GEOID DETERMINATION IN THE ITALIAN REGION

Riccardo BARZAGHI, Barbara BETTI, Alessandra BORGHI, Fernando SANSO', Giovanna SONA, Vincenza TORNATORE, Giovanna VENUTI

DIIAR - Politecnico di Milano - Piazza L. da Vinci 32 - Milano
tel. ++ 39-02-23996528 - fax 02-23996530 - e-mail: riccardo@geo.polimi.it

Abstract

Geoid estimate is nowadays one of the most relevant topic in Physical Geodesy.
Precise geoid estimates (few centimetre precision) are required on the oceans and on land areas. Having a reliable geoid estimate on the oceans allows SST computation which can give valuable information on currents dynamic that are related to climate changes.
Precise geoid estimates are also required over land to be used in connection with GPS observations to compute orthometric heights, thus replacing spirit leveling (although with lower precision).
In this paper, an overview on geoid determination in the Italian area, land and surrounding seas, is given and comparison with GPS/leveling is discussed to define the estimate precision.

Geoid computation in Italy

The altimetry research line has been developed significantly up to the 1999. Here achievements have been reached in

- methodology: the clarification of the split between time-varying and areawise signals along tracks by collocation and time-spectral analysis has been a major breakthrough which has not, to the date, been overwhelmed;
- the analysis of the rank deficiency problem in cross over adjustment for bounded areas (e.g. the Mediterranean) has been fully accomplished
- software has been produced to perform signal splitting (timewise – areawise) and the subsequent cross over adjustment.
  The software has been made operational and results have been numerically achieved, yet it has never reached a standard operability and as such it cannot be considered as finished;
- numerical results on Mediterranean SST have been derived which are still, to our knowledge, one of the two only internationally known acknowledged solutions.

Overall we can say that these researches had to be stopped due to lack of manpower; however the results obtained are still better than the international average level and in future the research could start again.
On the other side, the researches on geoid estimation and the comparison with GPS/leveling data have been carried out intensively in the 1998-2000 period. A new geoid estimate has been computed over the whole Italian area using the remove-restore procedure and fast-collocation. As it is well known, the geoid, i.e. the equipotential surface of the Earth gravity field which is close to the mean ocean surface, can be used, for instance, in combination with radar-altimetric data to get ocean currents. Furthermore, GPS observations together with geoid estimates can give orthometric heights. This is of particular relevance, since this can be done in a faster and cheaper way than using spirit leveling, although with lower precision (which is however sufficient in many practical applications). Hence, the estimate of a subdecimetric precision geoid over the whole Italian region is a primary task for the national geodetic community.

Many improvements have been introduced with respect to the previous Italian geoid estimate ITALGEO95 (Barzaghi et al., 1995). The Italian gravity data set has been enlarged introducing 4624 new gravity data in the area $45.3 \leq \varphi \leq 46.8$; $13.4 \leq \lambda \leq 16.8$, corresponding to Slovenia. In this way, the gravity data gap in this area, which is very close to the Italian boundaries, was filled so avoiding possible mismodelling in the quasi-geoid estimate in the Friuli Venezia Giulia area. Furthermore, the 7.5" x 10" Italian DTM (Carrozzo et al., 1982) has been carefully checked for outliers using the values extracted from an independent 100 m resolution DTM, supplied by I.G.M. (Istituto Geografico Militare). In this way, 327 outliers, distributed randomly in the whole Italian area, have been found and corrected.
New geopotential models have been also considered in computing the new geoid estimate. Since the ITALGEO95 computation, two new geopotential models have been made available: EGM96, complete up to degree 360, (Lemoine et al, 1998; IGeS Bulletin, 1997) and the high resolution model GPM98CR by Wenzel, complete up to degree 720, (Wenzel, 1998). These models were adopted to account for the long wavelength component of the geopotential field.
Based on the two global geopotential models EGM96 and GPM98CR, two quasi-geoid estimates have been computed in the Italian area.
In both cases, the classical "remove-restore" (Barzaghi et al.,1996) procedure has been used and the residual quasi-geoid components have been evaluated using the Fast Collocation approach (Bottoni and Barzaghi, 1993).

The computation of the quasi-geoid named ITG99_EGM96, based on the EGM96 global model, has been carried out on a regular 3' x 3' grid in the area $36° \leq \varphi \leq 47°$, $6° \leq \lambda \leq 19°$.
RTC has been computed up to 70 km from each computation point both in the gravity component and quasi-geoid effect.
Statistics of the "remove" step are listed in tab. 1. Point gravity values have been then gridded on a regular 3' x 3' geographical grid. GEOGRID program of the GRAVSOFT package (Tscherning et al., 1994) was used for such a step: statistics of the residual gridded gravity values Δg_r^G are shown in tab. 1. The empirical covariance of these values and the best fit model are represented in fig. 1.
As one can see, a very good fit is reached between the empirical values and the best fit model covariance which, in terms of anomalous potential T(P), is given by

$$COV_{TT}(P,Q) = \sum_{i=2}^{\infty} \sigma_i^2 \left(\frac{R^2}{rr'} \right)^{i+1} P_i(\cos\psi) \qquad (1)$$

where: $$\sigma_i^2 = \begin{cases} \varepsilon_i \text{ error degree variances} \\ \text{degree variances, e.g. } \dfrac{A}{(i-1)(i-2)(i-B)} \left(\dfrac{R_B}{R} \right) \end{cases}$$

R is the Earth radius
r, r' are respectively the radial distances of points in space P, Q
P_i the Legendre Polynomial of degree i
ψ the spherical distance between P and Q

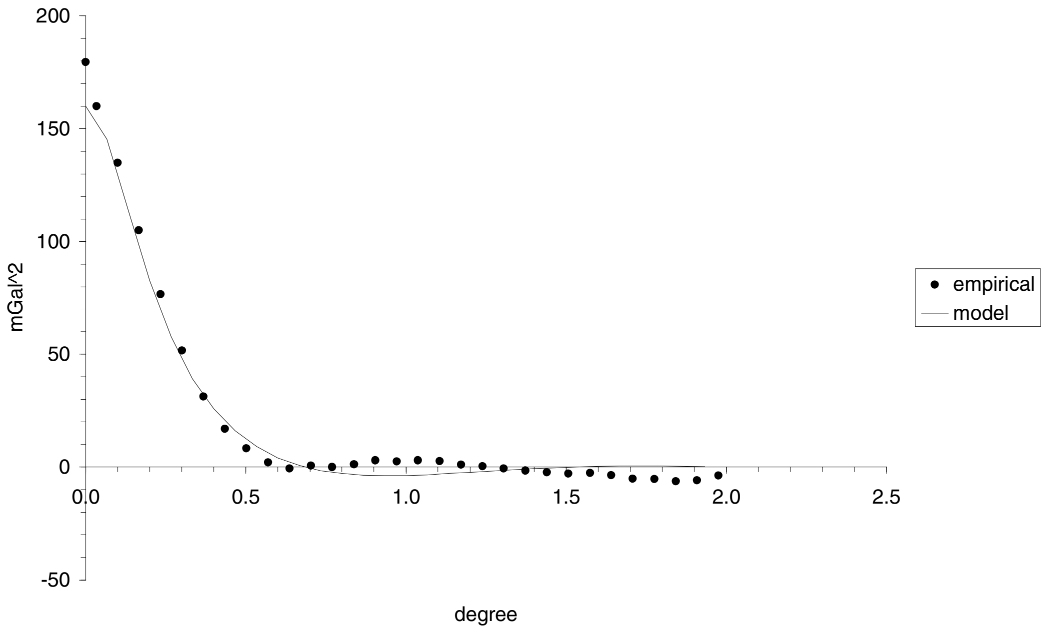


Figure 1 - Empirical and model covariance function of the gridded gravity residuals obtained with the global geopotential model EGM96

	Δg_0 [mGal]	Δg_0 - Δg_M [mGal]	Δg_r [mGal]	Δg_r^G [mGal]
n	109927	109927	109927	57681
E	11.92	-6.71	-2.37	-0.33
σ	61.71	30.75	15.57	12.97
min	-162.36	-253.33	-188.32	-116.13
max	269.71	187.95	109.38	102.86

Table 1 - Statistics of the "remove" step using the EGM96 geopotential model.

Δg_0: observed gravity values (free air) Δg_M: gravity geopotential model component
A_{rtc} :gravity terrain correction component $\Delta g_r = \Delta g_0 - \Delta g_M - A_{rtc}$ gravity residuals
Δg_r^G: gridded gravity residuals

The Fast Collocation solution giving ζ_r has been computed on the same 3'x3' grid used for Δg_r^G. The "restore" step was then accomplished: the ζ_{rtc} and the ζ_M component have been added to ζ_r, thus getting the final quasi-geoid estimate ITG99_EGM96. In tab. 2 and fig. 2, the statistics of the "restore" step and the contour lines of the quasi-geoid are shown.

	ζ_r [m]	$\zeta=\zeta_r+\zeta_M$ [m]	$\zeta=\zeta_r+\zeta_M+\zeta_{RTC}$ [m]
n	56781	56781	56781
E	-0.09	44.26	44.35
σ	0.52	4.96	5.09
min	-1.55	25.73	25.23
max	1.58	54.09	55.28

Table 2 - Statistics of the "restore" step using the EGM96 geopotential model

ζ_r: residual quasi-geoid

ζ_M: quasi-geoid geopotential model component

ζ_{rtc} :quasi-geoid terrain correction component

ζ: quasi-geoid

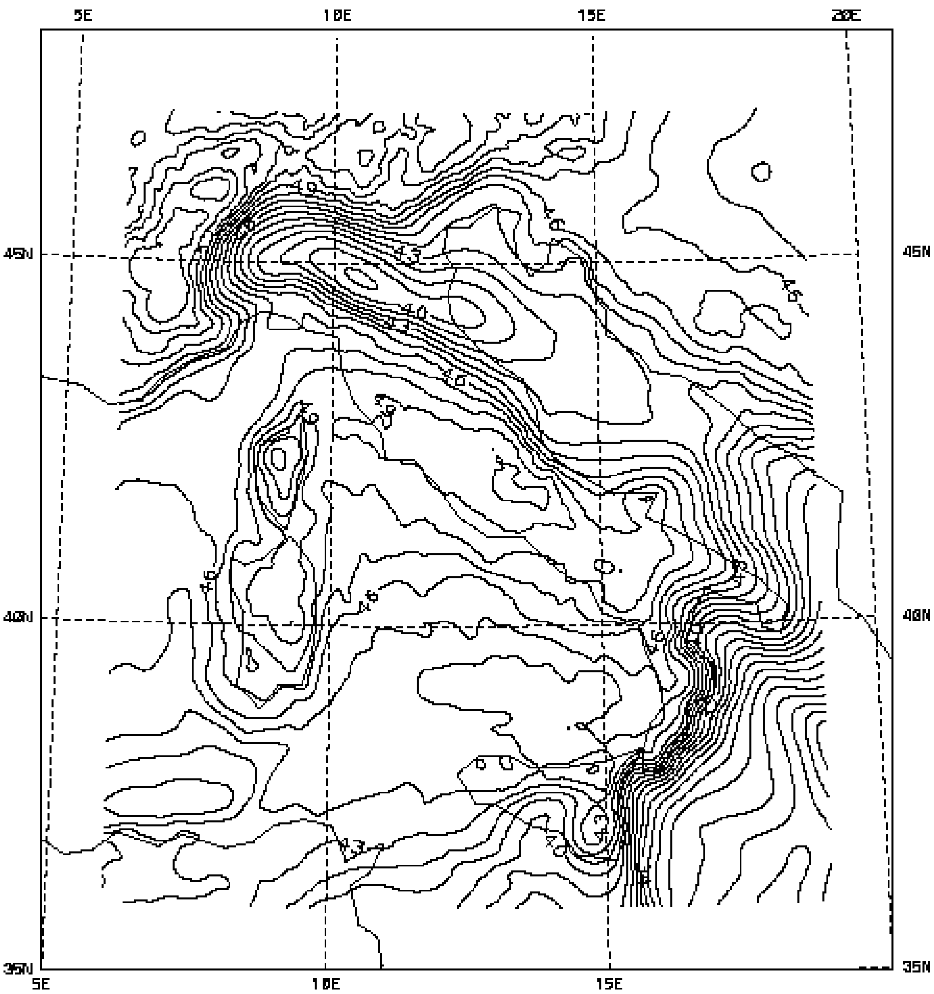


Figure 2 - The Italian quasi-geoid ITG99_EGM96 (equidistance = 1m)

Similarly, the high resolution geopotential model GPM98CR by Wenzel has been used up to degree 720 to get the ITG99_GPM98CR estimate.
Also in this case, the steps described in the ITG99_EGM96 computation have been performed.
Statistics of this "remove" step are given in tab. 3. Residual gravity values have been gridded on a 2'×2' regular geographical grid covering the same area used in the EGM96 based computation (their statistics are listed in tab. 3). The empirical covariance and the best fit model are shown in fig. 3.

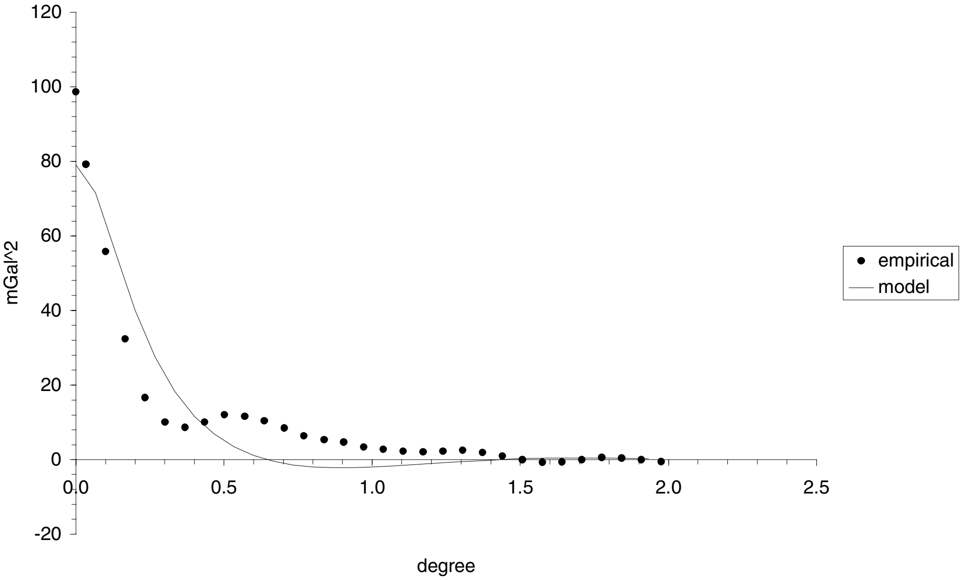


Figure 3 - Empirical and model covariance function of the gridded gravity residuals obtained with the global geopotential model GPM98CR

The empirical covariance is quite irregular but its amplitude is remarkably smaller that the one obtained in the EGM96 empirical covariance. This means that the GPM98CR model and the related RTC reduction can give a better representation of the local gravity data than EGM96 (this can be seen also in the statistics of the gravity residuals in tab. 3)

	Δg_0 [mGal]	$\Delta g_0 - \Delta g_M$ [mGal]	Δg_r [mGal]	Δg_r^G [mGal]
N	109927	109927	109927	129421
E	11.92	-6.07	-0.95	0.22
σ	61.71	25.20	11.15	9.66
Min	-162.36	-200.55	-191.81	-135.32
Max	269.71	164.85	92.37	89.32

Table 3 - Statistics of the "remove" step using the GPM98CR geopotential model.

Δg_0: observed gravity values (free air)
Δg_M: gravity geopotential model component
A_{rtc} :gravity terrain correction component
$\Delta g_r = \Delta g_0 - \Delta g_M - A_{rtc}$ gravity residuals
Δg_r^G: gridded gravity residuals

As in the previous estimates, Fast collocation was applied for computing ζ_r on the 2'x2' regular grid used for Δg_r^G evaluation. The statistics of the "restore" step related to the ITG99_ GPM98CR quasi-geoid are presented in tab. 4, while the plot of the estimate is shown in fig. 4.

	ζ_r [m]	$\zeta=\zeta_r+\zeta_M$ [m]	$\zeta=\zeta_r+\zeta_M+\zeta_{RTC}$ [m]
n	129421	129421	129421
E	0.05	44.40	44.43
σ	-1.63	44.44	5.06
min	0.49	5.06	25.45
max	1.65	25.45	55.27

Table 4 - Statistics of the "restore" step using the GPM98CR geopotential model

ζ_r: residual quasi-geoid
ζ_M: quasi-geoid geopotential model component
ζ_{rtc} :quasi-geoid terrain correction component
ζ: quasi-geoid

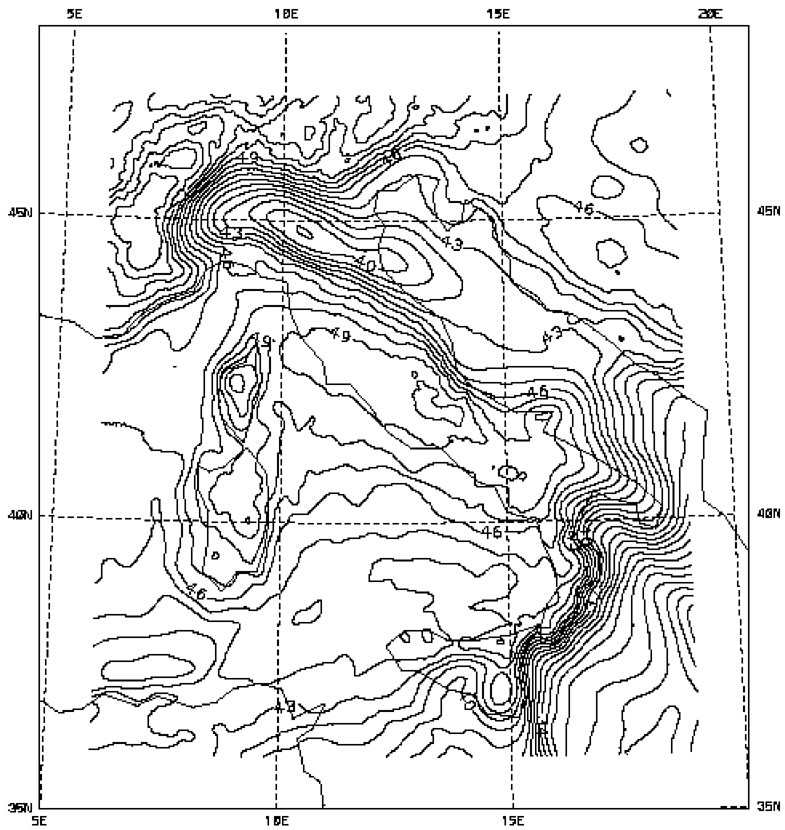


Figure 4 - Italian quasi-geoid ITG99_GPM98CR (equidistance = 1m)

The two gravimetric quasi-geoid estimates have been then compared on 583 points with GPS derived undulations.
In these 583 points, both h (ellipsoidal height) and H (orthometric height) are known so that $N_{GPS/lev} = h - H$ can be computed. The h values refer to the IGM95 GPS campaign whereas the H

values are obtained via spirit leveling (these double points belong to the so called GEOTRAV network and were supplied by IGM). To properly perform the comparison, a datum shift between the gravimetric quasi-geoid estimates and the $N_{GPS/lev}$ must be computed to reduce the data to the same reference system. While $N_{GPS/lev}$ is in the GPS reference system, ζ computed with the "remove-restore" method is in the reference system implied by the global geopotential model.
To this aim, the following formula, which accounts for a translation based datum shift in terms of geoid undulation, has been considered (Heiskanen and Moritz, 1990):

$$N_{grav} = N_{GPS/lev} + \Delta N(\theta, \lambda) =$$
$$= N_{GPS/lev} + dx \sin\theta \cos\lambda + dy \sin\theta \sin\lambda + dz \cos\theta$$

(dx,dy,dz) = translation between GPS and geoid reference systems
$\theta = 90 - \varphi$

(we remark that only translation is considered in this relationship between the two reference systems).
We also assume that $N_{grav} \sim \zeta$, being ζ the quantity which is effectively estimated: this can induce distorsions and perturbations specially in high mountain areas
The quantities (dx,dy,dz) were estimated by least squares; outliers rejection, in the hypothesis of normal distributed residuals and with significance level $\alpha = 1\%$, was also performed.
The datum shift estimate was done separately for the peninsular part of Italy, for Sicily and Sardinia.
This subdivision reflects the geographical difference of these three areas and also the possible discrepancies among their orthometric height systems (reference tide gauge problems). Hence, comparisons after datum shift computation were carried out separately on the above mentioned zones for the two quasi-geoid estimates.
The results are summarized in the following in tab. 5 and 6 and in fig. 5 and fig. 6.

	ITG99_EGM96: $\zeta - N_{GPS/lev}$ [m]		
	Continental Italy	Sicily Island	Sardinia Island
#	495	36	46
E	0.00	0.00	0.00
σ	0.17	0.08	0.09
Min	-0.44	-0.16	-0.17
Max	0.42	0.13	0.19

Table 5 - Statistics of the residuals between ζ_{ITG99_EGM96} and $N_{GPS/lev}$ after datum shift estimate

	ITG99_GPM98CR: $\zeta - N_{GPS/lev}$ [m]		
	Peninsular Italy	Sicily Island	Sardinia Island
#	496	36	45
E	0.00	0.00	0.00
σ	0.15	0.04	0.06
Min	-0.38	-0.08	-0.13
Max	0.38	0.06	0.15

Table 6 - Statistics of the residuals between $\zeta_{ITG99\_GPM98CR}$ and $N_{GPS/lev}$ after datum shift estimate

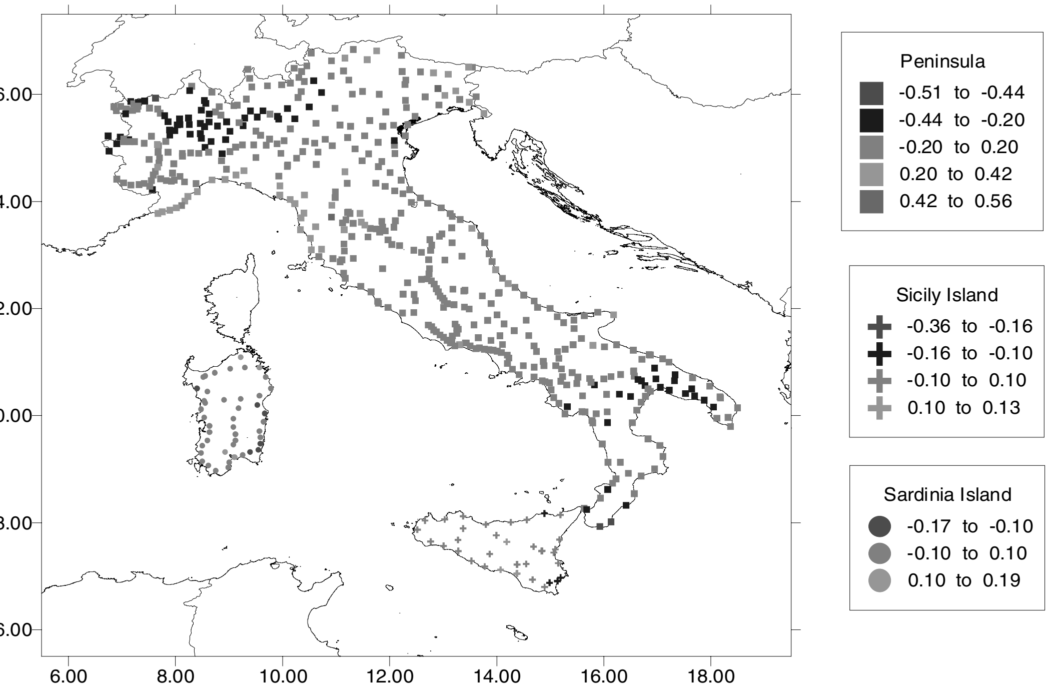


Figure 5 - Residuals between ζ_{ITG99_EGM96} and $N_{GPS/lev}$ after datum shift estimate (m)

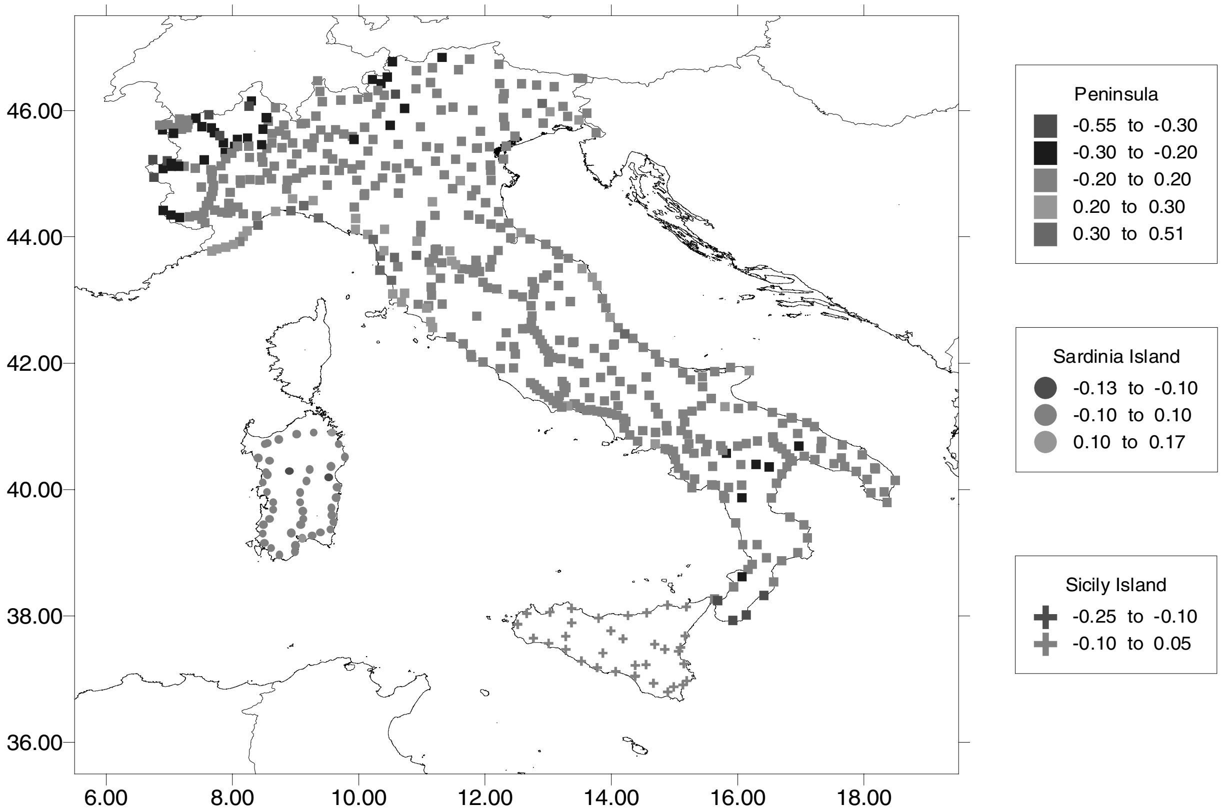
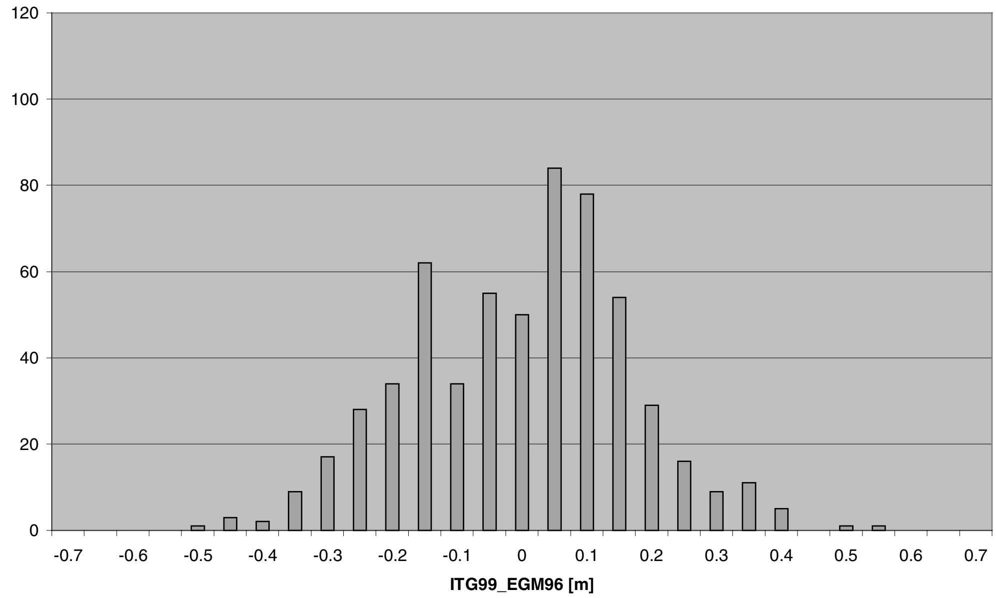


Figure 6 - Residuals between $\zeta_{ITG99\_GPM98CR}$ and $N_{GPS/lev}$ after datum shift estimate (m)

In both cases, the two models show a good agreement with $N_{GPS/lev}$ in Sicily and Sardinia while a more complex structure of the residuals is present in the Peninsular area.
This can be explained if we take into account that in this area sharp disomogeneities exist both in the orography and in leveling lines (while h is homogeneous in time and precision, this doesn't hold for H which has been measured in the continental part over a large time span).
Due to that, an error analysis on such a data set seems to be very complex. It is quite clear, and obvious, that part of the discrepancies are related to the quasi-geoid estimates. A comparison between fig. 5 and fig. 6 shows that the ITG99_GPM98CR quasi-geoid behaves better that the ITG99_EGM96 solution in the North-Western region.



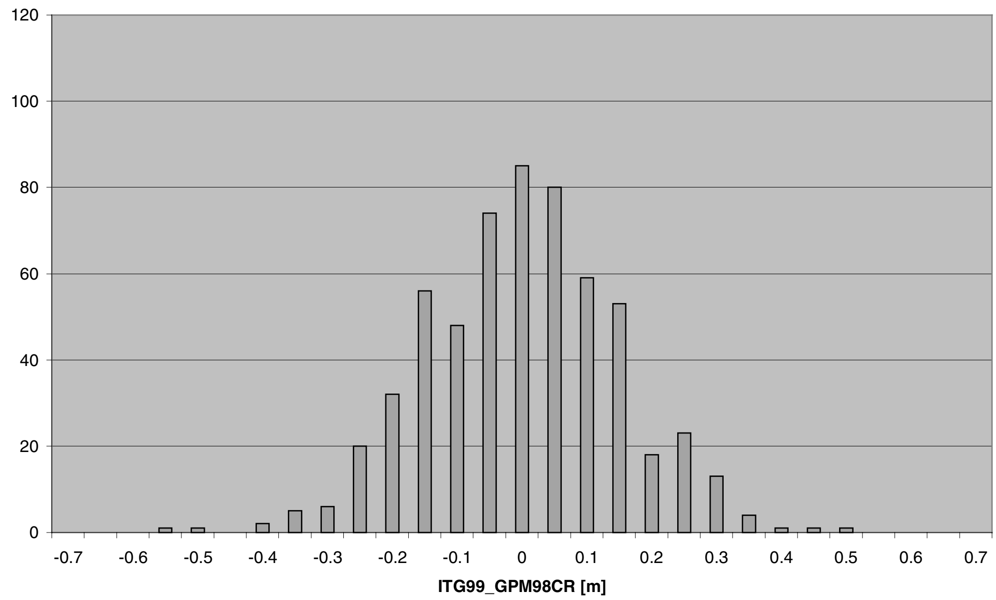


Figure 7 - Histogram of the residuals between quasi-geoid solution and $N_{GPS/lev}$ after datum shift estimate

However, in the same area, particularly along the Liguria coasts, sharp discrepancies are present between $N_{GPS/lev}$ and both quasi-geoid models, so that other possible error sources should be taken into account.
To further compare these solutions, histograms of the residuals between $N_{GPS/lev}$ and the two quasi-geoids here described are presented in fig. 7, grouping the results obtained in the three different areas.
This plots give a synthetic overview of the residuals and show that the ITG99_GPM98CR solution has the most symmetric histogram which is the only one significantly approaching a normal distribution (according to a χ^2 goddness-of-fit test with α=5%).
All these analyses prove the good quality of the ITG99_GPM98CR estimate which has been named ITALGEO99 and which is, at the moment, the most accurate quasi-geoid estimate for the whole Italy. Thus, this research task has been completely accomplished and this precision estimated geoid will be a sound basis for future researches on the altimetric datum in Italy.

Bibliography

Barzaghi R., Borghi A., Sona G. (2001), "New covariance models for local applications of collocation", *IAG Symposia, Vol. 122*: *IV Hotine-Marussi Symposium on Mathematical Geodesy,* Trento, 14-17 September 1998, pp. 91-101, Springer.

Barzaghi R., Borghi A. (1998), "The definition of a regional geopotential field for precise gravity field modelling", *International Geoid Service, Bulletin* No. 7, pp. 52-67, DIIAR, Politecnico di Milano, Italy.

Barzaghi R., Brovelli M.A., Sona G., Manzino A., Sguerso D. (1996), "The new quasi-geoid: ITALGEO95", *Bollettino di Geodesia e Scienze Affini*, No. 1, pp. 57-72.

Borghi A. (1999), *The Italian geoid estimate: present state and future perspectives,* Tesi di dottorato in Scienze Geodetiche e Topografiche, Politecnico di Milano, XII ciclo (1996-1999).

Bottoni G., Barzaghi R. (1993), "Fast Collocation", *Bulletin Géodésique*, Vol. 67, No. 2, pp. 119-126.

Carrozzo M.T., Chiarenti A., Giadam M., Luzio D., Margiotta C., Miglietta D., Pedone M., Quarta T., Zuanni F. (1982), "Data bases of mean height values and gravity values", *Proceeding of the 2nd International Symposium on the Geoid in Europe and Mediterranean Area.*

Heiskanen W.A., Moritz H. (1990), *Physical Geodesy,* Institute of Physical Geodesy, Technical University, Graz, Austria.

Lemoine F.G., Kenyon S.C., Factor J.K., Trimmer R.G., Pavlis N.K., Chinn D.S., Cox C.M., Klosko S.M., Luthcke S.B., Torrence M.H., Wang Y.M., Williamson R.G., Pavlis E.C., Rapp R.H., Olson T.R. (1998), "The development of the joint NASA GSFC and the National Imaginery and Mapping Agency (NIMA) geopotential model EGM96", *NASA Report* TP-1998-206861, Goddard Space Flight Center.

Surace L. (1997), "La nuova rete geodetica nazionale IGM95: risultati e prospettive di utilizzazione", *Bollettino di Geodesia e Scienze Affini*, anno LVI, n. 3, 1997.

The Earth Gravity Model EGM96: testing procedures at IGeS (1997) - IGeS, Bulletin No. 6, DIIAR, Politecnico di Milano, Italy

Tscherning C.C., Rapp R.H. (1974), "Closed Covariance Expressions for Gravity Anomalies Geoid Undulations, and the Deflections of the Vertical Implied by Anomaly Degree-Variance Models", *Reports of the Department of Geodetic Science*, No. 208, The Ohio State University, Columbus, Ohio, 1974.

Tscherning C.C., Knudsen P., Forsberg R. (1994), "Description of the GRAVSOFT package", *Geophysical Institute, University of Copenhagen, Technical Report*, 1991, 2. Ed. 1992, 3. Ed. 1993, 4. Ed. 1994.

Tscherning C.C. (1994), "Geoid determiantion by least-squares collocation using GRAVSOFT", *Lectures Notes of the International School for the Determination and Use of the Geoid*. IGeS, DIIAR, Politecnico di Milano, Italy.