Analytical approaches for flexural analysis of RC slabs strengthened with prestressed or non-prestressed CFRP laminates

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ABSTRACT

Analytical approaches are developed for predicting the flexural behaviour of reinforced concrete slabs strengthened with carbon fibre reinforced polymer (CFRP) laminates applied according to (i) near-surface mounted (NSM) and (ii) externally bonded reinforcement (EBR) as non-prestressed strengthening techniques, as well as according to the EBR with (iii) mechanical anchorage (MA) and with (iii) gradient anchorage (GA) systems as prestressed strengthening techniques. The approaches are based on a trilinear relationship of force-deflection curve, in which the formulations depend on the type of failure mode, strengthening type, and initial concrete and CFRP strains. In order to predict the ultimate load, different formulations existing in the literature for the calculation of the CFRP failure strain are applied and compared. Experimental results previously published by the authors are compared with the results from the developed analytical approaches and a good agreement is found.

Keywords: CFRP laminate, Flexural strengthening, Analytical modelling, Failure mode, Prestressed EBR.

1 INTRODUCTION

Carbon fibre reinforced polymer (CFRP) composites possess excellent mechanical properties, which make them reliable in different domains such as aeronautics and civil engineering. In civil engineering, there are different strengthening techniques using CFRP composites including the externally bonded reinforcement (EBR) and the near-surface mounted (NSM). These techniques can additionally be prestressed mainly to achieve the full usage of the CFRP strains, e.g., using mechanical anchorage (MA) or gradient anchorage (GA) systems [1]. In the EBR technique, CFRP composites are bonded on the tensile face of the element to be strengthened, and the adhesion between FRP and concrete substrate generally controls the ultimate capacity of the strengthened element [2]. Besides, the most common failure mode (FM) for EBR is by intermediate crack debonding (ICD) that starts from the intermediate region throughout the interface [2]. On the other hand, for the NSM technique, pre-cut grooves opened on the concrete cover are created in which the FRP bars or strips are inserted and bonded to concrete [3, 4]. It has been shown that the failure of an element strengthened according to NSM can be critical diagonal crack debonding (CDCD) or a combination of the CDCD with concrete cover separation [4]. According to the literature, NSM can increase the cracking, yielding and ultimate loads, and it has been shown that NSM techniques possess superior benefits than EBR [3, 4, 5]. Furthermore, EBR systems can be prestressed to delay the onset crack initiation, to relieve the internal steel reinforcement strains [6] and to reach higher FRP failure strain [7]. A number of studies on the analytical prediction of the flexural behaviour of RC elements with strengthening systems, e.g. [7, 8, 9, 10] have been conducted; however, there are some points that still need to be addressed. For example, in most of the studies, the trilinear relationship of force-deflection curves e.g. [8, 9] are generally established without including the effect of the FM and the prevailing FM. Additionally, the curvature required when calculating the deflection may depend on the FM. Besides, for the prestressed systems, the curvature may depend on the initial prestress at all loading stages. Finally, different formulations for predicting the CFRP failure strain exist, but little or no studies exist on the comparative perspectives of these formulations. The present works attempted to develop an analytical approach (for each of the mentioned strengthening techniques) that addresses all the above existing issues.

2 ANALYTICAL APPROACHES

The moment-curvature relationship for a cross-section of a strengthened RC slab can be idealized as a trilinear relationship representing pre-cracking, post-cracking, and post-yield stages. The developed analytical approach can detect the ultimate flexural capacity by adopting four different types of failure modes, namely: concrete crushing (CC), CFRP rupture (CR), end debonding (ED) and ICD.

2.1. Constitutive law of the materials

The concrete compressive behaviour is assumed linear in both pre- and post-cracking stages up to the yielding of steel. The concrete contribution after the steel yielding is simulated by a rectangular stress block mainly governed by a multiplier on the concrete compressive strength to determine the intensity of the stress distribution (k_1) and the ratio of the equivalent rectangular stress block to the neutral axis depth (k_2) as per fib90 [11]. Furthermore, the tensile behaviour of concrete is assumed to be linear up to the stress where the concrete tensile surface starts to have initial cracks as per ACI [12]. For the longitudinal steel bars, elasto-perfectly plastic behaviour is adopted. The CFRP laminate is modelled as linear up to its ultimate tensile strength.

2.2. Moment-curvature relationship

Using section analysis of the investigated RC slabs, performed based on the strain compatibility and force equilibrium for each of the governing stages (pre-cracking, post-cracking and post-yield stages), the moment-curvature (M - φ) relation is established. For more clarification, equations of one of the strengthening techniques (EBR) are established as follows.

The curvature and moment at pre-cracking ($\varphi_{cr,s}$, $M_{cr,s}$), post-cracking ($\varphi_{y,s}, M_{y,s}$), and post-yield ($\varphi_{u,s}$, $M_{u,s}$) stages are provided in Eqs. (1)-(3).

$$\begin{aligned} \varphi_{cr,s} &= \varepsilon_{cct}^{crs}/y_s \\ M_{cr,s} &= (\varepsilon_{cct}^{crs}\cdot E_c\cdot b\cdot {y_s}^2)/3 + (\varepsilon_{cr}\cdot E_c\cdot b\cdot (h- y_s)^2)/3 + \varepsilon_{st}^{crs}\cdot E_{s2}\cdot A_{s2}\cdot (y_s - d_{s2}) + \varepsilon_{sb}^{crs}\cdot E_{s1}\cdot A_{s1}\cdot (d_{s1}-y_s) + \varepsilon_{fb}^{crs}\cdot E_f\cdot A_f\cdot \left(d_f - y_s\right) \end{aligned} \tag{1}$$

$$\begin{aligned} \varphi_{y,s} &= \varepsilon_{cct}^{ys}/C_{ys} \\ M_{y,s} &= \left(\varepsilon_{cct}^{ys}\cdot E_c\cdot b\cdot {C_{yt}}^2\right)/3 + \varepsilon_{st}^{ys}\cdot E_{s2}\cdot A_{s2}\cdot \left(C_{ys}-d_{s2}\right) + \varepsilon_{sb}^{ys}\cdot E_{sb}\cdot A_{s1}\cdot \left(d_{s1} - C_{ys}\right) + \varepsilon_{fb}^{ys}\cdot E_f\cdot A_f\cdot \left(d_f - C_{ys}\right) \end{aligned} \tag{2}$$

$$\begin{aligned} \varphi_{u,s} &= \varepsilon_c/C_{us} \\ M_{u,t} &= A_{s1}\cdot \sigma_1\cdot (d_{s1}-k_2\cdot C_{us}) + A_{s2}\cdot \sigma_2\cdot (k_2\cdot C_{us}-d_{s2}) + A_f\cdot E_f\cdot \varepsilon_f\cdot (h-k_2\cdot C_{us}) \end{aligned} \tag{3}$$

where, ε_{cr}, $\varepsilon_{cct}^{crs}, \varepsilon_{st}^{crs}, \varepsilon_{sb}^{crs}$ are the strains in concrete's bottom fibre at crack initiation, in concrete's top fibre, in top steel bar, and in bottom steel bar in pre-cracking stage (Eq. (1)) respectively, while the same notation for strains is used in the post-cracking stage (Eq. (2)) but with the superscript "crs" replaced by "ys". Furthermore, y_s is the moment of inertia for un-cracked section and d_f is the distance from the centre of the CFRP section to the concrete top fibre, ε_c is the strain in the concrete top fibre at a certain level of loading. The value of ε_c is calculated depending on the type of FM. d_{s1} and d_{s2}, A_{s1} and A_{s2}, E_{s1} and E_{s2} are the distance from the centre of the bottom and top steel bars to the concrete's top fibre, bottom and top steel section area, and bottom and top steel modulus of elasticity, respectively; b is the section width and h is the section height. C_{ys} and C_{us} are the neutral axis depth in post-cracking stage

(Eq. (2)) and post-yield stage (Eq. (3)) stage, σ_1 and σ_2 are the stress in bottom and top steel bars, respectively; E_c, E_f, ε_f, and A_f are the elastic modulus of concrete, elastic modulus of CFRP, CFRP strain, and the CFRP section area, respectively. The formulations for other strengthening techniques (i.e., NSM, MA, and GA) follow the same procedure as that of EBR described above. For prestressed systems, new formulations for predicting the curvature are derived from Fig. 1, where the total curvature ($\varphi_{u,p}^T$) if FM is by CR can be calculated as in Eq. (4) or (5), or if is by debonding as in Eq. (6).

$$\varphi_{u,p}^T = \left(\varepsilon_{cct}^{ci} + \varepsilon_{fp}\right)/h + \varepsilon_c/C_{up} \quad (4)$$

$$\varphi_{u,p}^T = \frac{k_2 \cdot \varepsilon_{cu}}{(0.5 \cdot A_f \cdot \sigma_f / k_1 \cdot f_{cm} \cdot b)} \quad (5)$$

$$\varphi_{u,p}^T = \varphi_{ip} + \varepsilon_c / C_{up} \quad (6)$$

where,

$$\varphi_{ip} = \varepsilon_{cct}^{ci} / C_{up} \;\; or \; (\varepsilon_{cct}^{ci} + \varepsilon_{ccb}^{ci})/h$$

$$\sigma_f = \sqrt{\frac{\left(E_f \cdot \varepsilon_{cu}\right)^2}{4} + \frac{(k_1 \cdot k_2 \cdot f_{cm}) \cdot E_f \cdot \varepsilon_{cu}}{\rho_f}} - E_f \cdot \varepsilon_{cu}/2$$

$$\rho_f = A_f / b \cdot h$$

ε_{cct}^{ci} and ε_{ccb}^{ci}, f_{cm}, ε_{cu}, ε_{fp}, and C_{up} are top and bottom initial strains in concrete due to prestress, tensile strength of concrete, concrete's top fibre ultimate strain, the initial CFRP prestrain, neutral axis location at ultimate stage as shown in Fig. 1, respectively.

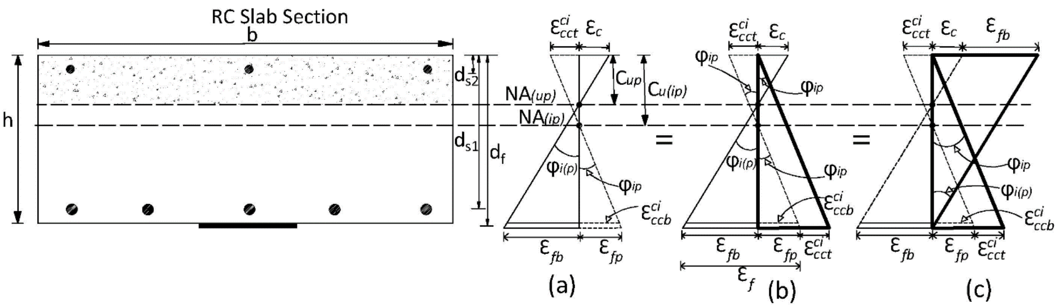

Figure 1. Prediction of curvature for prestressed systems: (a) strain profile, (b) and (c) are the geometric transformations of the strain profile shown in (a)

To predict the deflection, an approach in which the deflection is obtained in each loading stage by performing integration in the corresponding stage, as shown in eq. (7) [10], is adopted.

$$\delta_i = \int_{L_i}^{L_i+1} \varphi_i(x) \cdot x \cdot d(x) \quad (7)$$

where x is a variable along the region, L_i and L_{i+1} are the corresponding distances of the section boundaries for the region to the support; and φ_i is the curvature.

2.3. Reliability of the Developed Analytical Approaches

In the present work, experimental results previously published by the authors in [13] are used in order to assess the capability of the developed analytical approach. The main results for the force-deflection curves obtained from the experimental work for different tested RC strengthened slabs, namely the EBR (SL_EBR_T0), NSM (SL_NSM_T0), MA (SL_MA_T0), and GA (SL_GA_T0) are compared with those obtained from the developed analytical approaches. It is worth noting that these slabs were tested under four-point bending configuration, and the material properties and RC slab dimensions can be found in [13].

3 RESULTS AND DISCUSSIONS

The results from this work are plotted together with the experimental results in Fig. 2. A general observation is that the developed analytical approaches can predict well the force-deflection behaviour of the RC slabs investigated, as there is a very good agreement between analytical and experimental results. However, prior to digging deep into discussion of the results in Fig. 2, the notations shown in this figure are firstly defined: A: Eq. (4), B: Eq. (5), C: Eq. (6), D: $\varphi_u = \varepsilon_c / C_u$ (ε_c is as previously defined, C_u is the neutral axis depth at ultimate stage, for example, D is calculated as $\varphi_{u,s}$ in Eq. (3) for EBR system), FIB: ultimate capacity is predicted as per fib90 [11], FIB*: $\varepsilon_f = 0.9 \cdot \varepsilon_{fu}$, PW: $\varepsilon_f = 0.88 \cdot \varepsilon_{fu}$, BL: Eq. in [14] for CFRP strain at failure, BR: Equation in [3] for CFRP strain at failure, CNR: Eq. proposed in CNR [15] for calculation of CFRP strain at ICD. ACI: $\varepsilon_f = 0.7 \cdot \varepsilon_{fu}$, Pw: ε_{fu} is calculated as $\varepsilon_f = 0.8 \cdot \varepsilon_{fu}$. The constants in the PW and Pw formulations are obtained after conducting a parametric study involving the properties of RC slabs with NSM and MA systems, respectively, hence the validity of these formulations are pertinent to the RC slabs with the same properties as the above slabs. In the above formulations, ε_{fu} is the CFRP failure strain provided by manufacturer. An X_Y_Z notation is used in the legend of Fig. 2 to indicate a combination of different formulations in the developed approach, where X is the type of standard or formula adopted for the prediction of the CFRP failure strain (i.e., ε_f used in Eq. (3)), Y is the type of curvature formula adopted, and Z is when steel strain in post-cracking stage is calculated using steel ultimate strain.

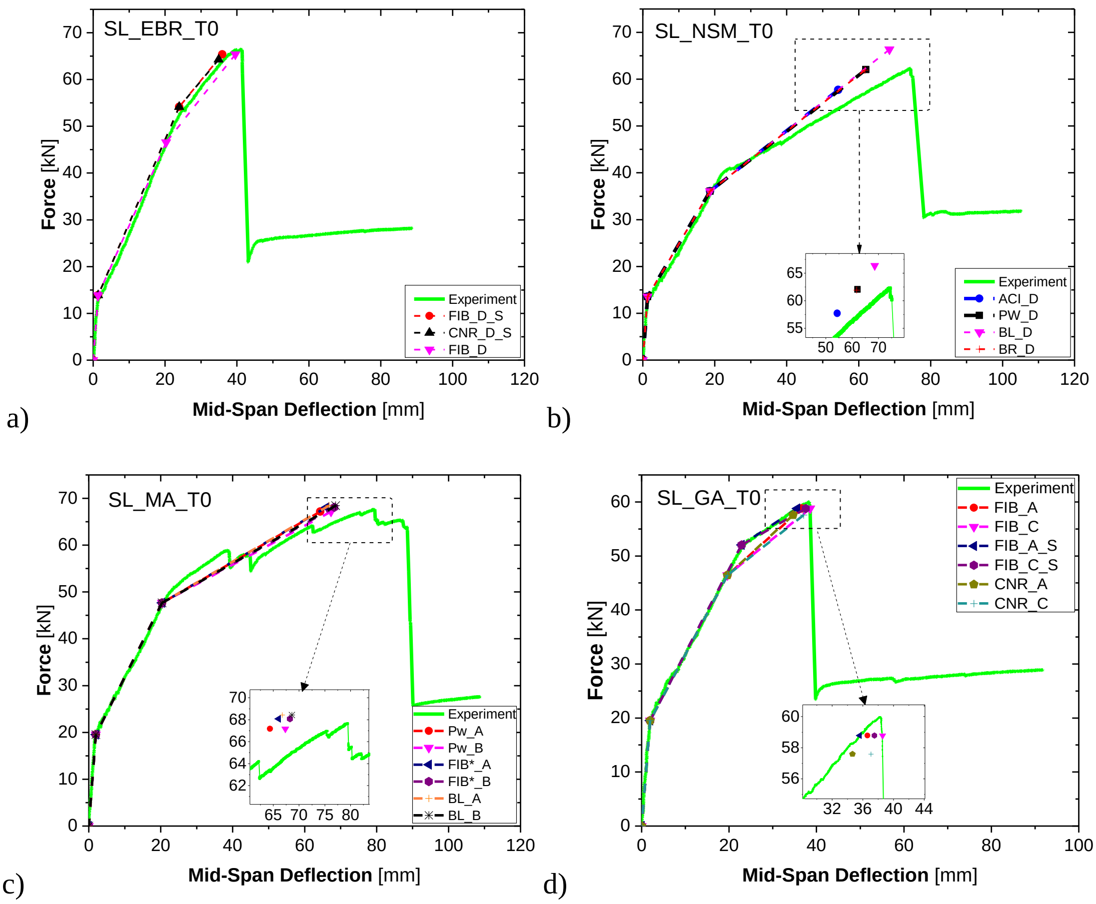

Figure 2. Analytical *vs*. experimental results: a) EBR, b) NSM, c) MA, and d) GA systems

A detailed discussion of the obtained results considering different formulae existing in the literature for the calculation of the CFRP failure strain is given as follows.

The force-deflection relationship for RC slab strengthened according to EBR (Fig. 2.a) shows that all attempted combinations provide promising results, and for the prediction of the CFRP failure strain either fib90 [11] or CNR [15] can be adopted.

Looking at the prediction for CFRP failure strain for NSM (Fig. 2b), the formulae proposed from this work (i.e. PW), from Barros et al. [3] and Balaguru et al. [14] can lead to promising results. This is probably because Barros et al. [3] formula includes the dependency of CFRP effective strain to the equivalent reinforcement ratio factor, and the formula in Balaguru et al. [14] also includes the reduction factor that accounts for possible debonding or delamination at failure. The formula from ACI [16] standard (i.e. ε_f= 0.7· ε_{fu}) adopted in this work is based on average values from different selected existing studies; however, according to this standard, ε_f may vary from 0.6· ε_{fu} to 0.9· ε_{fu} depending on many factors. Hence it is reasonable to infer that the properties of the investigated RC slab with NSM (in this work) are closely related to those that require the use of ε_f that falls in the above range (but not exactly 0.7· ε_{fu}). On the other hand, the stiffness in the post-yield stage is not well predicted. Further research can focus on the development of formulations that can predict not only the ultimate load but also the stiffness in the post-yield stage.

For prestressed systems, all the attempts made for MA can lead to reasonable agreement with the experimental results (Fig. 2c). However, it is observed that the combination of either fib90 [11] equation or equation proposed by Balaguru et al. [14], combined with the developed formulae in Eq. (4) or Eq. (5) can lead to slightly more promising results than those from the formula obtained after conducting a parametric study. The experimentally observed failure mode for MA system was by CFRP rupture, the fact that both the formulae in fib90 [11] and Balaguru et al. [14] address the CFRP strain at failure can be the reason for the observed promising results. Nevertheless, for all combinations, the failure load and deflection are slightly underestimated, hence further relevant studies with formulations that allow to increase both the failure load and deflection can be pursued. Additionally, for GA systems (Fig. 2d), it is observed that the curvature can be better predicted by Eq. (6), rather than Eq. (4) mainly because the experimentally observed GA failure mode was by ICD and Eq. (6) is developed to specifically address the curvature at debonding failure. Besides, a better prediction of the failure load is observed when the formula proposed in fib90 [11] is adopted as compared to that from CNR [15].

4 CONCLUSIONS

Analytical approaches for predicting the flexural behaviour of RC slabs strengthened with non-prestressed or prestressed CFRP laminates were developed. The key findings can be highlighted as follows.

1) The developed analytical approaches are found to provide the force-deflection curves that are in a good agreement with those from experimental program.
2) Regarding the prediction of the CFRP failure strain, the results from various formulations show that the performance of these formulations significantly depend on the strengthening technique and failure type. In fact, in the developed approach: fib90 [11] or CNR [15] predict well the CFRP failure strain for EBR at ICD failure; Barros et al. [3] and Balaguru et al. [14] equations lead to promising CFRP strain prediction for NSM failing by CFRP rupture; the combination of fib90 [11] or Balaguru et al. [14] with Eq. (4) or Eq. (5) may lead to promising ultimate capacity prediction for MA systems failing by CFRP rupture; and the formula from fib90 [11] and Eq. (6) may lead to promising ultimate capacity prediction for GA systems failing by ICD.
3) The newly developed equations for determining the curvature for prestressed systems are promising, mainly because these formulations incorporate the effect of the type of failure mode and are observed to effectively participate in providing the deflections that are in a very good agreement with those from experimental program.
4) Extension of the developed approach to include the formulations that provide better predictions of the stiffness in different loading stages can be highly recommended, particularly for NSM techniques.

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