships (11a) and (12a) for the estimation of its mean and variance, depending on the impermeability ratio of the catchment, were proposed. It is important to highlight that these relationships show a good agreement with experimental data, even if these data were recorded in catchments that are very different both for urbanization characteristics and for climatic contexts.

Equation (17b) shows that the random variability of the runoff coefficient makes the variance of design discharge to be greater than that of rainfall intensity and consequently makes $Q_{max}(T)$ to increase, other parameters being equal. This means that assuming a constant value of runoff coefficient, according to the usual engineering practice, leads to estimates of design discharge whose return period may be sensibly different from that of rainfall intensities from which they are obtained. In most of cases this error causes an underestimation of $Q_{max}(T)$ for high return periods and an overestimation for low ones (T< 2 years).

In conclusion, it seems important to take into account the randomness of runoff coefficient to achieve more reliable estimates in design and reliability analysis of drainage networks [Yen, 1970]. The proposed modification of the rational formula, expressed in equation (20), together with relationships (11a) and (12a) for estimation of mean and variance of runoff coefficient, can be considered a simple and useful tool to be used for achieving this task.

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