Introduction to Hydrology

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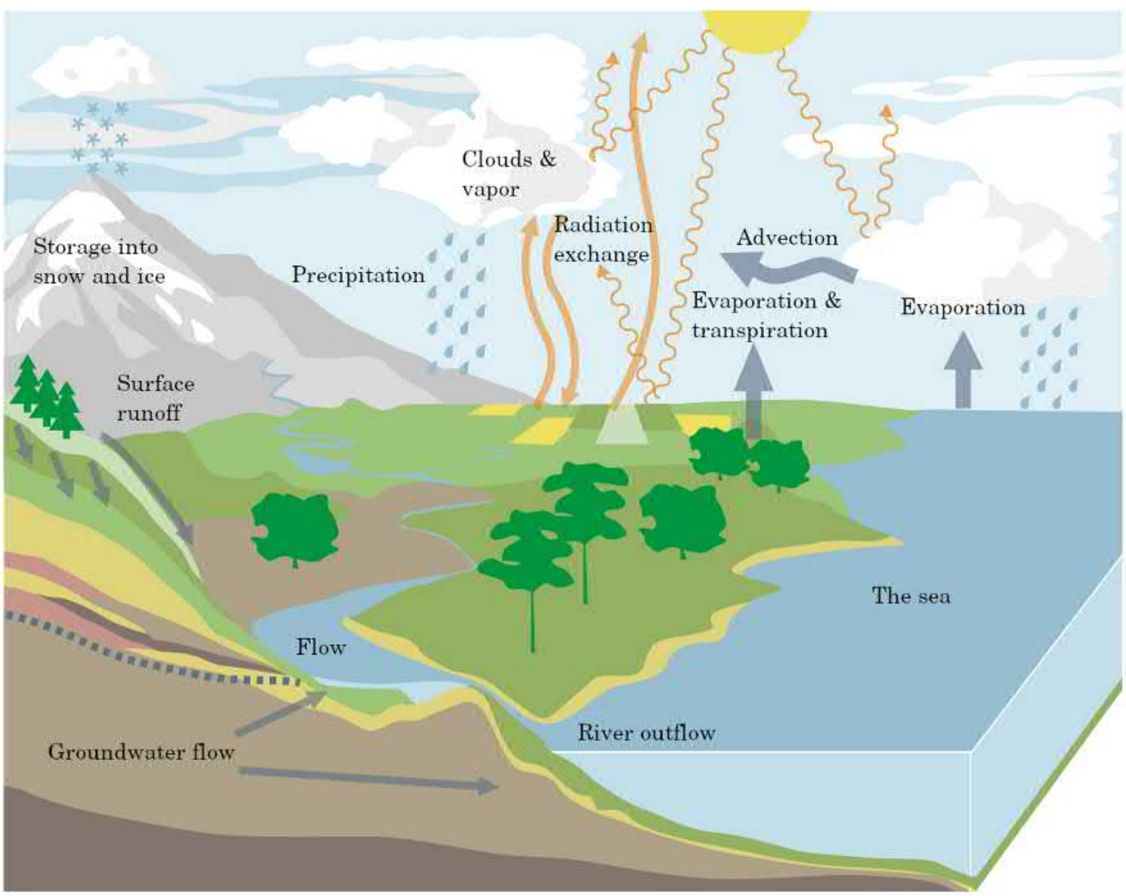

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Properties of fresh water and air

Fresh water (temperature +10 °C, pressure 1013.25 mbar)

Molar mass	m_0	18.016 g mol^{-1}
Density	ρ	999.70 kg m^{-3}
Dynamic viscosity	μ	$1.31 \cdot 10^{-3}$ kg m^{-1} s^{-1}
Surface tension	σ	0.0742 N m^{-1}
Compressibility	K	$4.82 \cdot 10^{-7}$ Pa^{-1} = $4.82 \cdot 10^{-5}$ bar^{-1}
Speed of sound	c_s	1 450 m s^{-1}
Thermal expansion coefficient	α	$0.88 \cdot 10^{-4}$ °C^{-1}
Specific heat capacity	c	4.19 kJ °C^{-1} kg^{-1}
Thermal conductivity	k	0.580 W °C^{-1} m^{-1}
Latent heat of vaporization	L_e	2.47 MJ kg^{-1}
Relative permittivity	ϵ	83.3

Freezing and ice (temperature 0 °C, pressure 1013.25 mbar)

Temperature of maximum density	T_m	3.98 °C
Freezing point	T_f	0 °C
Latent heat of freezing/melting	L_f	333.6 kJ kg^{-1}
Density	ρ_i	917 kg m^{-3}
Thermal expansion coefficient	α	$1.5 \cdot 10^{-4}$ °C^{-1}
Specific heat capacity	c_i	2.11 kJ °C^{-1} kg^{-1}
Thermal conductivity	k_i	2.14 W °C^{-1} m^{-1}
Relative permittivity	ϵ_i	91.6

Water vapor

Gas constant of water vapor	R_w	461.5 J °C^{-1} kg^{-1}

Air (temperature +10 °C, pressure 1013.25 mbar)

Mean molecular mass	m_a	28.97 g mol^{-1}
Gas constant of dry air	R_a	287.04 J kg^{-1} °C^{-1}
Density	ρ_a	1.250 kg m^{-3}
Dynamic viscosity	μ_a	$1.764 \cdot 10^{-5}$ kg m^{-1} s^{-1}
Specific heat capacity	c_a	1.004 kJ kg^{-1} °C^{-1} (at constant pressure)
Thermal conductivity	k_a	$2.501 \cdot 10^{-2}$ W m^{-1} °C^{-1}

List of symbols

a	absorption coefficient
a_y	absorption coefficient of CDOM
A	surface area
B	Bowen ratio
c	concentration, specific heat, attenuation coefficient
c_0	speed of light in vacuum
c_s	speed of sound
C	coefficient of friction
C_b	coefficient of basal friction
C_D	drag coefficient
C_E & C_H	transfer coefficient of heat, transfer coefficient of moisture
d	diameter
D	diffusion coefficient
e	vapor pressure (water)
E	evaporation, irradiance
E_p	potential energy, potential evaporation
f	friction, Coriolis parameter

F	force, friction
Fr	Froude number
g	gravitational acceleration
h	height, depth, thickness of ice or ground frost, Planck constant
H	depth, energy head
I	influx
k	thermal conductivity, wave number
k_B	Boltzmann constant
k_r	recession constant, thermal conductivity of ground frost
K	compressibility, turbulent diffusion coefficient, hydraulic conductivity
L	length, unit of length
L_E	latent heat of evaporation of water
L_f	latent heat of freezing for water
m	mass, water content
M	mass, Manning coefficient
n	porosity
n_e	effective porosity
N	cloudiness, Brunt-Väisälä frequency
N_A	Avogadro constant
O	outflow
p	pressure, probability density
P	precipitation, wet perimeter, cumulative distribution function
P_e	effective precipitation
q	specific humidity
Q	heat flux, discharge
Q_c	sensible heat flux
Q_e	latent heat flux
Q_g	heat flux into ground
Q_L	Long wave radiation of surface or atmosphere
Q_n	net heat flux

Q_P	precipitation heat flux
Q_s	short wave radiation
Q_{SC}	solar constant
r	radius
R	runoff, universal gas constant, Bowen ratio, hydraulic radius
R^*	interseption coefficient
R_a	gas constant for dry air
R_d	direct runoff
Re	Reynold's number
RH	relative humidity
Ri	Richardson's number
Ro	Rossby number
R_w	gas constant for water vapor
S	salinity, negative degree days, water storage, source/sink, slope
t	time
T	temperature, unit of time
T_F	freezing temperature
T_m	temperature of maximum density
u	eastward component of velocity
U	mean velocity, wind speed, unit of speed
v	northward component of velocity
V	volume, speed of sound, wave speed
w	vertical component of velocity
W	work
x	east coordinate
y	north coordinate
Y	snowdrift
z	vertical coordinate, depth
Z	solar zenith angle
α	albedo

β	slope
γ	fraction of visible light solar radiation, psychrometric constant, ρg
Γ	hypsographic curve
δ	flattening, declination
Δ	difference operator, change
ϵ	emissivity, relative permittivity
η	frazil ice thickness
Θ	thermal energy
θ	potential temperature, solar incidence angle
κ	attenuation coefficient of light, exponent describing the temperature – pressure relation of a gas
λ	wavelength
μ	dynamic viscosity
ν	kinematic viscosity, porosity
ρ	density
σ	electrical conductivity, Stefan – Boltzmann constant, surface tension
τ	shear stress, hour angle
ω	frequency
Ω	rotation rate of the Earth

Subscripts

a	air
i	ice
s	snow, saturation
w	water
0	surface, reference

Superscript

$'$ fluctuation (in turbulence)

Preface

Hydrology is the branch of geophysics on the fresh water resources and the cycle of water on Earth. Its basis is very practical: making liquid water available for use in households, irrigation and industry. This science is rooted in the usage and rationing of rivers in ancient Egypt, Mesopotamia, India and China. The importance of water was postulated by Thales of Miletus (636 – 546 BC), who considered it to be the origin of everything. Scientific view of the cycle of water was developed in the 17th century, and monitoring systems for water resources were founded in the following century. Water supply is a cornerstone of a functioning society. Nowadays, in addition to availability of water, flood protection, needs of the hydropower industry, transport, and water quality have become ever more important questions.

Hydrology concerns precipitation and evaporation (hydrometeorology), runoff, lakes and rivers, soil water and groundwater (geohydrology), glacial hydrology, and the water cycle which connects all these parts. Groundwater forms the most significant storage of liquid fresh water in our planet, and rivers are important transport routes of water and substances and also modifiers of landscape. Lakes form temporary storages of water, and especially in Finland lake hydrology has been a very important topic. Glaciers form storages of water as well, and melt water from some ice caps are used by people. For the research of water ecology, hydrology provides the physical background of the water resources and information on the linking of physics to the state and ecology of natural waters.

This book contains the basics of hydrology. As a prerequisite the reader should have a high school level knowledge on aquatic systems. There are plenty of other books available in English, of which for example Hendricks (2010) *Introduction to Physical Hydrology* is suitable for beginners. In Finland, university level teaching in hydrology is given at the Universities of Helsinki, Jyväskylä and Turku. Engineering hydrology is being taught at Aalto university and University of Oulu.

This book is based on the lecture notes of professor emeritus Juhani Virta for the course *Basic course in hydrology*, into which professor Timo Huttula and professor emeritus Matti Leppäranta have made their own additions. This book is used as a textbook on the course *Introduction to hydrology* at the University of Helsinki, where it is a part of the studies of geophysics, limnology and fisheries science. In general, this book is suitable as a university textbook, for self-study or as a small reference book. The contents of this book revolve around hydrology in the Finnish environment. The course is 5 ECTS credits, consisting of 30 hours of lectures and 15 hours of exercises.

We thank docent Esko Kuusisto for his help and photographs that were very useful in the making of this book. The assistants and students of the course *Introduction to hydrology* have also helped a lot by bringing up questions that we have tried to answer in this book. The original Finnish text has been checked by B.Sc. Jesse Heikkilä, B.Sc. Arttu Jutila, Ph.D. Niina Kotamäki, M.Sc. Anna-Riikka Leppäranta, Ph.D. Pekka Rossi and M.Sc. Cecilia Äijälä. Many of the figures were made by Ph.D. Salla Jokela. Pictures have been given to our use by Elonet National Filmography, Hämeenlinna Art Museum, Finnish National Gallery, Tomas Kohout, Petrina Köngäs, Mobilia-museum, The Regional Council of Päijät-Häme, professor Jouko Sarvala, professor Kari Suomalainen's heirs collectively, Finnish Environment Institute, Tretyakov Gallery in Moscow and M.Sc. Jouni Tulonen.

The finalizing of this book took place during the six month sick leave of the first author in 2016. This was possible thanks to the good care and treatment given by the staff at Päijät-Häme Central Hospital ward 33 and Meilahti Triangle Hospital ward 5B. The Association of Finnish Non-fiction Writers gave a grant for this project, for which we are very grateful.

Professor emeritus Juhani Virta passed away after a sudden illness on 6th of November 2016. This was, for us signatories and his students, a great loss and we will remember him with great warmth from the times of his lectures and field work. Teachings and thoughts of Juhani Virta can be found in many parts of this book.

Helsinki and Jyväskylä on 31.1.2017

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Translator's preface

As a student in the field of hydrology and geophysics, one is constantly in contact with the English language. Reading, writing and speaking. It has almost become my second mother's tongue, and sometimes I even find myself thinking in English. For this reason I thought that translating a book into English would be fairly easy. Something to be done over one summer while living on a boat. Sure enough, it was done, but it took a bit more effort than I first thought.

Translating a book into a language that is not your mother's tongue is bit tricky, especially for someone who is not a translator. It is difficult to get rid of all the structures of the original language. While they might be sensible for anyone Finnish, they might sound weird and even be blatantly wrong for anyone else. I hope that I was able to get rid of most of that, and that the reading experience is as pleasant as it is in the original language.

As a sidenote I must say, that for anyone wanting to learn the contents of any book I must recommend translating said book. It forces the reader to go through the book sentence by sentence. Sort of like disassembling and putting a book back together.

For translations of hydrological, meteorological and mathematical terms I have used *The Helsinki Term Bank for the Arts and Sciences* [1] and *The Nordic glossary of hydrology.* These were of great help, and I could have not finished the translation without these resources.

I was able to work on this project thanks to the funding from the Institute of Atmospheric and Earth System Research (INAR) Digiloikka (*Eng.: Digital Leap*) project and Ph.D. Laura Riuttanen, who obtained the funding, and trusted me with this work. Thank you for giving me free hands on the work. Thanks also belong to my girlfriend Ulrika Repokari, who captained our boat *Lunni* well for many long and short legs, allowing me to concentrate on the translation in the comfortable rocking motion of the Baltic Sea.

Helsinki on 9.1.2020

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[1] https://tieteentermipankki.fi/wiki/Termipankki:Etusivu/en

Chapter 1

Introduction

The hydrosphere of the Earth consists of oceans, ice and snow, water in the ground, lakes, rivers and the water in the atmosphere. Nearly all of the water (97 %) is saline sea water, but people need fresh water[1] for household use, agriculture and industry. This usable water is referred as the water resources. The occurrence and cycle of these resources have controlled the spread of life on our planet, the migration of peoples and the birth and development of urban centers throughout the history (Fig. 1.1). Thus a claim can be made, that the future of mankind and of all life on Earth depend on the availability of clean water.

The water on Earth is going through an eternal cycle. It is evaporated from the surface to the atmosphere, only to return as precipitation. Water is purified in this process, because evaporation takes only molecules of water. Upon falling back onto the surface, precipitated water drains along lake and river systems and ground toward the ocean, also evaporating from the surface along its way. In the Earth's hydrological cycle, the total amount of water remains fixed. A small fraction disappears out into space, but comets impacting Earth bring in new water. In hydrological applications the amount of water on Earth can be assumed constant up to the time scales of hundreds to thousands of years.

The hydrosphere consists of several large reservoirs (Table 1.1). The total amount of fresh water comes in at 35 million km^3, most of which ($^2/_3$) is bound to the large continental ice sheets (Antarctica and Greenland). The frozen phase storage is renewed slowly. Around one half of the ground water is fresh, and this makes up for $^1/_3$ of the total fresh water storage. From the human perspective groundwater is very important due to its liquid phase (Fig. 1.3). Lakes are a prominent storage of fresh water in lake districts, although their fraction of the total fresh water storage is small. Rivers form the transport network through which most of the hydrological cycle drains back into the ocean. They store a very small fraction of fresh water, about 2130 km^3, or about 2,3 % of the volume of lakes. The storage in the atmosphere is very small as well, but its capacity to transport water is remarkable, and, in addition, the atmosphere functions as a very efficient water purification route. The retention period of water in the atmosphere is about 10 days.

In 1995, 7.2 billion m^3 of fresh water was used by the Finnish industry. The majority of this (62 %) was used for the cooling of power plants. Pulp and paper industry used 17 % (for cooling and process water), and oil and petrochemistry took 12 % (mostly cooling).

[1] Fresh water refers to water, where the fraction of dissolved solids is less than 0.5 ‰

Figure 1.1: *Riparian areas have historically been the sites of major population centers due to their water resources and fertile soil. River mouths also act as natural transport hubs. Pictured here is the river Daugava, which flows through the city of Riga, Latvia. It is the third largest river flowing into the Baltic Sea, with a discharge of 20.8 km*^{3} *annually and catchment area of 87 900 km*^{2}*. The total river discharge into the Baltic Sea is 440 km*^{3} *annually. Photo: Matti Leppäranta.*

Other less significant users were chemical industry (4 %) and metal refinement (2 %).

Finnish households consume about 270 million m^3 of water (about 4 %), which translates into 150 litres per person per day (Table 1.2). This can divided into three roughly equally sized fractions: personal hygiene, flushing of toilets and laundry. Cooking and drinking account for 6 % of the total household consumption.

Hydrology is the study of the water resources and the water cycle. This includes the study of water in soil and ground (*geohydrology*[2]), the study of lakes (*limnology*), the study of rivers (*potamology*), and the study of atmospheric water (*hydrometeorology*). In addition, glacial hydrology concerns the glacial melt water and its usage.

Although a global approach to hydrology is on the rise, most hydrological research is still performed on a regional scale. Subjects that are especially on focus are the formation of runoff in the water cycle, floods, and water quality. In Finland, all of the main branches of hydrology have been under investigation. But, due to the nature of the Finnish environment, particular interest has been given to lakes, and snow and ice. The needs of the maritime community gave start to the study of the seas (*oceanography*), and that is considered as a separate issue from water resources (Myrberg & Leppäranta, 2014). Although, the division is

[2]In geology, the term *hydrogeology* is in use.

Table 1.1: *Earth's water resources according to UNESCO (1978). Surface waters refer to lakes and rivers.*

Storage	**Volume [km^3]**	**Volume [%]**	**Fraction of fresh water [%]**
Oceans	1 338 000 000	96.5	N/A
Ice sheets & glaciers	24 023 500	1.7	68.6
Fresh groundwater	10 530 000	0.76	30.1
Saline groundwater	12 870 000	0.93	N/A
Soil moisture	27 970	0.002	0.08
Seasonal ice & snow	340 600	0.025	1
Atmospheric water	12 900	0.001	0.04
Biospheric water	1 120	0.0001	0.003
Fresh surface water	93 120	0.007	0.27
Saline surface water	85 400	0.006	N/A
Total water	**1 385 984 610**	**100**	**N/A**
Fresh water	**35 029 210**	**2.5**	**100**

becoming grey since lately desalination of sea water for household use has been commenced in places where water resources are scarce.

Since the very early history, hydrological research has been concerned on the availability of water for households and irrigation. The nature of water was pondered upon during the classical antiquity, where water was considered as one of the four principal elements. Hydroengineering became quite advanced in the Roman empire, aqueducts being an iconic symbol of their skills. Scientific research of natural waters began in the renaissance era. Leonardo da Vinci's description of the water cycle was conceptually correct, and step by step the theory was made more accurate with the tools of the new scientific era. Da Vinci also studied the flow of rivers and designed canals. The physical basis of hydrology was laid down with the advent of thermodynamics and fluid dynamics in the 19th century. Then also began widespread experimental fieldwork and systematic collection of hydrological time series.

Hydrology took its first steps in Finland in the 18th century. The research was motivated by the need to clear rapids in order to make rivers more usable for log transportation, for lowering water level of lakes to claim more arable land, and to control flood damage and droughts. Frost was one of the most prominent research questions of the 19th century in Finland, as the European climate was colder than now and summer frost could destroy crops and cause famine. The damage from the Great Flood in summer 1899 was extensive that

Figure 1.2: *Lake Valkea-Kotinen (Evo, Finland) has been the subject of many hydrological studies since the 1980s. It represents boreal small and humic lakes. Pictured is a maintenance trip to a measurement raft on the lake. Photo: Matti Leppäranta.*

it led to founding of the Hydrografical Office (*Fin.:* Hydrografinen toimisto) in 1908. Its duty was to set up a network of hydrological monitoring. In 1928 the Ministry of Agriculture started the research of agricultural hydrology. In 1969 these two organizations were combined to form a new Hydrological Office (*Fin.:* Hydrologian toimisto).

Modern hydrology began to take shape in the 1960s, with the advent of powerful computers, automated measurement systems, and remote sensing capabilities. Longest hydrological time series reached a ripe age of 100 years. Also, numerical modeling of hydrological systems saw daylight in the 1960s. New challenges to hydrological research were posed by problems in water quality and environmental issues. Nowadays, environmental issues form a complex set of problems, and new challenges posed by the connections between changing climate and water resources have created very important questions to be answered. In particular, hydrological forecasting for practical applications is of great political and economical interest. In 1995, the state hydrological research was concentrated to the new Finnish Environment Institute (SYKE) and regional environment centers that are now part of the regional Centres for Economic Development, Transport and Environment (ELY). Hydrology in Finland is currently taught and studied in the universities of Helsinki, Jyväskylä and Turku, and technical applications of hydrology are taught in the Aalto University, and the universities of Oulu and Tampere.

Technological development has increased and diversified the need for hydrological research. In order to guarantee the supply for the increasing demand for clean fresh water, the dynamics of the circulation and flow of lakes and rivers, the movements of ground waters, and the quality of the water resources need to be understood. The physics of natural waters have

Table 1.2: *Usage of water in Finnish households in litres [L]. Source: Lahti Aqua (http://www.lahtiaqua.fi).*

Application	Amount [L]
Cooking and drinking	6
Washing dishes	20
Cleaning	5
Personal hygiene	55
Flushing toilet	40
Laundry	20
Other	4
Total	**150**

become relevant also in the fields of aquatic ecology and fisheries, since it tells of the living conditions in water bodies. Acquisition of our basic knowledge on natural waters has required long-term time series that are still being updated. Hydraulic engineering, the use of natural waters, acidification, problems arising from regulation of lakes and rivers, and pollution of the environment still increase the need for hydrological basic research. This book presents the basics of hydrology with an emphasis on research questions that are characteristic for the Finnish water environment.

Chapter 2 presents the basic research tools used in hydrology with examples of their usage. This foundation will be used later on in the book. Chapter 3 discusses water as a geochemical compound and its physical characteristics. Also, the principles of the water cycle are presented, with additional details laid out later on so that by the end we have a holistic picture of the cycle. Chapter 4 contains a description of hydrometeorology, i.e. rain and evaporation will be in the focus. Snow hydrology is discussed in this chapter as well. Chapters 5 & 6 concern the surface waters. Lake hydrology introduces the concept of water balance, and the thermodynamics, dynamics and water quality are discussed as well. River hydrology concentrates on discharge and its effects on erosion and transport of matter. Chapter 7 is about geohydrology, the properties and dynamics of ground water and aquifers. Soil heat balance and frozen ground will receive attention as well as glacial hydrology. Chapter 8 ties previous chapters together and describes runoff as a part of the water cycle, as well hydrological models. Chapter 9 focuses on the future of water resources in the changing climate.

Figure 1.3: *The spring of Narkissos in Monrepos park (Vyborg, Russia). According to folklore, its waters have health promoting properties. Locals have referred to it as "The Eye". Springs are fed by the flow of groundwater. Nowadays, they are protected by the Finnish environmental legislation. Photo: Matti Leppäranta.*

Chapter 2

Hydrological research methods

This chapter presents the methods and tools used in hydrological research. These are supplemented by relevant examples and exercises. The methods have been arranged in four categories: fieldwork, statistical hydrology, dimensional analysis, and modelling. Traditionally, hydrological questions have been tackled with field experiments and statistical analysis. Dimensional analysis has a dedicated part in this chapter, as it is a relatively simple, yet powerful tool for empirical research of natural waters. Mathematical and physical models are discussed in a superficial manner, with the goal of introducing their basic concepts to the reader. Supplemental mathematical information is provided in the appendix of this book. A reader who has a university level basic knowledge in mathematics and statistics can discard this appendix.

2.1 Field measurements

Typically, hydrological questions and systems are complex due to topography, soil properties, land use, vegetation and weather. This complexity makes the usage of theoretical and mathematical models problematic. True knowledge of a hydrological system can only be acquired with well performed field measurements, which then are also used in calibrating, tuning and controlling of models. Fieldwork is at the core of hydrological research (Fig. 2.1), but due to their high costs, field experiments are rather limited in their numbers.

Public institutions have founded hydrological monitoring systems for the use and protection of water resources. Monitoring is done partly automatically and partly manually (Fig. 2.2). Finnish Environment Institute (SYKE) operates a wide network of monitoring stations, which provide information on the following variables:

Figure 2.1: *Preparations for the deployment of recording current meters in Lake Päijänne, summer 1987. Visible in the photo are Timo Huttula (foreground) and Jorma Koponen. Photo: Juhani Virta.*

Variable	**Measurement strategy**
Snow thickness and water equivalent	Snow stations and snow lines
Water level	Limnigraphs
State of lakes	Temperature, water quality, ice stations
State of rivers	Temperature, water quality, ice stations
State of groundwater	Water level, water quality
Ground frost	Prevalence, thickness

The state of water resources is strongly dependent on meteorological conditions. For this, operative meteorological observations are utilized in water resource monitoring. In addition to providing data for basic research, the society benefits from relevant information on, for example, flooding and extent of safe river and lake ice cover. Such monitoring also provides the means for the forecasting of hydrological hazards.

Field experiments are used to collect more detailed data sets of physical quantities and mechanisms to improve the usefulness of long-term monitoring networks. Examples of such experiments include direct measurements of evaporation, circulation of lakes, river discharge, groundwater aquifer yield, and variations in the properties of snow layers.

Remote sensing refers to observations of an object or area of interest without direct physical contact. Usually, this is considered to cover airborne or space-borne observations

Figure 2.2: *Niittyjoki measurement weir, built in 1958, standing near Kouvola, Finland. Weir can be used to determine the flow rate of a river. Catchment area of Niittyjoki is 29.7* km^2 *and mean flow rate is 0.25* m^3 s^{-1}*. Photo: Finnish Environment Institute (Kuusisto 2008).*

with electromagnetic signals. Remote sensing can also be performed from the surface of the Earth, as is the case with ground penetrating radars. These instruments are used to detect abrupt changes in the vertical structure of the underlying ground, such as the presence of water. Underwater acoustic sensing has been successfully employed in the mapping of river and lake bottom topography as well as sediment properties.

Since the 1960s, the use of remote sensing methods has grown steeply within the hydrological community (Fig. 2.3). First, aircrafts were employed, but in the last 30 years satellites have become the prominent measurement platform as their spatio-temporal resolution has increased sufficiently. Satellites observe the Earth's surface via electromagnetic radiation. Several radiation bands are useful for hydrological applications: visible light (0.4 – 0.7 μm), near-infrared (0.7 – 1.0 μm), thermal radiation (5 – 15 μm) and microwaves (1 – 100 cm). By employing these windows, the Earth's surface can be observed but hardly anything beneath.

Mapping the coverage and properties of seasonal snow has been one of the most prolific applications of remote sensing in hydrology from 1970s forward (Kuittinen 1988). Satellites can only see the extent of the snow cover, but the thickness and water equivalent of snow can only be estimated by indirect means. Since the year 2000, high-resolution operative satellite data has been easily available that has broadened the possibilities for real-time mapping. Remote sensing of the surface temperature of lakes, ice cover and algal blooms have all become part of the hydrological toolbox. Still, hydrological applications of remote sensing inherently include large uncertainties in them, and thus these will not be completely replacing in-situ field measurements and monitoring.

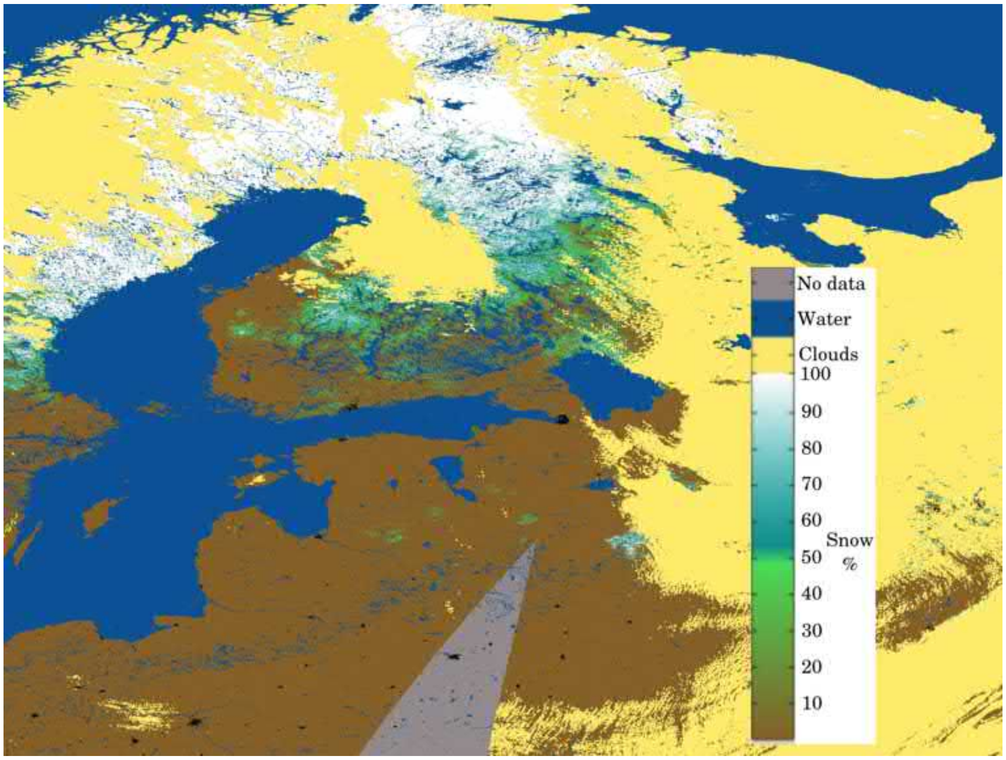

Figure 2.3: *Snow coverage in Northern Europe on 21 April 2013, as determined by MODIS (Moderate Resolution Spectroradiometer) camera on-board Aqua/Terra satellite. Shades from brown to white represent snow cover extent (0 – 100 %), yellow is cloud cover and blue represents water. Photo: Finnish Environment Institute, original observation ©NASA.*

2.2 Statistical hydrology

2.2.1 Classical statistics

Classical statistics refers to the study of a set of one or more independent random variables. This includes resolving distributions of variables, estimation of parameters, testing hypotheses and comparing data sets. Often, independent observations are reached by examining variations or by having long enough time intervals. Multiple variable analysis considers relationships between random variables. To answer, whether X and Y are interdependent, one must ask 'if I knew the value of X, does it give me any additional knowledge about the value of Y?'. Discovering dependencies offers possibilities for producing forecasts and estimates. Statistical correlation means a linear dependence between two variables but it does not imply a causal physical relationship. For the application of statistical methods, there exists plenty of literature and software tools, such as *R, Matlab, Excel, SPSS & SAS*.

Example 2.1
a) Water level in a lake follows a yearly cycle, where the mean values of any two consecutive months are correlated, but deviations from long-term mean or changes between months are weakly correlated at most. Similarly, soil surface temperatures within one month are correlated, but yearly averages are not.

b) Lottery draws of two consecutive weeks are completely independent, assuming that the machine is working properly. Thus, old lottery draws give no information on the future draws.

c) During spring, the sale of ice cream and the number of common chaffinches (*lat.: fringilla coelebs*) are clearly statistically correlated, but causation between these two does not exist.

d) One can assume, that the surface current in a lake depends on the wind speed. With observations, a more precise connection can be established for practical applications.

Statistical variables are divided into four levels, each having their own set of permissible arithmetic operations:

Level	**Permissible operations**	**Example**
Nominal level	Total number by class	Terrain type
Ordinal level	Organisation	River order
Interval level	Addition & subtraction	Celsius-scale
Ratio level	All arithmetic operations	Speed

Variables of the nominal level have a name, and only their occurrence can be studied. These variables cannot be arranged into any meaningful order. The names of people in a group form a set like this. The occurrence of any specific name can be studied and the most common name of this group would be its mode. The global mode of human surnames would be the Chinese name *Li*. Variables in the ordinal level can be arranged, but the difference between two members of this set cannot be discerned. An excellent example of the ordinal scale is the famous tango festival in Seinäjoki, Finland where singers are arranged by their skills, but the differences between the singers' skills cannot be quantified.

Variables in the interval and ratio level have numerical values, but they have one crucial difference. On the ratio level, all arithmetic operations are allowed, while the interval level only allows addition and subtraction. Temperature is a perfect example to illustrate this. The Celsius scale, developed by Anders Celsius (1701 – 1744) is at the interval level, and kelvin scale, developed by William Thomson, a.k.a. lord Kelvin (1824 – 1907), is at ratio level. Most of the common quantities, like speed and mass, are on the ratio level as well.

Example 2.2
One rather important interval level variable widely used in hydrology is the Celsius scale.

Differences of Celsius-degrees can be measured, but their proportions are meaningless. Therefore, if temperature increases from 5 °C to 6 °C, it can be said that the temperature increased by 1 °C, but it does not make sense to say that the temperature increased by 20 %. In Kelvin scale one could say that the temperature increased by 0.36 %, but to our intuition that does not say much.

Random variables follow certain types of distributions, and the knowledge of them lays the foundations for statistical analysis. Distribution describes the probabilities of a random variable, and it can be represented as a histogram or as a probability density function. The horizontal axis contains the possible values of the variable, and the corresponding probabilities are found on the vertical axis. Ordinal level variables are not feasible for a histogram presentation. For interval and ratio scales there exists a multitude of various theoretical distributions with open parameters.

A rather widely used distribution used in statistics is the Gaussian distribution (also known as the normal distribution). Statistical validations and tests provided by statistical software are typically based on the Gaussian distribution, therefore it is important to know, whether this is the right distribution for the specific application. Gaussian probability density p is written as

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2\sigma^2}(x-\mu)^2 \right] \tag{2.1}$$

where x is the random variable, μ is the mean and σ is the standard deviation (σ^2 is variance). This distribution is symmetric around its mean and has the characteristic bell shape (Fig. 2.4). The standard deviation describes how tightly the values are packed around the mean. A useful rule is, that the range $\mu \pm 2\sigma$ contains 95 % of the distribution's probability mass.

The probability $P[a, b]$ that a variable is within the range $[a, b]$ is the integral of the probability density over this range: $P[a, b] = \int_a^b p(x)dx$. Gaussian distribution cannot be integrated in closed form, but values of this integral are readily available in statistical tables and software. According to the *central limit theorem* in most cases the sample distributions approach Gaussian distribution when the sample size becomes large.

On interval and ratio levels, so-called fractiles x_p can be constructed. Those are defined as

$$x_p : P(x \leq x_p) = p. \tag{2.2}$$

As such, p-fractile x_p is the limit below which x is found with a probability p. For example, the probability of x being less than fractile $x_{0.1}$ (which can represent a practical lower limit of the variable) is 0.1. Fractiles are used, for example, to describe the size distribution of soil particles, and to present values in the analysis of extremes. The median of a distribution is its midmost value, so that $P(x \leq x_{md}) = P(x \geq x_{md}) = \frac{1}{2}$. Fractile $x_{0.5}$ represents the median, by definition.

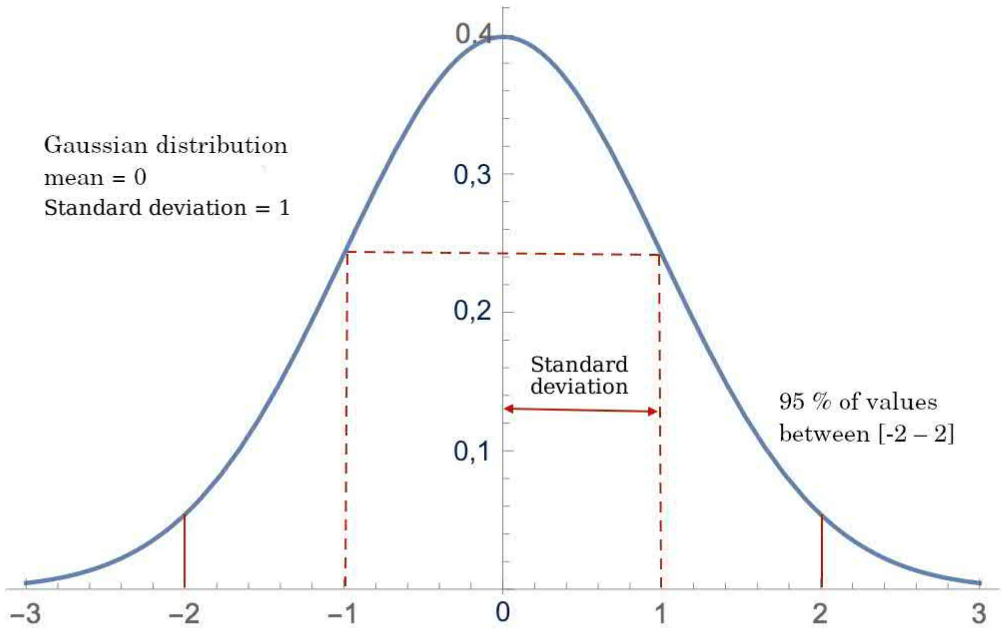

Figure 2.4: *Gaussian distribution, with a mean $\mu = 0$ and standard deviation of $\sigma = 1$*

Example 2.3
Exponential distribution is widely used in geosciences. Its probability density function is $p(x) = \lambda e^{-\lambda x}, x \geq 0$. It has its maximum at origin, and the function descents monotonously as x grows. This distribution can describe the prevalence of extreme and rare events. In that case, x is the time passed since the previous incident. This function has also been applied to describe the queue time of a customer, in which case λ describes the strategy of the business. This density function can be integrated, and the probability that the variable is smaller than x is $1 - e^{-\lambda x}$. x_p fractile is acquired from the equation $p = 1 - e^{-\lambda x_p}$.

Probability distributions have characteristics, or statistics, whose values need to be estimated. These include, for example: mean or expected value, variance, minimum and maximum. Even if it is known that the data set follows a Gaussian distribution the expectation and variance need to be determined from observations. The most important requirements for an estimator are that it should be unbiased (i.e., that the expected value is equal to the statistic in question) and that it should have as high accuracy as possible (i.e., as small variance as possible). The optimal estimators for expected value $\bar{x}$ and variance s^2 are

$$\bar{x} = \frac{1}{n}\sum_{k=1}^{n} x_k \ , \ s^2 = \frac{1}{n-1}\sum_{k=1}^{n}(x_k - \bar{x})^2 \tag{2.3}$$

where $x_1, ..., x_n$ form the data set in question. In the equation for variance, the divisor in the square sum is $n - 1$ due to the fact that upon computing the mean, one degree of freedom[1] is lost, and thus the number of independent deviations from the mean is $n - 1$. Variance

[1]In classical statistics, a *degree of freedom* refers to the number of linearly independent observations.

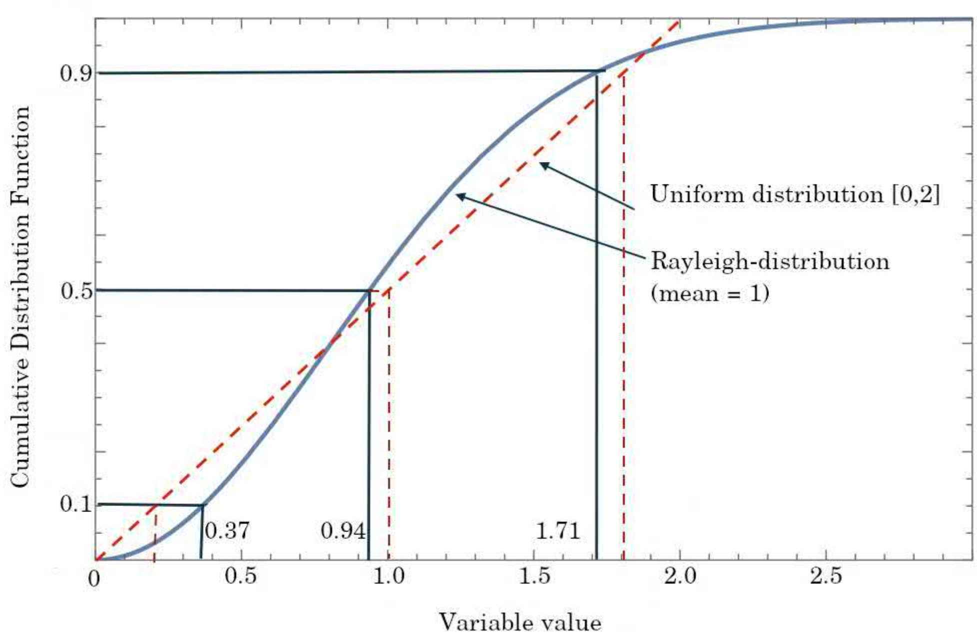

Figure 2.5: *Defining fractiles from probability density function. In this example, fractiles 0.1, 0.5 and 0.9 are marked, the solid lines representing Rayleigh distribution and dashed representing uniform distribution. The distributions have been normalized so, that their mean* $\mu = 1$. *The probability density function of Rayleigh distribution is* $p(x) = \frac{\pi}{2}x\ exp(-\frac{\pi}{4}x^2),\ x > 0$, *and for uniform distribution* $p(x) = 0.5,\ 0 \leq x \leq 2$.

is used in statistical analysis due to its mathematical properties. *The law of large numbers* tells that a sample average approaches the expected value as the number of observations n increases, and that the standard deviation of a sample average is $s/\sqrt{n}$. Standard deviation has the same dimension as the variable itself and therefore it is a natural quantity to describe the variation.

It is common in hydrological research that one needs to estimate parameters of extreme events, such as the maximum water level in a flood occurring on average once every 50 or 1 000 years. There is a branch of statistics regarding the analysis and extrapolation of extremes. Typically, if there are n independent observations $x_1, ..., x_n$ of a variable X, the distribution of maximums can be determined from the cumulative distribution of the variable itself

$$P(max(X_k) \leq x) = P(X_k \leq x)^n \tag{2.4}$$

Example 2.4
Water level X at a small bridge during a spring flood is distributed uniformly between 0 – 5 m over the mean water level. The cumulative probability function is then $P(X \leq x) = 0.2x$ and so

$$P[max(X_k; 1 \leq k \leq n) \leq x] = (0.2x)^n.$$

This shows us that the maximum is 4 m at most with a probability 0.33 when $n = 5$, or 0.11 when $n = 10$. Often it is also required to provide the maximum value x_{max}, with which $P(max(X_k) \leq x_{max}) = P^*$. This is acquired from the equation of this aforementioned probability: $x_{max} = 5\sqrt[n]{P^*}$. When $P^* = 0.9$ and $n = 5$ (10), $x_{max} = 4.90$ m (4.94 m). In fact, uniform distribution is unrealistic for situations regarding extremes, but it is sufficient to demonstrate the method.

Figure 2.6: *In Kallavesi, Finland, the history of water level has been inscribed in rock on the shore. The Great Flood of 1899 is clearly the highest. Photo: SYKE (Kuusisto 2008)*

In the study of extremes, the observed extreme events of the past control the estimation of distributions. These events are always exceptional, and the properties of the distributions of extremes have plenty of uncertainties. For example, the Great Flood of 1898 – 1899 in Finland was a remarkable extreme deviation from normal, and people have made markings of that water level in various locations in Finland (Fig. 2.6). In practice, the recurrence of abnormal events has to be extrapolated beyond the observed data set. A distribution is fitted into the data set that allows us to perform this but extrapolation brings a lot of uncertainty into the results. Gaussian distribution stretches into the infinity on both the negative and positive sides, but in hydrological systems there are physical upper and lower boundaries that the variable in question approaches with ever decreasing probabilities. The problem is that these boundaries are usually not well known. It has turned out that the Gaussian distribution is not particularly well-suited for the examination of extreme events, but other distributions tailored for this use have to be applied.

Dependencies between random variables are typically examined by correlation analy-

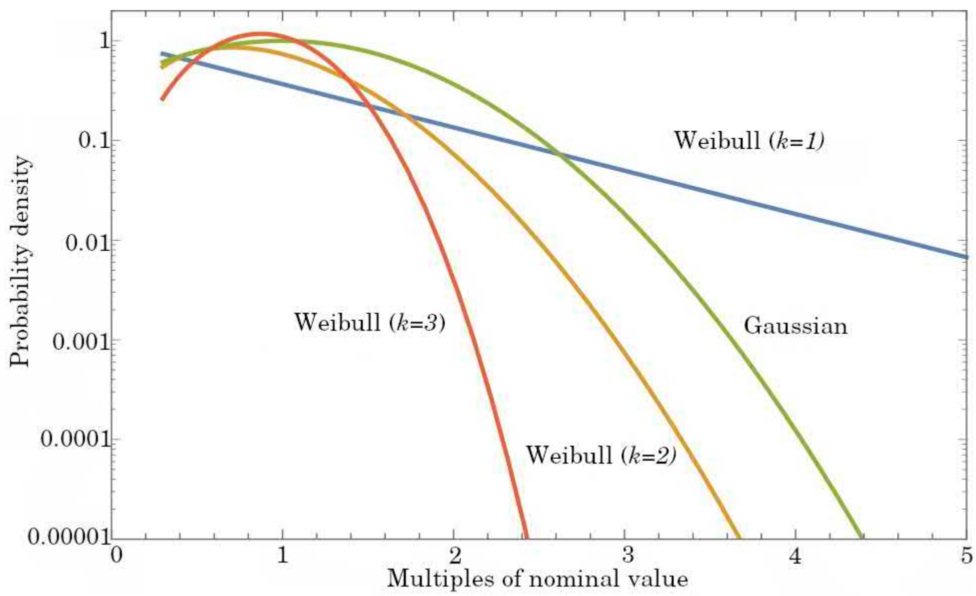

Figure 2.7: *Weibull distribution of extremes, compared to Gaussian distribution. Here the nominal value refers to scale of the mean, and for example multiple = 3 tells what value of probability density there is at three times the mean. When Weibull k-parameter increases, the distribution fits less and smaller extremes. The probability density functions of these distributions are: Weibull* $= \frac{k}{a}(\frac{x}{a})^{k-1} exp[-(\frac{x}{a})^k]$ *and Gaussian* $= \frac{1}{\sqrt{2\pi}\sigma} exp[-\frac{(x-\mu)^2}{2\sigma^2}]$.

sis. Correlation coefficient r_{xy} is the covariance (s_{xy}) of X and Y divided by the standard deviations:

$$r_{xy} = \frac{s_{xy}}{s_x s_y} \ , \ s_{xy} = \frac{1}{n-1}\sum_{k=1}^{n}(x_k - \bar{x})(y_k - \bar{y}) \tag{2.5}$$

It should be kept in mind though, that correlation is only a measure of linear dependency. If, for example, $X^2 + Y^2 \approx 1$, X and Y are strongly dependent on each other, but correlation between them is zero. Dependencies can be quantified through regression analysis, in which case we are examining relationships

$$Y = F(X_1,\, X_2,\, \dots\, , X_m; a_1,\, a_2,\, \dots\, ,\, a_p) + \epsilon, \tag{2.6}$$

where a_1, a_2, ..., a_p are relationship parameters and ϵ is the deviation related to the observation site. Variable Y is called the *regressand*, or *depended variable* and variables X_k are *regressors* or *independent variables*. Often, the so-called linear regression is used, which in the case of two variables provides the equation

$$Y = a + bX + \epsilon. \tag{2.7}$$

This method is also called the least squares method, since it attempts to minimize the sum of the squares of the deviations from the regression line (i.e., it minimizes the variance of the error term). The variables X and Y can be transformed from their originals (like X^n), as long as the equation is linear regarding the parameters a and b. Especially, if the hypothetical dependency is $Y = ae^{bX}$, this can be linearized into the form $\log(Y) = \log(a) + bX$, and analogously $Y = aX^b$ can be linearized as $\log(Y) = \log(a) + b\log(X)$.

Example 2.5
a) Often it is known that X and Y are proportional to each other, so $a = 0$. In that case, the coefficient b can be estimated more accurately by employing the boundary condition $a = 0$. In the least squares method the parameter b is determined so that the sum of the squares of the deviations $\sum_k (y_k - bx_k)^2$ reaches its minimum. The solution is $b = s_{xy}/s_x^2$.

b) It is common in regression analysis that we want to know the accuracy of the regression coefficients. In the case of the two variable model (Eq. 2.7), the covariance matrix of the coefficients a and b is

$$Cov(a,\, b) = \sigma_\epsilon^2 \begin{pmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{pmatrix}^{-1} \tag{2.8}$$

where σ_ϵ^2 is the variance of the error ϵ.

Upon testing hypotheses, a null hypothesis H_0 is established to represent a common perception, or the current situation, and it is complemented by a counter hypothesis, H_1. The decision table then is as follows:

	H_0 ***is true***	H_1 ***is true***
Accept H_0	Correct	Error II
Accept H_1	Error I (P)	Correct

The aim of statistical analysis is to figure out whether to accept or reject H_0. If H_0 is rejected when in reality it is true, we run into error I. The probability of this happening is the significance limit P, and it needs to be set to a sufficiently low level. Typically, 5 % is considered to be statistically significant, but sometimes when very strong proof is required, an even lower percentage is used. Error II is not considered as critical as error I, but in the mathematical formulation of the test it is kept low. The size of error II is not automatically known.

For a given data set, test statistics are calculated based on the probability distributions of the variables in question. Upon testing whether the data set (x_1, ... x_n) mean deviates from the given mean μ, a test statistic is computed:

$$t_{n-1} = \frac{\bar{x}}{s/\sqrt{n}} \ , \ \bar{x} = \frac{1}{n}\sum_{k-1}^{n}(x_k - \mu) \ , \ s^2 = \frac{1}{n-1}\sum_{k=1}^{n}(x_k - \bar{x})^2 \tag{2.9}$$

Here, t_{n-1} follows the Student's t-distribution with $n-1$ degrees of freedom. The larger the test statistic is, the more clearly the data set deviates from the null hypothesis. For example, one may study if a bridge has an effect on the flow rate of a river. The null hypothesis would be 'no effect'. Establishing hypotheses and testing them is an important part of hydrological research.

2.2.2 Time series

Time series analysis is needed, when there are temporal dependencies in the data set. This method also applies to one-dimensional spatial analysis. Important questions regarding hydrological time series analysis are time response, recurrence, persistence and extremes. As was mentioned above, transform from time series into a series of independent variables can be made by making the time interval between observations sufficiently long (i.e., longer than the correlation time scale). Thus, yearly values of hydrological variables can be usually treated with the methods of classical statistics.

Example 2.6
Let us take a look at a process $Y_{n+1} = aY_n + e_{n+1}$, where $a < 1$ is a constant, and e_n's are independent random variables whose mean is 0 and variance σ^2. This process can be written as $Y_n = \sum_{k=0}^{\infty} a^k e_{n-k}$, and its mean and variance are

$$\bar{Y_n} = 0\ ,\ \sigma_Y^2 = \left(\sum_{k=0}^{\infty} a^k\right)\sigma^2 = \frac{\sigma^2}{1-a}.$$

Autocovariance that has a delay of k is $\sigma_k = a^k \frac{\sigma^2}{1-a}$. Autocorrelation then is the autocovariance divided by the variance, or $r_k = a^k$. Correlation disappears gradually, as k increases. The memory of a system can be defined by setting a lower limit to correlation. If r_0 is chosen, memory is acquired from the equation $a^k = r_0$. A more objective way to to achieve this is to define the integral time scale of correlation:

$$\sum r_n = 1 + a + a^2 + ... = \frac{1}{1-a}.$$

If $a = 0.9$, the length of the memory becomes 10 time steps. As a practical example, one can imagine Y representing the summertime average surface temperature and e representing weather related disturbances. Parameter a then represents how far into the future these disturbances have an effect.

Hydrological time series are usually rather short, spanning just a few decades. The river Nile forms a special case in this regard, as the annual floods have been recorded there for over 2 000 years. In Finland, a few time series longer than 100 years have been collected. These are recordings of water level, flow rate and ice conditions. Observations of water level were started upon the construction of canals. In addition to systematic observations, many exceptional floods have been recorded into markings carved onto rocks by local people.

As is typical for time series, hydrological time series have some regular, temporal variation in addition to random variations. This structure is caused by both internal properties and external forcing. The length of the memory usually defines how far the internal properties reach. External forcing, such as changes in land use, weather and climate, typically affect the system with longer periods. The basic components of a time series are:

1. *Trend* refers to a monotonic change over time. In hydrology this typically takes place in the scale of decades and is usually caused by changes in climate. For example, the freezing date of lakes was delayed by about a week during the last century. Changes in land use (forest cuttings, drying of swamps, changes in crops etc.) can also have a long term effect.

2. *Periodicity* means regularly recurring phenomena. The basic periods are one year and one day. The retention time of water in lakes forms an internal period. In flow dynamics other shorter periods can be found, such as the seiche oscillations in lakes, typically ranging from minutes to hours (for example, 12 minutes for Lake Jyväsjärvi and 5 hours for Lake Päijänne). Climatic periods longer than one year are in the scale tens of thousands of years and are caused by oscillations in the orbital parameters of the Earth. These help to explain the ice age cycles and are called the Milankovich cycles. The 11-year-long sunspot cycle is not visible in hydrological time series.

3. *Shift* manifests itself as an abrupt change in the nominal level. This can be caused by a change of the measurement site, a change in the surrounding environment or human action, such as the start of river flow regulation or ditching of a swamp.

4. *Random component* is usually rather strong in hydrological time series. It is unpredictable, and tells about the uncertainty included in hydrological forecasts made based time series analysis. Nevertheless, properties, like variance, of the random component can be studied.

A trend in a time series can be resolved using various fitting tools. The simplest, linear trend can be resolved by linear regression. Periodical components can be discovered using spectral or harmonic analysis. For example, in the time series compiled from monthly precipitations numbers, annual periodicity and random component are typically the most important parts. In time series formed of the monthly averages of flow rate, the annual cycle is a large part of it as well, but some other, shorter periods can be found in the residual, in addition to the random component.

Return period is frequently of concern in hydrology, to which researchers approach with methods of classical statistics and time series analysis. It means the average time it takes to repeat a given phenomenon. This phenomenon can be, for example, a specific height of the water level. The inverse of the return time is *recurrence*. This can be determined by placing observations into order by their size, $y_1 > y_2 > ... > y_n$. Then, $\frac{N}{k}$ is the return time of the variable y_k, and $\frac{k}{N}$ is recurrence. Fig. 2.8 has an example of determining the return time. The *persistence* of a phenomenon means its duration. For example, how long is the water level going to stay over a certain value?

Upon designing a construction project, whether it is a pier at a summer cottage, a large bridge, or a hydroelectric power station, and planning for any use of water resources, such

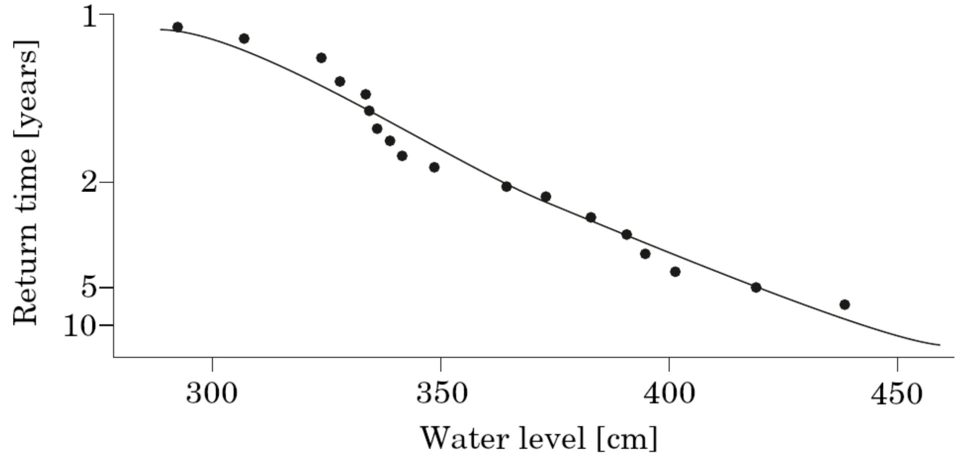

Figure 2.8: *The annual maximum water level in lake Saimaa during the years 1958 – 1977. Source: Mustonen (1986), modified.*

as the usage of natural water or sewage treatment, the influence of the project on water resources needs to be known. These influences may concern surface and groundwater levels, changes in river flow rates, and the sufficiency and quality of water resources. The usage of water resources, regulation of flow, and the release of water are determined by the amount of water needed, but also by the amount of water sustainability. The mean flow rate and water level only describe the average state and are not sufficient as a basis of design.

The return time of a limiting value chosen as the basis of any design work is defined by the planned lifetime of the construction and by the damage sustained if the set limit is exceeded. One may also take the return time of a low water level and flow rate as the design basis. Persistence is the duration of a flood over a certain water level, or just as well, the time flow rate or water level spends below a certain limit. Low-water period is closely related the extraction of water and the release of sewage. The persistence curve illustrates the fractional times the flow rate or water level spends above or below a certain limit.

Other important statistics are the lowest water level in harbours and waterways during the navigation season and the lowest level during the log driving season (Fig. 2.9). Regarding the fishing industry, an interesting statistic would be the lowest water level during the spawning time of fish. All statistics are to be determined from multiyear observational data. If a data set of sufficient length is not available, data from comparative water systems, statistical distributions, or hydrological models are used in the preparation of estimators of the statistics. The return time often has to be estimated for a lot longer time scale than the observational data spans.

2.2.3 Spatial statistics

When the value of a variable depends on its location, we are dealing with spatial analysis. This method is usually applied to two-dimensional surfaces, but in principle this applies to three-dimensional structures as well. In one-dimensional cases methods of time-series analysis can be applied. The methods of spatial statistical analysis spawned from the prospecting

Figure 2.9: *Log driving (the transportation of timber via rivers) has been an integral part of the Finnish water industry. Log driving had its heyday during the early 20th century, and the surrounding culture had a large effect even on Finnish art. Pictured, is a poster for the 1954 film 'Kaksi vanhaa tukkijätkää' (Two old loggers). ©Elonet kansallisfilmografia.*

of ores and minerals but have since spread widely into nearly all branches of geosciences. Upon dealing with two or three dimensions, or arguments, the data set needs to be large in order to draw any significant conclusions.

In two-dimensional spatial analysis, variables $f(x, y)$ are investigated, where x and y are coordinates for a location. One can think of this as a extension of time series analysis into two dimensions. Thus, we can investigate the structure and spatial variations in variable f by fitting surface functions and trend surfaces, and by studying autocorrelation and patchiness (Fig. 2.10). With multiple spatial variables their connections are investigated. For example, the spatial dependencies between runoff, water quality and land usage can be studied.

Example 2.7
Variogram is one of the basic components of spatial analysis. Let us examine a two-dimensional field variable $Z = Z(x, y)$. Variogram describes how variations in Z depend on the distance between any two points. In a stationary case, the variogram is a function of their distance only. A practical example would be a gold mine, where two samples taken

very near to each other have nearly the same gold content, but samples taken from further apart have more variation between them. Semivariogram is a variogram divided by two, and it is defined as

$$\mu(h) = \frac{1}{2} Var[Z(x + h_x, y + h_y) - Z(x, y)] \ , \ h = \sqrt{h_x^2 + h_y^2}.$$

Two important properties of (semi)variograms are the *nugget effect* and the *correlation distance*. When $h \to 0$, semivariogram has a limit

$$\mu(h) \to \mu_{0+} \geq 0$$

while $\mu(0) = 0$. Variable μ_{0+} is called the nugget effect, and it describes the rapid growth of semivariogram upon stepping out of point (x, y). Figuratively, it could be said that one step away the conditions differ from what you have under your boot. When $h \to \infty$, the limit is $\mu(h) \to Var(Z)$, so the semivariogram approaches the variance of Z. This means, that the information about any one point is meaningful only so far. Correlation distance can be determined from the semivariogram so that at that distance the semivariogram is still significantly smaller than the variance of Z. This limit is reached, when h is larger than spatial correlation distance.

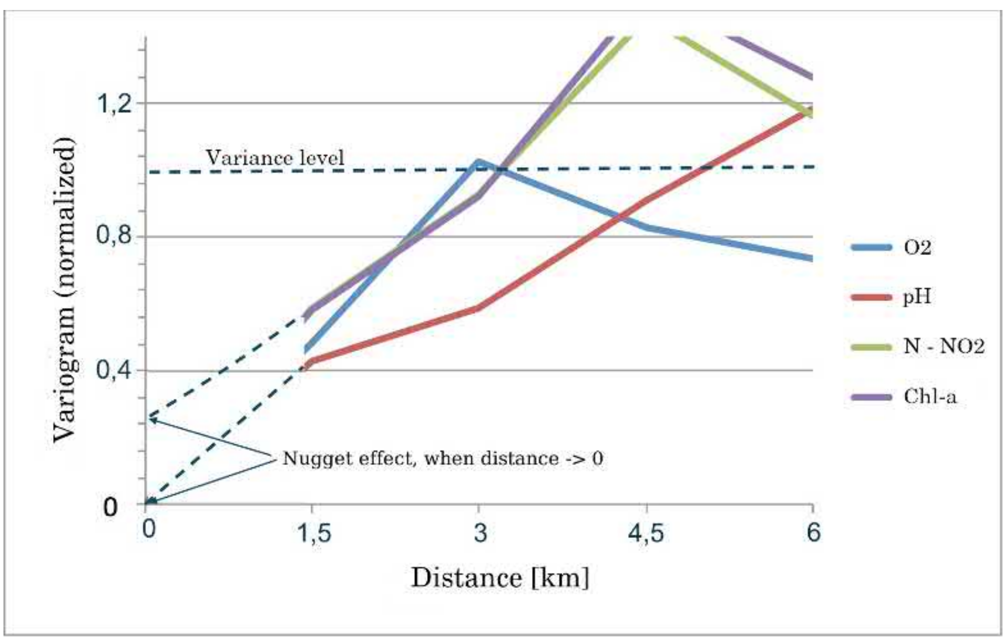

Figure 2.10: *Semivariograms from summertime values of dissolved oxygen, pH, nitrate, and chlorophyll a from Lake Vanajavesi. Semivariogram describes the level of variation as a function of distance. It reaches the value of total variance at correlation distance. Figure: Leppäranta et al. (2017), modified.*

For spatial analysis, GIS (Geographic Information Systems) software is available. With GIS one can perform statistical analysis and examine different types of maps at the same time. GIS software has plenty of applications in all geosciences. In hydrology, they can used for example, to examine water quality together with vegetation, land use and climate, and to obtain estimates on the changes in the environment due to anthropogenic impact on the system.

2.3 Dimensional analysis

2.3.1 Principles

Physical quantities have a *dimension*. When a sufficient number of fundamental dimensions are chosen, all others can be derived from them. In classical mechanics, three basic dimensions exist: mass, length and time, as M, L and T, respectively. The dimensions of the quantities (Q) in mechanics can thus be written as:

$$dim(Q) = M^A L^B T^C \tag{2.10}$$

where the exponents A, B and C are integers or fractions. The dimension of a dimensionless variable is 1 ($A = B = C = 0$). All formulas in physics must be dimensionally consistent, which means that all terms have the same dimension. Dimensions are also written with square brackets, $dim(Q) = [Q]$. Each quantity has a unit, and correspondingly the units of all derived quantities can be acquired from the units of the fundamental quantities.

Example 2.8
Dimension of some of the most common quantities:

$$dim(velocity) = \frac{L}{T}\ ,\ dim(acceleration) = \frac{L}{T^2}\ ,\ dim(volume) = L^3\ ,$$
$$dim(density) = \frac{M}{L^3},\ dim(force) = \frac{ML}{T^2},\ dim(pressure) = \frac{M}{LT^2},\ dim(angle) = 1$$

According to Newton's II law $F = ma$, where F is force, m is mass and a is acceleration. Thus, we can see that $[F] = MLT^{-2}$, $[m] = M$, $[a] = LT^{-2}$. Both sides of the equation have the same dimension, and so it can be said to be consistent. In SI-system the basic unit of mass is kilogram (kg), length is meter (m) and time is second (s). The unit of acceleration is then m s^{-2}, and the unit of force is kg m s^{-2}.
Note, that the dimension of angle is one. Angle means the ratio of the arc of the angle to the radius, and the 'unit', like degree (°) or radian (rad) only tells the scale in use. The angle of a whole circle is called a round or a cycle. Thus, 1 round = 2π rad = 360°.

Dimensional analysis is an useful tool for checking that your calculations are consistent. But above all that, it is a simple, yet powerful tool for understanding connections and relationships between quantities. Through dimensional analysis one can simplify problems by decreasing the number of variables by using dimensionless quantities. One does not need to know the underlying laws of physics to discover connections with dimensional analysis, but one must have a intuition to see which quantities are relevant for the problem at hand. The following examples are meant to clarify the concept of dimensional analysis.

Example 2.9
a) Let's check the consistency of the catchment water balance equation. The balance is written as

$$\frac{dV}{dt} = (P - E)A - O$$

where on the left hand side is the rate of change of the volume V of the water storage, and on the right hand side of the equation there are precipitation P, evaporation E, catchment area A and runoff out of the catchment O. Dimensions are $[\frac{dV}{dt}] = \frac{L^3}{T}$, for evaporation and precipitation LT^{-1}, surface area L^2 and runoff $L^{-3}T$. Here, we can see that the terms have the same units, and the equation is consistent.

b) Let us consider the dimensions in the classical Chezy's river flow velocity equation. It is generally written as $U = C\sqrt{R\beta}$, where $R = {}^{A}/_{P}$ is the hydraulic radius of the river, A is the surface area of the cross section, P is the wet perimeter, β is slope of the river and C is the adjustment factor. Clearly, $[R] = L$ and $[\beta] = 1$, so in order to acquire velocity as the dimension here the dimension of the adjustment factor needs to be $[C] = L^{1/2}T^{-1}$. These kinds of dimensions are common in parameters of semi-empirical equations. When using such equations, one needs to be careful with units and dimensions, since in past especially in Europe the cgs-system was in use, where the basic units are centimeter, gram and second. Similarly, in the Anglo-Saxon world the feet-pound-second system was used for long and is still in use in the U.S.A. In most countries SI-system was taken as the standard by the 1970s.

c) Actually, Chezy's river flow speed equation can be derived with dimensional analysis. Homogeneous mean river flow speed U can be thought to be the result of acceleration due to gravity g, river slope β and river cross section R working together. Gravity in the direction of the flow is $g\beta$. Here, we can derive the dimension equation $U \propto R^A(g\beta)^B$, to which the solution is $U = constant \cdot \sqrt{g}\sqrt{R\beta}$. Chezy's adjustment factor thus is of the form $C = constant \cdot \sqrt{g}$, $[C] = L^{1/2}T^{-1}$. The relationship of the parameter with other river parameters can be further investigated.

d) Let us take a look into the period t of a pendulum. One can think that it is affected by the length of the wire l, the mass of the pendulum at the end of the wire m, and the acceleration due to gravity g. Dimensions are $[t] = T$, $[l] = L$, $[m] = M$ and $[g] = {}^{L}/_{T^2}$. So it has to be that [2] $t \propto m^A l^B g^C$. On the basis of dimensions we have only one solution: $A = 0,\ B = 1/2,\ C = -1/2$, or $t = constant \cdot \sqrt{lg^{-1}}$, $dim[constant] = 1$. The period is not dependent on the mass of the pendulum. To find out the constant, one would need to perform one pendulum experiment (from the study of mechanics we know, that this constant

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[2] The symbol $\propto$ means proportionality, so that $y \propto x$, if $y = constant \cdot x$

has a value of 2π).

Examples **2.9 c – d** showed how dimensional analysis can be used to derive simple relationships between physical quantities. When there are n fundamental dimensions and $m > n$ derived dimensions affecting a system, $m-r$ dimensionless quantities can be derived, where r is the number of independent derived quantities. In the case of the pendulum $n = 3$, $m = 4$ and $r = 3$, and the problem is reduced to an equation with one unknown dimensionless parameter $\Pi = t\sqrt{g/l}$. Then, the solution is reached upon setting $\Pi = constant$.

If the resulting number of dimensionless quantities is two, $\Pi_1 \& \Pi_2$, we get a function $\Pi_1 = f(\Pi_2)$ and so forth. The number of dimensionless quantities required can be solved by mathematics. The choice of quantities is decided by the researcher. This method is best suited for stable situations, where the physical system has reached an equilibrium. The group of fundamental dimensions chosen depends on the problem at hand. In mechanics, for example, one can take force as a fundamental dimension instead of mass, in which case mass becomes an derived quantity $M = FL^{-1}T^2$.

In fluid dynamics dimensional analysis has been widely applied. Flow fields are described by characteristic speed U, length scale L and fluid properties. As the length scale, an applicable dimension of the basin or related topographical feature, such as canal depth, lake length or a mound in the river bed, can be used.

2.3.2 Examining a group of quantities

With the help of the general theory of dimensional analysis, one can find out the number of dimensionless quantities required to solve the problem at hand. Assume there are n basic dimensions, P_i. The number of derived quantities Q_k is $m > n$. The dimension of the derived dimension can represented as a product

$$[Q_k] = [P_1]^{b_{1k}}[P_2]^{b_{2k}} \, ... \, [P_n]^{b_{nk}} \, , \; k = 1, 2, \, ... \, , m. \tag{2.11}$$

From this equation it is possible to construct dimensionless quantities, Π_j, so that exponents of the basic dimensions become zeros:

$$[\Pi_j] = [Q_1]^{k_1}[Q_2]^{k_2} \, ... \, [Q_m]^{k_m}. \tag{2.12}$$

By solving the exponents k_j, the number of linearly independent derived quantities r can be obtained, after which we can choose convenient $m - r$ dimensionless quantities. Then, we can choose the primary quantity of interest Π_1, and construct an equation

$$\Pi_1 = f(\Pi_2, \, ... \, , \Pi_{m-r}) \tag{2.13}$$

If $m - r = 1$, then $\Pi_1 = constant$. The actual choice of the dimensionless quantities is left to the researcher, because there are multiple ways to make the choice. Below, examples will be given. Dimensional analysis is typically applied to problems that have no more than five dimensionless quantities. Then, the interpretation of the results is still simple and the derivation of the equations is still easy to perform.

Example 2.10
In the pendulum example (example **2.9 d**) there were $n = 3$ basic quantities and $m = 4$ derived quantities (t, l, g, m). The goal was to investigate, how the period of the swing of the pendulum is determined. The basic dimensions and the dimensions of the derived can be arranged as a matrix

	t	l	g	m
M	0	0	0	1
L	0	1	1	0
T	1	0	-2	0

Columns give us the dimensions of derived quantities: $[t] = T$, $[l] = L$, $[g] = LT^{-2}$ and $[m] = M$. This matrix lets us derive an equation for dimensionless quantities:

$$[\Pi_j] = [t]^{k_1}[l]^{k_2}[g]^{k_3}[m]^{k_4}.$$

The condition of dimensionlessness provides the following equations:

$$\begin{array}{ll} M & 0 = k_4 \\ L & 0 = k_2 + k_3 \\ T & 0 = k_1 - 2k_3 \end{array}$$

This results in three linearly independent solutions: $k_2 = -k_3$, $k_3 = {}^1\!/\!_2 k_1$ and $k_4 = 0$, and k_1 remains unassigned. Thus, $r = 3$ and there is one dimensionless quantity that describes the system:

$$\Pi = t^{k_1} l^{-1/2 k_1} g^{1/2 k_1} m^0.$$

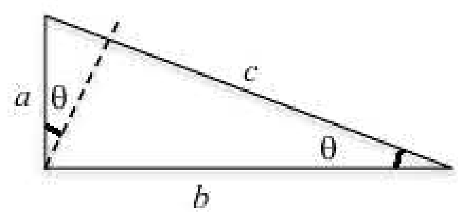
If we take k_1 as 1, we get $k_2 = -1/2$ and $k_3 = 1/2$. This dimensionless quantity is $\Pi = t\sqrt{g/l}$ and due to the fact that there is only one of them, it follows that $\Pi = constant$. Even if we would have chosen k_1 differently, the end result would have been the same. The choice is made on the basis of simplicity and elegance.

Example 2.11
a) Let us have a look at the time t it takes to cover a distance s while accelerating at a constant acceleration a. Other factors are not taken into account. In this problem, $n = 2$ and $m = 3$, and the dimension matrix is

	t	s	a
L	0	1	1
T	1	0	-2

Here we find two linearly independent derived quantities, so there should be $m-r = 1$ dimensionless quantities. As in the previous example we get $[\Pi] = [t]^{k_1}[s]^{k_2}[a]^{k_3}$, where $k_2 = -k_3$ and $k_3 = 1/2k_1$, and thus $\Pi = t\sqrt{a/s} = constant$. From the basic laws of mechanics we know that $t = \sqrt{2s/a}$ and so $constant = \frac{1}{2}$.

b) The Pythagorean theorem can be derived with the help of dimensional analysis. According to the theorem, the length of the hypotenuse c and the lengths of the cathetus b and c of a right triangle are related to each other by the equation $c^2 = a^2+b^2$. When investigating right triangles in general, we notice that their surface area is determined by the length of the hypotenuse and the angle between the hypotenuse and a cathetus: $A = f(c, \theta)$.



The dashed line divides the triangle into two smaller similar right triangles, and their areas are $A_1 = f(a, \theta)$ and $A_2 = f(b, \theta)$, respectively. Because $[\theta] = 1$, dimensional consistence demands that $f(c, \theta) = c^2 g(\theta)$, from which it follows that

$$c^2 g(\theta) = a^2 g(\theta) + b^2 g(\theta).$$

Dividing all terms by g we arrive to the Pythagorean theorem.

Example 2.12
A very interesting, although a rather complex, example comes from the field of rowing. The aim here is only to illustrate the powerful idea behind dimensional analysis. Let us take a look at a rowing boat with N rowers on board. We then assume:

- the boats are geometrically similar
- the volume of a fully laden boat per rower = constant = G
- power output of a rower = constant = A

We are interested here of the speed of the boat, v. Surface friction F is working against the rowers, which has a friction coefficient of λ. The length scale of the wet surface is l. Equilibrium condition between rowing output and friction is

$$AN = vF = \lambda \rho v^3 l^2.$$

As the basic quantities of this problem, we can take U = speed, N = number of rowers, l = length and R = density, so $n = 4$. In this case it is convenient to choose density as the fundamental quantity instead of mass, and the number of rowers is its own fundamental quantity. The number of derived quantities is $m = 5$, and their dimensions are

$$[v] = U,\ [N] = N,\ [G] = \frac{P}{N},\ [\rho] = R,\ [A] = \frac{1}{N} R U^3 l^2.$$

It can be seen now that $r = 4$. There is only one independent dimensionless quantity, so it has to be a constant. We can choose

$$\Pi = v \frac{\rho^{1/3} G^{2/9}}{A^{1/3} N^{1/9}} = constant.$$

When A, ρ and G are constants, $v \propto N^{1/9}$. In the figure below are shown the competition results from boats of 1, 2, 4 and 8 rowers. The track is 2 000 m, and the time is proportional to the number of rowers as $N^{1/9}$. The result acquired from dimensional analysis fits the actual competition results very well, which is a verification of the power of this method (Fig. 2.11). Noteworthy is also the fact that increasing the number of rowers very slowly increases the speed, due to the exponent being so small, $1/9$. Thus, the speed achieved by 10 (100) rowers is only 1.29 (1.67) times the speed reached by a single rower.

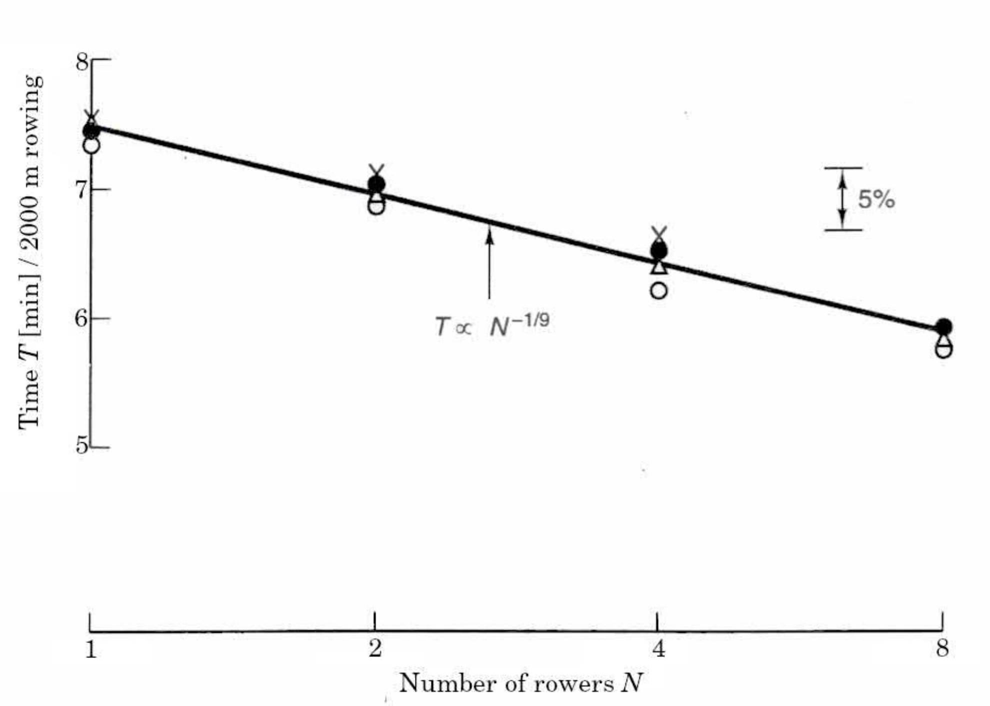


Figure 2.11: *Time for 2 000 m rowing as a function of the number of rowers. The solid line represents the model derived above, and competition results have been made in calm wind conditions.* Δ *– 1964 Tokyo Olympics,* $\bullet$ *– 1968 Mexico City Olympics,* $\times$ *– 1970 Ontario Rowing World Championship,* $\circ$ *– 1970 Lucerne Rowing Championship. Source: McMahon (1971), modified*

2.4 Hydrological models

2.4.1 Physical models

Physical models refer to the study of a problem through a scale model in a laboratory. For example, the drag an object experiences in a fluid flow can be studied in a tank. Physical scale models are smaller than the corresponding natural basins, and thus the physics of the flow do not scale easily and problems with scaling are common. Most physical models are built to study flow dynamics, but one could also study the transport of heat through ground, or the transmission of light through water. In this chapter, models built to study flow dynamics are discussed.

Scales of length, time and flow velocity are (L, T, U) and (L_m, T_m, U_M) in nature and in model, respectively. In a simple case, only the geometry of the basin or the object is affecting the flow. In that case it is sufficient to have the same shape for both basins, then

$$\frac{U_m T_m}{L_m} = \frac{UT}{L}. \tag{2.14}$$

When the length and velocity scales are fixed, time passes at rate of $\frac{T_m}{T} = \frac{L_m}{L}(\frac{U_m}{U})^{-1}$. In general, when setting up a physical model one needs to be aware of the key dimensionless quantities, and to make sure that they are similar in the natural system and in the model. Geometrical scaling alone does not get one very far in physical modeling of flow. If some physical phenomenon, like friction, is important for the problem at hand, it is important for the model to be *similar* to nature regarding this aspect. These questions can be studied with dimensional analysis.

In the flow of natural waters there are two key effects: viscosity and waves in the body of water. Two dimensionless parameters are used to describe these: Reynolds number $Re = \frac{UL}{\nu}$ and Froude number $Fr = \frac{U}{\sqrt{gL}}$, where ν is the kinematic viscosity and g is the gravitational acceleration. These numbers shall receive more scrutiny in section 3.3.1. For the similarity condition to be fulfilled, Reynolds and Froude numbers need to be the same in both in nature, and in the model:

$$viscosity: \ \frac{U_m}{U} = (\frac{L_m}{L})^{-1} \ \& \ waves: \ \frac{U_m}{U} = \sqrt{\frac{L_m}{L}}. \tag{2.15}$$

From this we can see that in order to keep the viscosity similar, the flow velocity needs to be increased from the natural values, but the condition to keep the waves similar requires the exact opposite. The situation is conflicting. One could choose a completely different fluid for the experiment that would have a completely different viscosity, or perform the experiment under a different gravitational acceleration, but either way one runs into practical difficulties very easily. Physical models work well if either waves or viscosity is important, but not both (Fig. 2.12).

Physical models have been used in single parameter problems in flow dynamics. These kinds of problems are, for example, flow of fluid past an obstacle or the drag experienced by the hull of a ship. In more complicated problems, like the development of flow field in a lake, physical models offer very limited help. Rotating tanks have been used with some success while studying the effect of the rotation of Earth on ocean currents. Here, the similarity condition is $\frac{U_m}{U} = \frac{\Omega_m}{\Omega} \cdot \frac{L_m}{L}$, where Ω_m is the rotation rate of the model basin and Ω is the rotation rate of Earth.

2.4.2 Mathematical models

Mathematical models are based on solving the equations that govern the phenomenon in question. Calibrating and tuning the model means optimizing the parameters of the model in order to make it replicate measurements as accurately as possible. Parameters acquired from calibration need to be functional in all situations in order for the model to work universally.

Mathematical models can divided into four categories: analytical, numerical, stochastic and statistical. Analytical models give general relationships between different quantities and 'rules of thumb', while numerical models give detailed information on the physics of hydrological systems. When a random component is added into an analytical or a numerical model, we are speaking of a stochastic model. Statistical models are based only on observations, and they best describe average dependencies. Statistics also give knowledge on the

Figure 2.12: *In the flow of rivers, shallow water waves play a dominant role in the dynamics. Therefore, its effects can be studied with a physical model. Pictured is a 1 km long model of a section of the Yellow River. Photo: Matti Leppäranta.*

accuracy and reliability of the results.

Example 2.13
Lets have a look at predicting the surface temperature $T = T(t)$ of a lake. The basic equation is

$$\frac{dT}{dt} = k(T_a - T)$$

where T_a is the air temperature and k is the heat exchange coefficient. From physics we know that this coefficient is approximately inversely dependent on the mixed layer depth H_1, $k \sim {}^1\!/\!_2 H_1$ m^{-1} d^{-1}.

a) If $k = constant$, the equation can be solved analytically:

$$T(t) = \int_{-\infty}^{t} k \exp\left[k(s-t)\right] T_a ds$$

Disturbances in the system are damped within the relaxation time k^{-1}.

b) If $k = k(...)$ is dependent, for example, on the turbulence occurring in the surface layer, the problem usually has to be solved via a numerical model.

c) In a stochastic model external forcing has had a random element ϵ added into it

$$\frac{dT}{dt} = k[(T_a + \epsilon) - T].$$

Although the stochastic term forcing disturbances into the temperature is on average zero, upon integrating it accumulates as a variable extra term in the temperature.

d) A statistical model to forecast the surface temperature of tomorrow can be written as

$$T(t_n) = \sum_{j=0}^{m} a_j T_a(t_{n-j}) + e_n,$$

where T_a is air temperature, m describes the span of the memory of the system, e_n is a residual term and a_j's are statistically estimated coefficients. Therefore, surface temperature is the weighted sum of temperature history of air. It is evident, that $a_0 > a_1 > a_2$ ... $\sum_j a_j \approx 1$.

Mathematical models describe the quantities in question and their changes in space and time. A point in space-time is defined as $(x, y, z; t)$, where x is the east-coordinate, y is the north-coordinate, z is the vertical coordinate and t is time. In three-dimensional models the zero-level of the vertical coordinate is set at ground, or water surface or below the volume of interest. It is taken as an altitude, with positive direction being up. In pure vertical models, when working in (z, t) space it is sometimes more convenient to take z-coordinate as depth, positive direction being down, and keep the zero-level at the surface.

Hydrological models have significant uncertainties. These are especially caused by the movement of water through soil-vegetation system and the parameterization of associated phenomenon, and complex geometry of basins and rivers. Thus, model results alone are not a sturdy enough basis for decision making, and more information has to be gathered by field measurements. Components of a mathematical model are listed in the table below. For clarification, an example from a lake circulation model has been included for each row.

General description	Example: Lake circulation model
model equations	momentum & continuity equations, conservation of heat
model quantities	flow velocity, water level, temperature
physical parameters[1]	friction and heat exchange coefficients
numerical parameters[2]	grid size, time step
model input	meteorological data
model output	circulation, water level, temperature

[1] Properties of the system, with which the model is tuned to function properly

[2] Parameters dependent on the computational capacity. They define the numerical accuracy of the model.

Analytical models saw most use during the era before computers, although they are still useful in 'seeing the forest from the trees' and constructing simple rules of thumb. Analytical models are algebraic or differential equations, that can be solved by analytical means. One of the most famous analytical hydrological models is Chezy's formula, which describes the flow velocity of a stable turbulent stream, and how it is a balance between gravity and friction between water and the channel bottom (see Example 2.8).

Example 2.14
A simple model for the propagation of light in water is based upon the Beer - Lambert law. If the irradiance [3] of light is $E = E(z,\, \lambda)$, where z is the depth and λ is the wavelength of light, then $\frac{dE}{dz} = -\kappa E$, where $\kappa = \kappa(z,\, \lambda)$ is light attenuation coefficient. The solution of this is

$$E(\lambda,\, z) = E(\lambda,\, 0) \exp\left[- \int_0^z \kappa(z')dz' \right].$$

Coefficient κ still needs to be chosen. Values measured in the field can be used, or a realistic estimate made. When $\kappa = \textit{constant}$ the solution can be written as $E(\lambda,\, z) = E(\lambda,\, 0) \exp(-\kappa z)$.

Numerical models aim to solve the model equations by computers. It is possible to reach quantitative results with these methods. The basic equations in hydrology are differential equations. The construction of a solution begins from the *grid*, where for each grid node the values of the unknown quantities are calculated (Fig. 2.13). The grid is typically fixed to

[3] Irradiance is the total amount of radiation entering the subject from a given half sphere (or the whole sphere). In this case, it is all the radiation shining from all sky above the surface.

certain locations (x_i, y_j, z_k; t_p), and the solution progresses through time t one *time step* at a time. The basic equations describe the functions $f_l(x, y, z; t)$ to be solved, and in numerical problems unknowns are their values $f_l(x_i, y_j, z_k; t_p)$ at all grid cells.

In the method of finite differences derivatives are replaced by difference quotients. Time derivative of a function $f = f(x, y, z; t)$ is commonly approximated with forward difference and spatial derivative by central difference. From the definition of a derivative we get

$$\frac{\partial f}{\partial t} \approx \frac{f(x, y, z, t + \Delta t) - f(x, y, z, t)}{\Delta t}, \tag{2.16}$$

$$\frac{\partial f}{\partial x} \approx \frac{f(x + \Delta x, y, z, t) - f(x - \Delta x, y, z, t)}{2\Delta x} \text{ and} \tag{2.17}$$

$$\frac{\partial^2 f}{\partial x^2} \approx \frac{f(x + \Delta x, y, z, t) - 2f(x, y, z, t) + f(x - \Delta x, y, z, t)}{(\Delta x)^2} \tag{2.18}$$

where Δt is the time step and Δx is the grid cell length. Other derivatives can be handled in a similar manner. When difference quotients replace derivatives in the model basic equations an algebraic system of equations is formed to solve the unknowns $f_L(x_i, y_j, z_k; t_p)$. The number of grid points is usually very large, so the computational capacity required is large as well.

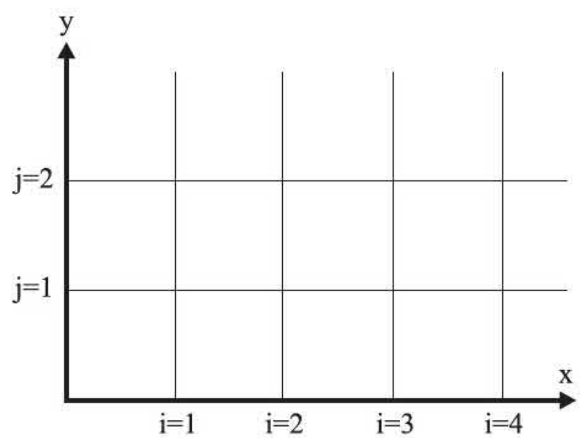


Figure 2.13: *Numerical models employ grids, where the desired values are solved for the nodes.*

The size of grid cells is defined by the problem and the desired accuracy of the solution, but in the end, typically the limits of computational capacity set the boundaries for the scale of the model. Smaller grid cell size brings with itself a more accurate solution, but takes up more computational resources. When the spatial grid is set, the choice of the time step depends on the numerical method in use. The length of the time step has to be short enough to resolve the impact of external forcing. This means, that the time step length has to be at most around 1/10 of the time scale of the external forcing. On the other hand, stability requirements of the model can set an upper boundary for the time step as well.

Example 2.15
Let us consider the vertical temperature model of a lake. The basic equation is

$$\frac{\partial T}{\partial t} = K\frac{\partial^2 T}{\partial z^2} + K^* \frac{Q}{\rho c}\exp(-K^* z),$$

where K is the heat diffusion coefficient, $Q = Q(t)$ is the solar radiation at the surface, κ is the light attenuation coefficient in water and c is the heat capacity of water. In accordance to Euler's method, this equation can approximated with finite differences, as:

$$\frac{T(t_{p+1}, z_k) - T(t_p, z_k)}{\Delta t} = K\frac{T(t_p, z_{k+1}) - 2T(t_p, z_k) + T(t_p, z_{k-1})}{(\Delta z)^2} + \frac{Q(t_p)}{\rho c}e^{-\kappa z_k}$$

where the time step $\Delta t = t_{p+1} - t_p$ and spatial grid cell size $\Delta z = z_{k+1} - z_k$ are constants. z_0 is set at the surface of the lake and z_N is at the bottom. When temperature at the time t_p is known, the equation solves it for the time t_{p+1}, etc. Solving the temperature $T(t_{p+1}, z_k)$ requires, that temperatures at points $T(t_p,\, z_{k-1})$, $T(t_p,\, z_k)$ and $T(t_p,\, z_{k+1})$ are known. Temperatures at the edges, z_0 and z_N, have to be acquired from the boundary conditions.

This system is controlled by heating from the solar radiation, and heat exchange at the boundaries. For boundaries we can set constant temperatures, $T(t_p, z_0) = T_0$ and $T(t_p, z_N) = T_b$. the number of time steps depends on the temporal scope of the model run. Number of vertical grid cells, N is defined in advance. If depth is 50 m, then a good grid cell size could be 0.5 m, and $N = 100$. The stability criterion of Euler's method declares that $K < \frac{1}{2}\frac{(\Delta x)^2}{\Delta t}$. If we take $K = 0.01$ m^2 s^{-1}, the time step requirement will be $\Delta t < 12.5$ s.

Numerical models provide the means to describe hydrological systems rather well, like the development of the water storage in a catchment area, and to provide drought and flood warnings. Other often modeled phenomena are the circulation of water in lakes and ground and the structure and thickness of snow and ice (Fig. 2.14). Circulation models can also take into account the transport of matter with the water, if the input and output of the desired material is parameterized.

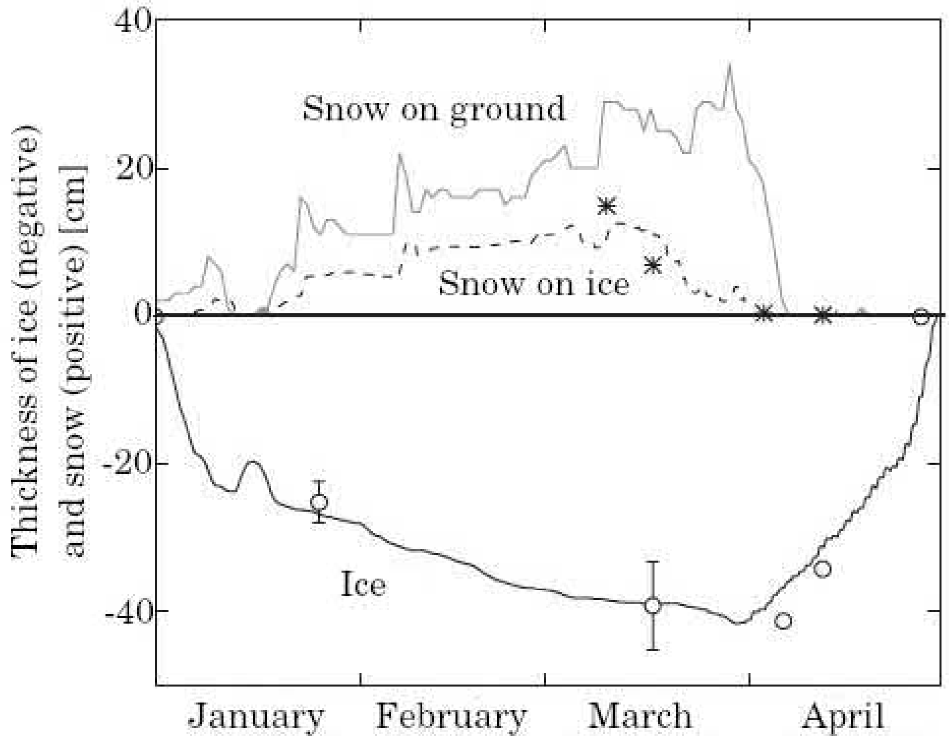


Figure 2.14: *Results from a model describing the development of ice and snow cover on lake Vanajavesi for the winter of 2009. Snow cover thickness on ground drawn for reference. Asterisks describe measurements of snow thickness, and circles ice thickness.*

Statistical models can be derived from observed statistical dependencies. Their basis is that the data set itself contains all the required information, and no knowledge on the actual underlying physical laws is required. Statistical models were particularly popular in the era before electronic computing, but they have pertained themselves in the modern times as well due to their simplicity. Linear regression, for example, is still very popular method in many fields. Other methods include empirical orthogonal functions, autocovariance models and Markov chains. On the other hand, with the help from statistical models one can compress data sets to find the underlying physical connections.

Example 2.16
The drift velocity (u, v) of a floating object (where u is the eastern component of velocity and v is northern component) is often modeled with a linear model

$$u = au_a - bv_a + e_x$$
$$v = bu_a + av_a + e_y$$

where u_a and v_a are the eastern and northern components of wind velocity, e_x and e_y are error terms and a and b are the parameters to be solved that shorten (a) and rotate (b) wind to fit better to the observed drift velocities. The values of these parameters are determined by the shape and displacement of the object in question. They can be determined by linear regression from observations by minimizing variance of the error. The solution achieved is $2-5\%$ of the wind speed with a deviation to right (on the northern hemisphere) of $0-30°$ from the direction of the wind, depending on the geometry of the object in question.

A very important application of time series analysis has been the forecasting of river flow rates with a linear multiparameter model. In very large catchment areas that react slowly to, for example, input from precipitation, a long timestep of up to a month has been employed. The forecast has been composed with least squares method, where monthly flow rate is the regressand and precipitation from the month in question and a few moths before that are the regressors. Nowadays, forecasts can be performed with more powerful computers and with more advanced models.

A stochastic model refers to either analytical of numerical, physics based model that has a random component introduced into it. Random variables are introduced into the basic physical equations to represent inaccuracies or natural variation. Stochastic models are typically differential equations that have deterministic and stochastic components assigned to variables, and possibly even for the parameters. The idea is to solve the role of random components in a physical system through the physical laws themselves. In a non-linear system, adding random components can have a significant effect on deterministically retrieved results.

Figure 2.15: *The Monte Carlo casino in Monaco, which John von Neumann used as namesake for the method he invented. This method is based on random numbers, first investigated in Los Alamos during the Second World War. Now, Monte Carlo method is used in a wide variety of computational fields. Photo: Leena Leppäranta.*

In the Monte Carlo method (Fig. 2.15) the effects of random components are modeled with a random number generator. A manual example of this would be the determination of a lake surface area by throwing markers randomly on the map. The surface are of the lake would be the % of markers that hit the lake times the total area of the map. Monte Carlo method is very useful in many hydrological problems.

Various mathematical models are readily available on the Web, and not all researchers need to write their own models from the ground up. But, everybody should still understand the basics of the type of thinking behind models and the possibilities and limitations presented in them. Even if you do not write the model yourself, it is important to run diverse tests on the model in order to truly understand its properties, not unlike experiments in nature. Models are not perfect. They have simplifications and parameters that sometimes are specific only to some geographic location.

This book is not about constructing models. This is why models are not discussed more in depth, but results obtained from models are presented throughout this book. In the next chapter we will dive into the foundations of hydrology, as we investigate the properties of water and the basics of hydrological cycle.

Chapter 3

Water and the hydrological cycle

3.1 Properties of natural waters

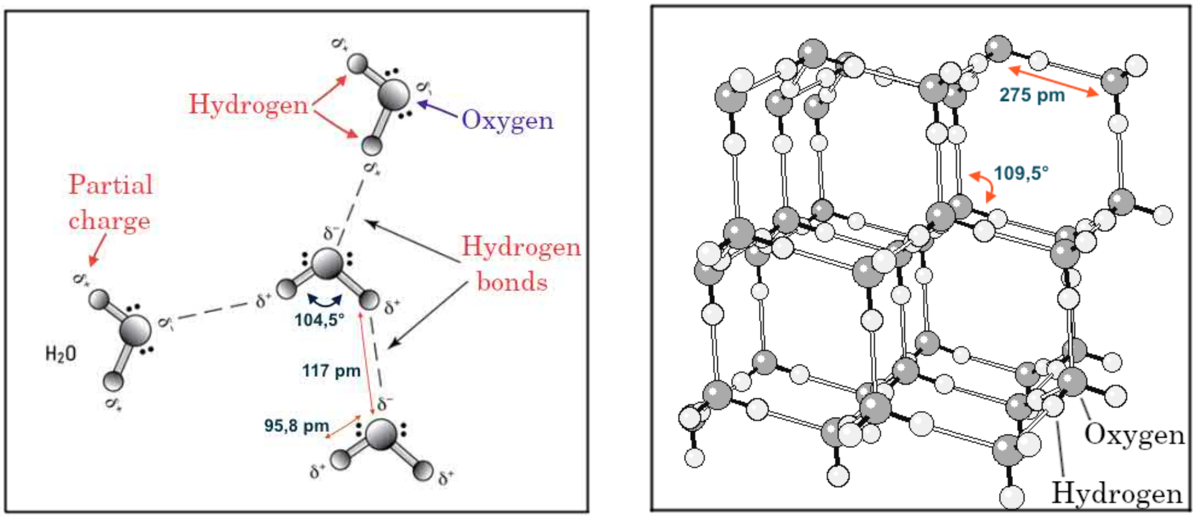
3.1.1 Pure water

A molecule of water consists of two hydrogen atoms and a single atom of oxygen (Fig. 3.1a). Both hydrogen atoms are bound to the oxygen atom by a *covalent bond*, where a hydrogen atom shares a pair of electrons with the oxygen atom. The length of these bonds is 95.8 pm, and they are asymmetrically positioned into a concave 104.5° angle. Due to this asymmetry, molecules of water form an electrical dipole: the side with oxygen atom has a negative partial charge and the side with the hydrogen has positive partial charge. This is called *polarity*, and many of water's peculiar properties, like its great ability of dissolve, spring from this.

Molecules of water, on the other hand, are bound to each other by *hydrogen bonds*. These bonds are due to the aforementioned polarity and are formed between an oxygen atom and a hydrogen atom of a neighbouring molecule. Hydrogen bonds have a length 117 pm, and are much weaker than covalent bonds. Hydrogen bonds in water are still relatively strong, and it explains how the high melting and boiling points are possible. In liquid form, molecules of water form linear chains that are held together by hydrogen bonds. These chains are constantly being formed and broken up, so in liquid form, the structure of water is dynamic. The solid form of water, ice, is made up of crystals. On the surface of Earth, and in pressures and temperatures occurring 10 km below and above it, ice crystals form a hexagonal structure, known as *ice* I_h[1] (Fig. 3.1b). Each molecule of water is connected to four neighbouring molecules by hydrogen bonds, forming tetrahedral structures. Pairs of electrons are evenly spaced on their perimeters, and the concave angles of H-O-H bonds are 109.5°, as the tetrahedral geometry suggests. The tetrahedrons are positioned in a hexagonal symmetry, which is the namesake for this arrangement.

In the structure of ice I_h, the molecules of water are arranged in a less dense fashion, and for this reason it is also significantly less dense (8.3 %) than liquid water at 0 °C. At the macroscale, the hexagonal structure is often apparent in shapes of crystals and in the six 'arms' (also called dendrites) of snowflakes. The decrease in density in phase change from

[1] Many crystal structures and geometries of the solid phase of water are known, but anything other than *ice* I_h are formed only at very high pressures or at very low temperatures, and they are not present in Earth's hydrology. In the upper atmosphere, small amounts of cubic *ice* I_c can be found.



(a) (b)

Figure 3.1: *Molecular structure of pure water.* ***a)*** *In liquid form. Covalent oxygen-hydrogen bonds have a length of 95.8 pm and their concave angle is 104.5°.* ***b)*** *Ice Ih. Molecules form a network so, that each molecule has four other molecules attached to it by hydrogen bonds, forming a hexagonal structure. The concave angle formed by the covalent hydrogen-oxygen bonds is in this case 109.5°.*

liquid to solid is extremely rare among materials[2], and no naturally occurring substances exhibit this behaviour as strongly as water. Later in this book, 'ice' refers always to hexagonal ice.

The triple point of water is at a temperature of 273.16 K (0.01 °C) and at a pressure of 611.73 Pa. At the triple point, all phases are in balance, and very small deviations in temperature and pressure can provoke a change of all water in the system into steam, liquid or ice.

Water is the only substance that can be found at all phases on Earth's nature: gaseous (steam), liquid, and solid (ice). Molecules of water also transit from one phase to another (Fig. 3.2). Additionally, a large amount of water is bound into chemical compounds as water of crystallization. In the surface layer of Earth, water is the most abundant substance, and its liquid phase at normal pressure and temperature is exceptional among inorganic compounds found there. Thus it is not odd, that water has been considered as one of the basic elements, and that many deities have been attributed to it (Fig. 3.3).

Table 3.1 presents the key properties of pure water, regarding hydrology. These are weaktly dependent on temperature[3] and pressure. Properties of natural waters are also affected by dissolved substances, but the values presented can be applied to many fresh water hydrological applications. When impurities are dissolved into water, its electromagnetic properties change rapidly. Electrical conductivity increases, which is often used as an indirect

[2]Gallium, bismuth and germanium have similar properties.

[3]Temperature in hydrology is typically presented in the Celsius scale, which is a interval scale unit. With such scales, only addition and subtraction are allowed, and multiplication and division are not defined. Absolute temperature is presented in Kelvin scale (273.15 K = 0 °C)

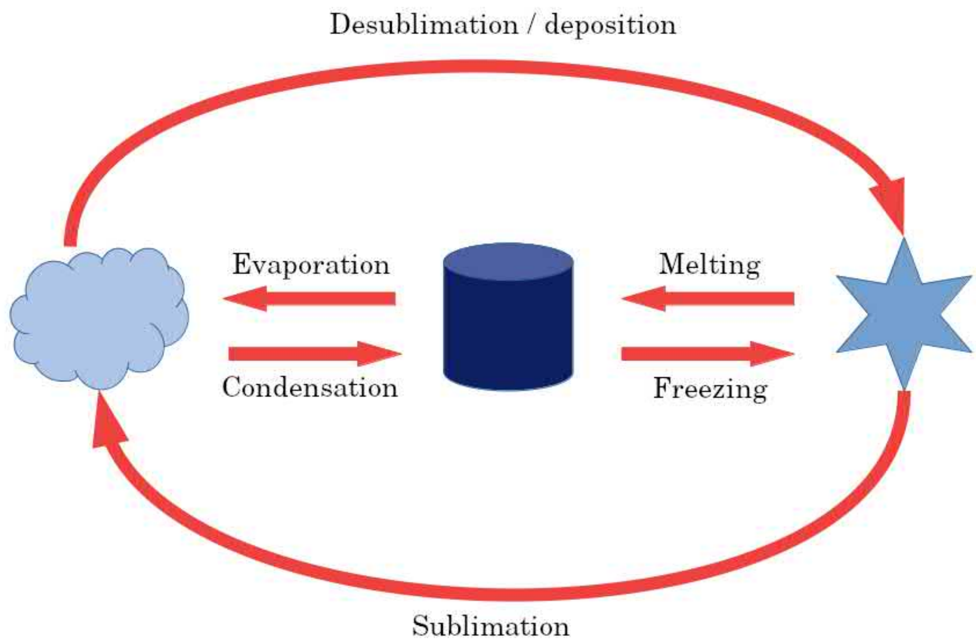


Figure 3.2: *Phase changes of water.*

Figure 3.3: *Vellamo is the goddess of the water in ancient Finnish folklore. She, and her spouse Ahti are still part of Finnish culture. The county of Päijät-Häme has placed Vellamo in their coat of arms. It was designed by professor Tapani Aartomaa in 1997.*

method for measuring the concentration of dissolved substances. Optically active impurities affect the absorption and scattering of light already at very small concentrations.

Table 3.1: *Physical properties of pure water and ice at normal atmospheric pressure of 1 013.25 mbar and at different temperatures*

	Temperature				
	Liquid			**Ice**	
Quantity	**0 °C**	**10 °C**	**20 °C**	**0 °C**	**Unit**
Density ρ	999.8	999.7	998.2	916.8	kg m^{-3}
Dynamic viscosity* μ	$1.8 \cdot 10^{-3}$	$1.31 \cdot 10^{-3}$	$1.01 \cdot 10^{-3}$	$\sim 10^{15}$	kg m^{-1} s^{-1}
Thermal expansion coefficient** α	$-0.68 \cdot 10^{-4}$	$0.88 \cdot 10^{-4}$	$2.1 \cdot 10^{-4}$	$1.5 \cdot 10^{-4}$	$°C^{-1}$
Surface tension γ	0.076	0.074	0.072	n/a	N m^{-1}
Specific heat capacity c	4.22	4.19	4.16	2.11	kJ $°C^{-1}m^{-1}$
Thermal conductivity k	0.56	0.59	0.62	2.14	W $°C^{-1}m^{-1}$
Latent heat of fusion L_f	333.6	n/a	n/a	333.6	kJ kg^{-1}
Latent heat of vaporization L_E	2.49	2.47	2.45	2.82	MJ kg^{-1}
Relative permittivity ϵ_r	87.9	83.8	80.1	91.6	1

*Ice behaves like a viscous liquid under long term load, like in the flow of glaciers.

**Expansion of volume.

Density of natural waters is dependent on temperature, concentration of dissolved matter and pressure. Density is a key quantity used to describe natural waters, since it defines the vertical stratification. As density changes, water finds its way to a corresponding layer, and in the development of stratification even very small variations in density are important. If, for example, surface water density increases due to cooling, the water sinks down to a depth with a similar density. This vertical circulation caused by changes in density is called *convection.*

Many properties of water are exceptional among similar kinds of chemical compounds (Table 3.2). Its ability to dissolve is strong, latent heat of evaporation and surface tension are very high, strong expansion upon freezing is unique, and thermal capacity and latent heat of fusion are among the highest. Relative permittivity is also very high.

Table 3.2: *Exceptional properties of water and their significance regarding hydrology.*

Property	**Consequence**
Great dissolving ability	Nearly all elements are found dissolved in natural waters
Freezing and boiling points are relatively high	The Earth has water in all phases. Life based on liquid water is possible.
Large specific heat capacity	Temperature of water is slow to change, which brings thermal inertia to weather and climate, and limits the extremes.
Latent heat of fusion and evaporation are high	Freezing / melting buffer the temperature of water, and evaporation consumes plenty of heat.
Temperature of maximum density is above the freezing temperature (for fresh and brackish water)	Winter stratification develops in fresh water lakes in such a way, that upper layer is at the freezing temperature and lower layers are warmer ($\leq$ 4 °C)
Significant expansion upon freezing	Ice floats on water and creates pressure. Frost heaving deforms ground.
Great relative permittivity	Pure water is a poor conductor, and concentration of dissolved matter affects conductivity greatly.
Strong surface tension	Capillary action and the consequent rising of water in vegetation and cavities in ground. Formation of rain droplets.

Acidity of water solution is measured in pH scale, which indicates the amount of active hydrogen ions (H^+) in the solution. The scale is base ten logarithmic. When the pH number drops by one, the acidity of the solution increases tenfold. pH of neutral solution is 7.0, acidic solution have smaller pH and alkaline solutions have higher pH. pH of natural waters is typically between 4 and 10. For example, vinegar has a pH of 3 and laundry detergent 12.5.

3.1.2 Snow and ice

In cold climate zones part of the water is found as ice and snow (Fig. 3.5). These can be classified as follows:

Environment	**Type of ice**	**Origin**
Atmospheric ice	Snowflakes	Formed in clouds from water vapour
Winter terrain	Snow cover	Accumulation of snowfall
Ice cover on bodies of water	Ice and snow	Freezing of water and snowfall
Glaciers	Snow and ice	Snowfall and metamorphosis of snow
Frozen ground	Ice and soil	Soil water freezing
Ground surface	Frost	Deposition of water vapour onto ground
	Rime ice	Freezing of supercooled water onto ground
	Glaze	Freezing of water on ground

Glaciers and permafrost contain old ice, and retention time for them is long. In other environments snow and ice are seasonal and limited to the cold season, apart from a few exceptions. For example, in dry valleys of Antarctica there are lakes with year-round ice cover.

Ice and snow are chemically similar. Their differentiation is based upon visual observation and various physical properties. Snow contains plenty of air filled pores, up to 90 % of its volume, while gas bubbles in ice usually fill no more than 1 % of total volume. For this the **macro-properties**[4] of snow and ice differ significantly. Pores filled with gas scatter light strongly and all colors equally, due to their size being much larger than the wavelength of

[4]Macro-property refers to sufficiently large elements, where the combined effect of crystals and pores is significant.

the light being scattered. This makes snow appear white under sunlight. Gas pores increase the thermal insulation of snow and decrease its strength.

Properties of pure ice are presented in table 3.1. Thermodynamic properties are of the same order of magnitude with liquid water, but mechanical properties have significant differences. Water is a viscose liquid with a small viscosity, for which reason water moves and flows effortlessly. Ice is a rather firm viscoelastic solid material. Under short term loads ice behaves like an elastic material, up until its ultimate strength. The strength of ice is $\sim$ 1 MPa and modulus of elasticity $\sim$ 1 GPa. Pressure caused by the expansion ice upon freezing can reach its breaking point. Under long term loads, like gravity pulling a sheet of ice on slope of a mountain, very slow viscous flow happens, like how a clump of dough flattens when left to stand on a table.

An important basic property of snow is its density, which in Finland typically has values of 200 – 300 kg m^{-3}. Density can written as $(1 - \nu_a)\rho_i$, where ν_a is the porosity (relative gas volume) of snow and ρ_i is the density of pure ice. Heat capacity of snow is its density multiplied by the specific heat capacity of ice and heat conduction is proportional to the square of density. Mechanical properties of snow are even qualitatively different from ice. Carrying capacity of snow indicates, for example, how wide a skis are needed to move on the snow, and its internal strength tells about the risk of avalanches and of the difficulty of digging into the snow. Properties of snow change over time due to the metamorphosis of snow.

Figure 3.4: *An example of ice in nature: frost flowers on a window. Photo: Matti Leppäranta.*

3.1.3 Impurities in natural waters

In the physics of natural waters attention needs to be paid to the geochemical properties as well, first because these have an effect on the physical properties and behavior of water,

Figure 3.5: *An example of ice in nature: Collecting drinking water from supraglacial lake Suvivesi near the Finnish Antarctic research station Aboa. Photo: Matti Leppäranta.*

and second, because geochemical information is required for the applications of hydrology (Fig. 3.6). Impurities are found in natural waters as dissolved and insoluble (suspended matter). Additionally, natural waters contain *colloids*[5] that can be caught in other insoluble matter during filtration, or they can seep through the filter, depending on the pore size. Automatic probings can provide data on impurities, but accurate measurements require laboratory analysis (Fig. 3.7).

Concentration of dissolved matter affects the density of water significantly, especially in brackish and saline lakes. Nutrients (mostly phosphorus and nitrogen) are used as indicators of biological primary production, which is why their balances are taken into account in physical studies. Suspended matter exists in a body of water as particles of various sizes, and they can be extracted from the water by filtration. Pore size of a typical filter is in the scale of a micrometer (for example, $0.4 - 1.0$ μm for the measurement of chlorophyll content, $0.45 - 2.0$ μm used by the environmental administration). Suspended matter is caught in the filter, and dissolved matter can then be extracted by evaporating water out of the filtered sample. If there is plenty of suspended matter, the mixture of water and suspended matter can be considered as a binary mixture. Depending on the type of use the water is intended for, public authorities have set limits for impurities as quality standards.

The dimension of the concentration of dissolved and suspended matter is typically mass

[5]Colloid is an intermediate of homogeneous and heterogeneous medium, which consists of dispersed phase and continuum phase. Dispersed matter consists of small (1 – 1 000 nm) particles, which are evenly distributed within a continuum phase. Examples of such mediums are fog (droplets of water in air) and milk (droplets of fat in water).

Figure 3.6: *In lake research, studies are carried out by automated instruments, but also by taking water samples for biogeochemical analysis. On large lakes, research vessels are employed. Pictured, is the Russian Academy of Sciences' research vessel Ekolog on Lake Onega. Photo: Matti Leppäranta.*

per volume, with the unit being milligrams per liter (mg L^{-1})[6]. Natural waters also have gases dissolved in them, of which undoubtedly oxygen is the most important for life. The concentration (c) of dissolved matter is typically measured from electrical conductivity (σ). Its dimension is current divided by voltage and distance, and its unit is S m^{-1} (siemens per meter). Electrical resistance is the inverse of electrical conductivity, $S = \Omega^{-1}$. Electrical conductivity is also dependent on temperature, $\sigma = \sigma(c, T)$. This is usually reduced to a reference temperature, normally set at 25 °C:

$$\sigma(c,\, 25\,^\circ\mathrm{C}) = \sigma(c,\, T) + \Delta\sigma_{25\,^\circ\mathrm{C}}, \tag{3.1}$$

$$\Delta\sigma_{25\,^\circ\mathrm{C}} = \sigma(c,\, T)\cdot 0.0191\,^\circ\mathrm{C}^{-1}\cdot(25\,^\circ\mathrm{C} - T), \tag{3.2}$$

where $\Delta\sigma_{25\,^\circ\mathrm{C}}$ is the temperature correction for the conductivity. Concentration of dissolved matter can then be estimated on the basis of equation (3.2):

$$c = 6.7\cdot\frac{\sigma(c,\, 25\,^\circ\mathrm{C})}{\mathrm{mS\ m^{-1}}}\cdot\frac{\mathrm{mg}}{\mathrm{L}}. \tag{3.3}$$

In oceanography, the amount of dissolved matter is expressed as *salinity* (S), which is defined

[6]Liter is denoted by a capital L in this book.

Figure 3.7: *Processing of water samples at Lammi biological station water laboratory during the 'Optics of natural waters' course. Photo: Matti Leppäranta.*

as the ratio of the masses of dissolved matter and solution, and thus it is dimensionless. Salinity is expressed as per mille (‰), but according to latest recommendations it is not necessary to write out the permille symbol ($S = 35$ is read as $S = 35$ ‰). In this book the symbol is nevertheless used in order to avoid confusion. Natural waters are classified by their salinity in the following way:

$S < 0.5$ ‰	Fresh water
0.5 ‰ $\leq S < 25$ ‰	Brackish water
25 ‰ $\leq S < 40$ ‰	Seawater
$S > 40$ ‰	Hypersaline water

Because a liter of fresh water weighs fairly accurately 1 kg, concentration of dissolved matter in mg L^{-1} has the same numerical value as salinity in parts per million (ppm). As salinity increases, so does density, which makes the mass of one liter of saline water more than 1 kg.

In Finnish lakes, rivers and ground waters the concentration of dissolved matter is typically under 100 mg L^{-1}, so salinity is less than 0.1 ‰. These are rather small values. In practise, the salinity of drinking water needs to be below 0.5 ‰. The salinity of human blood is 9 ‰, so water less saline than this could theoretically be used as drinking water, although the taste of salt becomes uncomfortable when salinity reaches 0.5 ‰. Salinity of ocean water is around 35 ‰, and salinity of the Baltic seawater at the Finnish coastline is 4 – 8 ‰. Some salt lakes are hypersaline, for example, the Dead Sea, which has a salinity of around 250 ‰.

Suspended matter contains both organic and inorganic compounds. It is studied from the matter caught by filters. Fraction of organic matter can be found out by burning. Organic carbon is in dissolved and particle form. Living organic flora (like phytoplankton) is measured by the concentration of chlorophyll, typically having a unit of micrograms per liter (μg L^{-1}). The same unit is used for the concentration of the two main nutrients required for biological primary production[7], nitrogen (N) and phosphorus (P).

Suspended matter is detached from ground or deposited from the air. Detaching from the bottom due to intense waves or currents is called *resuspension*. Particles of suspended matter transport via currents and sink according to Stoke's law:

$$w = \frac{(\rho_s - \rho) g d^2}{18\mu}, \tag{3.4}$$

where w is the vertical velocity, ρ_s is the particle density, ρ is the density of water, $g = 9.81$ m s^{-2} is the gravitational acceleration, d is the particle radius and μ is the dynamic viscosity of water. Sinking velocity is strongly dependent on the particle size. In the derivation of the law, an assumption has been made on the spherical shape of the particles, which of course is not always true in nature. In turbulent river flow, intense eddies help particles stay suspended in water longer.

Example 3.1
Let's assume, that in a lake particles have a radius of d and density $\rho_s = 2\,500$ kg m^3 and water temperature at 10 °C, when viscosity is $\mu = 1.3 \cdot 10^{-3}$ kg m^{-1} s^{-1}. Clay particles ($d = 0.2\,\mu$m) sink at a rate of 2.2 mm day^{-1}, and grains of sand ($d = 1$ mm) sink at rate of 63 cm s^{-1}. Particles of clay stay suspended for a long time, but sand settles at the bottom very fast. Clay particles have a flat shape, which causes them to glide in water, making them sink even slower in reality.

Water is transparent to visible light, which makes photosynthesis possible below the surface of water. Light is absorbed and scattered by water itself and so called *optically active substances*: colored dissolved organic matter (CDOM), suspended matter, and chlorophyll *a*. Pure water allows sunlight to penetrate for up to 100 m, but in turbid lakes penetration is less than one meter. Passage of light in water is investigated by absorption and scattering spectra. The color of water can also be determined by this kind of analysis.

In limnology, the color of water is determined from a filtered sample by comparing it to a platinum-cobalt solution. It tells about the amount organic colored matter. Previously, a

[7]Primary production refers to the amount of energy or organic matter produced by green plants via photosynthesis.

colored disc was used for comparison, but nowadays an absorption spectrum produced by a spectrophotometer is used, and the absorption at 420 nm or some other short wavelength is used to determine the color. The color describes the browness and humus content of the water. The result is given in units of mg Pt L^{-1}. Clear waters have values of 5 – 10 mg Pt L^{-1}, while humic waters have values of 50 – 100 mg Pt L^{-1}. This value does not take suspended matter into account, which is why the result will not always correspond with a visual observation by a human eye. If the amount of suspended matter is low, humic lakes seem brown to the naked eye.

Suspended matter usually makes water more gray, and it can be indirectly estimated with turbidity measurements. The unit of turbidity is *FTU (Formazin Turbidity Unit)*, which compares the water in question to a standard solution. Formazin is a polymer, which exists as suspended matter in the standard solution. Turbidity of clear water is less than 1.0 FTU, slightly turbid is 1 – 5 FTU. Absorption bands of chlorophyll *a* are found in blue (430 – 440 nm) and red (660 – 690 nm). These bands can be used to confirm the presence of chlorophyll and estimate its concentration.

Requirements for drinking water are strict and concern concentrations of substances and bacteria, and sensations. Table 3.3 presents key quality parameters of drinking water, and gives values for the drinking water in Helsinki. Each harmful substance has its own limits. Generally, Finnish drinking water fulfills these requirements well, and in fact, Finnish tap water is higher in quality and purity than bottled water sold at grocery stores (Fig. 3.8). Water for residents of Helsinki and surrounding municipalities comes from Lake Päijänne. Connecting the lake to the city is the longest continuous tunnel bored in bedrock, Päijännetunneli, which has a length of 120 km. Water flows in it at a depth of 30 – 100 m, and provides water for the daily needs of around 1 million people.

Table 3.3: *Quality requirements for drinking water in Finland. As an example, data for the drinking water in Helsinki is used. The water for the residents of Helsinki and surrounding municipalities is transported via the Päijänne tunnel. Data: Helsinki Region Environmental Services Authority (HSY)*

Quantity	**Effect**	**Quality requirement**
Conductivity	Salinity	< 2.5 mS cm^{-1}
pH	Acidity	6.5 – 9.5 (HSY: ~ 8.0)
Turbidity	Clarity	No requirement (HSY: 0.06 FTU)
Calcium & magnesium	Hardness of water, causes lime deposits	No requirement (HSY: soft)
Alkalinity	The ability of water to resist acidification	No requirement
Fluoride	Beneficial in small amounts, improves dental enamel	< 1.5 mg L^{-1} (HSY: 0.2 mg L^{-1})
Iron	No health hazard	< 0.2 mg L^{-1} (HSY: 0.02 – 0.08 mg L^{-1})
Coliform bacteria	Risk of disease	0 cfu (colony forming unit) / 100 ml
Smell and taste	Sensation	Sufficient dilution

3.1.4 Oxygen in water

Water oxygen content is an important quantity for water quality. It is given in absolute amount (mg L^{-1}) or relative to the saturation level (%). Oxygen ends up in water mostly from the water/atmosphere interface, and to a lesser degree, as a byproduct of photosynthesis. It is depleted in biochemical processes. Water oxygen content affects the taste of water, and 3 mg L^{-1} is considered to be a good lower limit for drinking water. In the ecology of natural waters, oxygen content is a central quantity describing their health.

Dissolution of oxygen is proportional to temperature, so that it increases as temperature drops. Saturation points are listed in table 3.4. Cold water can thus retain more oxygen than warm. When water oxygen content drops below 2.0 mg L^{-1}, creatures requiring oxygen can no longer live in it. In waters with oxygen deficiency, decomposing releases hydrogen

Figure 3.8: *Finnish tap water is higher in quality than industrially bottled water. Globally speaking, this is a luxury. Photo: Matti Leppäranta.*

sulfide, which smells like rotten eggs.

Table 3.4: Oxygen saturation concentration in water as a function of temperature.

Temperature [°C]	**O_2 saturation point [mg L^{-1}]**
0	14.6
10	11.4
20	9.2
30	7.7

Oxygen situation in lake surface waters during summer remains good, as oxygen from the atmosphere is constantly available. Oxygen is depleted all along the water column and in the bottom sediment, and deeper parts of lakes can run out of oxygen if replenishment is slow. When the density of surface water increases, free convection brings mixing to increasingly larger depths. When this convection hits the bottom, the lake is said go through an overturn, and atmospheric oxygen is replenished all throughout the lake. Pure convective overturn happens when the whole lake is at the temperature of maximum density, but depending on wind speed the autumn overturn may happen within a wide temperature band, 1 – 7 °C. During long winters Finnish lakes commonly face oxygen depletion which may result in fish die offs.

3.2 Equation of state for natural waters

3.2.1 Density of water

Quantities defining the state of pure water are temperature T, density ρ and pressure p. When studying natural waters, salinity S must also be considered. These quantities are linked together by the equation of state, which is usually written out as

$$\rho = \rho(T,\, S,\, p). \tag{3.5}$$

The effect of salinity will become significant in concentrations of over 1 ‰, but even smaller quantities will make a difference in stratification in situations with very weak currents, such as in ice covered lakes. Pure water has a very abnormal property in that the temperature of maximum density is above the freezing point (Fig. 3.9). Freezing point is at 0 °C, and maximum density is reached at 3.98 °C. The reason behind this behaviour is that the molecules of water start organizing towards the tetrahedral structure of ice crystals as they are approaching the freezing point. Below the temperature of maximum density this effect is decreasing the density more than the decrease in thermal vibrations of molecules is increasing it.

Density of pure water under normal atmospheric pressure p_0 is a polynomial function of temperature $T(°\mathrm{C})$ (UNESCO, 1981)

$$\begin{aligned}\rho(T, 0, p_0) = {} & 999.842594 + 6.793952 \cdot 10^{-2} T - 9.095290 \cdot 10^{-3} T^2 \\ & + 1.001685 \cdot 10^{-4} T^3 - 1.120083 \cdot 10^{-6} T^4 + 6.536332 \cdot 10^{-9} T^5.\end{aligned} \tag{3.6}$$

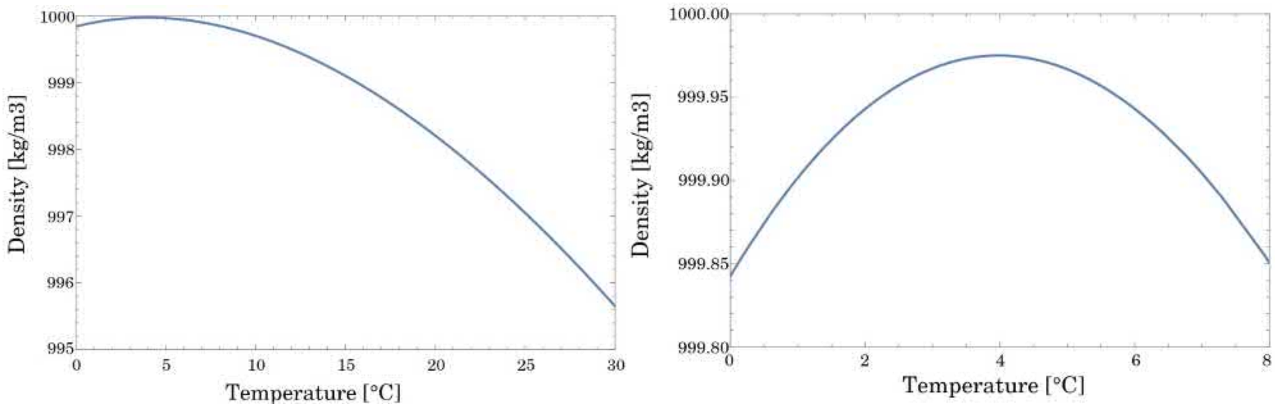


Figure 3.9: *Density of pure water at atmospheric pressure as a function of temperature between 0 °C and 30 °C. On the right, a more accurate view of the temperature range between 0 °C and 8 °C.*

In many applications it is enough to make simple second degree fit into equation (3.6):

$$\rho(T, 0, p_0) = \rho_m - \alpha(T - T_m)^2, \tag{3.7}$$

where $\rho_m = 999.975$ kg m^{-3} and $T_m = 3.98$ °C are the maximum density and the corresponding temperature, α is a fitting parameter, that depends on the temperature range in question. For fresh water, like the type that is typically found in Finnish surface and ground waters, the equation for pure water can be used. Its density is 997 – 1 000 kg m^{-3}, when temperature is 0 – 25 °C and pressure is 1 bar. The SI-unit of pressure is Pascal (Pa), but in hydrology bar is also used extensively, 1 bar = 1 000 hPa ≈ 1 atmospheric pressure.

Example 3.2
Let's assume cold water ($T < 10\,°$C). Parameter α can be defined in such a way that the approximation (3.7) is accurate at 0 °C. The resulting parameter is then $\alpha = 8.333 \cdot 10^{-3}$ kg m^{-3} °C^{-2}. When $T = 10\,°$C, we acquire a value of 999.675 kg m^{-3} for density, while the accurate value calculated from equation (3.6) is 999.702 kg m^{-3}.

Water is a compressible fluid, but the effect of pressure becomes in any means significant only at depths below hundred meters. Pressure can be calculated from the basic equation of hydrostatics:

$$\frac{dp}{dz} = \rho g, \tag{3.8}$$

where z is the depth coordinate. If we assume constant density, we get

$$p = p_0 + \rho g z, \tag{3.9}$$

where p_0 is the atmospheric pressure at surface. From this it can be seen, that a layer of water 10 m in thickness exerts a pressure of ∼1 bar below it, so about one atmosphere. Pressure increases density circa 0.5 % every 100 bars. This results in a density of 1 005 kg m^{-3} at the bottom of lake Baikal, which has a depth of over 1 km. As pressure increases the temperatures of freezing point (T_f) and maximum density (T_m) decrease a bit, at a depth of 1 km they get values of $T_f = -0.75\,°C$ and $T_m = 1.90\,°C$. When studying the stability of vertical stratification of very deep waters, an accurate empirical formula has to be used, because in large depths even small differences in density are significant. However, this book seeks not to tackle these questions.

Salinity affects density in a similar manner to pressure: as salinity increases, density increases and temperatures of freezing and maximum density decrease. Approximate formula for the density of saline water at atmospheric pressure is

$$\rho(T,\, S,\, p_0) \cong \rho(T,\, 0,\, p_0) + S \cdot 0.82\,\mathrm{kg\ m^{-3}} \tag{3.10}$$

where salinity is presented in permils. When salinity is $S = 0.5\,‰$ the density of water is 0.41 kg m^{-3} larger than for corresponding pure water. In Great Salt Lake (Utah, USA) salinity of the water varies significantly, but is in the order of 100 ‰, for which the corresponding density is in the order of 1 100 kg m^{-3}. When salinity is greater than 1 ‰, equation (3.10) is not accurate enough to describe vertical stratification, and equations with higher order terms need to be included. This book does not go further with saline waters, as those are addressed in other literature (Myrberg & Leppäranta, 2014).

Equation of state for seawater is well defined to suit the needs of oceanographic research. Nowadays, the TEOS-2010 standard is in use, replacing the UNESCO 1981 equation of state. TEOS-2010 brought with it small updates into the calculation of salinity[8]. The equation of state is applicable for salinities $0 \leq S \leq 40\,‰$ and it can be applied in the research of fresh, brackish and saline water. If the chemical composition of the water differs significantly from seawater, accuracy of the equation is decreased. In order to be exact, one would need to construct an equation of state for every body of natural water, especially in the case of hypersaline waters.

In fresh natural waters relative changes in density are within a few thousandths, but these variations are nevertheless important for the mixing of lakes. In theory it is possible to place heavier water on top of lighter water, but in reality the situation would be very unstable and would break easily. Thus, assumption can be made that stratification in natural waters is always either neutral (density is the same at all depths) or stable (density increases with depth). Stratification in freshwater lakes is dictated by temperature structure. In flowing rivers the turbulent motion of water keeps the water column always well mixed and homogeneous, and no stable stratification is formed.

Example 3.3

The effect of salinity and pressure on the temperatures of maximum density and freezing point can be estimated with the following equations (Leppäranta, 2015):

[8]In oceanography, salinity S is nowadays written without the '‰' symbol: $S = 35$ is thus read as $S = 35\,‰$. In this book the ‰ symbol is used for clarity.

$$T_m[°C] = 3.982 - 0.2229 \cdot S - 0.02004 \cdot p \cdot (1 + 0.00376 \cdot S) \cdot (1 + 0.000402 \cdot p) \tag{3.11}$$
$$T_f[°C] = -0.0575 \cdot S + 1.710523 \cdot 10^{-3} S^{3/2} - 2.154996 \cdot 10^{-4} S^2 - 7.53 \cdot 10^{-3} p \tag{3.12}$$

Where pressure is given in bars salinity in permils.

3.2.2 Speed of sound in water

Sound propagates through water well. According to the theory of hydromechanics, the speed of sound is $V = \sqrt{K/\rho}$, where $K \sim 10^9$ Pa is the compressibility of water. For the quantities K and ρ there exists empirical multiparameter formulae, but the speed of sound can be calculated from a simpler formula with sufficient accuracy. A common equation used in fresh and seawater applications is

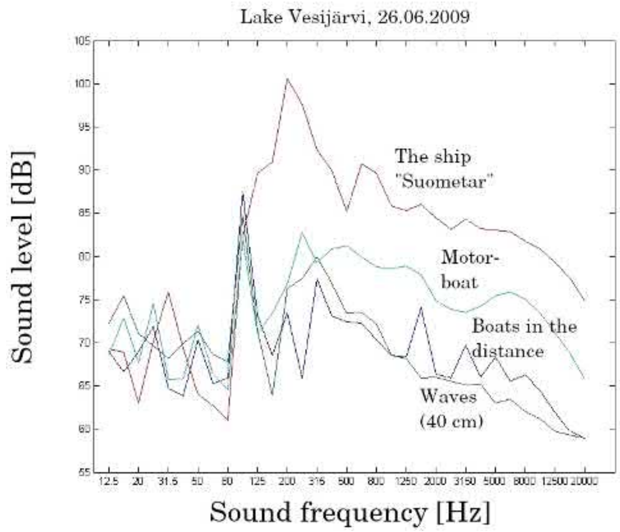
$$V = V(T,\, S,\, z) = V_0 + c_S(S - 35) + c_T T + c_{T2} T^2 + c_z z, \tag{3.13}$$

where the speed of sound is acquired in meters per second, when salinity is given in permils, temperature as Celsius and depth z in meters. The constants in the formula are

$$\begin{aligned} V_0 &= 1449 \text{ m s}^{-1}, \\ c_S &= 1.4 \text{ m s}^{-1}, \\ c_T &= 4.6 \text{ m s}^{-1}\,°\text{C}^{-1}, \\ c_{T2} &= -0.55 \text{ m s}^{-1}\,°\text{C}^{-2}, \\ c_z &= 0.017 \text{ s}^{-1}. \end{aligned}$$

If $S = 0$ and $z = 0$, the speed of sound is 1 400 m s^{-1} if $T = 0$ °C, $V = 1$ 391 m s^{-1} if T = 10 °C, and $V = 1$ 272 m s^{-1} if $T = 20$ °C. Echo sounding is based on reflecting sound from the target, as used in depth mapping, and by alternating the frequency of the signal, information on the quality of the bottom can be received. Side-scan sonars can produce two-dimensional images of the bottom topography. Even side-scanning sonar are entering the consumer market, but mostly the applications of sonar in consumer market are limited to spotting schools of fish. In recent years, studies on the acoustic environment of lakes has been made (Fig. 3.10), the effect of which on the ecosystem and environment is only vaguely understood.

(a)



(b)

Figure 3.10: **a)** *Hydrophone measurements being done at lake Vesijärvi (Southern Finland).* **b)** *Sound spectra from Vesijärvi, from which natural sounds can be told apart from man made sounds. Photo: Leppäranta & Merkouriadi (2017).*

3.3 The cycle of water and water balance

In this section, we begin by looking at the basics of fluid dynamics and the yearly cycle of weather. They are the necessary background for the water cycle, and they concern the movement of water and interplay between the atmosphere and the water resources. After that, the water cycle is presented along with the necessary concepts. Water storages involved in the water cycle and the pathways between them are investigated with more detail in the next chapter.

3.3.1 Basics of fluid dynamics

Fluid dynamics attempts to solve the flow of viscous fluids. This problem involves four basic equations of dynamics: the equation of motion, the continuity equation, the equation of state and the conservation of heat. With this set of equations, the flow velocity, pressure, temperature and density of water can be solved. Rules of thumb and orders of magnitude can be acquired with analytic methods, and with numerical methods the problem can be solved in a chosen grid cell. If salinity plays a significant role, the conservation of salt must also be included.

In fluid dynamics, a fluid is considered as a continuum where the fluid consists of infinitely small particles. In this case the methods of calculus can be employed. Newton's II law of mechanics, 'acceleration is the sum of forces', applied to the case of dynamics of natural waters, writes out as

$$\frac{d}{dt}(u,\, v,\, w) = -g - \frac{1}{\rho}\nabla p + \frac{1}{\rho}F, \tag{3.14}$$

where u, v and w are the east, north and vertical components of flow velocity, respectively. t is the time, p is pressure and F is the internal friction. Pressure affects the flow by its gradient[9] ∇p, by pressure differences. Thus, if the surface of the water is tilted, pressure gradient drives flow from high to low pressure.

In the flow of natural waters friction usually plays a significant role. In stable river flow, gravity and friction are in balance. In a wind driven flow in a lake, bottom friction and force of wind are in balance. As a stable situation develops, the pressure gradient also becomes significant when the masses of water displaced by the wind begin to modify the pressure field. In a flow through ground, the 'theory of flow through porous material' is implemented. These flows are slow, driven by pressure, and pressure gradient and friction are in balance.

The basic equations of fluid dynamics are the equation of motion and the continuity equation, also called the conservation of mass. The flow of natural waters is usually considered incompressible, continuously distributed material, or a continuum. The fluid equation of motion, the *Navier-Stokes equation*, is based on Newton's II law, and is generally written as

$\frac{\partial \boldsymbol{u}}{\partial t}$	$= -\boldsymbol{u}\cdot\nabla\boldsymbol{u}$	$-\,\boldsymbol{g}$	$-\,\frac{1}{\rho}\nabla p$	$+N\nabla^2\boldsymbol{u}$,
⇑	⇑	⇑	⇑	⇑
Change of velocity	Advection	Gravity	Pressure differential	Internal friction

where $\boldsymbol{u} = (u,\, v,\, w)$ is the flow velocity, $\boldsymbol{g} = (0,\, 0,\, g)$ is gravity and N is viscosity, which is molecular in laminar flow ($N = \nu$), and turbulent in turbulent flow ($N = K$). This equation describes, how the flow velocity changes due to transport (or *advection*), gravity, pressure differentials and internal friction. The effect of wind is implemented via a boundary condition in the surface, from where viscosity takes the momentum deeper.

In a rest state, the pressure field of a basin is horizontally constant. But if the pressure has horizontal differences, the pressure differential spawning from this drives the water into motion. If the properties of a flow field are not changing in time the flow is stationary, or steady. Otherwise, the flow is non-stationary, or changing. Boundary conditions are an integral part of the solution of the flow. In the theory of frictionless flow, a slip condition is used, where the flow is parallel with the wall near the wall. In a flow with friction the continuity of stress is applied, according to which force acting on the wall corresponds to the internal friction of the flow field at the boundary.

In questions regarding hydrological flows, water can be considered as an incompressible fluid. Then, according to the continuity equation the volume of water is conserved. *The continuity equation* is then based on the assumption that water conserves its volume:

[9] With the nabla operator, $\nabla = (\frac{\partial}{\partial x},\, \frac{\partial}{\partial y},\, \frac{\partial}{\partial z})$, derivations can be denoted in a compact manner.

$\frac{\partial u}{\partial x}$	$+ \frac{\partial v}{\partial y}$	$+ \frac{\partial w}{\partial z}$	$= 0.$
$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$
Change in eastern direction	Change in northern direction	Change in vertical direction	$= 0$

When water flows horizontally towards one spot, the water level on this spot raises as a consequence. If the water is homogeneous, its density is constant, then the above equations are enough to produce a solution. In a general case, the equation of state (Eq. 3.6) has to be taken into account since density differences influence the flow and also the conservation of heat has to be included

Horizontal flow speed in water is in the order of 1 m s^{-1} in rivers and as wind driven current 10 cm s^{-1} in lakes. Vertical velocity component is orders of magnitude smaller than the horizontal one. Usually, the equation of motion is solved only for horizontal velocity, and the vertical component is then calculated from the continuity equation. Pressure can be treated as hydrostatic in hydrological questions (Eq. 3.9). Flow of groundwater changes radically depending on ground properties, but it is significantly slower than flow in lakes and rivers.

Flow can be either laminar or turbulent. *Laminar flow* is smooth, and moves like the flow of traffic on a well behaved multi-lane highway. Streamlines move neatly around obstacles and in curves. *Turbulent flow* on the other hand is chaotic and very irregular. Varying sizes of eddies are characteristic for it. Good examples of this would be the flow of rivers, or the smoke rising from a factory smokestack. The type of flow is determined through the *Reynolds number*:

$$Re = \frac{UL}{\nu} \tag{3.15}$$

where U is the scale of velocity, L is the length scale and $\nu = \mu/\rho \approx 0.01$ cm^2 s^{-1} is the kinematic viscosity of water. Reynolds number is one of the fundamental dimensionless numbers of fluid mechanics. It describes the effect of viscosity: the smaller it is, the more significant is the role of viscosity. A flow is typically laminar, when $Re < 10^3$ and turbulent when $Re > 10^4$. Between these there is the transition zone, where flow transforms from one state to another. For example, if $U = 1$ cm s^{-1} and $L = 1$ m, $Re \approx 10^4$.

Flow in rivers and lakes is turbulent, with the exception of small, ice covered lakes. In small scales, like near the bottom, or at the surface of plants and animals, flow is laminar. This has great significance in ecology. Groundwater flow is usually laminar, due to small velocities and the small size of the pores.

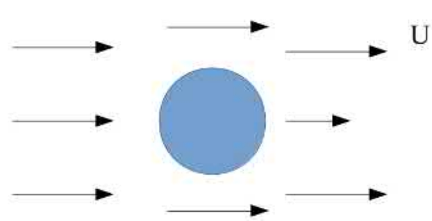
In laminar flow the internal friction of water is molecular, which is a poor transporter of momentum. Time scale of momentum transfer scales as $T \sim L^2\nu^{-1}$. For example, transport over a distance $L = 1$ m takes 10 days. In turbulent flow, the eddies are a very efficient way of transport. If l is the length scale of the eddies, and u_* the characteristic speed of turbulent fluctuations, mixing takes place in the scale $T \sim lu_*^{-1}$. In the vertical mixing of lakes and rivers, the length scale is $l \sim 1 - 10$ m, $u_* \sim 0.1 - 1$ cm s^{-1}, so the time scale becomes $T = 10^2 - 10^4$ s. This comparison between laminar and turbulent transport is valid for typical properties of water, like heat and concentration of dissolved matter. Turbulent mixing is much more effective than molecular (laminar). As is easily observed, coffee cools down in a cup much faster, when it mixed with a spoon.

In vertical turbulent mixing, the transport of quantity a B over a unit area and time is $Q = -Bw$, where w is the vertical flow velocity. Quantities B and w change rapidly over time, and they can thus be written as $B = \langle B \rangle + B'$ and $w = \langle w \rangle + w'$, where $\langle \cdot \rangle$ denotes time average and $'$ means the momentary fluctuation from this average. Time averages are usually calculated every 10 minutes and the fluctuations, whose time average is zero, are measured several times a second. The vertical transport of the quantity B is then

$$\langle Q \rangle = -\langle Bw \rangle = -\langle B'w' \rangle \approx K\frac{dB}{dz}, \tag{3.16}$$

where K is the turbulent diffusion coefficient. When the average of the transport Bw is calculated, only the covariance $\langle B'w' \rangle$ is left, because the time average of fluctuations is zero, as well as is the average of vertical speed (assuming that the water level stays fixed). This covariance can be written with the help of correlation r_{Bw} and standard deviations σ_B and σ_w: $\langle B'w' \rangle = r_{Bw}\sigma_B\sigma_w$. Equation (3.16) is based on Prandtl's turbulent transport model, which is analogous to the molecular transport $\nu\frac{dB}{dz}$. Based on experimental work performed, natural waters have turbulent diffusion coefficients in range of $K \sim 1 - 100$ cm^2 s^{-1}, $K \gg \nu$. When examining the net effect of transport in a small volume, we subtract the outgoing from the incoming, and end up in the second derivative. So, the net effect of diffusion in vertical mixing in a molecular case is $\nu\frac{d^2B}{dz^2}$ and for turbulent case $K\frac{d^2B}{dz^2}$.

Example 3.4
Let's consider the drag imposed on a cylinder in a flow using dimensional analysis.



The fluid with speed U flows past a cylinder with a diameter of d. What is the drag F imposed on the cylinder, when the fluid density is ρ and viscosity μ?

Starting with these quantities we can write

$$F \propto d^A U^B \rho^C \mu^D.$$

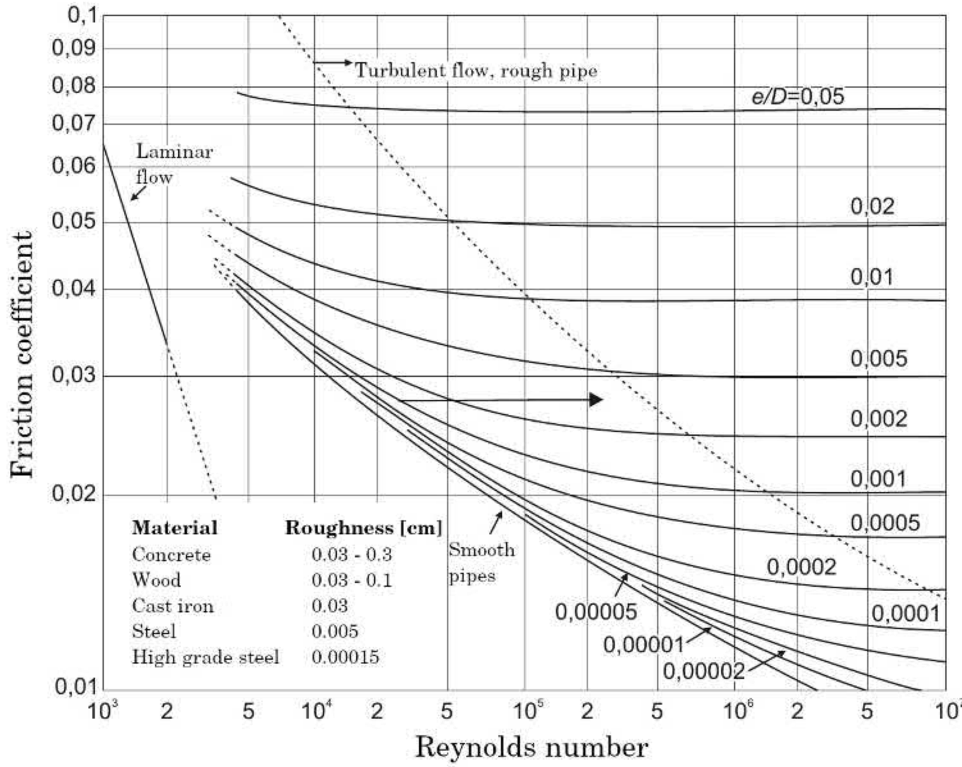


Figure 3.11: *Friction coefficient f in pipe flow as a function of Reynolds number Re, pipe diameter D and surface roughness e. In laminar flow, $f \propto Re^{-1}$. In a turbulent flow, f is not dependent on Re, but it is very strongly dependent on the surface roughness and the pipe diameter. Figure: Moody (1944), edited.*

The dimensions of our quantities in this case are $[F] = MLT^{-2}$, $[d] = L$, $[U] = LT^{-1}$, $[\rho] = ML^{-3}$ and $[\mu] = ML^{-2}T^{-1}$. Exponents A, B, C and D can be solved from the system of equations

$$\begin{aligned} M: &\quad 1 = C + D \\ L: &\quad 1 = A + B - 3C - D \\ T: &\quad -2 = -B - D \end{aligned}$$

In this system we have four unknowns in three equations. The number of dimensionless quantities will then be two. In the solution there will be one free variable, which we will assign as A. Thus, we will get $B = A$, $C = A - 1$ and $D = 2 - A$. The quantity $\rho d^2 U^2$ has the dimension of force, and it is handy to take it as the scale of F in one dimensionless quantity

$$\frac{F}{\rho U^2 d^2} = U^{A-2} d^{A-2} \rho^{A-2} \mu^{2-A} = \left(\frac{\rho U d}{\mu}\right)^{A-2} \quad \text{or}$$

$$\frac{F}{\rho U^2 d^2} = f\left(\frac{\rho U d}{\mu}\right) = f(Re).$$

The dimensionless force on the left side and the Reynolds number on the right are the natural dimensionless quantities of this problem. By performing measurements the function f can be constructed.

This problem can be further expanded by taking into account the surface roughness of the cylinder. If the characteristic roughness of the cylinder is e, $[e] = L$, we will get a dimensionless roughness of e/d, and

$$\frac{F}{\rho d^2 U^2} = f\left(Re, \frac{e}{d}\right).$$

Now the function f is dependent on two arguments, but it is still rather easy to construct with a proper experimental setup (Fig. 3.11).

The above example underlines a key difference between laminar and turbulent flow: laminar drag is proportional to flow speed, while turbulent drag is proportional to the square of flow speed. This relation is universal, and concerns all shapes of objects and also the bottom friction of flow.

Waves are periodic phenomena that changes over space and time. They are born out of disturbance, that gravity overshoots upon restoring, and this keeps the system in motion[10]. This is analogous to a pendulum being set into motion. The mathematical basis for waves is the sine wave. Let's take a look into a wave propagating into the x direction at time t:

$$f(x, t) = A \sin(kx + \omega t + \phi), \tag{3.17}$$

where A is the amplitude of the wave, k is the wave number, ω is the frequency and ϕ is the phase. Wave height is the vertical distance between the crest and trough of the wave, or $2A$, $\lambda = 2\pi/k$ is the wavelength and $T = 2\pi/\omega$ is the wave period. In the linear wave theory, waves can be added on top of each other, and in this way describe phenomena with multiple periods. Inversely, any periodic function can be returned back to its basic one component sine functions by harmonic analysis.

In some problems of fluid dynamics, a frictionless model can be applied. This can be used to solve properties of the flow field imposed by the geometry of the basin, but in order to take in to account wind and bottom friction, friction has to be included in the model. Frictionless models have been used in lake and river research to study waves. Water is not transported with the waves, but rather they are moved back and forth with the wave in elliptical trajectories.

Waves are divided into three basic types, according to the ratio between wavelength λ and water depth H (Fig. 3.12).

[10] In capillary waves the restoring force is surface tension. Wavelength is less than 2 cm and period shorter than 0.1 s.

- *Shallow water waves* or long waves ($\lambda > 20H$) that 'feel' the bottom. Particles of water move back and forth in elliptical trajectories in the whole water column.
- *Deep water waves* or short waves ($\lambda < 2H$) that do not 'feel' the bottom. Particles of water move in circular trajectories, and this motion is strongly damped with depth. At a depth of $1/2\lambda$ the waves are not felt anymore.
- A mixture between these two ($2H < \lambda < 20H$), where properties of both types are found.

Wave speeds are

$$\text{Shallow water waves:} \quad V_1 = \sqrt{gH} \text{ and}$$
$$\text{Deep water waves:} \quad V_2 = \sqrt{\tfrac{1}{2\pi} g\lambda}.$$

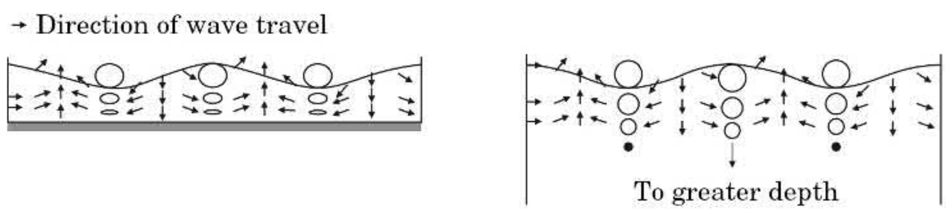


Figure 3.12: *Schematic presentation shallow water waves (left), and deep water waves (right). Waves travel from left to right. The waving motion of the shallow water wave flattens towards the bottom, while deep water waves abate never reaching the bottom.*

The speed of shallow water waves is only dependent on the depth, and that is why they are non-dispersive. The speed of deep water waves is dependent on the wavelength of the wave, and they are dispersive. Alongside Reynolds number, a second key dimensionless number for fluid dynamics is the *Froude number*

$$Fr = \frac{U}{\sqrt{gH}}. \tag{3.18}$$

Froude number illustrates the importance of gravity, and it describes the relation of flow speed and the shallow water wave speed. The flow is subcritical (critical) when $Fr < 1$ ($Fr > 1$). This is meaningful in, for example, river studies. When the flow is critical, waves cannot propagate upstream.

In geophysical flows, the rotation of Earth plays a significant role. The effect of this rotation is described with the Coriolis parameter $f = 2\Omega \sin\phi$, where Ω is Earth's rotation rate and ϕ is the latitude. From this, we can derive the *Rossby number*

$$Ro = \frac{U}{fL} \tag{3.19}$$

which is related to the length scales of geophysical flows, like layer depth and the size of eddies. When $Ro \ll 1$, the Coriolis effect is significant. This is true in large lakes, and in smaller lakes under ice cover.

3.3.2 The cycle of water

Water forms storages: the ocean, the atmosphere, groundwater, soil water, seasonal snow, and glaciers. The size of these storages is usually given as volume. Here, the density of water can be assumed as 1000 kg m^{-3}, so volume is easily converted in mass. In the cycle of water, water is transported from one storage to another, while the total amount of water stays constant (Fig. 3.13).

Water in continents forms catchment areas and water systems. *Catchment* is the basic component of a water system. It is an area containing land and water, where precipitation falls and runs off to a common outlet. Catchments are divided by *watersheds*. A catchment can contain groundwater reserves, rivers and lakes. *Water system* is a larger entity, and can be formed of one or several catchments. For example, the Vanajavesi catchment contains most of the waters of Tavastia proper (in Southern Finland) and is a part of the river Kokemäenjoki water system. Watersheds can be defined from a map by following high terrain, springs and stream courses. In unclear situations topographical measurements at the site are used. The most important flux quantities in the water cycle are

Precipitation	The amount of water that falls liquid or solid on ground from water condensed and desublimated in the atmosphere.
Interseption	The fraction of precipitation that is caught in the foliage.
Evaporation	The evaporation and transport of water from land, water or plant surface into the atmosphere.
Transpiration	Transport of water from the body functions of plants (roots, stem and leaves) via evaporation.
Evapotranspiration	Evaporation + sublimation + transpiration.
Condensation	The reverse of evaporation, where atmospheric water vapor is condensed onto ground surface. When vapor condenses and supercools in cold air and droplets freeze onto ground and other structures, *hard rime* is formed.
Sublimation	Phase change of ice into vapour.
Deposition	Phase change of water vapour onto a surface or plants, called *frost*.
Runoff	Flow that transports water out of a certain area over a certain time, on or in ground. Runoff can happen via groundwater flow (groundwater runoff) or river flow (surface runoff).
Flow	Transport of water in rivers, streams and canals.
Infiltration	The penetration of precipitation through ground surface.
Percolation	Penetration of soil water into ground water.

Water is evaporated from the surface of the Earth and transported in the atmosphere as vapor, which condenses into droplets that precipitate onto the surface again in liquid or solid phase. Small amounts of vapor condenses as dew or hard rime or deposits as frost on the ground surface. Part of the precipitation, interseption, gets stuck on the foliage of plants and trees and evaporates from there, without ever reaching ground.

Precipitated water is evaporated back, infiltrated into ground or run off into surface waters. In the top layer of the ground there is the soil water layer, where the voids between soil particles are filled with both air and water. Below that, there is the groundwater layer, which is saturated with water. Drawn by gravity, the soil water goes straight down into the groundwater storage, where it flows towards lower pressure. From the surface waters and groundwater, water is slowly transported back into the ocean or evaporated into the atmosphere. Seasonal snow cover acts as a seasonal storage of water, from where the spring melt brings it back into the cycle. In mountain glaciers, water is trapped for centuries, and into continental ice sheets for hundreds of thousands of years. From the surface of snow and glaciers some amount of sublimation is observed, depending on the local weather.

The water cycle can be local, called the short cycle. Then evaporated water returns as precipitation rather quickly. In the long cycle, water is transported through several storages. The bulk of water transported into the atmosphere (c. 80 %) comes from the oceans. Often it is necessary to separate surface water and groundwater watersheds. Watersheds of the surface water can determined from maps, as was shown above, while groundwater watersheds do not necessarily follow shapes of the terrain. If the surface water watershed resides on a coarse soil type that conducts water well, the groundwater watershed might differ from the surface conditions significantly.

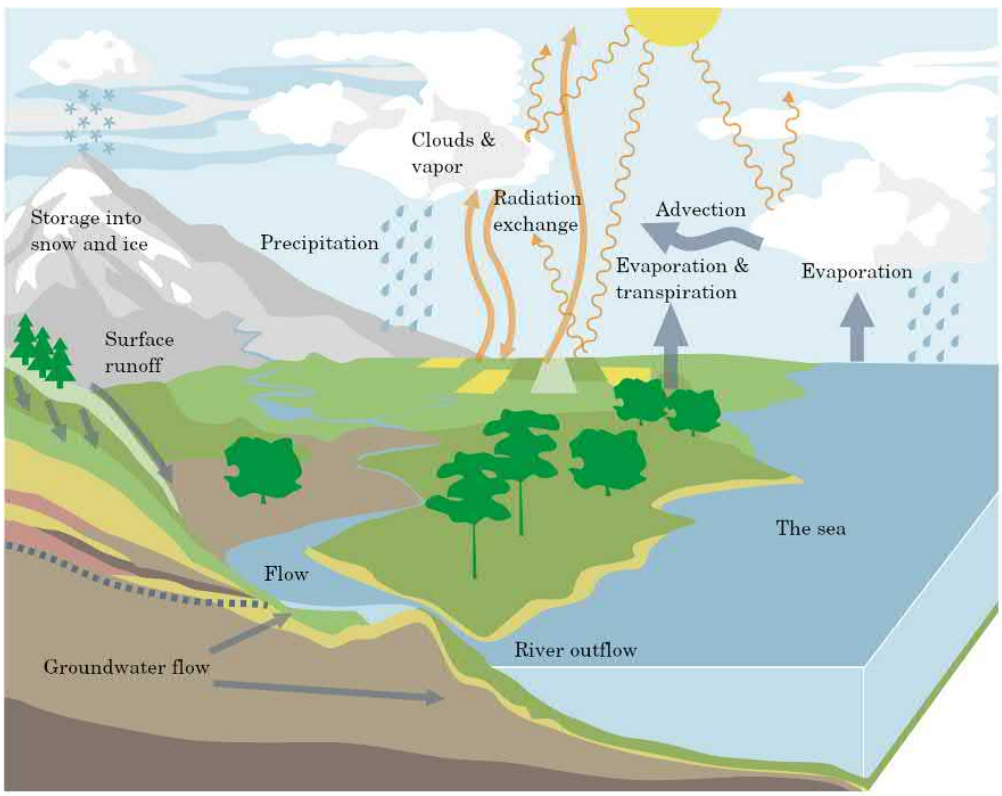


Figure 3.13: *The hydrological cycle. Water is evaporates from the surface into the atmosphere, where it is carried on by wind to eventually precipitate back to the surface. From ground, water is returned back into the atmosphere and transported forward as surface and groundwater runoff into surface waters and the sea. Figure: Salla Jokela.*

In this cycle water moves from one storage to another as *fluxes*, or volume of water transported per unit of time (discharge) or unit of time and surface area (Fig. 3.14). Flux describes the volume of water coming and going from one storage to another. The dimension of discharge is written as volume over time (L^3T^{-1}), and other fluxes as volume over time and surface area ($L^3T^{-1}L^{-2}$). The common unit of discharge is cubic meters per second, and for others the unit is millimeters per day, month, or year.

Example 3.5
a) If 10 mm of precipitation falls onto an area of 100 km^2 in one day, the total volume of water is 10^6 m^3. The rate of transport is then 12.6 m^3 s^{-1}. River Amazon is the biggest of all rivers by discharge: 209 000 m^3 s^{-1}. The biggest rivers in Finland (Kemijoki and Vuoksi) have discharge of c. 500 m^3 s^{-1}, and river Vantaanjoki running through Helsinki has a discharge of 15 m^3 s^{-1}.
b) A lake with a surface area of 100 km^2 has river inflow of 50 m^3 s^{-1}. During one day, the river brings in $4.3 \cdot 10^6$ m^3 of water, which evenly spread on the lake would raise the water level by 43.2 mm.

Figure 3.14: *Flow rate measurements in Valkeakoski, 1913. Measurements are done with a paddle-type current meter. The current moves the paddles, and the rate of their turning can be translated into flow velocity with proper calibration coefficients. Measurements are done at several spots, and based on them the discharge rate of the river is discerned. Photograph: SYKE; Kuusisto, 2008.*

The hydrological environment is described by its surface area, shape and slope, fraction of area covered by surface waters and location, different terrain types (field, wetland, forest) and their areas and locations, soil type, land use (agriculture, peat extraction, water use by population centers) and the length of waterways (Fig. 3.15). The boundaries of water systems have shaped greatly the nature of human settlement, and the movement of tribes and peoples.

The temporal and spatial distribution of precipitation affect the formation of runoff. If precipitation is very narrow in time, so will be the following runoff. If precipitation is even in time, evaporation can remove a large fraction of the rainfall from the system. A part

Figure 3.15: *Lake Tuusulanjärvi catchment area in Southern Finland. Surface area of the lake is 6 km^2 and mean depth is 3.2 m. The lake discharges into river Tuusulanjoki, and from there eventually into river Vantaanjoki. Figure: Salla Jokela.*

of the runoff happens soon after the rain as surface runoff (immediate runoff) and part is infiltrated into the ground, slowly seeping towards groundwater. The fraction of surface runoff is determined by the intensity of precipitation, surface layer water content, vegetation and soil type. Groundwater runoff is important in areas where ground top layer is coarse (for example, gravel eskers) and water flows well in. On high latitudes, like in Finland, precipitation is stored temporarily as snow cover, and the short melt season is followed by an intense runoff peak.

Annual precipitation is the highest in moist, oceanic climate zones, between 2 000 and 3 000 mm. In boreal zones it is 400 – 600 mm, and in deserts and polar areas it is under 200 mm. The amount of evaporation is dependent on humidity, temperature, and wind. On land areas it smaller than precipitation, in forested ares 20 – 70 % of precipitation. In deserts, evaporation almost equals precipitation. The difference between precipitation and evaporation is the runoff, most of which is typically surface runoff.

3.3.3 Weather

The atmosphere is directly linked to the surface water resources. For this reason, hydrological models require detailed information about the local weather conditions and climate, especially on temperature, humidity, wind and radiation balance. Table 3.5 is an example of Jokioinen (Southern Finland) weather station monthly mean values. Solar radiation and temperature are strongly connected to the cycle of seasons. Relative humidity and cloudiness are the smallest and precipitation the largest during summer.

Atmospheric storage of water is a part of the hydrological cycle. Absolute humidity, which is the amount of water vapor mixed in air, is denoted by q. This is the fraction of the total air mass that is water. q is dimensionless ($[q] = 1$), and it has values in range of $10^{-3} - 10^{-2}$. Sometimes, $\text{g kg}^{-1} = 10^{-3}$ is given as a 'unit' for q, which makes clear the fact that this is a mass fraction. Air humidity is expressed both as absolute and relative. Humidity can also be given as partial pressure of air, e, which denotes the fraction of total atmospheric pressure exerted by water vapor. Specific humidity and vapor pressure have a relation between them

$$q = 0.622 \cdot \frac{e}{p}. \tag{3.20}$$

Saturation humidity limits the amount of vapour in air. It is strongly dependent on temperature (Fig. 3.16). Relative humidity is defined as the fraction of saturation humidity, or how many percents the humidity is from the saturation humidity. The ability of cold air to hold vapor is much less than than that of warm air. When the vapor pressure is less than saturation pressure, more moisture can be evaporated into the air. Oversaturated vapor on the other hand can condense into droplets or desublimate into crystals of ice, to form clouds. Condensation and desublimation can also happen near ground as dew, frost or hard rime. In weather reports, humidity is usually usually expressed as the relative humidity.

Table 3.5: *Monthly mean meteorological parameters from Jokioinen weather station (60° 48'N, 23° 30'E) during 1981 – 2010 (Pirinen et al., 2012). Cloudiness is from the Finnish Meteorological Institute 1971 – 1980 statistics.*

	Pressure	**Solar radiation**	**Temperature**	**Humidity**	**Wind speed**	**Cloudiness**	**Precipitation**	**Snow cover thickness**
Month	**mbar**	**W m^{-2}**	**°C**	**%**	**m s^{-1}**	**1/8**	**mm/month**	**cm**
1	1 009	10.8	–5.6	89	3.7	6.4	46	14
2	1 012	37.3	–6.3	87	3.6	6	32	20
3	1 012	87.4	–2.4	82	3.7	5.5	32	24
4	1 014	151.6	3.5	72	3.6	5.6	30	1
5	1 014	207.6	9.8	65	3.6	5.1	41	–
6	1 011	215.3	14	68	3.4	4.9	63	–
7	1 011	218	16.7	71	3.2	5.3	75	–
8	1 012	162.4	15	77	3.2	5.6	80	–
9	1 012	97.2	9.9	83	3.4	5.9	58	–
10	1 012	40.7	4.9	88	3.7	6.2	66	0
11	1 011	13.1	–0.2	91	3.8	6.7	57	1
12	1 010	6	–3.9	91	3.8	6.5	47	8
Yearly	**1 012**	**105**	**4.6**	**80**	**3.6**	**5.8**	**52**	–

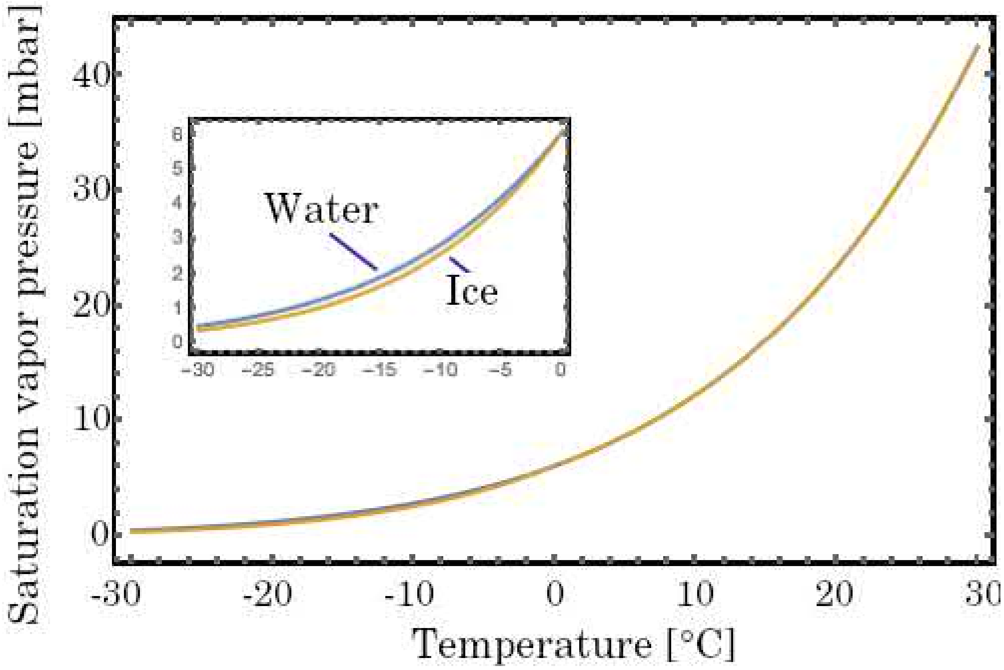


Figure 3.16: *Saturation pressure as a function of temperature. Air can hold water vapor until the saturation pressure, which is strongly dependent on temperature. Below the temperature freezing point the saturation pressures over ice and water surfaces differ slightly from one another.*

When the air temperature is expressed in the Celsius scale and the unit for vapor partial pressure in millibars, the saturation humidity e_s above water surface ($e_{s,w}$) and over ice ($e_{s,i}$) is

$$\log_{10}(e_{s,w}(T)) = \frac{0.7859 + 0.03477 \cdot T}{1 + 0.00412 \cdot T} \qquad \text{and} \tag{3.21}$$

$$\log_{10}(e_{s,i}(T)) = \log_{10}(e_{s,w}(T)) + 0.00422 \cdot T \quad \text{when} \quad T \leq 0\,°\text{C}. \tag{3.22}$$

Example 3.6

In normal air pressure and at a temperature of 10 °C, the saturation humidity is $q_s = 7.5$ g kg^{-1} and at a temperature of -10 °C it is $q_s = 1.8$ g kg^{-1} (above water surface) and $q_s = 1.6$ g kg^{-1} (above ice surface). Air density can be calculated from the equation of state of ideal gas,

$$\rho_a = \frac{p}{R_a T},$$

where $R_a = 287.04$ J kg^{-1} K^{-1} is the gas constant for air. One cubic meter of air at Earth's surface thus has a mass of 1.2 – 1.3 kg, depending on the temperature. So, the water vapor mass per cubic meter of cold air ($T < -15$ °C) at saturation pressure is less than 1 g, and in

Figure 3.17: *Waters from Eastern Finland and Russian Karelia get transported via the River Neva into the Gulf of Finland. Photo: Matti Leppäranta.*

warm air ($T > 25$ °C) it is over 20 g. If water vapor fraction at 10 °C is 5.0 g kg^{-1}, relative humidity is 66.7 %. Vapor partial pressure is $e = 8.1$ mbar.

3.3.4 Water balance

The water balance equation of a catchment or a water system describes the developments of water storage V over time. It is written as

$$\frac{dV}{dt} = (P - E)A - R, \tag{3.23}$$

where P is precipitation, E is evaporation, A is the surface area of the system and R is runoff. All water coming into a catchment or a water system comes as precipitation and all water going out is runoff or evaporation (Fig. 3.13). The dimension of the water storage V is volume of water, and the terms in equation (3.23) describe the change of this volume over unit of time. Water balance equation can be generalized for any area, when incoming flow of water I is added to the right side of the equation.

In the long term, usually no significant changes result in the storage size, and V can be set as a constant. For example, the annual net balance is typically zero, so $(P-E)A \approx R$. In a water system with no outlet rivers $R = 0$, and in order for the water storage to conserve, $P = E$. In a water balance of a continent, runoff flows into the ocean (table 3.6). In

Finland, precipitation is 500 – 650 mm per year, and evaporation is 400 mm per year, so the net exchange with the atmosphere is 100 – 200 mm per year.

Table 3.6: *Water balance of the continents and of the whole Earth.*

	Area	**Precipitation**	**Evaporation**	**Runoff**
	[10^6 km^2]	[mm]	[mm]	[mm]
Europe	9.8	734	–415	–319
Asia	45	726	–433	–293
Africa	30.3	686	–547	–139
North America	20.7	670	–383	–287
South America	17.8	1 684	–1 065	–583
Australia	8.7	736	–510	–226
Antarctica*	14	150	–50	–100
Continents, total	**158.1**	**834**	**–540**	**–294**
Oceans	351.9	1 120	–1 250	130
Whole Earth	**510**	**1 020**	**–1 020**	**0**

*Rough estimate due to low number of observations.

The water balance equation described above is also applicable for glacial water systems, where the water storage is mostly in solid form (Fig. 3.18). In glacial hydrology the term 'iceshed' is used analogously to the watershed. Marine terminating glaciers have to include calving of icebergs into the outgoing runoff. Antarctic continental runoff is almost completely due to calving of icebergs.

With the water balance equation the effect of human water usage on the water system can be estimated. With the help of water regulation, surface water storages can be kept within acceptable limits. Groundwater usage affects groundwater storage, therefore the volume of the storage has to be kept track of. Excessive water usage has led to great environmental disasters. A famous incident is the case of Aral Sea in the 1960s, when the incoming water was diverted into irrigation of cotton fields. This has led to a dramatic decrease in lake volume and area, and the following increase in salinity has triggered a local ecocatastrophe. Aral Sea is also saline, so changes in water volume are reflected in the salinity.

During the last 100 years, the world ocean water level has risen 1 – 2 mm per year[11], while at the same time, air temperature has gone up 0.3 – 0.6 °C. The shrinking of glaciers has increased the volume of water in the ocean, on top of which the thermal expansion of seawater has increased the water level. The melting of all current glaciers and ice sheets

[11]The global change in the ocean water level due to melting glaciers and thermal expansion of seawater is called *eustatic* change in water level.

Figure 3.18: *Lake has formed in front of the Vatnajökull glacier in Iceland. Photo: Matti Leppäranta.*

would add up to 70 m of water level rise.

In this chapter the physical properties of natural waters were presented, along with the basics of fluid mechanics and the water cycle. Water moves from one storage to another during its cycle. Its total volume is conserved, and by visiting the atmosphere it is purified. In the following chapters these concepts are used when the various storages and pathways between them are taken into closer examination (Fig. 3.19). Next, we will look the atmospheric water storage and its renewal more closely. Also, the seasonal snow cover and temporary storage due precipitation on land are covered in this context.

Figure 3.19: *Hydrological excursion to Estonia in autumn 1997. The Finnish-Estonian co-operation program SUVI regarded the optics of surface waters and water quality. On the left: program coordinators Dr. Helgi Arst, Tiit Kutser, prof. Juhani Virta, on the right Antti Hellevi and Alberto Blanco Sequeiros.*

Chapter 4

Hydrometeorology

4.1 Surface heat balance

4.1.1 Basic equation

For many hydrological applications, the ground surface heat balance needs to be estimated accurately. This is necessary, for example, when studying evaporation or frost (Fig. 4.1). Let's now set the z-axis as the height, positive upwards, and its origin to the surface. The surface can for this case be soil, vegetation, snow, water or ice. For a general case, the surface heat balance is written as

$$k\frac{\partial T}{\partial z}\bigg|_{z=0^-} = Q_s - Q_r + Q_{La} - Q_{L0} + Q_c + Q_e + Q_p, \tag{4.1}$$

where k is the heat conductivity of the surface medium, Q_s is the incoming *solar radiation*, Q_r is the solar radiation reflected and scattered from the surface, Q_{La} is the incoming, atmospheric *thermal radiation*, Q_{L0} is the thermal radiation emitted by the surface, Q_c is the *sensible*[1] *heat flux*, Q_e is the evaporated or *latent heat* and Q_p is heat carried along precipitation. The $|_{z=0^-}$ on the left side of the equation (4.1) marks that the product of heat conductivity coefficient and vertical derivative of the temperature is taken immediately below the surface, in order to acquire the heat exchange with the ground. On the right hand side of the equation, the transfer of heat is positive when the heat is heading down (i.e., into the ground).

The sum of the radiation terms, $Q_R = Q_s - Q_r + Q_{La} - Q_{L0}$, is called the *radiation balance*. The radiation here is based on Planck's law of black body radiation:

$$I(\lambda; T) = \frac{2hc_o^2}{\lambda^5}\frac{1}{\exp\left(\frac{hc_0}{\lambda k_B T}\right) - 1}\ , \tag{4.2}$$

where λ is the wavelength, T is temperature in the Kelvin scale, $h = 6.6261 \cdot 10^{-34}$ J s is the Planck constant, $c_0 = 2.9979 \cdot 10^8$ m s^{-1} is speed of light in vacuum and $k_B = 1.3807 \cdot 10^{-23}$ J

[1]'Sensible' here refers to the fact that the heat is transported via temperature difference. Heat moves from hot to cold.

Figure 4.1: *The village of Toppari had a weather station built for the evaporation measurements at Lake Pyhäjärvi, in Tampere (here pictured in 1912). Evaporation pans were installed in water near the shore, behind the peninsula visible in the picture. Photo: SYKE hydrological archive (Kuusisto 2008).*

K^{-1} is the Boltzmann constant. According to this law, the intensity of emitted radiation is dependent on the wavelength and the temperature of the emitter, in such a way that higher temperatures lead to shorter wavelengths at the maximum intensity, and the total radiation summed over all wavelengths is stronger. Solar radiation is also called 'shortwave radiation' (most of the radiation falls into a spectral band of 0.3 – 3 μm) and the thermal radiation of the atmosphere and the Earth's surface is called the long wave radiation or the terrestrial radiation (most of it in the spectral band between 5 – 15 μm).

Solar radiation can be computed based on the solar constant, time, place, and atmospheric effects. To compute the terrestrial radiation, Planck's radiation law is employed, which is integrated over all wavelengths to get the total radiation

$$Q_L(T) = \int_0^\infty I(\lambda; T) d\lambda = \sigma T^4, \tag{4.3}$$

where $\sigma = 5.6704 \cdot 10^{-8}$ W m^{-2} K^{-4} is the Stefan-Boltzmann constant. This integrated form is called the Stefan-Boltzmann law. A *gray body* radiates spectrally like a black body, but with a lesser intensity, dictated by the emission coefficient $\epsilon\,(0 < \epsilon < 1)$ of the medium.

The sum of sensible heat flux and latent heat flux, $Q_c + Q_e$, is the turbulent heat exchange between Earth's surface and the atmosphere. These terms can be approximated with the basic equation of turbulent exchange (Q_T)

$$Q_T \propto (B_a - B_0) U_a, \tag{4.4}$$

where B is the quantity under scrutiny (in this case, temperature or humidity). The subscript 0 refers to surface and a to atmosphere, and U_a is the wind speed. The dimension of heat flux is $[Q_T] = MT^{-3}$, thus in the relation (4.4), the proportionality coefficient has to have the dimension of $ML^{-1}T^{-2}[B]$. The heat contained in precipitation is written as

$$Q_p = \rho[c(T - T_0) + I_f L_f]. \tag{4.5}$$

Here, $c \approx 4.2$ kJ kg^{-1} °C^{-1} is the specific heat capacity of water, T is the temperature of the precipitation, $L_f = 333.6$ kJ kg^{-1} is the latent heat of fusion of ice and I_f is the phase change indicator. If liquid precipitation is frozen upon hitting the surface, $I_f = 1$. Conversely, if solid precipitation (snow or hail) melts on the surface, $I_f = -1$. If there is no change of phase, $I_f = 0$. Heat flux from precipitation is significant when $I_f \neq 0$.

Example 4.1
Rainwater is falling down on a snow covered surface. Temperature of this rainwater is 5 °C and intensity is 20 mm per day. If the rain is cooled down to its freezing point, heat is transferred to the snow cover with a power of 4.9 W m^{-2}. If this water is then frozen, a rather large heat flux of 77 W m^{-2} is released into the snow.

4.1.2 Adiabatic change in temperature

The vertical profile of air temperature can be examined by the *adiabatic model*. The first law of thermodynamics states that the change in energy, dU, of an infinitesimally small volume of gas dV is the sum of the work done on the volume (dW) and work done by the pressure

$$dU = dW - pdV \tag{4.6}$$

In the theory of adiabatic change, the external work is left out, $dW = 0$. In this case, all changes in the internal energy of the volume are temperature variations caused by changes in pressure.

Atmospheric pressure drops with increasing altitude. This is due to the simple fact that higher up in the atmosphere there is less air pressing over the parcel of air[2]. As a parcel of air is brought up adiabatically, its volume is increased as pressure drops. According to the equation of state for ideal gases, we have

$$\frac{pdV}{T} = constant. \tag{4.7}$$

Based on the adiabatic assumption it can be shown, that in the case of ideal gases the relation between temperature and pressure can be described with the equation $p(dV)^\kappa = constant$,

[2] Altitude of, for example, an airplane, can be measured by measuring the surrounding atmospheric pressure, if a reference pressure is known.

where κ is a gas specific constant. For air $\kappa = 1.4$. By entering this result into the equation of state we get

$$T = T_0 \left(\frac{p}{p_0} \right)^{\frac{\kappa - 1}{\kappa}}, \tag{4.8}$$

where the subscript 0 refers to the ground surface. It can be shown that $\frac{\kappa-1}{\kappa} = \frac{R_G}{c_p}$, where R_G is the gas constant and c_p is specific heat capacity at a constant pressure. The temperature profile given by equation (4.8) is called *the potential temperature* with respect to the surface. This profile describes the stability of the atmosphere. If the potential temperature is constant with regards to altitude, the change in temperature follows the adiabatic cooling and the stratification is then said to be neutral. If the potential temperature increases (decreases) along with the altitude, the stratification is said to be stable (unstable). Equation (4.8) can also be used to estimate the temperature on mountains.

Atmospheric pressure as a function of altitude z can be acquired from the basic equation of hydrostatics:

$$\frac{dp}{dz} = -\rho_a(z)g, \tag{4.9}$$

$$\rho_a = \frac{p}{R_a T}, \tag{4.10}$$

where ρ_a is the air density and $R_a = 287.04$ J kg^{-1} °C^{-1}. The equation for the density of air (Eq. 4.10) is one form of the equation of state (Eq. 4.7). The density of air in the lowest 1 km is $\rho_a = 1.2 - 1.3$ kg m^{-3} and air pressure decreases by about 10 Pa for each meter of altitude. When a change in pressure is converted to a change in temperature, in the lowest few kilometers the dry adiabatic lapse rate (that is, the decrease in temperature of dry air due to lowering of the pressure) is 1.0 °C and wet lapse rate is 0.6 °C for every 100 m of altitude. In dry lapse rate, there is only change in temperature. In the wet lapse rate there is also condensation of vapor into water, which releases heat into the air. Typically, adiabatic cooling is around 0.65 °C per 100 m.

Example 4.2
Lake Kilpisjärvi (in Western Lapland) resides at an altitude of 473 m. The Saana fell standing next to it reaches a height of 1 029 m above mean sea level (MSL). According to the normal adiabatic lapse rate, the mean temperature at lake Kilpisjärvi is 3.1 °C lower than at sea level, and the mean temperature at the top of Saana is 3.6 °C lower than on Lake Kilpisjärvi.

4.1.3 Solar radiation

Solar radiation follows approximately Planck's radiation law. Surface temperature of the Sun is 5 900 K, and the radiation intensity peak is found at a wavelength of 500 nm (Fig.

4.2). The radiative power is described by the solar constant $Q_{sc} = 1.367$ kW m^{-2}, which is defined as the yearly mean radiation on a surface perpendicular to the direction of radiation just outside the Earth's atmosphere. In addition to the direct sunlight, the surface of the Earth baths in radiation scattered by the atmosphere and reflected by clouds, buildings, trees etc. Together, they form the *global radiation.*

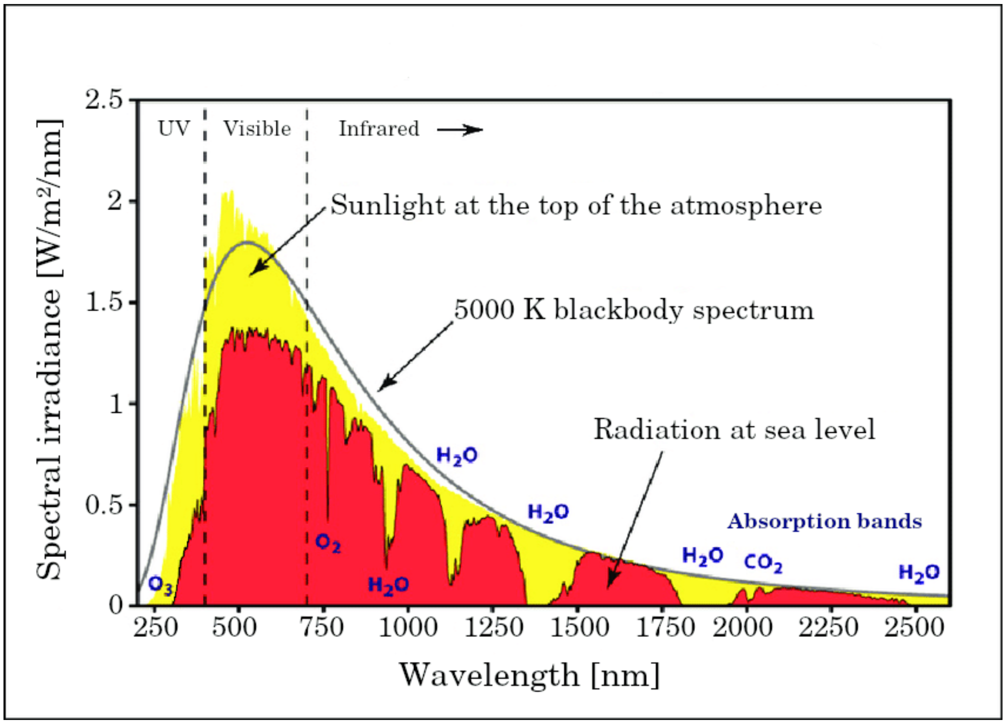


Figure 4.2: *Spectrum of solar irradiance as a function of wavelength above the atmosphere (yellow) and at sea level (red). Atmosphere absorbs and scatters back about half of the total radiation. Greenhouse gases in particular, like water and carbon dioxide, absorb radiation on their absorption bands. Figure: Robert A. Rohde (adapted).*

The amount of incoming solar radiation depends on the solar elevation angle θ, cloudiness $(N, 0 \leq N \leq 1)$ and concentration of greenhouse gases. A common equation to calculate the radiation downwelling on a horizontal surface is

$$Q_s = F(N, \theta) \cdot \mathcal{T}(e, \theta) \cdot \sin\theta \cdot \left(\frac{r_0}{r}\right)^2 Q_{sc} \tag{4.11}$$

where $F(N, \theta)$ is the cloudiness correction, $\mathcal{T}$ is the transmission coefficient of clear atmosphere and r and r_0 are the current and mean distance, respectively, between the Earth and the Sun. Zillman's (1972) transmission coefficient (Eq. 4.12) and Lumb's (1964) cloudiness correction (Eq. 4.13) are

$$\mathcal{T} = \frac{\sin\theta}{1.085\sin\theta + \frac{e}{\text{mbar}}(2.7 + \sin\theta) \cdot 10^{-3} + 0.01} \quad \text{and} \tag{4.12}$$

$$F \approx 1 - (0.6 - 0.1\cos\theta)N. \tag{4.13}$$

With these equations, correction formulae can be constructed for regular weather stations to complement radiation measurements made relatively close by. For a clear weather, the solar radiation can be solved rather accurately with Zillman's coefficient (Eq. 4.12), but there are many uncertainties in the cloud correction, due to the thickness and type of cloud cover. The solar elevation angle can solved with spherical trigonometry:

$$\sin\theta = \sin\phi\sin\delta + \cos\phi\cos\delta\cos\tau, \text{ and} \tag{4.14}$$

$$\sin\delta = \sin\epsilon_c \sin\left(2\pi \cdot \frac{j-80}{365}\right), \tag{4.15}$$

where ϕ is the latitude, δ is the declination, τ is the hour angle in solar time (at midday $\tau = 0$, at 6:00 $\tau = -90°$, 18:00 $\tau = 90°$ and so forth...), $\epsilon_c = 23°27'$ is the inclination angle of the ecliptic and j is the number of day from the beginning of the year. Midday of the solar day in UTC[3] is

$$t_{UTC} = -\frac{\Lambda}{15°}\mathrm{h} + \Delta t, \tag{4.16}$$

where Λ is the longitude (positive towards east), and Δt is a time correction due to the fact that Earth's orbit around the Sun is elliptical. h refers to the unit; hours. The time correction has an absolute value of $|\Delta t| < 0.25$ h, and there is no simple equation for it. Values can be found tabulated, for example in the almanac published by the University of Helsinki almanac office.

Example 4.3
The geographic coordinates of University of Helsinki Kumpula campus are 60° 12′N 24° 58′E. During the summer solstice, the solar elevation angle is $\phi + \epsilon_c = 53°\ 15'$. Solar time midday is 10:20 UTC + Δt. Daylight saving time in Finland is UTC+3 hours, so in Helsinki the solar midday is at 13:20 +Δt. According to the almanac, for summer solstice the time correction is $\Delta t = 2$ min.

From the incoming solar radiation Q_s, a part is absorbed by the surface and a part is reflected and scattered back. The returning fraction is αQ_s, where α $(0 \leq \alpha \leq 1)$ is the *albedo*[4]. If the surface is transparent, part of the net radiation is absorbed below the surface. The fraction that is absorbed by the surface [5] is $\gamma(1-\alpha)Q_s$, where $\gamma(0 \leq \gamma \leq 1)$ is surface absorption coefficient. Thus, the absorbed fraction of solar radiation is further divided up to two parts: radiation absorbed by the surface (Eq. 4.17) and radiation penetrating through the surface (Eq. 4.18).

$$Q_{s0} = \gamma(1-\alpha)Q_s \text{ and} \tag{4.17}$$

[3] *Coordinated Universal Time*, also known by its older name, GMT (*Greenwich Mean Time)*. It is the solar time at longitude 0°.

[4] 'Albedo' has its etymology in Latin, meaning 'whiteness', cf. albino

[5] Actually a thin near-surface layer

$$Q_{s0} = (1-\gamma)(1-\alpha)Q_s \tag{4.18}$$

On land, $\gamma \approx 1$ and on water $\gamma \approx 1/2$ (visible light penetrates into water, and about half of solar radiation consists of this). Snow and ice surfaces behave similarly to the surface of water. Values of albedo vary widely (Table 4.1).

Table 4.1: Typical values of albedo for various surface types.

Surface	**Albedo**
Water	0.05 – 0.10
Dense forest	0.10 – 0.15
Crop field	0.10 – 0.20
Dense grass	0.25
Wet ice	0.20 – 0.30
Dry ice	0.5
Wet snow	0.50 – 0.70
Dry snow	0.80 – 0.90
Global average	0.34

Solar radiation incident on a surface consists of direct and scattered radiation. On a clear summer's day most of the radiation is direct, and on an overcast day scattered light dominates. Reflection from the surface depends on the angle of incidence, so that radiation coming in at a small incidence is more likely to get reflected than light from a large angle. For this reason, the albedos for scattered and direct radiation are different. The mean albedo of the surface of water is 0.07. Snow is a very efficient scatterer. The albedo of snow decreases, as the liquid water content at the surface layer increases. Especially in springtime the albedo of snow gets progressively smaller during melting, absorbing sunlight more and more, and accelerating the melting.

4.1.4 Terrestrial radiation

Thermal radiation of Earth's surface and atmosphere can be approximated by using the gray body model, $Q_L = \epsilon\sigma T^4$, which is based on the Stefan-Boltzmann law of black body radiation (Eq. 4.3). At the wavelength of terrestrial thermal radiation, the emission coefficient of the surface ϵ_0 is very close to 1 ($\epsilon_0 = 0.96 - 0.98$) and varies only a little.

The radiation emitted by the atmosphere is a much more complex a phenomenon. It is emitted by different layers, with differing temperatures, and the emissivities can vary widely due to greenhouse gases and cloudiness. The gray body model cannot be, at least exactly, employed in this very heterogeneous situation, but the model is still often used based on the principle of analogy. Atmospheric temperature at a height of 2 m is used as a reference, and emissivity is replaced by an *effective emissivity coefficient*, ϵ_a. A widely used form of this is

$$\epsilon_a = \epsilon_a(N, e) = (a + b\sqrt{e}) \cdot (1 + cN^2), \tag{4.19}$$

where $a = 0.68$, $b = 0.036$ mbar$^{-1/2}$ and $c = 0.18$ are empirical coefficients. According to Kirchoff's law, emission and absorption are balanced and equal. Snow, ice and water emit thermal radiation well, but absorb light at much lower level.

Example 4.4
The temperature of both, surface and air, is 10 °C. The thermal radiation emitted by the surface is according to the gray body model ($\epsilon = 0.97$) 354 W m^{-2}. If the partial pressure of vapor is 10 mbar, cloudiness is 50 % and effective emission coefficient (Eq. 4.19) of the atmosphere is 0.830, then the long wave radiation emitted by the atmosphere is 302 W m^{-2}. Net long wave radiation is thus -52 W m^{-2}.

Usually, the net thermal radiation of the atmosphere is strongly negative, in the order of -50 W m^{-2}, because atmosphere is not as effective a radiator as the surface is. This net sum varies quite little. Net solar radiation is at the highest in summer and can reach up to 700 W m^{-2} in southern Finland (61°N). At its lowest at winter solstice it hovers just a bit above zero. Net radiation balance is positive from April to September, and negative from October to March. In the following table some key radiation components (in Finland) are presented (Laitinen, 1970):

	Southern Finland (61°N)		**Northern Finland (67°N)**	
	January	July	January	July
Solar radiation	8.7 W m^{-2}	230 W m^{-2}	3.4 W m^{-2}	223 W m^{-2}
Albedo	0.44	0.14	0.45	0.13
Radiation balance	-27 W m^{-2}	126 W m^{-2}	-34 W m^{-2}	124 W m^{-2}

Example 4.5
The radiation balance of a planet orbiting the Sun can be written as

$$\frac{1}{4}(1-\alpha)\left(\frac{AU}{R}\right)^2 Q_{sc} = \epsilon\sigma T^4,$$

where R is the distance of the planet from the Sun, and AU is the astronomical unit ($149.6 \cdot 10^6$ km, the mean distance between the Sun and the Earth). On the left side is the solar radiation, and on the right is the planet's thermal long wave radiation. The factor 1/4 is due to the rotation of the planet, which makes only part of the planet lit at any given time. From this equation we get the surface temperature of the planet

$$T = \sqrt[4]{\frac{(1-\alpha)}{4\epsilon\sigma}\left(\frac{AU}{R}\right)^2 Q_{sc}}\ .$$

For Earth, this gives a temperature of $T = 254$ K when $\alpha = 0.3$ and $\epsilon = 1$. For Mars ($R = 227.9 \cdot 10^6$ km) $T = 217$ K, when $\alpha = 0.15$ and $\epsilon = 1$, and for Venus ($R = 108.2 \cdot 10^6$ km) $T = 252$ K, when $\alpha = 0.65$ and $\epsilon = 1$. Observed values are 288 K for Earth, 210 K for Mars and 736 K for Venus. The differences between the theoretical and observed values are due to the greenhouse effect, which for Earth is 34 K, for Mars ~ 0 K and for Venus ~ 500 K.

4.1.5 Turbulent heat exchange

In the layer of air closest to the ground, called the *boundary layer*, wind causes turbulent mixing. This mixing transports heat, vapor, kinetic energy, and particles between the surface and the atmosphere. Turbulent eddies always bring new air next to the ground, and thus the surface is never insulated (Fig. 4.3). Over a wide, open field, transport is mostly vertical. If there is a temperature gradient between the air and the surface, heat is transported from hot to cold. Latent heat is transported away from the surface as water is evaporated, or the surface can receive latent heat, as vapor condenses on the surface. Ice sublimation and deposition take away and give latent heat to the surface respectively. These phase transition processes are linked to the humidity of air, which was covered in section 3.3.3.

Fluxes of sensible heat (Eq. 4.20) and latent heat (Eq. 4.21) can be approximated with bulk aerodynamic formulae:

$$Q_c = \rho_a c_a C_H (T_a - T_0) U_a \text{ and} \tag{4.20}$$

$$Q_e = \rho_a L_{E*} C_E (q_a - q_0) U_a, \tag{4.21}$$

where L_{E*} is the latent heat of evaporation or sublimation, $C_H \approx C_E \sim 1.5 \cdot 10^{-3}$ are the exchange coefficients of sensible and latent heat and U_a is the wind speed[6]. The density of air is dependent on the temperature, pressure and moisture content, and near the surface it is ± 10 % from 1.23 kg m^{-3} ($T = 15$ °C, $p = 1013.25$ mbar). Usually, relative humidity R_h and temperature T_a are acquired from meteorological observations, and then the specific humidity can be solved from its definition

$$q_a = R_h q_s(T_a). \tag{4.22}$$

The surface humidity is not recorded at typical weather stations. If the surface is snow or ice, relative humidity can be assumed to be 100 %, and thus $q_0 = q_s(T_0)$. If $q_a - q_0 > 0$, $Q_E > 0$ and condensation happens. The ratio between sensible and latent heat fluxes is called *the Bowen ratio*, which has stability in certain circumstances. If the Bowen ratio is known, the whole turbulent heat exchange can be solved, if one component is known.

[6] The standard measurement height of temperature and humidity is 2 m, and for wind speed it is 10 m.

Figure 4.3: *Exchange between the surface and the atmosphere is studied in boundary layer research. Pictured is the Lotus-station on Lake Kilpisjärvi. Photo: Matti Leppäranta.*

Example 4.6
The temperature difference between surface and air is 2 °C, the difference in humidity is 3 g kg^{-1} and wind speed is 5 m s^{-1}. With equations (4.20) and (4.21) one can calculate that the sensible heat flux is 18 W m^{-2} and that the latent heat flux is 78 W m^{-2}. Bowen ratio in this case is 0.26.

Turbulent exchange coefficients are dependent on the surface roughness and stratification of the atmosphere, which can be stable, neutral or unstable. In neutral stratification, air temperature decreases with altitude, following the adiabatic lapse rate. In a stable situation, the decrease is smaller, and turbulent transport is damped, and in an unstable situation the situation is reversed. In these cases, the turbulent exchange coefficients are estimated as a function of Richardson number, Ri

$$Ri = \frac{g}{\overline{\theta}} \cdot z \frac{\theta_a - \theta_0}{U_a^2}, \tag{4.23}$$

where θ_a and θ_0 are the potential temperatures of air and surface, $\overline{\theta}$ is the surface layer reference temperature (in Kelvin scale) and z is the measurement height. Richardson number describes the ratio of buoyant and flow shear mixing. When $Ri > 0$ ($Ri < 0$), the stratification is stable (unstable). In Fig. 4.4 the turbulent exchange coefficient is plotted against Richardson number. Compared to a neutral situation, the coefficient can be half of this value in a stable situation, and double in unstable cases.

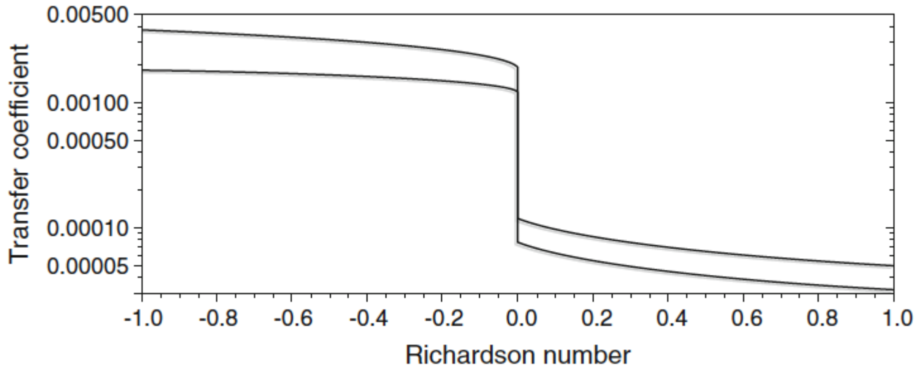


Figure 4.4: *Turbulent exchange coefficients as a function of Richardson number. The lower curve represents a relatively smooth surface, like snow. The upper curve is for a bit rougher surface, like grass.*

4.2 Precipitation

4.2.1 Formation of rain

Rain begins to form, when vapor is condensed or desublimated into droplets or small crystals of ice. If they keep on growing and clouds form, droplets and crystals are forced down by gravity when they are large enough. The amount of precipitation is given as the volume of water over a unit of surface area over a unit of time. The geographical distribution of precipitation follows wind, surface topography and land-sea relations.

There are three prerequisites for the production of rain: cooling of air, availability of condensation/desublimation nuclei, and the growth of droplets and/or crystals. When air is cooled, its relative humidity grows (Fig. 3.16) and can reach the saturation point. This point is at a lower temperature over snow or ice surface, in comparison to water surface, and for this reason plenty of snow/ice crystals are formed. Impurities in air function as condensation/desublimation nuclei. When droplets collide with each other, they grow bigger. Eventually, they will grow massive enough so that gravity will overtake the vertical wind, and precipitation is initiated.

Air going up is adiabatically cooled. Now, the relative humidity increases, which combined with condensation or desublimation can produce rain. This ascent of air can be caused by convection, mechanical forcing or by two air masses coming together. Based on their method of formation, precipitation are divided into three groups:

1. *Convective rain.* When solar radiation heats a spot of terrain more than it heats its surroundings, the air above this spot will become warmer and begins a convective rise. Pressure of this ascending air drops, and consequently the air cools and after some specific height, droplets are formed due to condensation. This kind of unstable situation is possible, if prior to the heating of the surface the temperature drops by more than 1 °C per 100 m of altitude. Convective rains are typical for the tropics. Additionally, when cold air blows over a warm sea, the heating by the surface can cause

convection. In Finland, convective rains are very local and are commonly associated with thunder in summertime. The amount of precipitation can be large.

2. *Orographic rain.* When a stream of air encounters an obstacle, for example a large mountain range, the air is forced to climb up. This climb is again associated with cooling of the air, and all the prerequisites for rain are fulfilled. Precipitation of this type is concentrated on the side of the obstacle where the climb happens. For example, heavy rains typical for the climate of coastal Norway are due to the orographic effects caused by the Scandinavian mountains. In Finland this is felt in the fact, that western winds are drier than they would be without the mountains and associated orographic rains. It has been observed, that in Finland even small hills have a measurable effect on the local precipitation patterns.

3. *Frontal rain.* The third mechanism for rain is associated with the frontal activity in low pressure systems. In the area of the front, cold and more dense air is pushed under a less dense, warm air mass. The lighter air is then brought up, and thus cooled. The warm air mass can also be the one on the move, but the end result is the same: rain. Precipitation events caused by frontal activity are large in size, and their duration is longer than those of convective rain, often 6 – 12 hours. Most rains in Finland are due to frontal activity.

Annual total of precipitation ranges from under 100 mm in deserts to over 3 000 mm at the equator. In Finland it is 500 – 650 mm. Areas of great precipitation are also found in the monsoon areas, and in the western coasts of continents in the mid-latitudes. Very dry areas are found in the subtropics, where some of biggest deserts lie, like Sahara and Kalahari. Very high latitudes are very dry as well, due to the fact that cold air cannot hold much vapor at all in itself. So, in the central plains of Antarctica, yearly precipitation is under 100 mm, which is in the same range with Sahara.

4.2.2 Measuring precipitation

Precipitation is measured directly, and it is not much affected by the surface conditions. This is why in geohydrological and surface water research rain is considered as a completely external factor. Classical instruments used in measuring precipitation are presented in Fig. 4.7. A regular rain gauge is the most important of them. Rain accumulates in a vessel, which is emptied every day, usually at 6:00 UTC[7]. The opening of the gauge has a surface area of 100 cm^2, surrounded by a shield, to lessen the errors caused by wind. In addition to knowing the amount of precipitation, it is also often necessary to differentiate between liquid and solid phase. The Finnish rain gauge network was renovated between 1981 and 1982, when the older Wild-type wind shield was replaced with the more effective Tretyakov-type shield. A recording rain gauge is able to measure precipitation continuously with the help of a float. The float chamber is emptied with a siphon when necessary. The Finnish Meteorological Institute operates the Finnish rain gauge network, which is made up of about 800 gauges.

[7] Most meteorological observations are done every three hours in UTC time (00 UTC, 03 UTC etc.), in accordance to international conventions. Rain gauges are checked once or twice a day.

Measuring precipitation does not suffer from the problems that indirect measurements suffer from. This does not mean that measuring would be without its own sources of error: evaporation, condensation and wind. Due to the fact that a rain gauge is emptied twice a day, water can spend hours in it. Even if there is a shield against evaporation, it still happens. Evaporation error is estimated to be c. 1 mm per month in summer. Another error source is the wetting of the vessel: not all of the collected water goes to bottom of the vessel, but some is retained on the walls of the vessel. In Finland, this error is estimated to be 0.2 mm per 24 h of rain. During strong winds droplets can fly past the gauge. To mitigate this, the gauges have wind shields, and they are placed in a position sheltered from wind, sometimes even into the ground, so that only the top of the vessel is above the ground surface. Wind error is estimated to be about 5 %, but in the case of snowfall, the error can reach 50 – 75 %. All of the aforementioned errors make the measured result smaller than the actual precipitation. The Tretyakov-type rain gauge has an error of 6 % in the summer, and 30 – 40 % in winter (Mustonen, 1986). The yearly error is estimated to be 20 %.

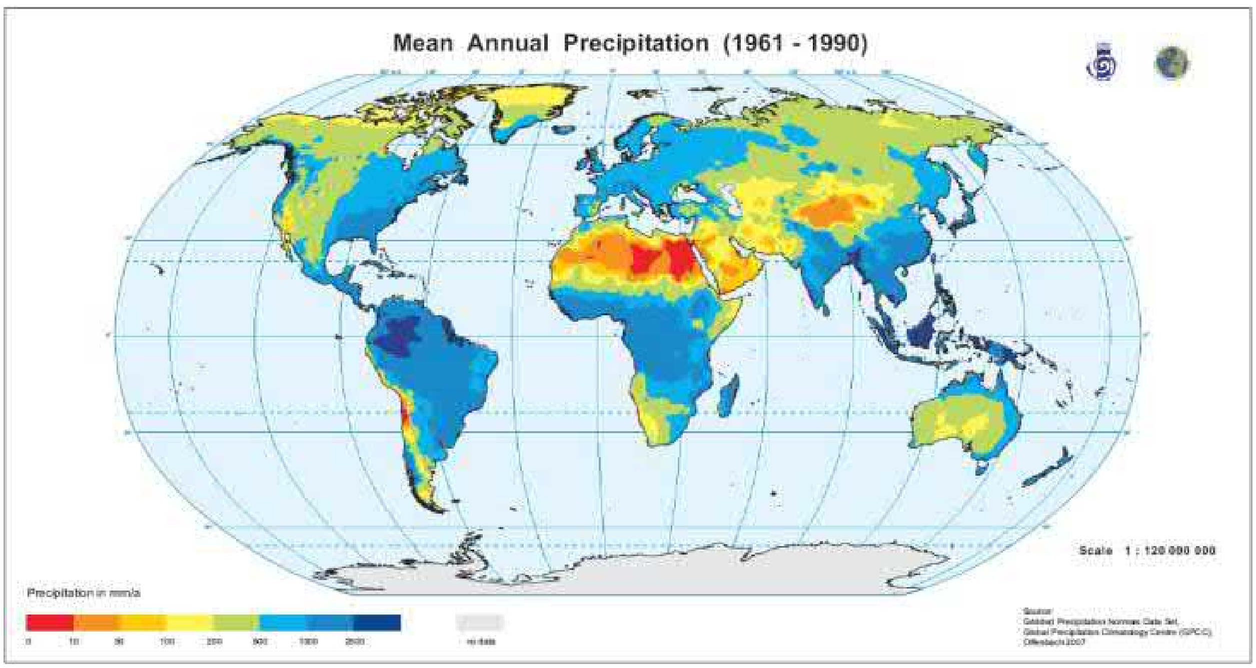


Figure 4.5: *Mean annual precipitation in millimeters (1961 – 1990). Source: Global Precipitation Climatology Centre / Deutcher Wetterdienst.*

Precipitation can also be measured with a weather radar. This method is based on the fact that backscattering of the radar signal is a function of the droplet concentration of the target and therefore an indicator of the intensity of the rain. The benefit of this method is that a single radar can map distribution and movement of rain on a very large area (radius of ∼75 km), and that this result is immediately usable, for example to make runoff forecasts. Weather radar is an excellent tool for meteorology, but due to some intrinsic errors in the method, the network of rain gauges still forms the backbone of the quantitative mapping of precipitation.

Interseption, which is the fraction of precipitation that is trapped on the surfaces of

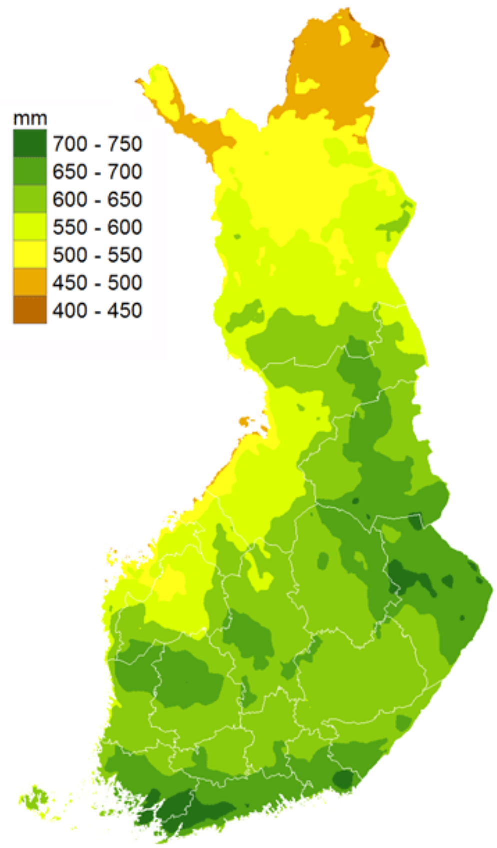


Figure 4.6: *Mean annual precipitation in Finland (1981 – 2010). Source: Finnish Meteorological Institute.*

vegetation, evaporates back into the atmosphere without reaching the ground (Fig. 4.8). It is dependent on the quality of the vegetation. Foliage has a certain maximum capacity to hold water, which depends on the total surface area of the foliage. For this reason, the fraction of rain intercepted decreases rapidly after the onset of rain. Due to the difficulties in measuring foliage surface area, interception is usually estimated just by tree species and total tree volume. According to research, the fraction of interception varies between 0.1 – 0.5, depending on the type of forest and precipitation (Table 4.2).

Regional data on precipitation is often necessary for hydrological research. For this reason, deriving the regional distribution of precipitation from measurements made on several surrounding locations is important. *Kriging-interpolation* is commonly used in spatial statistics. Here, the spatial distribution of a given quantity is calculated as a weighted mean of observations. The weights are determined by the autocorrelation structure of the quantity, and so they are dependent on the distances from the points of observation.

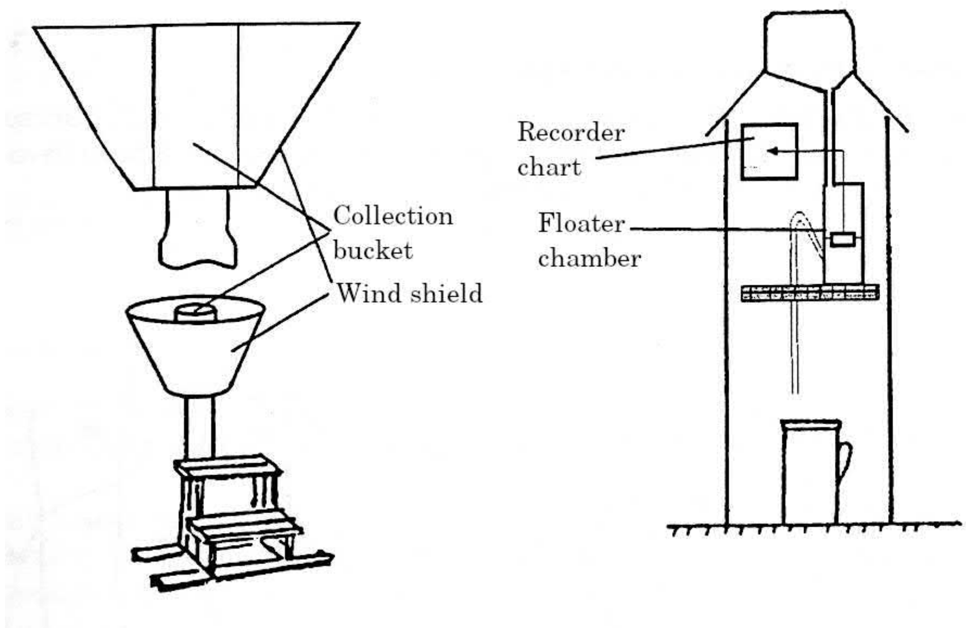


Figure 4.7: *Instruments used to measure precipitation. On the left, a regular rain gauge with a collection bucket and a Wild type wind shield. On the right, a recording rain gauge is displayed.*

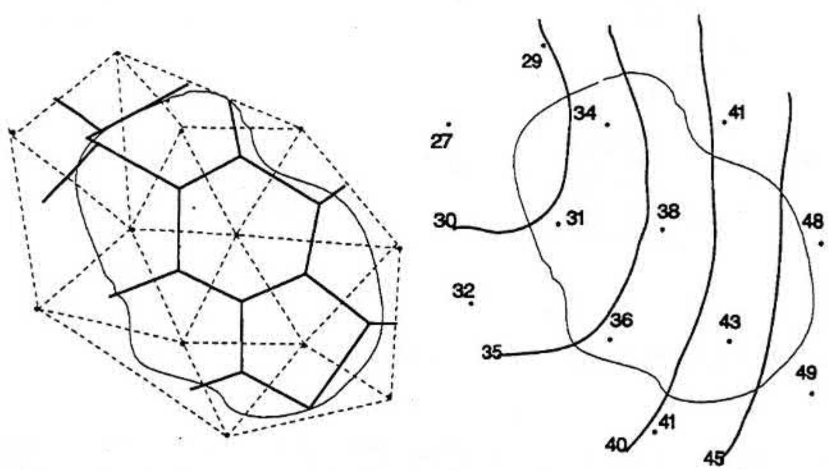


Figure 4.9: *Calculating regional values for precipitation by using Thiessen polygons (left) and isohyetal method (right).*

Table 4.2: Intercepted fraction of rain on different tree species (Päivänen, 1966).

	Type of precipitation	
Type of forest	Light rain	Heavy rain
Birch	0.45	0.1
Pine	0.45	0.2
Spruce	0.5	0.3

Figure 4.8: *Interception on branches. Photo: Matti Leppäranta.*

There are many methods to derive regional values for precipitation:

- *Arithmetic mean.* Regional value is calculated as a mean of surrounding point values.
- *Thiessen polygons.* Polygons are drawn around the measurement points. The sides of these polygons are normals passing through the middle points of lines connecting two nearby points (Fig. 4.9). A regional value is acquired when a weighted average is calculated, where the weighs are the relative sizes of these polygons.
- *Isohyetal method.* Based on the observations, isohyets (that is, isolines of precipitation) are drawn first. After this, the surface areas between the contour lines are measured. Regional value of precipitation is acquired as a weighted mean, where the weights are the relative areas of the areas between the contours. Precipitation in the areas between the contour lines is the mean of the values at the contour lines.

These methods are based on the idea behind the Kriging-interpolation, but in these cases the weights are chosen without any knowledge of the autocorrelation structure. Arithmetic mean is suitable, when the observation network is sufficiently dense and regular. Often, the value for a water system and its subsystems (or catchment areas) is calculated at the same time. Then, the precipitation at these smaller catchments can be estimated directly from the isohyets, based on their centers, and the precipitation of the larger area can be estimated as a weighted mean of these smaller subsystems.

Ouryvaev & Toebes (1970) estimated the regional precipitation with the help of autocorrelation. The accuracy of this estimator depends on the size of the area in question, the

		Max error [%]		
		Period length [days]		
Area [km^2]	**Number of stations**	**1**	**10**	**30**
1 000	10	22	9	5
1 000	100	4	2	1
100	1	70	28	16
100	10	14	6	3

regional variation in precipitation, the density of the observation network and the frequency of observations. The values given below describe the relative error in the regional estimate of frontal rain, on a 25 % limit of error.

These error estimates do not take into account any systematic instrumentation errors. During convective rain, the errors might double, or even triple. Estimating regional precipitation is especially difficult in mountainous regions, where local variations can be large due to topography. Then, in the calculation of regional precipitation values one must take into account not only the local changes in elevation, but also the wind direction in single precipitation events. Annual precipitation can also be estimated as a residual of mean annual runoff and evaporation.

4.3 Evaporation

4.3.1 Physics of evaporation

In evaporation, liquid water is turned into vapor. The dimension of evaporation is LT^{-1}, and it usually has a unit of millimeters per day, month or year, similar to precipitation. Evaporation can take place from an unobstructed surfaces of soil, water or vegetation. Evaporation happening as a result of plant functions, where water is transported through the root-stem-leaf system, is called *transpiration*. Together, they form *evapotranspiration*. Because in hydrology it is usually difficult to separate these two, the term evapotranspiration is used when speaking of regional evaporation. Vapor formation on a surface of ice is called *sublimation*.

The opposite of evaporation is condensation, which can be observed especially on late summer evenings as the formation of dew. The crystallization of vapor is called deposition or desublimation, which can be observed in winter as frost. Later, when speaking of evaporation, sublimation is included in it, unless otherwise stated.

Over a surface where evaporation happens, there are plenty of free water molecules moving around. Some of them end up back onto the surface, but the surface is constantly giving up more to the air. If an equilibrium exists, the number of molecules returning to the surface equals the number of leaving molecules, and the air is saturated with vapor. To be exact, besides air temperature evaporation is also dependent on the curvature of the surface and impurities present in the water.

For evaporation to be possible, humidity must be larger at the surface than above. To increase the partial pressure of water vapor, the velocity of water molecules in the vapor must be increased, i.e., energy has to be added to the surface. It is also possible, that the surface itself is heating the water, in which case evaporation brings down the surface temperature (Fig. 4.10). Difference in vapor pressure between the surface and the air above is not enough to sustain evaporation: turbulent eddies are required to transport the vapor further up in the atmosphere. Evaporation and sublimation require large amounts of energy, which significantly limits their occurrence. By definition, the consumption of energy in evaporation is

$$Q_e = -\rho L_{E*} E, \tag{4.24}$$

where E is the evaporation, $L_{E*} = L_E$ is the latent heat of evaporation or $L_{E*} = L_E + L_f$ sublimation, where L_f is the latent heat of freezing. From equation (4.24) it can be seen, that to evaporate (sublimate) 5 mm of water over the course of one day, 145 W m^{-2} (150 W m^{-2}) heating is required, which is quite a lot.

Figure 4.10: *The cold feeling on your skin that can be felt after getting up from water is due to the evaporation happening from your skin, and the associated flux of heat from skin into latent heat. Photo: Matti Leppäranta.*

The distribution of evaporation depends on one hand on the available energy, and on the other hand on the available water. For this reason, evaporation is very small in deserts but very large in warm and wet equatorial areas and in the ocean, especially over warm ocean currents (Fig. 4.11). Close to the polar areas, evaporation decreases, because of the low saturation vapor pressure of cold air. Estimating evaporation is difficult, because it is not visually observable, unlike precipitation.

The regional evaporation in Southern Finland is estimated at 400 – 500 mm per year, depending on the percentage of lakes in the area. Over areas with plenty of lakes, evaporation is 500 – 600 mm per year, and the mean daily evaporation during summer is 3 mm. In Northern Finland evaporation is less, 150 – 200 mm per year.

Physically, the difference between evaporation and transpiration is small. Both require about the same amount of energy, and the mechanism of transport from the surface is similar. However, plants have the ability to control transpiration to some degree. Their water usage is linked to photosynthesis and respiration, ergo, the formation of organic matter. Transport of CO_2 from the atmosphere to the plant happens via small droplets of water in stomata (small openings on the leaves). Because evaporation dries up these droplets, more water is needed constantly. This is part of the flow of water through the root-stem-leaf system as transpiration, which also functions as a cooling system, similar to sweating. Some parts of plants also require water for their structural integrity. During nights and cloudy weather, organic matter is not produced, and the need for water is smaller. Many plants can ration and control their water usage by stomata on the undersides of their leaves. This has an effect on the diurnal cycle of transpiration. Second important feature of transpiration is that plants take the water they need through their roots from a wide area underground. Drying up of the surface does not then prevent plants from using water, but it does stop evaporation almost completely.

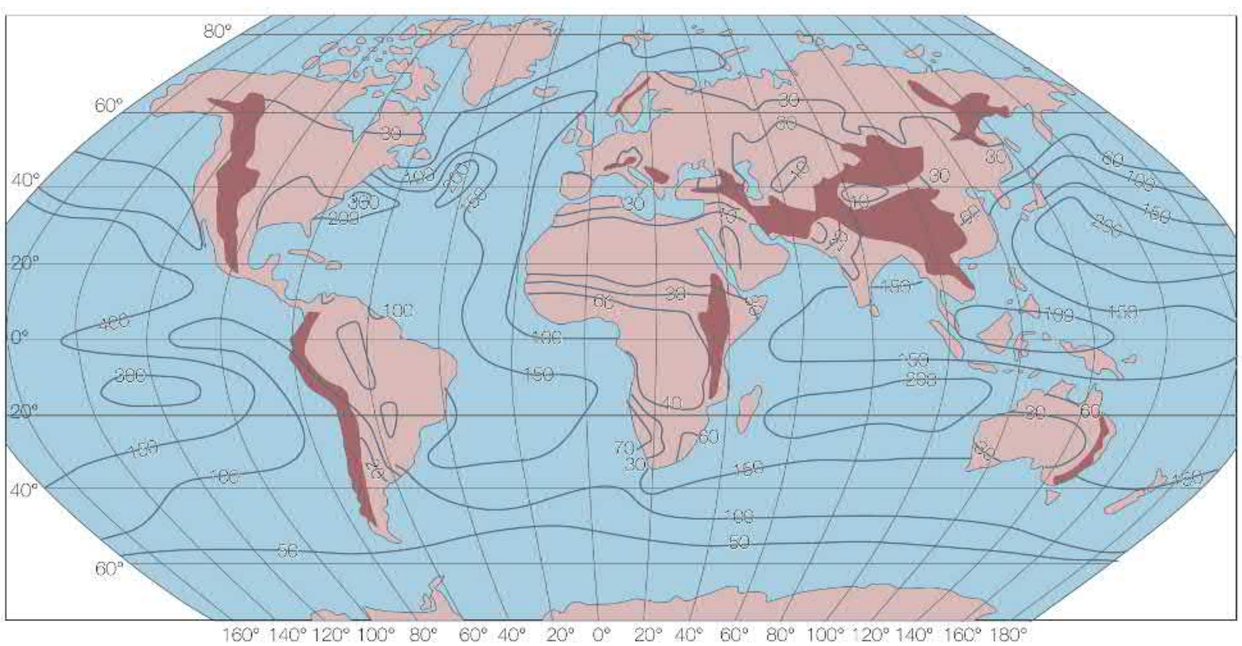


Figure 4.11: *Global distribution of evaporation (centimeters per year). Largest values are attained on tropical seas, 300 cm or 3 000 mm. Source: Budyko et al. (1962).*

Upon moving through soil and plants into the atmosphere the energy of a molecule of water changes. Changes in energy can be seen as changes in potential energy, $\delta E_p = \rho g \cdot \delta h$, where δh is the change in the height of the water column equal to the change in potential energy (table 4.3). Osmosis of the roots, and the capillary action of stomata play a significant role in transpiration. From the values in the table, one can notice that the energy required for evaporation is significantly more than what is needed for any other process listed.

Table 4.3: Change in potential energy per unit of mass ($\frac{\delta E_p}{\rho}$) and a corresponding change the a water column height (δh) as water moves through a plant. The sign describes whether the change increases or decreases energy. Source: Eagleson (1970).

Event	**Sign**	$\frac{\delta \mathbf{E_p}}{\rho}$ **[kJ kg^{-1}]**	δ**h [mm]**
Dislodging from the soil	-	0 – 1.5	0 – 150
Move into roots due to osmosis	+	0 – 1.5	0 – 150
Move into leaves	-	0 – 0.15	0 – 15
Move into stomata	+	1 – 100	100 – 10 000
Evaporation	+	2 500	N/A

4.3.2 Potential evaporation and true evaporation

Potential evaporation corresponds to true evaporation in a case where there is no shortage of water to evaporate. For this quantity, many empirical formulae have been developed. They do not take the surface and root layer moisture content into account. There are weaknesses in the definition of potential evaporation itself as well. Evaporation is affected by several factors the models do not take into account, like the properties of the surface and the vegetation type, thickness, height and its phase of growth. According to another definition, potential evaporation is evaporation from a thick field of grass, cut short and not suffering from any lack of water. This definition is more unambiguous than the first one, since the properties of the ground are defined better. On a lake and other water surfaces, evaporation always equals potential evaporation.

The following concepts are critical in understanding evaporation from soil:

- *Field capacity* is the greatest possible amount of water bound to the soil.
- *Wilting point* is the soil water content, when plants can just barely extract water.
- *Available water* is the fraction of water usable to plants = field capacity - wilting point.

Depending on the soil type and quality, field capacity is reached within a few hours or days from the last rain. When the water content drops below the wilting capacity, adhesive forces between soil particles and water become too strong for plants to overcome, and they can no longer extract water from the soil.

The connection between potential (E_p) and true (E) evaporation is dependent on the soil water content m

$$E = f\left(\frac{m}{m_*}\right)E_p \tag{4.25}$$

where m_* is the available water. Examples of the function f are presented in Fig. 4.12. This function has many forms for different climate zones and soil types, and their course is mainly defined by precipitation. In a moist climate, $f > 0.8$, when $\frac{m}{m_*} > 0.2$, and in dry zones $f \sim \frac{m}{m_*}$. Limits are $f \to 0$, when $\frac{m}{m_*} \to 0$, and $f \to 1$ when $\frac{m}{m_*} \to 1$. Curve 5 in Fig. 4.12 depicts a dry zone, where due to low precipitation there is no evaporation from interception and no evaporation from surface.

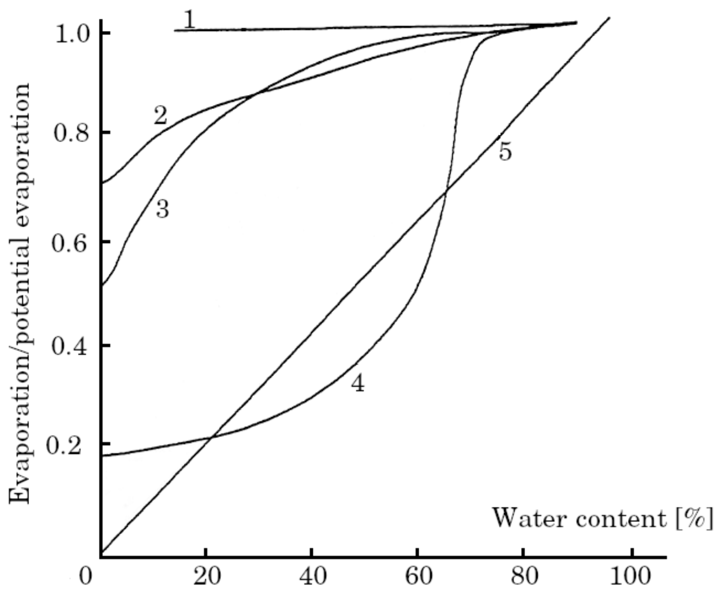


Figure 4.12: *Ratio of evaporation over potential evaporation as a function of soil water content (% of field capacity). Curves 1 – 3 represent moist zones and 4 – 5 drier zones.*

Measuring evaporation directly is very challenging. Nevertheless, evaporation is an integral part of so many hydrological and meteorological phenomena that often many independent and indirect methods can be used simultaneously to achieve reliable estimators. Furthermore, it has been shown that on timescales longer than a day, very simple computational methods can be successfully implemented.

Evaporation can be measured by an evaporation pan or a lysimeter (Fig. 4.13). *Evaporation pan* is a vessel filled with water. It can be on land or installed onto a floating platform. *Lysimeter* is a container that is filled with as natural a soil as possible, and it is topped off with the vegetation that is under investigation. Evaporation is determined from the water balance of the instrument. There are lysimeters of many sizes, ranging from 500 cm^2 to several tens of square meters.

The water content of smaller lysimeters is measured by weighing. In larger ones, it is done by measuring the soil moisture content. In Lahti, Finland, there is the soil research center *Soilia*, which offers various realistic environments for soil research. This center is owned by the Lahti Area Development Center (LADEC Oy), which operates it in cooperation

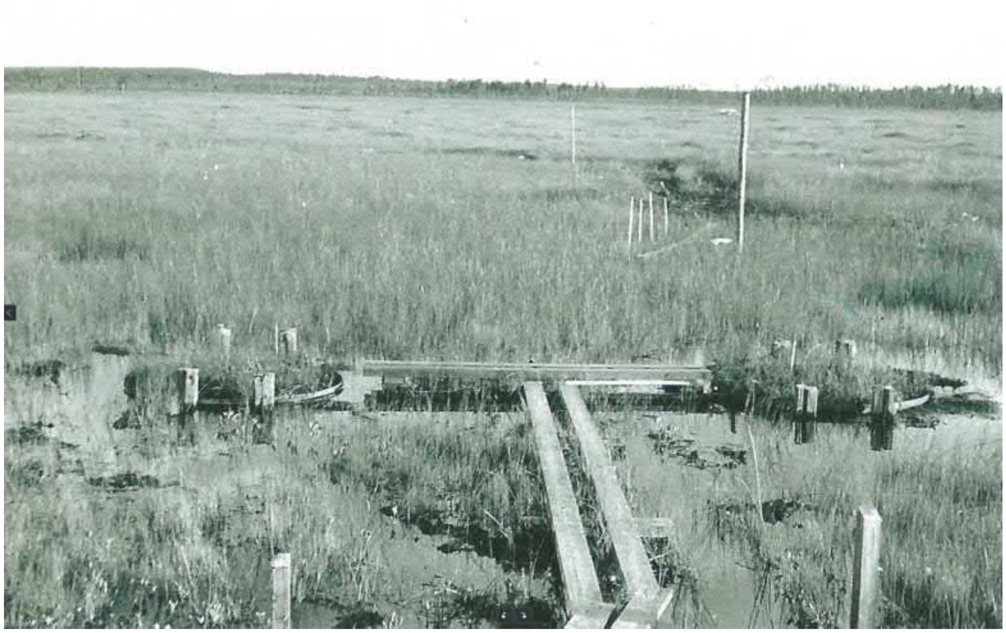


Figure 4.13: *Measuring evaporation over Möksy fen (South Ostrobothnia, Finland) using a lysimeter. Photo: Courtesy of Juhani Virta.*

with the University of Helsinki Faculty of Biological and Environmental Sciences. Direct measurements of evaporation require great care in handling of the pans, because the artificial filling of the container changes the vertical motion and temperature profile of the water in it. Even large systematic errors are possible, due to large local variations present in soil, like the oasis effect.

4.3.3 Determining evaporation by computational methods

Water balance method gives evaporation of an area as the residual of the water balance equation

$$E = P - R - \frac{dV}{dt} \tag{4.26}$$

where R is the runoff. The storage of ground water and soil water is important but hard to measure for the whole water system. For this reason the usage of the water balance method is limited to long term averages, few years or more, of evaporation. In that case the change in storages can be assumed to be small. A similar equation can be applied to large volumes of air. Instead of runoff the equation has horizontal transport of vapor. This way, precipitation transports water away from the control volume and evaporation increases the water content. The storage of vapor manifests as increase in humidity and cloudiness. This method has been applied for Southern Finland and the Baltic Sea (Palmén & Söderman, 1966).

Evaporation needs remarkable amount of heat (Eq. 4.24) that offers a means for estimation. In the *heat balance method* evaporation is defined as the residual of the heat balance

Figure 4.14: *Soilia lysimeter field in Lahti. The foreground of the left image is covered by active lysimeters. The photo on the right depicts underground lysimeter containers under the field. Photos: Leena Leppäranta.*

(Eq. 4.1). The term for the heat contained in precipitation can be neglected, since no evaporation takes place during rain. We can write

$$E = \frac{1}{\rho L_{E*}}\left(Q_s - Q_r + Q_{La} - Q_{L0} + Q_c - k\frac{\partial T}{\partial z}\bigg|_{z=0^-}\right). \tag{4.27}$$

Data for all the terms on the right side of this equation are not always available. The biggest problem is the lack of surface temperature data. If this is the case, net terrestrial radiation is estimated with air temperature and for turbulent exchange the Bowen ratio is used. Then, we get

$$E = \frac{1}{\rho L_{E*}(1+B)}\left[(1-\alpha)Q_s + \epsilon_0\sigma(\epsilon_a - 1)T_a^4 - k\frac{\partial T}{\partial z}\bigg|_{z=0^-}\right]. \tag{4.28}$$

The bulk aerodynamic method employs the theory of turbulent boundary layer. Vapor is transported in turbulent eddies, and the net transport is from moist to dry air. Using the bulk aerodynamic formula of latent heat exchange (Eq. (4.4)) and the definition of energy needed for evaporation (Eq. 4.24) we get

$$E = -\frac{\rho_a}{\rho}C_E(q_a - q_0)U_a. \tag{4.29}$$

This method gives a good result, if the atmospheric conditions near the surface are known. When evaporation or sublimation is happening ($E > 0$), the evaporated energy is negative ($Q_e < 0$). If $E < 0$ condensation or desublimation take place. Järvinen & Huttula (1982) have used the aerodynamic method for determining evaporation over a lake.

4.3.4 The Penman method

The biggest problem in general in estimating evaporation by computational means is the lack of surface humidity data. Weather stations measure humidity at a height of 2 m, and evaporation is determined by wind speed and the difference in humidity between the surface and the air just above. For the computation of potential evaporation, it is enough to know the surface temperature, since then the air is assumed to be saturated with vapor. With the help of the Penman formula, one can solve potential evaporation from the data available from regular weather stations. The idea behind this method is the elimination of surface temperature from the equation by employing two different formulas for evaporation. The aerodynamic formula (Eq. 4.29) is used as the basis for the vertical transport of vapor, and using the energy balance a correction is performed for the surface temperature.

The aerodynamic formula can be written as $E = f(U_a)[e_s(T_0) - e_a]$, where e_s is the vapor saturation pressure (Fig. 3.16). Heat conduction into the soil is usually left out of the heat balance equation (Eq. 4.28) as it usually is small. Two necessary auxiliary quantities are the psychrometric constant $\gamma = 0.66$ mbar °C^{-1} and the temperature sensitivity Δ of vapor saturation pressure, which are acquired from the Bowen ratio and saturation pressure:

$$B = \frac{Q_c}{Q_e} = \frac{\rho_a c_a C_H (T_a - T_0) U_a}{\rho_a L_{E*} C_E (q_a - q_0) U_a} = \frac{c_a}{L_{E*}} \frac{p}{0.622} \frac{(T_a - T_0)}{(e_a - e_0)} = \gamma \frac{T_a - T_0}{e_a - e_0} \tag{4.30}$$

$$\Delta = \frac{e_s(T_0) - e_s(T_a)}{T_0 - T_a}. \tag{4.31}$$

From the heat balance (Eq. 4.28) we now get the temperature correction, and the potential evaporation given by the Penman method is

$$\overline{E_p} = \frac{1}{\rho L_{E*}} \frac{Q_{R0} + \gamma \Delta^{-1} f(U_a)[e_s(T_a) - e_a]}{1 + \gamma \Delta^{-1}}. \tag{4.32}$$

The Penman formula has been repeatedly shown to produce upwards biased results, that is why a correction coefficient has been developed for it: $E_p = r \cdot \overline{E_p}$, $r \approx 0.7 - 0.9$. This method has also been further developed to solve transpiration (Vakkilainen, 1982).

Radiometric measurements of temperature have been advancing lately, and satellites can be used to measure the surface temperature. This reduces the need to use the Penman method, but when using data from regular weather stations, it is still a very viable method.

4.4 Snow

4.4.1 Snow climatology

In the Finnish climate, part of the annual precipitation falls down as snow, forming a temporary storage of water. During winter, the storages of surface, soil and groundwater get smaller. In spring, the water stored in snow is released, forming a strong peak in surface and groundwater runoff. This is evident, for example, in the decrease of water level in wells and lakes in winter and as the rapid raise during spring and early summer.

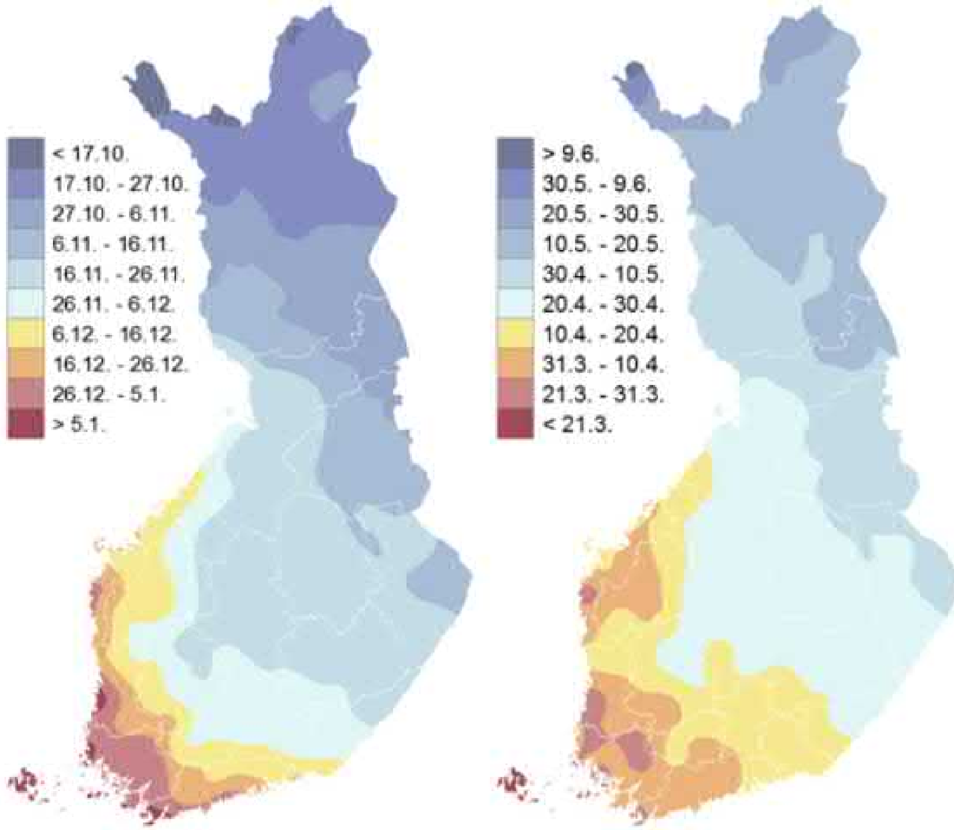


Figure 4.15: *Average date of the formation of snow cover and its melting in Finland during 1981 – 2010. Source: Finnish Meteorological Institute.*

On average, snow cover forms over Northern Finland in late October (Fig. 4.15). In mid-November, the snow line has reached Tornio – Nurmes line, and in early December it has reached Kokkola – Haapamäki – Lappeenranta line. By the end of the year, the whole of Finland is covered in snow, apart from possibly the south western corner and the archipelago. The final melting of snow begins in Southern Finland at the end of March, and in the tundra in May. Snowcover lasting for more than a year can only be found in small isolated spots, mostly in the municipality of Enontekiö (North West Lapland). If there is new snow added every year over old snow, glaciers are eventually formed, as snow is pressed into ice under its own weight.

Finland has three major types of snow environment: tundra, taiga and ephemeral zones. Northern Lapland belongs to the arctic tundra snow zone, most of the country is in the taiga snow zone, and the southwestern part is in the ephemeral zone, where snow accumulates and melts several times during the winter. The greatest mean maximum annual snow depth is found in Northern Lapland in March, in excess of 80 cm (Fig. 4.16), while in South West Finland the mean snow depth is around 10 cm.

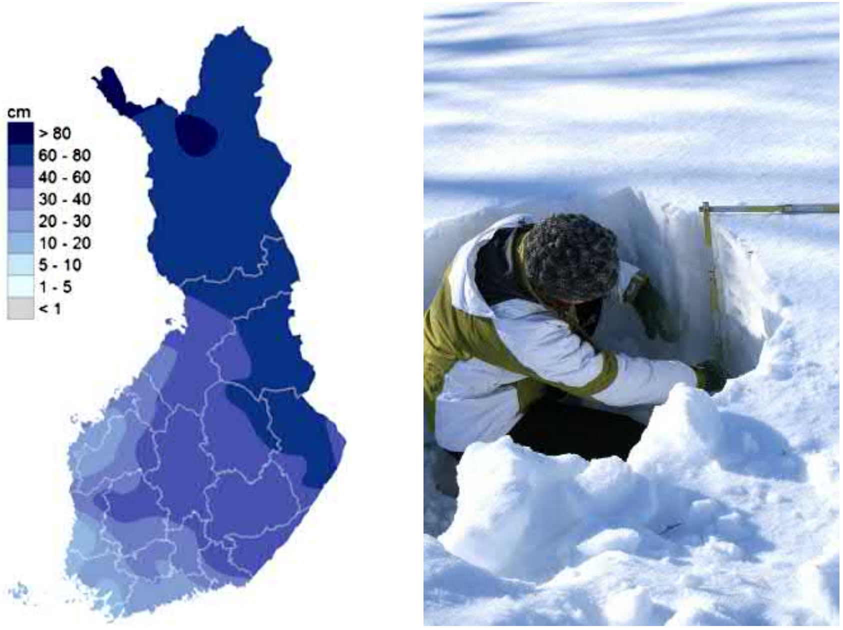


Figure 4.16: *Mean snow cover thickness on 15th of March, between 1981 – 2010. On the right, studies of the snow cover properties are done by digging a snow pit. Map: Finnish Meteorological Institute FMI. Photo: Matti Leppäranta.*

4.4.2 Properties of snow

Snow is a difficult material to handle and model. It consists of ice crystals, liquid water, gas pores, and impurities in particulate form. The well known thought of the Greek philosopher Heracleitos (535 – 475 B.C.) 'no one can step into the same stream twice' can be applied for snow as well, 'no one can step into the same snow twice'.

The most important physical properties of snow are its density, crystal size and shape, and the concentration of liquid water. Gas pores are plentiful, and their amount dictates the density of snow:

$$\rho_s = (1 - \nu_a)\rho_i, \tag{4.33}$$

where ν_a is the relative volume of gas pores, and ρ_i is the density of pure ice. Crystal size varies in the range 0.1 – 10 mm, and typically the biggest crystal of a snow sample is used to characterize the sample crystal size. On the surface, the crystal size varies a lot, but deeper down they are rounded to a more narrow distribution. The fraction of liquid water is defined as its relative volume to the total sample. During melting, the fraction of liquid water in snow increases, and when it is about 5 %, making snowballs becomes very easy. When air temperature is below 0 °C, the fraction of liquid water gets negligible.

Density, crystal size and liquid water tend to define other physical properties of snow as well, like its load-carrying capacity, heat conduction, albedo and light attenuation. Heat conduction of snow is mostly dependent on the density of snow: $k_s = \kappa\rho_s^2$, where κ is an empirical coefficient. When density of snow is around 250 kg m^{-3}, its heat conduction is approximately 0.15 W m^{-2} °C^{-1}, an order of magnitude less than for pure ice.

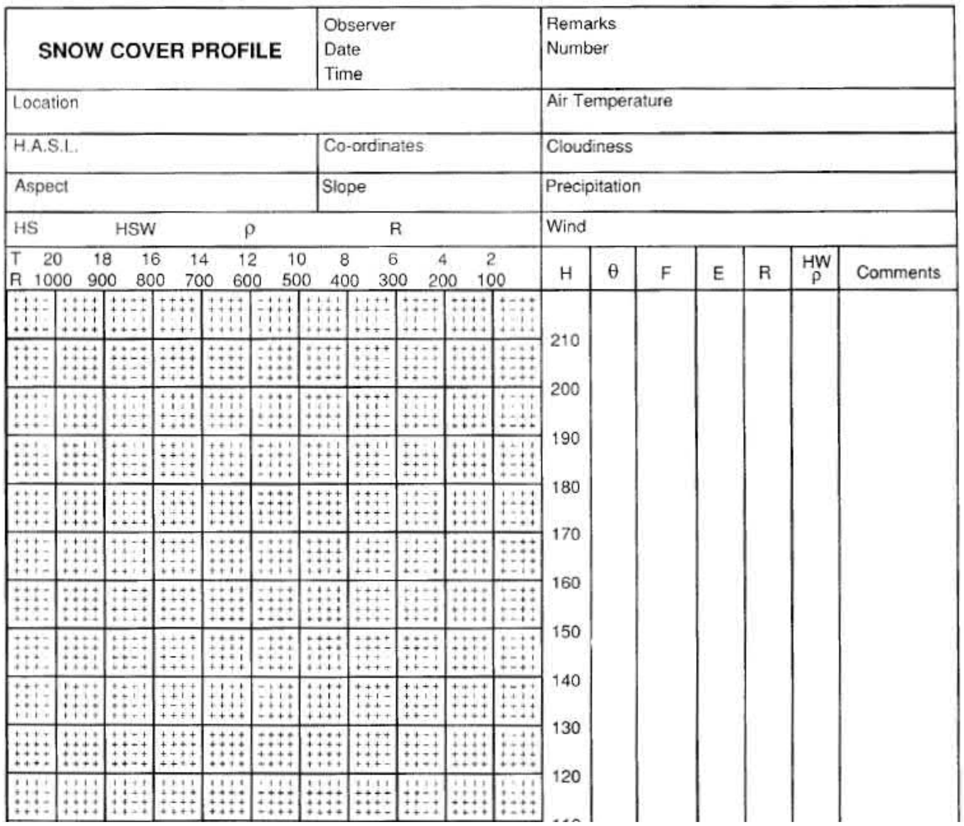

SNOW COVER PROFILE
Observer
Date
Time
Remarks
Number
Location
Air Temperature
H.A.S.L.
Co-ordinates
Cloudiness
Aspect
Slope
Precipitation
HS HSW ρ R
Wind
T 20 18 16 14 12 10 8 6 4 2
R 1000 900 800 700 600 500 400 300 200 100
H θ F E R HW/ρ Comments
210
200
190
180
170
160
150
140
130
120

Figure 4.17: *Standard snow cover profile form. Sections HS, HSW, P, and R are for snow thickness, water content, mean density and strength. The grid on the left is for profiles of temperature, strength or density, depending on the application. Columns on the right are meant for liquid water content (θ), crystal shape (F), crystal size (E), strength (R) water content/density (HW/ρ). Source: Fierz et al. (2009).*

Chemistry of snow cover is primarily investigated from melted snow samples. Impurities can significantly alter the physical properties of snow. For example, dirty snow is much more efficient absorber of solar radiation than clean snow. Snow also stores atmospheric deposition during the winter, to be released into the water cycle during the melting season.

Snow cover evolves during the winter, depending on the weather conditions (Fig. 4.17). This is called the metamorphosis of snow cover. In mechanical metamorphosis snow is pressed tight under its own weight, and packed by the wind. Water vapor is at saturation pressure inside of the gas pores. Diffusion and mass balance of water vapor significantly alter the structure of snow cover. Vapor is transported from convex surfaces to concave due to sublimation-deposition, with the help of that snow crystals become bound to each other. If there is a vertical temperature difference in the snow cover, vapor is transported from warmer layers to colder ones. In spring, the structure of snow changes rapidly due to the freezing-melting metamorphosis.

Snow density in Finland is typically 200 – 300 kg m^{-3} (Fig. 4.18). In these conditions, a 100 mm layer of snow will produce a 20 – 30 mm layer of water upon melting. Fresh snow can have a density of about 100 kg m^{-3}, while on an open tundra wind can pack snow into densities of up to 400 kg m^{-3}. The snow cover is at its thickest in March, and the water equivalent of snow is on average 100 – 200 mm (Fig. 4.18).

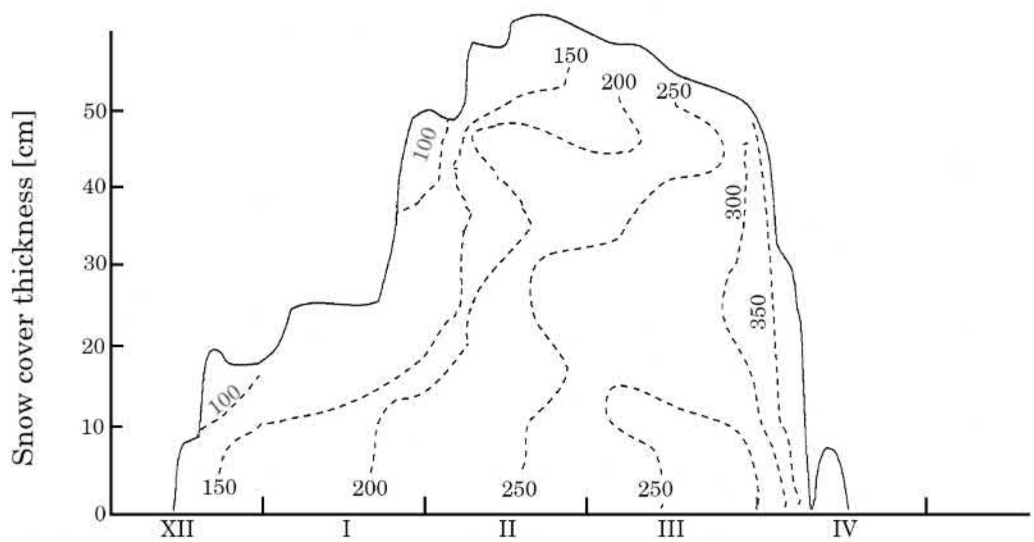


Figure 4.18: *Snow cover thickness (in cm) and density (in kg m^{-3}) from December 1968 to April 1969 in Southern Finland. Source: Lemmelä (1970), modified.*

4.4.3 Water equivalent of snow

In the hydrology of snow cover, an important quantity is its water equivalent, which indicates how much water the snow would produce upon melting without any losses due to evaporation or runoff. Water equivalent of snow cover is expressed as a height of an equivalent water column over a unit of area

$$h_w = \frac{\rho_s}{\rho} h_s, \tag{4.34}$$

where h_s is the snow cover thickness. The unit of water equivalent is usually millimeters. It is used in forecasting how much water can be expected to enter water systems during the melting season, which is helpful in predicting floods and runoff. The water equivalent can also be used to estimate the load on structures caused by snow. In the field, water equivalent can be measured with a snow scale (Fig. 4.19). It consists of a cylinder that is pressed into the snow cover and a scale, from which the value of water equivalent can directly be read from.

Snow lines form the backbone of the snow water equivalent measurement network. A snow line is a 4 km long track, along which ten density and 80 snow cover thickness measurements are performed. Results from several lines are combined to compute regional values for water equivalent. Water equivalent measurements are usually done twice a month in Finland.

Water equivalent can be measured continuously at snow stations with a snow pillow (Lemmelä, 1970). It consists of a rather large (10 m^2) flat rubber pillow, placed over ground prior to snowfall. The pillow is filled with a non-freezing liquid. Changes in the water equivalent can be continuously recorded as changes in the pressure of the fluid. Water equivalent can also be measured by remote sensing. A method based on the absorption of gamma rays emitted naturally by the ground has been in use for a long time. This absorption is a function of the water equivalent of the snow cover, which can be solved if the background gamma radiation has been measured during a snow free time. With this method, the water equivalent of a rather large area can measured quickly. This method has been used in the River Kemijoki water system during the period of maximum snow cover thickness (Kuittinen, 1988).

Example 4.5
If there is a snowfall of 50 mm, and the density of this snow is 200 kg m^{-3}, the water equivalent of the layer will be 10 mm. Then, the mass density of snow will be 10 kg m^{-2} and the snow will exert a pressure of $mg \approx 100$ Pa on a horizontal surface.

Figure 4.19: *Measuring water equivalent of snow with a cylinder and steelyard. If the thickness of the snow cover is also measured, mean density of the snow pack is acquired. Pictured are students on the course 'Geophysics of snow and ice' at Lammi Biological Station. Photo: Matti Leppäranta.*

Satellite imagery has been shown be a viable method for snow observations (Fig. 4.20). They are especially useful in determining the extent of snow cover, but for water equivalent only coarse, indirect results can be acquired. For this reason, satellite remote sensing has not replaced the in-situ field measurements of the water equivalent. Microwave instruments have proved to be most suitable for snow research. They give results of the surface, regardless of weather and time of day, and with them the whole Earth can be mapped every day. Radiometric observations on their own are not very accurate, because all the required information for the interpretation of the data is seldom available. To improve the results from remote sensing, they can be assimilated with in-situ field measurements.

A fraction of snowfall is intercepted by trees and other vegetation. The amount of snow on the ground is thus dependent on the type of forest cover. Accumulation of snow is also affected by the surface topography, wind and the properties of the snowfall (Kuusisto, 1984). Regional value for the water equivalent of snow is calculated similarly to precipitation. These calculations have to take into account the fact that snow accumulation is different on different environments.

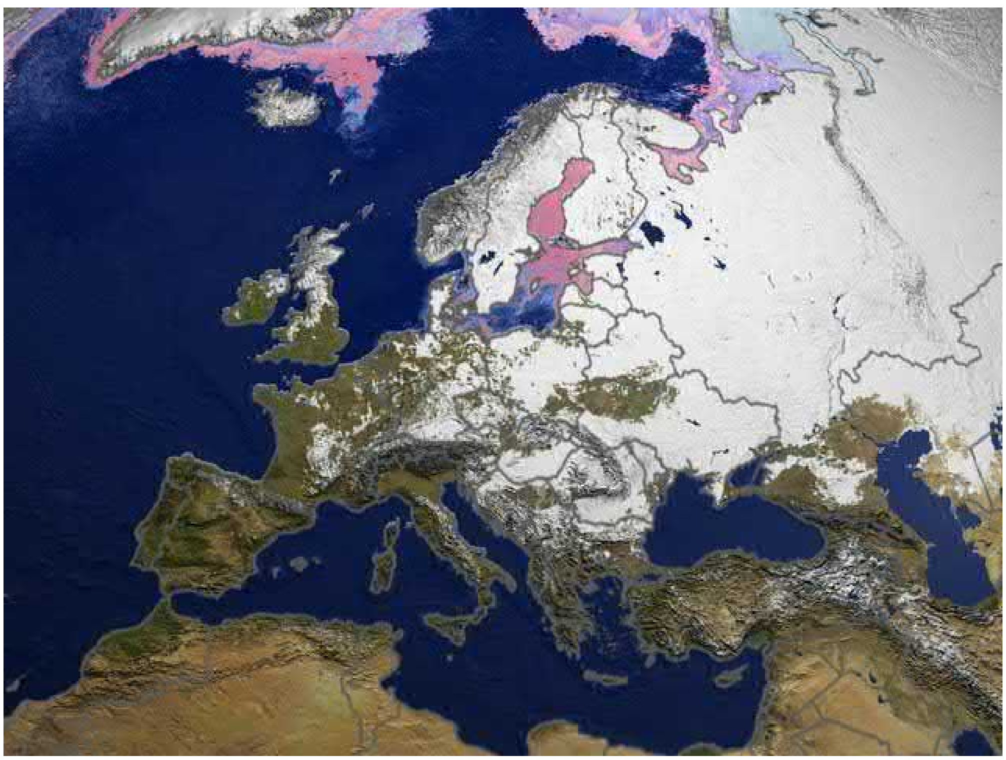


Figure 4.20: *Snow cover in Europe on 7.2.2003. Open sea is deep blue, and different shades on ice covered sea indicate surface temperature.*

Highest values of water equivalent are usually measured on forest clearings and sparse forests, where the effect of wind is minimized. Reaching the highest value of water equivalent of snow marks a certain turning point in winter. According to Kuusisto (1984), on stationary snow stations this point is reached on average in Finland on 20th of March on the Pori – Kotka line, 30th of March on Raahe – Pieksämäki – Kitee line, 10th of April on Kolari-Kuusamo line and finally the maximum reaches North West Lapland (Enontekiö) on 20th of April.

Snow disappears by two different ways: melting and sublimation. These are controlled by the surface heat balance, humidity and wind. Melting always dominates in Finland, but in some springs sublimation can form a significant fraction of the removal of snow. Interaction between snow and air is dependent on the terrain type, of which the following table presents a few examples:

Terrain type	**Characteristics of interaction between snow and atmosphere**
Fully forested	Thermal radiation from trees due to difference in temperature causes condensation
Half forested	In addition to the previous case wind also plays a role.
Forest clearing	In addition to the previous cases, atmospheric thermal radiation has an effect.

Another important factor is liquid precipitation, which brings heat and wetness with itself into the snow, making it less effective at reflecting solar radiation. From topographic properties the slope of the terrain has to be mentioned as well, as snow melts significantly faster on slopes inclined towards south.

Melted snow is at first bound to ice crystals in the snow cover due to adhesion. When liquid water content of snow goes above 2 – 5 % of total snow volume, the *retention capacity* of snow is exceeded and water starts to flow out of the snow. Snow cover melts on average so, that by the end of April South West Finland is free of snow, by mid-May the snow free zone reaches Oulu – Joensuu line, and by the end of May all snow has melted, except from the top of fells in Lapland.

This chapter has addressed hydrometeorology and snow hydrology. The atmosphere is a quickly replenishing storage of water, which also works as an effective method of transportation and purification of water. Density of water vapor is less than that of air, which is why vapor is able to escape from ground and into the atmosphere, where it is transported with the wind. Upon reaching the saturation pressure, water vapor starts to condensate into droplets or desublimate into ice crystals, which fall back to ground as precipitation. During the cold season, precipitation accumulates on ground as snow, which causes a peak in runoff during the melt season. In the next chapter we take a look at lakes, which form a significant storage of surface water. The diversity of lakes is great, and they form an important habitat and a place for recreational activities.

Chapter 5

Lakes

This chapter takes a look at the diversity, hydrology and water quality of lakes. Lakes function as storage of surface water in water systems. Heat balance and circulation play a crucial role in the evolution of their state. This chapter also concerns physical questions of limnology, especially of light conditions, which are discussed towards the end.

5.1 Morphology and water balance

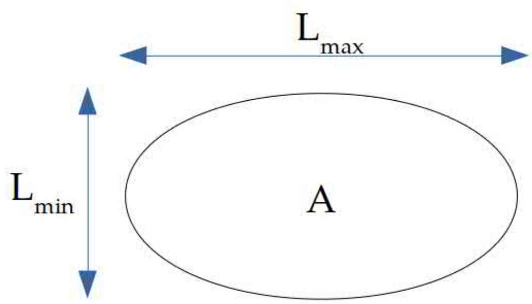
Lake is a depression in terrain, which contains water, and is connected to a catchment area that replenishes it. Water enters a lake as precipitation or runoff and exits through evaporation and outflow. Some lakes located in dry climate zones do not have outlet rivers. Lakes smaller than 0.01 km^2 are usually considered as ponds (Järnefelt 1958), and a lake is classified large when its area is 100 km^2 or more. These terms correlate with the language used in everyday life. Largest lakes have a surface area of $10^4 - 10^5$ km^2.

In hydrological sense, even small puddles are lakes. Ponds replenished by the flow of groundwater are called springs. There are lakes whose water balance is dominated by the flow of groundwater, like Lake Vesijärvi in Päijänne-Tavastia and Lake Sääksjärvi in Hyvinkää (both in Southern Finland). Most lakes consist of fresh water. There are also saline lakes, e.g. the brackish Aral Sea and the hypersaline Dead Sea. Lakes are classified by their mechanism of formation into categories (Fig. 5.1):

- Lakes of tectonic origin. The basin is formed into the bedrock during tectonic movements (The Caspian Sea, Lake Tanganyika, Lake Baikal).
- Lakes formed in depressions caused by the glacial retrieval during ice age (most lakes in Finland).
- Lakes formed in depression having volcanic origin (Lake Shikotsu in Hokkaido).
- Lakes formed in meteorite craters (Lake Lappajärvi in Western Finland, Kaali crater lake in Saarenmaa, Estonia).
- Lakes dammed by landslides (Lake Attabad in Pakistan).

- Glacial lakes: epiglacial lakes found around the edges of glaciers (Lake Mandrone, Italy), supraglacial lakes formed on top of glaciers (Lake Suvivesi, Queen Maud Land, Antarctica) and subglacial lakes under glaciers (Lake Vostok, Antarctic central plateau).
- Artificial lakes (Lokka reservoir in Finnish Lapland, Rybinsk reservoir in the Volga water system).

Morphology of lakes (i.e., their shape and size) is described by several quantities and distributions. Size is described by the surface area A and volume S. They define the mean depth H so, that $S = AH$. As for the horizontal scale, it is useful to employ the characteristic diameter, $L = \sqrt{A}$, and the dimensions of the smallest rectangle possible to cover the lake, L_{max} and L_{min}. The shape of a lake can be described by ratio of the dimensions, $\frac{L_{max}}{L_{min}}$ and by surface area ratio $\frac{A}{L_{max}L_{min}}$. Other horizontal characteristics are the shape of the shores and percentage of islands. The mean depth of lake basins is much smaller than its horizontal scale: when $L > 1$ km, $H/L \sim 10^{-3}$.



Depth distribution is described by the *hypsographic curve* (Fig. 5.2):

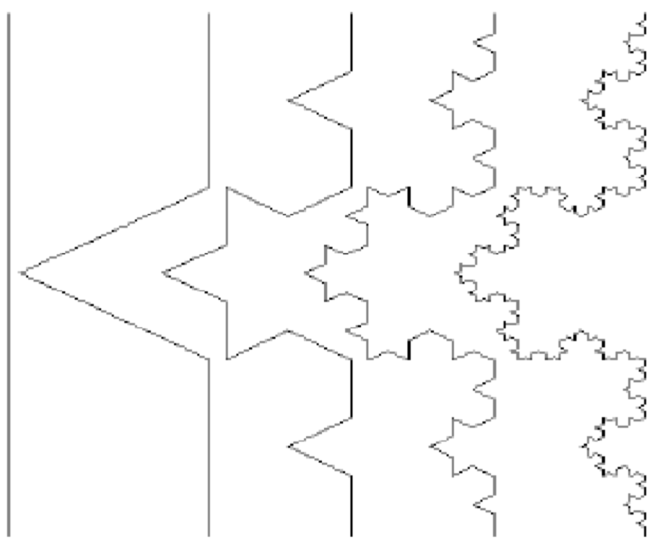
$$\Gamma(h) = \int_A I(h' - h)dA \tag{5.1}$$

where h is depth, h' is an integration variable describing depth and I is the so-called Heaviside function, $I(z) = 0(1)$ when $z < 0(z \geq 0)$. Hypsographic curve at point h describes how large area of the lake is deeper than h. So, $\Gamma(0) = A$ and $\Gamma(h_{md}) = A/2$, where h_{md} is the median depth of the lake, and $\Gamma(h) = 0$, when $h > h_{max}$.

Geometrically speaking, lakes are $fractals$. Fractals are common structures in the morphology of our planet. Fractals are self-similar, meaning that they have geometric patterns that are repeated when the scale is changed. This means that some structures are preserved as the absolute scale changes[1]. The fractal nature of lakes means that not all geometric properties are well defined, which is why such quantities must be taken with a grain of salt. Examples of these are the length of the shoreline and the number of lakes. Upon expressing them, it is necessary to state the length scale used in the measurements.

[1] Due to self-similarity, it is always important to state the scale when documenting fractal structures in pictures.

Figure 5.1: *Various types of lakes: lake in an ice age depression (Lake Puulavesi, Southern Savonia, Finland), meteorite crater lake (Kaali crater, Saaremaa, Estonia), supraglacial lake (Lake Suvivesi, Queen Maud Land, Antarctica) and geothermal lake (Lake Hévíz, Hungary). Photos: Matti Leppäranta.*



Shoreline is a fractal, and its length L can be measured with a suitable yardstick of length Δ, either in-situ (for example, by counting steps) or from a map. Then, $L = L(\Delta)$. In the case of fractals $L \to \infty$, when $\Delta \to 0$. More precisely:

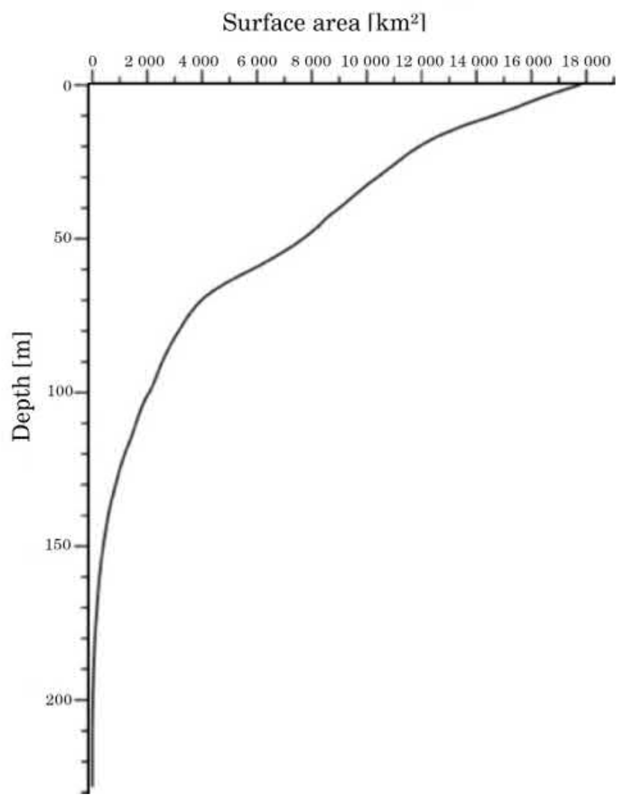


Figure 5.2: *Hypsographic curve of Lake Ladoga. Source: Naumenko (2013).*

$$L \propto \Delta^{D-1},\ when\ \Delta \to 0, \tag{5.2}$$

where the parameter D is the *fractal dimension* or *Hausdorff dimension*, $1 < D < 2$. It can be solved by measuring the length of the shoreline with various yardsticks, and by fitting the results into the equation (5.2). Correspondingly, the the number of lakes in a given area is fractal in nature. The number of lakes $n(A)$ larger than A follows the Korcak-Mandelbrot equation

$$n(A) = k\left(\frac{A}{A_0}\right)^{-d/2} \tag{5.3}$$

where k is a parameter of the distribution, $A_0 = 1$ km^2 and $d \sim 1.5$. On the other hand, the surface area of a lake is a well defined quantity, which approaches the correct, finite value upon measuring with increasing resolution.

Example 5.1
Counting the number of lakes in Finland has been attempted on several occasions. Mustonen (1986) presented the following distribution.

Lake surface area [km^2]	**Number**
> 100	47
10 – 100	279
1 – 10	2 263
0.1 – 1	13 114
0.01 – 0.1	40 309
0.0005 – 0.01	131 876
Total ($>$ 0.0005)	**187 888**

This result follows equation (5.3) quite well, when $k = 1\ 535$, and $d = 1.32$. So, the number of lakes is strongly dependent on the scale of the map used for this calculation. Halving the minimum size of a lake increases the number of lakes by a factor of 1.6. The term coined by the Finnish author Zachris Topelius in the 1800s, '*The land of thousands of lakes*', is numerically accurate, if 0.5 – 1 km is assumed as the lower limit of the lake size.

The Caspian Sea is the biggest lake in the world, about the same size as the Baltic Sea, but triple its depth. Lakes larger than 10 000 km^2 are rare. The largest lake in Finland is Suur-Saimaa. The depth scale of the largest lakes is typically in the range of tens of meters. Lake Baikal and Lake Tanganyika are exceptionally deep. Lake Balkash in Kazakhstan is very wide, but quite shallow.

	Surface area [km^2]	**Mean depth [m]**	**Max. depth [m]**
Caspian Sea	436 000	182	946
Lake Superior	83 300	145	307
Lake Victoria	68 800	40	79
Lake Huron	59 600	59	229
Lake Michigan	58 000	85	281
Lake Baikal	31 500	730	1 741
Lake Tanganyika	34 000	572	1 470
Lake Erie	25 700	19	64
Lake Ontario	19 000	86	244
Lake Ladoga	18 000	47	230
Lake Balkhash	17 000	6	26
Lake Vostok	14 000	150	344
Suur-Saimaa	4 400	17	82
Lake Päijänne	1 100	17	104
Lake Inari	1 100	14	100

Within the catchment area of the Baltic Sea, there are many great lakes: Ladoga, Onega, Vänern, Saimaa and Peipus (Fig. 5.3). 9 000 years ago, the Baltic Sea basin formed a large lake, called the Ancylus Lake, which had a surface area of over 400 000 km^2. Due to postglacial rebound, the Bothnian Bay is projected to diverge into a separate lake in 500–1 000 years, with a surface area of around 30 000 km^2.

In hydrological sense, lakes act as buffer reservoirs, temporary storage of water. The water balance of a lake can be written as

$$\frac{dV}{dt} = (P - E)A + R - O \tag{5.4}$$

where R is the runoff coming from lake's catchment area and O is the outflow, which depends on the lake water level. The outflow also reflects the evening out and delay of the incoming runoff (Fig. 5.4). Changes in the water balance are projected into changes in the water level (Fig. 5.5). Because $V = AH$, changes in the lake volume can be transformed into changes in mean depth and surface area with the formula

$$\frac{dV}{dt} = H\frac{dA}{dt} + A\frac{dA}{dt} = \left(A + H\frac{dA}{dH}\right)\frac{dH}{dt}. \tag{5.5}$$

Surface area as a function of depth is acquired from the hypsographic curve. If $PA + R >$

$EA + O$, then $\frac{dH}{dt} > 0$, and water level increases. Then the outflow grows as well, and eventually an equilibrium (O', A') is reached, where $O' = (P - E)A' + R$ and water is at its highest level. This way, a lake can store water. Conversely, as water level drops, outflow and surface area subside as well.

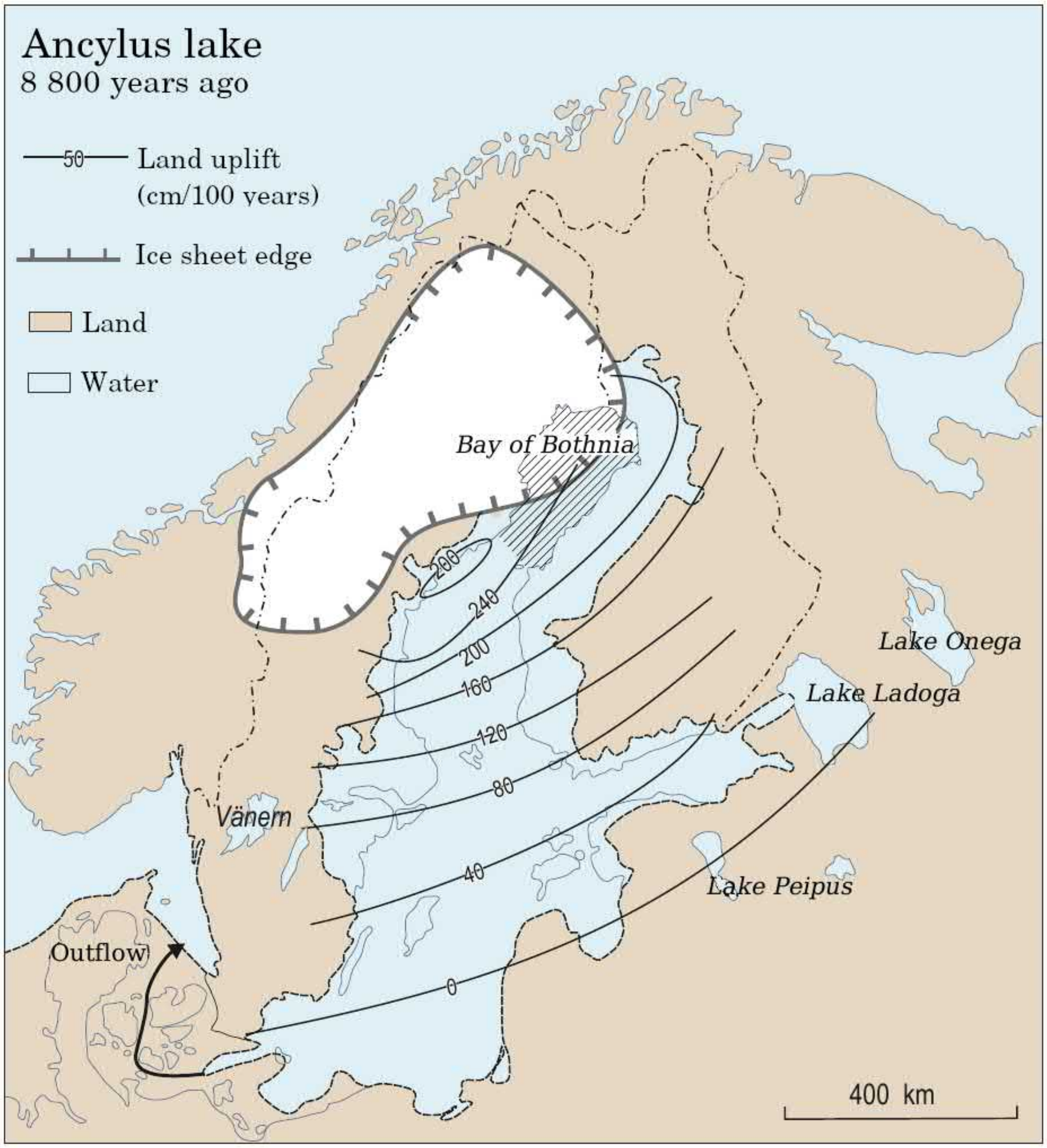


Figure 5.3: *Ancylus Lake and land uplift 8 800 years ago. Due to the uplift, the Bay of Bothnia will become the Europe's greatest fresh water lake in 500 – 1 000 years.*

Example 5.2
Aral Sea has the rivers Syr Darya and Amu Darya flowing into it but no outlet rivers. Equilibrium in water balance is between runoff, precipitation and evaporation. In 1960s, when the water from the rivers began to be used for the irrigation of cotton fields, the water balance of the Aral Sea turned negative and the water storage began to decrease. As the

surface area got smaller, the evaporation decreased, and new equilibrium $A = R/(E - P)$ was reached. Because Aral Sea had wide and shallow shores, the surface area of the lake got smaller fast. Nowadays, the Aral Sea is divided into three separate lakes.

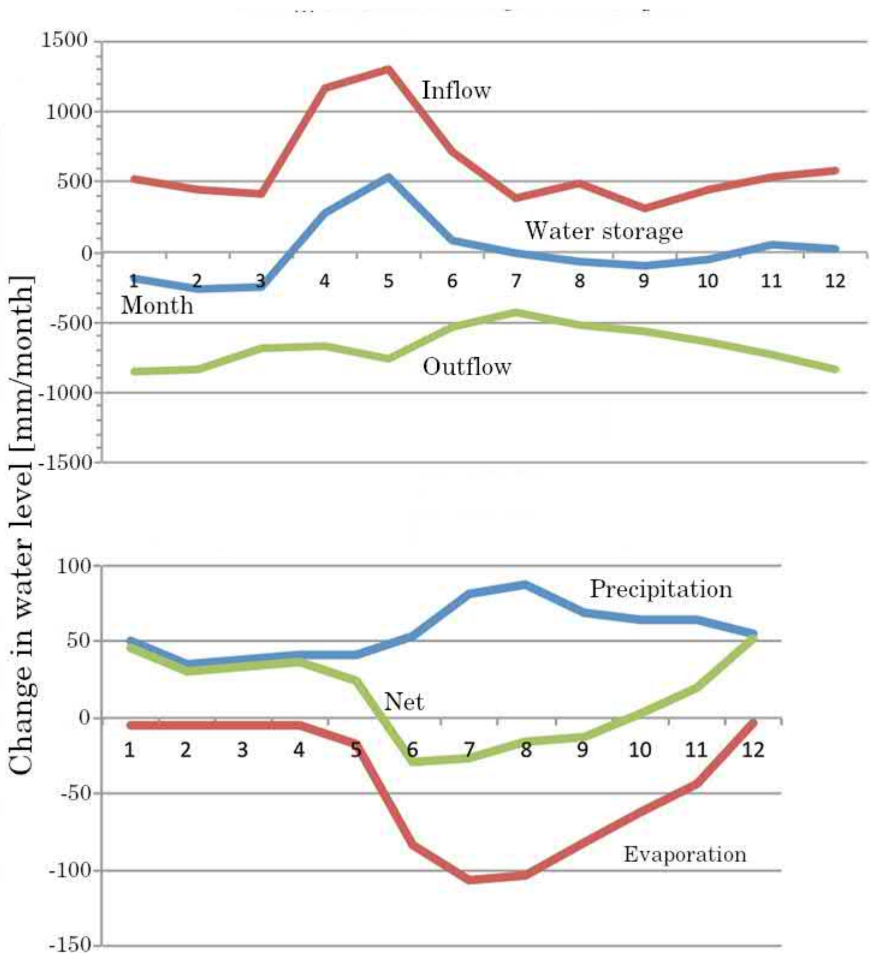


Figure 5.4: *Water balance of Lake Vanajavesi. In the upper figure is the monthly water balance, inflow (runoff) and outflow. In the lower figure is the precipitation, evaporation, and their net value. Source: Jokiniemi (2011).*

Example 5.3

a) If a lake has vertical walls, its surface area remains constant, and any change in water storage only manifests as a change in the lake's depth, $\frac{dV}{dt} = A\frac{dH}{dt}$.

b) In a more general sense, we can have a look at a conical lake, having a depth of H, and a surface radius of r. Then, $A = \pi r^2$, $V = \frac{1}{3}\pi r^2 H$ and the slope of the lake wall is $\beta = \frac{H}{r}$. As the water storage changes, the slope remains a constant, and thus

$$\frac{dr}{dt} = \frac{1}{\beta}\frac{dH}{dt}, \ \frac{dA}{dt} = \frac{2\pi H}{\beta^2}\frac{dH}{dt}, \ \frac{dV}{dt} = \pi\left(\frac{H}{\beta}\right)^2\frac{dH}{dt}.$$

If $r = 5$ km and $H = 10$ m, the length of the shoreline is 31.4 km, $\beta = 2 \cdot 10^{-3}$, $A = 7.9 \cdot 10^7$ m^2 and $V = 2.6 \cdot 10^8$ m^3. As the depth of the lake increases by one meter, its radius increases by 500 m. Surface area grows by $1.6 \cdot 10^7$ m^2 and volume by $0.79 \cdot 10^8$ m^3. The shoreline will be 3.14 km longer.

The effect of lakes on the hydrology of a water system primarily manifests via lake water storage, which evens out changes in runoff and discharge. Due to this, the low water flow (i.e., discharge during low water level) of the outlet rivers in a water system increase and the high discharge decrease as a function of lake fraction, which is the fraction of lakes covering the water system. Additionally, evaporation increases with the lake fraction, which is why the mean discharge out of the water system decreases along with lake fraction. This is apparent in the annals of the Hydrological Office.

Table 5.1: MHQ is the mean value of annual highest discharge rates and MNQ is the same, but for the lowest. Their ratio describes annual variation of discharge.

	Property of the water system			
	Surface area [km]	**Lake fraction [%]**	**MHQ/MNQ**	**Month of maximum**
Tainiokoski rapids	61 300	20	1.49	VII – VIII
Lake Kemi outlet	27 300	2.4	18.6	V

Lake water is typically fresh, having a salinity of less than 0.5 ‰. In Finnish lakes, salinity is less than 0.1 ‰(meaning, that the concentration of dissolved matter is less than 100 mg L^{-1}). In freshwater lakes, salinity is not usually taken into account in physical processes. Lakes consisting of brackish water are, for example, Lake Balkhash (salinity 1 – 6 ‰) and the Caspian Sea (5 – 10 ‰). Hypersaline lakes are for example the Dead Sea (250 ‰) and Don Juan pond in Antarctica (440 ‰).

Retention time is the time that a particle of water (or something carried with the water) spends in a lake. Flushing time means the time it takes to completely change the water in a given lake. These concepts are important when examining the progression of pollutants and impurities in lakes and water systems. When the volume V of a lake remains constant, the mean retention time can be estimated from inflow or outflow by using the equation

$$T = \frac{V}{O + E} = \frac{V}{R + P}. \tag{5.6}$$

This does not take into account the geometry or the stratification of the lake, due to which the change of water is not the same everywhere. Bays and deeps form spots, where the flushing of water is weaker. The magnitude of flushing time is usually the same as is the mean retention time.

When observing the flushing of a substance, the concentration in the lake, and both in the inlet and outlet, have to be known. Evaporation transports only water, but otherwise equations analogous to (5.5) and (5.6) can be used. So, the retention time of a substance

Figure 5.5: *Changes in water level in Finland are not typically large, but still they cause intense discussions every now and then. Pictured, is an example from Lake Saimaa during low water. During the period of 1984 – 2004 the water level varied within 1.46 m in Lake Saimaa. Photo: Timo Huttula.*

entering the lake purely via runoff is $T = \frac{C}{C_R} \cdot \frac{V}{R} = \frac{C}{C_0} \cdot \frac{V}{O}$, where C is the concentration, and C_R and C_0 are the concentrations in runoff and outflow, respectively.

5.2 Heat budget

5.2.1 Heat balance

Heat is transported into a lake mostly through its surface. Solar radiation heats the surface and the water below it until the *Secchi depth*[2]. Long wave radiation, turbulent heat transfer and precipitation all only exchange heat at the water surface. Lakes exchange heat also through their bottom. Heat is stored into the sediment during summer and released during the winter, which is important especially in shallow lakes. For geothermic lakes, heat flux from bottom is significant all throughout the year. Additionally, organic matter production binds energy from solar radiation, which is eventually released back via oxidation. This source of energy is weak, but it might be important during the winter.

Surface heat balance in general was addressed in the previous chapter. In the case of

[2]Secchi depth means the depth up to which an observer can see a white disk, 30 cm across (the *Secchi disk*) from the surface.

lakes, the following points need to be taken into account:

- During the open water season, albedo is low and quite stable, while in the ice-on season, albedo changes a lot. A value of 7 % is typically used for open water albedo. Of the solar radiation entering the lake about half (infrared part) is absorbed at or near the surface (10 cm), and the other half (visible light) penetrates the near-surface layer, ice or snow all the way until the Secchi depth (see Eqs. (4.17 & 4.18), $\gamma \approx 1/2$). Secchi depth in Finnish lakes is 0.5 – 10 m, in ice it is 1 – 3 m and in snow 10 – 20 cm.

- Emission coefficient for long wave radiation on water, ice and snow surfaces is $\epsilon_0 \approx 0.96 - 0.98$ (see Eqs. 4.17 & 4.18).

- Lake surface is either liquid or frozen water, so it can be assumed that the air just above it is always saturated with vapor (see Eqs. 3.21 & 3.22 and Fig. 3.15). Exchange coefficients for heat and moisture, C_H and C_E, are typically $\approx 1.5 \cdot 10^{-3}$ (see Eqs. 4.20 & 4.21).

- Rain can enter into snow or brittle ice and promote heating and melting there.

Radiation balance is the leading part of the lake energy balance. During summer, it is positive and during winter it is negative. In Finland, the mean daily value of net solar radiation can be as high as 300 W m^{-2} in summer, and next to zero in the dead of winter. Because atmosphere is not as efficient an emitter of long wave radiation as is the lake surface, the net long wave radiation is strongly negative, in the order of -50 W m^{-2}. the net long wave radiation changes very little from season to season. Turbulent heat exchange smooths temperature differences between the lake surface and the atmosphere so, that the difference between the two does not get very high. When the lake surface freezes over, heat is transported into the atmosphere as the latent heat of freezing which is conducted through the ice cover. This conduction is quite slow, for which reason the ice cover shelters the heat storage of the lake in the winter.

The heat content Θ of a lake is dependent on the water temperature, ice cover and lake volume. It is useful to keep the energy of liquid water at 0 °C as the reference Θ_0. Then

$$\Theta = \Theta_0 + \int_V \rho(cT - L_f J)dV, \tag{5.7}$$

where V is the volume of the lake, L_f is the latent heat of freezing, and $J = 0\,(1)$ if the phase of a water particle is liquid (solid). The heat content therefore consists of sensible heat, and in the case of ice, also latent heat of freezing. Solar radiation and heat exchange with the atmosphere and the bottom sediment change the heat storage. This change can be calculated by the formula

$$Q = \frac{d\Theta}{dt} = \int_{A_0 \cup A_b} [Q_0 + Q_b + (1-\alpha)(1 - e^{-\kappa H})\gamma Q_s]dA, \tag{5.8}$$

where A_0 and A_b are the surface and bottom of the lake, respectively, Q_0 and Q_b are heat fluxes through surface and bottom, H is the depth and κ is attenuation coefficient of light.

Therefore, transfer of heat happens at the surfaces, excluding solar radiation, which also heats the lake interior. When $H \gg \kappa^{-1}$, solar radiation is completely absorbed in water. Otherwise, a fraction $(1-\alpha)\gamma e^{-\kappa H}$ penetrates the water column to be absorbed into the bottom sediment. Because the transport of heat is mostly vertical in lakes, it is considered as a vertical process (Fig. 5.6).

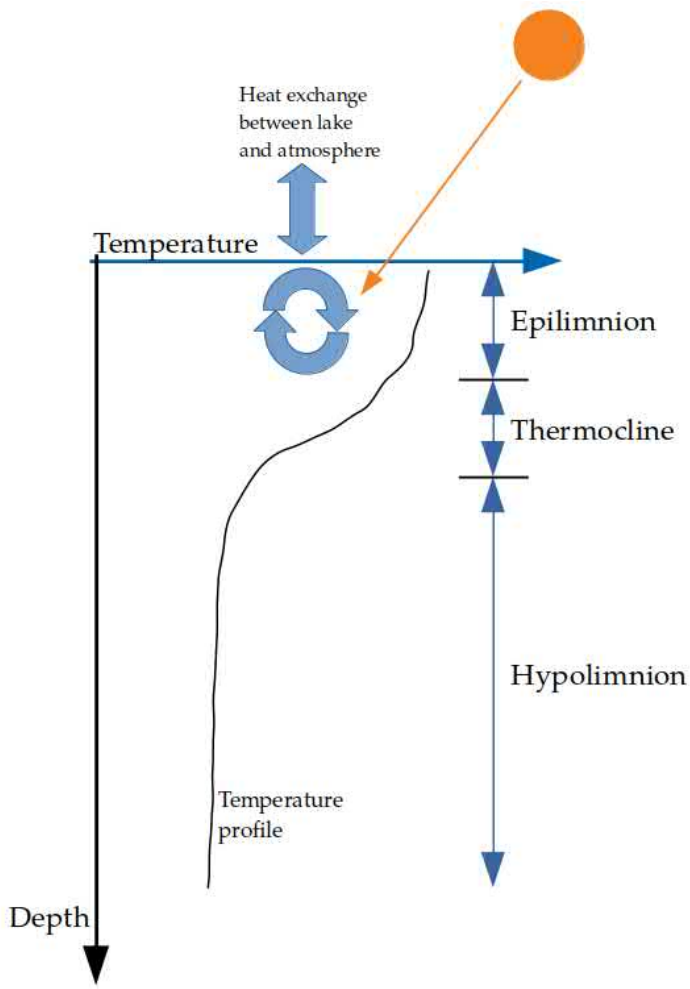


Figure 5.6: *Vertical transport of heat in the lake water column.*

Example 5.4
Temperature in a homogeneous layer of water increases at a rate of $\frac{Q}{\rho c H}$, Q is the heating power and H is the layer thickness. If $Q = 100$ W m^{-2} and $H = 10$ m, temperature increases by 0.21 °C per day.

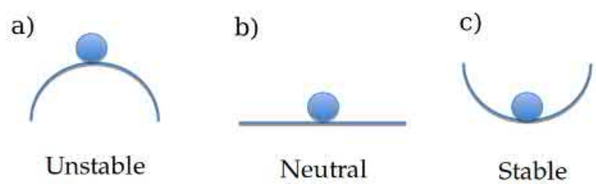
5.2.2 Stratification

Stratification of a water column refers to its vertical density structure. The type of stratification is defined by how a particle of water would react if its position were to be disturbed slightly in the vertical direction.

- If the particle returns to its original position, the stratification is considered to be *stable*.

- If the particle stays in the position where it was taken, the stratification is *neutral*.

- If the particle continues to get further from the original position, the stratification is *unstable*, or *labile*.



Analogous situation would be a ball on a (a) concave, (b) smooth or (c) convex surface. In theory, all of the above situations are possible in a lake, but the unstable situation is very prone to break and collapse into a neutral stratification. In practise, soundings of lakes have proven that the water column almost always is either neutral or stable, ergo, the density of water is either a constant or it increases along with depth. The following can be held as a empirical rule of the thumb: *The water column of a lake is always neutral or stable.* This result is useful in constructing mathematical models, or as a control, when interpreting results from field measurements.

In saline lakes, the effect of salinity on the density of water has to be taken into account when studying stratification. When salinity increases, the temperature of maximum density decreases. If salinity is 0.1 ‰, $T_m \approx 3.96\,°C$. If it is 1 ‰, then $T_m \approx 3.72\,°C$. In very deep lakes pressure also plays a role in the stratification of the water column.

In freshwater lakes, cooling of the surface (when $T > T_m$) or heating (when $T < T_m$) increases the density of the surface water, which causes vertical mixing, or convection, into a depth, that has the same temperature as the cooled surface water. When the surface water reaches the temperature of maximum density, the whole water column is mixed from surface to bottom. This is called an *overturn*; a process turning the water column isothermal. Lakes can be classified according to the occurrence of their overturns.

Type	Overturn	Region
Amictic	None	Polar regions, where temperature is always below +4 °C, or or in tropics, where temperature is always above +4 °C.
Oligomictic	Seldom	Polar regions, tropics
Cold monomictic	Once per year	Polar regions, but temperature is above +4 °C for some part of the year.
Warm monomictic	Once per year	Subtropics, temperature is below +4 °C for some part of the year.
Dimictic	Twice per year	Temperate zone, like Finland
Polymictic	Many times per year	Weak stratification, wind can easily mix the water column. High latitudes.
Meromictic	None	Salinity stratifies the water column so strongly, that temperature differences cannot break it.

In climate zones where Finland resides, lakes undergo two overturns per year, in spring and in autumn, and thus they are dimictic. Monomictic lakes can be found in polar or subtropical climate zones. They undergo overturn annually, when water is at its densest, and their temperature does not necessarily reach +4 °C. Overturns in oligomictic lakes are rare, and can happen due to exceptionally strong winds or cold precipitation. An example of a meromictic lake is a saline lake, or a bay, to where a river carries light, fresh water that stays on top of the dense seawater. If the stratification of salinity is sufficiently strong, surface water cannot, even when cooled to its maximum density, become as dense as the saline water below it. Stratification in such a situation is permanent. In Finland there are a few meromictic lakes, like Lovojärvi in Lammi (Southern Finland), Pakasaivo in Muonio (Northern Finland), which have had a saline bottom water layer (or hypolimnion) formed over a long time.

A stratified freshwater lake typically has three layers: an almost isothermic surface layer, *epilimnion*, a lower layer, *hypolimnion*, and a layer between them, *metalimnium* (also known as *thermocline*), where temperature changes steeply over a short vertical distance. In dimictic lakes, three different kinds of stratification can be observed (Fig. 5.7).

During the *summer stratification*, wind is able to mix the surface layer up to a depth of 5 – 20 m, depending on the lake morphology and weather conditions. The epilimnion is exposed to the mixing effects of wind, and temperature changes only little in the vertical direction. Below the thermocline there is the hypolimnium consisting of nearly isothermic and cool water, where mixing is also weak and depends on the circulation in the lake.

During the *winter stratification* the lake has an ice cover, which eliminates the effect from the wind and holds the water just below the ice cover at the freezing point. In shallow lakes the lower layer is heated by the sediment, releasing the heat stored there during the summer. In spring, sunlight penetrating the ice cover heats up the lake. Hypolimnium is at a temperature of 0 – +4 °C, depending on the lake depth and the autumn overturn. Thermocline is usually higher up in the water column than in the summer.

During the spring and autumn overturns, the whole water column is at +4 °C. When approaching this temperature, the stratification weakens and eventually breaks. This lets

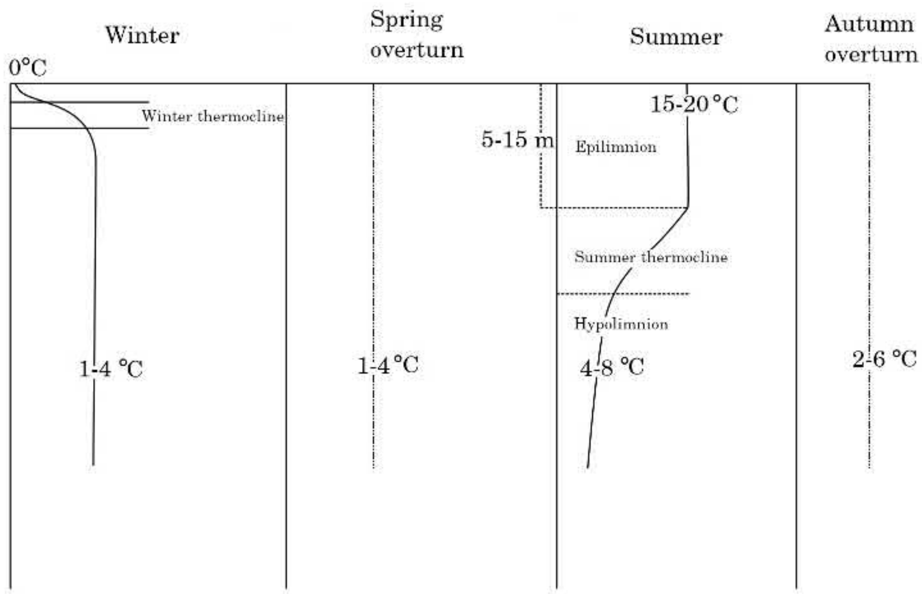


Figure 5.7: *Thermal stratification of lakes in Finland. Temperatures given are for reference, and they can vary depending on the lake bathymetry.*

convection reach from surface to bottom, and the whole water column is mixed. Near the temperature of maximum density differences in density are very small, and mechanical forcing from wind and waves can easily penetrate deep into the lake. On windy autumns, wind-induced mixing can continue until the whole lake is below +1 °C or so, in which case the lake is filled with cold, oxygen-rich water. Spring overturn begins during the melting of the ice cover, or sometimes even before it, if there is enough sunlight penetrating the bare ice cover to heat the surface water. Due to this heating, the surface water can rapidly reach temperatures above +4 °C, and summer stratification can begin to form, leaving the spring overturn short. In the overturns, deep water oxygen reserves are replenished. Unless a lake is 'ventilated' sufficiently often, oxygen can run out in the deeps. In Finland, long winters and nutrient rich waters cause problems with oxygen sufficiency.

The flow of ground water is able to raise the temperature at hypolimnion. In addition to stratification due to seasonal changes, lakes exhibit stratification due to diurnal changes, and sometimes changes in the weather conditions cause changes in the synoptic scale, 5 – 10 days.

5.2.3 Mechanical mixing

In addition to convection, in a neutral or stable water column *mechanical mixing* can occur, if sufficient energy for increasing the total potential energy of the water column is available. Usually, mechanical mixing is forced by wind or surface waves, but internal waves[3] can

[3]Internal waves can propagate in the lake's thermocline, and upon approaching shores they can break and mix water masses.

also force mixing when they break. Mechanical mixing is also called forced convection, or turbulent mixing.

Mechanical mixing needs energy to raise the center of mass of the water column higher. The energy required raises quickly along with increase in the mixed layer depth. This is why a thermocline is formed below the mixed layer. Potential energy of a parcel of water is calculated by multiplying its mass by the gravitational acceleration and depth. It is handy to set reference of depth at lake surface ($z = 0$). Then, the potential energy of a water parcel is $dE_p = -\rho g z dV$ (the negative sign is caused by the fact that depth increases when heading down). If a lake is stratified so that the density is only dependent on depth, then potential energy per unit of surface are is written as

$$E_p = -\frac{g}{A_0} \int_0^h \rho(z) A(z) z dz \tag{5.9}$$

where $A(z)$ is the surface area of the lake at a depth z, and $A_0 = A(0)$. If the variation in density is small, and the water is well above +4 °C, the density of water can be assumed to be linearly dependent on the temperature. Then, the water level of a lake remains constant even if there are internal changes in temperature.

Let's assume a well-mixed lake, that is at a temperature T_1 and has a corresponding density ρ_1. Heat brought into the lake from the atmosphere and the Sun first raise the temperature in the epilimnion. Wind mixes the surface layer, and in this manner transports part of the heat deeper into the lake, forming a vertical temperature profile $T = T(z)$ and correspondingly, a density profile $\rho = \rho(z)$. Potential energy of the lake is increased by

$$\delta E_p = \frac{g}{A_0} \int_0^h (\rho - \rho_1) A z dz. \tag{5.10}$$

Wind causes horizontal shear stress τ_a in the lake surface, which transports kinetic energy into the lake, following a square drag law:

$$\tau_a = \rho_a C_a U_a^2, \tag{5.11}$$

where U_a is the wind speed, and $C_a \approx 1.5 \cdot 10^{-3}$ is the drag coefficient of the air/water interface. The work done by this shearing is $\tau_a U_{w/a}$, where $U_{w/a}$ is the projection of surface current into the direction of the wind. This work is fed into currents, waves and water column potential energy of the lake. When surface current and wind are moving in the same direction, $\tau_a U_w \propto U_a^3$. In other words, the work done by the wind shear is proportional to third power of the wind speed. Because increasing the potential energy increases the thermocline depth, the rate of this change is also proportional to the third power of the wind speed.

Example 5.5

Let's begin from a situation, where a cylindrical lake is at $T_0 = +10\,°C$. First, a surface layer of $h_1 = 1$ m is heated to a temperature of $T_1 = +15\,°C$, after which wind mixes the lake up to $h_2 = 2$ m. In the end, the mixed surface layer is at a temperature of

$T_2 = +12.5\,^\circ$C. The density of water then is: $\rho(T_0) = 999.702$ kg m^{-3}, $\rho(T_1) = 999.102$ kg m^{-3} and $\rho(T_2) = 999.441$ kg m^{-3}.

As the surface layer is heated, its thermal energy is increased by $\rho c(T_1 - T_0) = 2.1 \cdot 10^7$ J m^{-2}. This increases the potential energy by

$$\delta E_p = g \int_0^{2\,m} (\rho_2 - \rho_0) z dz = g \left[\frac{1}{2}(\rho_2 - \rho_0) + \frac{3}{2}(\rho_2 - \rho_1) \right] \text{ m}^2 = 3.71 \text{ J m}^{-2}.$$

If the wind speed is 5 m s^{-1}, and the surface current is travelling in the same direction at a speed of 10 cm s^{-1} (2 % of wind speed), then the work performed by the wind is 0.0045 J m^{-2} s^{-1}. If all of this work were to put into deepening the thermocline, the mixing (from 1 m to 2 m) would happen in 14 minutes. In reality, work done by the wind is also consumed into waves and currents.

The *stability* of a water column indicates the energy per unit of surface area required to mix the column completely. Stability of mixed water column is zero. The stronger the stratification is, the stronger is the stability and more mechanical energy is required for mixing.

5.3 Winter in lakes

5.3.1 Ice structure and prevalence

In the boreal zone, tundra and high up on mountains, lakes freeze when air temperature stays below the freezing point for a few weeks or longer. Geothermal, saline and very deep lakes form exceptions. Lakes in Finland are covered by ice for 4 – 7 months each year. Typically, ice cover is formed in November or December, and ice melts in April or May.

Lake ice cover consists of three layers: snow, snow-ice and congelation ice. Congelation ice grows from the bottom surface downwards into the lake water, snow-ice forms of wet snow, from the top surface upwards. Congelation ice has large crystals and looks clear, while snow-ice has small crystals and, due to the large number of gas bubbles and pores, has a milky, opaque appearance. These layers are easy to discern from a sample (Fig. 5.8). Congelation ice makes up 10 – 100 % of the ice cover, depending on the amount and timing of snow accumulation. On average, about ⅔ of the ice cover is snow ice. Ice floats, as is dictated by Archimedes' principle, upward buoyant force = mass of the fluid displaced:

$$\rho_i h_i + \rho_s h_s = \rho_w h_w \tag{5.12}$$

where ρ_i is the density of ice, h_i is the thickness of ice, ρ_s is the density of snow, h_s is the thickness of snow, ρ_w is the density of water and h_w is the draft, or the thickness of ice cover that is underwater. The density of snow varies widely, usually it is in the range of 150 – 300 kg m^{-3}, and density of ice is 910 kg m^{-3} $\pm 1\,\%$. From this, it can be seen that the ice/snow interface is at water level ($h_w = h_i$), when $h_s = \frac{\rho_w - \rho_i}{\rho_s} h_i \approx \frac{1}{3} h_i$. When there more snowfall than this, the top surface of ice sinks below water level, and cracks in the ice cover let lake water flood the layer of snow, creating slush.

Figure 5.8: *Layered structure of lake ice. Ice thickness is 33 cm, of which the upper 10 cm is opaque snow-ice and the lower 23 cm is clear congelation ice. Congelation ice also shows some layers of gas bubbles. Photo: Matti Leppäranta.*

Excluding the very start and end of the ice-on season, the ice cover of Finnish lakes is static, i.e. the cover is intact and solid. On some springs, wind causes small ice displacements to occur in large lakes. In very large lakes, like Ladoga, Peipus or Vänern, drift ice occurs, which can move with strong winds and currents.

5.3.2 Ice thickness

The growth of ice can be estimated on the basis of a simple heat conduction law. Upon freezing, heat is released, which is conducted through ice and snow and further delivered from the ice surface into the atmosphere. On the basis of continuity, these fluxes of heat are equal, and we end up with a model

$$\rho_i L_f \frac{dh_i}{dt} = k_i \frac{T_f - T_i}{h_i} = k_s \frac{T_i - T_0}{h_s} = Q_0 \geq 0. \tag{5.13}$$

Here, L_f is the latent heat of freezing, k_i and k_s are heat conductivities of ice and snow, respectively, T_f is the freezing temperature, T_i is the ice surface temperature, T_0 is the snow surface temperature, T_a is the air temperature and $Q_0 = Q_0(T_a - T_0)$ is the transport of heat from ice into the atmosphere. This system of equations is valid for when the ice cover is growing, when $T_f \geq T_i \geq T_0 \geq T_a$. Heat always moves from warm to cold. In fresh water lakes, $T_f = 0$ °C.

Figure 5.9: *Hummocked ice, driven by wind on Lake Vanajavesi. Photo: Matti Leppäranta.*

From equations (5.13) we can see, that the growth of ice cover is controlled by air temperature and snow accumulation. In a general case, this cannot be analytically solved but a numerical solution can progress, for example day by day. The simplest case is the *Stefan-Zubov model*, where snow is not taken into account. When ice thickness starts at zero, we get

$$h_i(t) = \sqrt{a^2 S + b^2} - b, \tag{5.14}$$

$$a = \sqrt{\frac{2k_i}{\rho_i L_f}},\ S = \int_0^t [T_f - T_i(\tau)] d\tau, \tag{5.15}$$

where $a \approx 3.3$ cm °C$^{-1/2}$ day$^{-1/2}$ and $b \approx 10$ cm are the model parameters and S is the sum of freezing-degree-days. This sum is calculated by adding together the daily mean temperatures of those days, where the mean temperature was below freezing, the result having a unit of °C·day. Parameter a describes the conduction of heat through ice, and parameter b represents the effective insulation of the air above the growing ice cover. Accumulation of snow slows down the growth of ice, and to take it into account, an empirical coefficient

a^*, $\frac{1}{2} < \frac{a^*}{a} < 1$ is used instead of a (Fig. 5.10).

Example 5.6
If there are 100 days with a mean temperature below freezing with the mean of -10 °C, the sum of freezing-degree-days is S = 1 000 °C·day, which results in h_i = 95 cm in Stefan-Zubov model. If we assume the empirical coefficient $a^* = 2.5$ cm·(°C·day)$^{-1/2}$, ice thickness is h_i = 70 cm. In Lapland, the sum of freezing-degree-days can reach up to 1 500 °C·day, and according to observations, there the ice thickness can reach 110 – 120 cm.

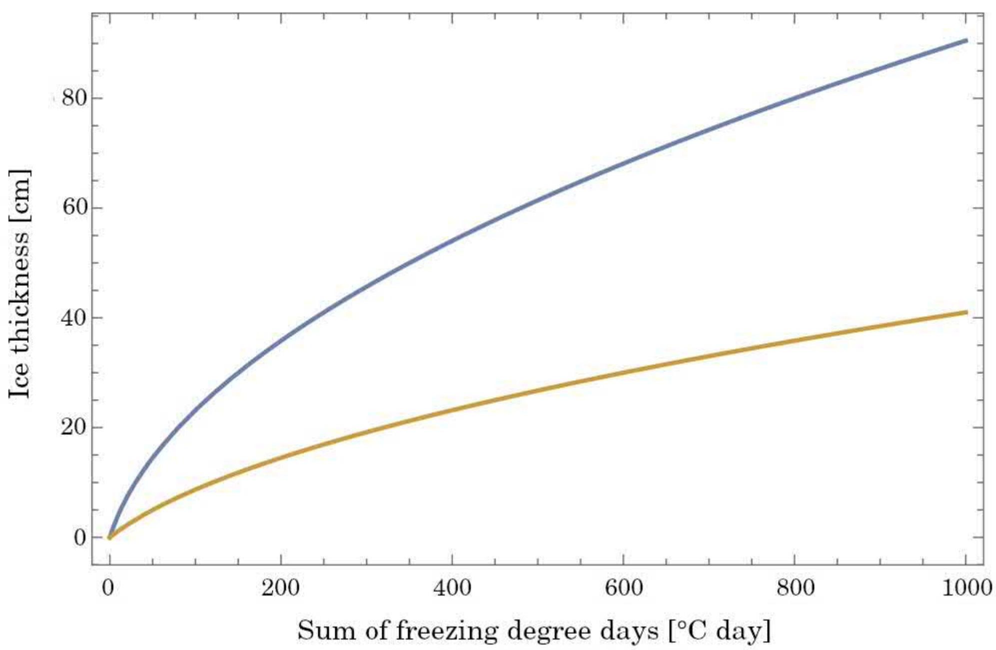


Figure 5.10: *Growth of lake ice according to the Stefan-Zubov model. Blue curve plots the growth of ice with no snow cover ($a^* = a$) and the yellow one is growth with snow on top of the ice ($a^* = 1/2 a$).*

Melting of the ice cover begins in spring, when the heat balance Q_n (heat balance at both, upper and lower surfaces of the ice cover + radiation absorbed into the ice) turns positive. The progress of melting can be estimated directly from the heat balance. Melting is often be approximated with an empirical formula based on air temperature:

$$\frac{dh_i}{dt} = -\frac{Q_n}{\rho_i L_f} \approx -AT_a \; ; \; Q_n, \; A \geq 0 \tag{5.16}$$

where $A \sim 0.5$ cm °C^{-1}·day^{-1} is the empirical degree-day coefficient. If the heat balance is 100 W m^{-2} for one day, 2.8 cm of ice can be melted. Formula (Eq. 5.16) gives the same result, if $T_a = 5$ °C. Estimating the heat balance is usually difficult, and a simpler method is often used. It results in a formulation similar to the sum of freezing-degree-days:

$$h_i(t) = h_i(0) - A \int_0^t max(0,\, T_a) d\tau \approx h_i(0) - A \sum_{k_0}^{n} max(0,\, T_a) \Delta t \tag{5.17}$$

where the melt season length is $t = n\Delta t$, usually $\Delta t = 1$ day. Ice melts proportional to the sum of positive degree days. Coefficient A depends on time, and it can be adjusted during the melt season for practical applications.

5.3.3 Environmental impacts of ice cover

Ice cover fundamentally affects the conditions in the water column. In Finnish lakes, sediment releases heat into the water, and a small amount of heat leaks out of the lake through its ice cover. All in all, the temperature structure is rather stable. Geothermic lakes can remain ice free for the whole year. During the spring, Sun heats up the water just under the ice cover that initiates convection that will eventually bring heat into the whole water column.

Wind is not able to drive circulation in lakes that have an ice cover, which is why current velocities then are low, in the order of 1 mm s^{-1}. Circulation is thermohaline, i.e., the development of the density structure of the lake is driving it. In the dead of winter, sediment heat flux affects the structure of the density distribution, and in spring, solar radiation becomes the main driving force of the lake physics. When there is over 10 cm of snow on the ice, practically no light can make its way into the water.

Ice cover also acts as an effective thermal insulator and, consequently, ice cannot grow very thick. In Siberia, 2 m of lake ice is pretty much as thick as is possible for a seasonal ice cover. Since snow-covered ice does not let light pass through itself, primary production grinds to a halt. Ice can be as transparent as the lake water itself. One of the most critical questions of the ice-on season is the situation regarding dissolved oxygen, since oxygen in water is mostly replenished via the air/water interface. During long winters, Finnish lakes often suffer from fish kills (Fig. 5.11). To remedy this, many lakes are 'resuscitated' by oxidation rafts, that pump air into the deep parts of the lake or force vertical mixing with pumps. Precipitation, and anything falling with it, is stored into the snow cover on the ice and then released into the lake during the short melting season.

Ice causes loads into structures in and around the lake, like piers and navigation beacons. Ice pushed onto the shores cause erosion. The most important practical question is the load bearing capacity of ice. A person or a vehicle moving on ice causes it to bend, and the pressure of water is acting against it. If the bending gets too steep, ice breaks. Load bearing capacity P is proportional to the square of the ice thickness, and it can be estimated by a semiempirical formula

$$P = \beta h_i^2, \tag{5.18}$$

where β is coefficient that depends on the quality of the ice, and is proportional to the strength of the ice. In the case of congelation ice $\beta \approx 5$ kg cm^{-2}; then 5 cm of ice will carry a human (125 kg) and 20 cm will carry a car (2 000 kg). Snow-ice is markedly weaker than congelation ice. When an ice cover consists of both, congelation and snow-ice, the thickness

Figure 5.11: *Fish kill on lake Äimäjärvi in Southern Finland in March of 2003. In autumn of 2002 ice cover formed early, and oxygen ran out from many lakes during that winter. Photo: Jouni Tulonen.*

of congelation ice gets added by half of the thickness of the snow ice, and the coefficient for congelation ice is then used in equation (5.18).

5.4 Currents and waves in lakes

5.4.1 Surface currents and flow driven by the pressure field

In the research of the dynamics of lakes, one is often interested in the flow of water and wave motion, which often are driven by wind. Transport of water changes the water level, which causes horizontal pressure differentials, which in turn drive circulation. Other possible forcing factors can be the flow of water through the lake from inlet to outlet, changes in the atmospheric pressure and horizontal gradients in the density structure. As an example of the last item is the cooling of lakes in autumn (when water is still above +4 °C: water cools faster in the shallow near-shore areas, becomes more dense and flows along the bottom to the center of the lake, and water to replace this is flowing from the center of the lake towards the shores.

Two types of currents in lakes can be distinguished: wind induced current and *thermohaline circulation*, which is flow due to changes in the density structure of a lake. Because wind changes radically in the time scale of days, wind induced flow is mostly irregular and short lived. Thermohaline circulation is initiated due to heating or cooling at the surface or at the bottom. On saline lakes, precipitation and evaporation can also trigger water flow. Currents are also affected by the friction between water and the bottom, and in sufficiently large lakes by the Coriolis force, which deflects the flow direction to the right (left) on the northern (southern) hemisphere. Even the largest lakes are too small for any meaningful tidal flow.

The basic equations of flow mechanics are the equation of motion and the continuity equation, or the conservation of mass (section 3.3.1). The horizontal component (u, v) of the lake flow can be solved from the equation of motion, and then the vertical component w can be solved from the continuity equation of incompressible fluid. Currents in lakes are anisotropic in such a way, that horizontal flow velocities are much greater than vertical ones. Horizontal flows are typically in the order of 1 – 100 cm s^{-1}, while vertical flows are less than 1 mm s^{-1}.

Viscous flow theory is applied for wind-driven flow. Wind blowing over a lake causes shearing stress on the lake surface (Eq. 5.11), which transports wind's kinetic energy into the lake and starts surface flow. Turbulent friction of water transports kinetic energy deeper into the lake, and at the bottom, friction between it and water in turn absorb this kinetic energy. The physics of bottom friction is similar to the turbulent drag at the surface. Equation for the bottom friction is written as

$$\tau_b = \rho C_b u^2 \tag{5.19}$$

where $C_b \sim 10^{-3} - 10^{-2}$ is the bottom drag coefficient, and u is the water current speed. In shallow lakes, wind-driven current flows in the same direction as the wind itself and is balanced by the friction at the bottom. The surface flow velocity is

$$u_0 = \sqrt{\frac{\rho_a C_a}{\rho C_b}} U_a. \tag{5.20}$$

Surface current is 1 – 3 % of the wind speed, depending on the depth of the lake and type of bottom. The geometry of the lake also controls the direction of the flow. In a stationary state, the current profile can be considered nearly linear. In deep lakes, at its strongest wind induced current reaches to a depth of 30 m.

Due to the finite size of lakes, water level eventually begins to rise on the downwind shore. The lake surface becomes inclined, and a pressure gradient $\nabla p = -\rho g \beta$ is formed, where β is the inclination of the surface. In a stationary state this balances with the wind:

$$\rho g H \beta = \tau_a, \tag{5.21}$$

where H is the lake depth. The pressure effect is multiplied by depth, because pressure is a body force acting on the whole water body. In an equilibrium, the net transport of water has to be zero. Surface water is carried by the wind, and deeper down there is a return current replenishing the water dislocated by the wind, driven by the difference in pressure (Fig. 5.12).

Example 5.7
If the wind speed is 10 m s^{-1}, then $\tau_a \approx 0.2$ N m^{-2}. This shear stress is subjected to the surface of the lake. If $H \approx 10$ m, the pressure differential force is 0.2 N m^{-2}, when the surface inclination is $2 \cdot 10^{-6}$, which corresponds to a 2 mm rise in water level per kilometer of horizontal distance. Because typically $\tau_a < 1$ N m^{-2}, inclination of the water level on a 10 m deep lake is less than 1 cm per kilometer. Inclinations this small cannot be seen with a naked eye.

In a stratified lake the wind driven surface current tilts the thermocline, which in turn drives a return current at lower edge of the epilimnion. Pressure differential drives an opposite circulation in the hypolimnion (Fig. 5.12). In the beginning, wind only drives a current on the epilimnion, that goes in the same direction as the wind does. The pressure differential force associated with the tilting of the water level forms a return current going against the wind in the lower parts of epilimnion, and again opposite of this, a circulation is initiated in the hypolimnion. Pressure gradient has little effect in hypolimnion, since the tilting of the thermocline cancel out most of it.

Figure 5.13 presents changes in the vertical current profile due to a change in the wind direction in Hyrkönsalmi on Lake Päijänne (Pulkkinen, 1989). Temporally, they correspond to the situation in Fig. 5.12b. Comparatively large current velocities in the hypolimnion are associated with horizontal temperature (and thus, density) differences.

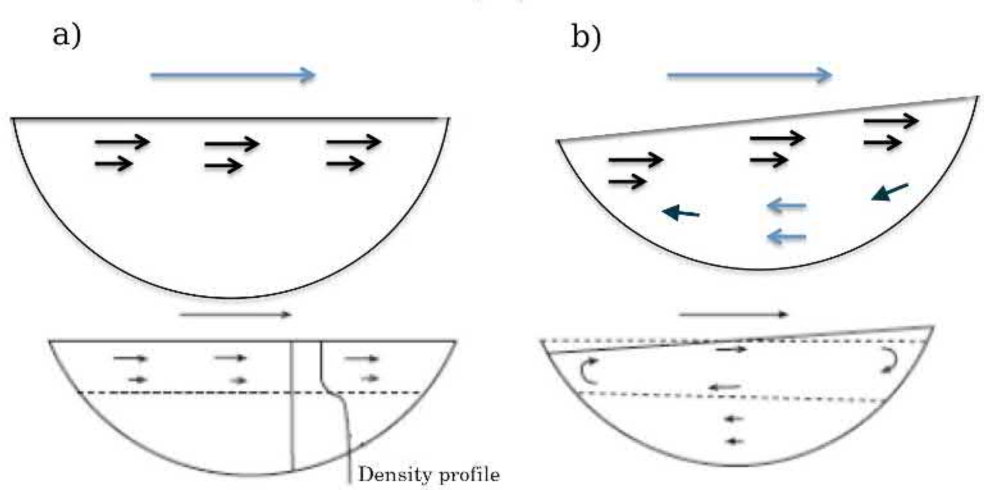


Figure 5.12: *Development of wind induced flow on a homogeneously stratified lake (top) and stratified lake (bottom).*

5.4.2 Wave motion

Wave motion (see section 3.3.1) occurs on the surface of lakes as well as deeper in the water body in zones of large density gradients (also known as *pycnocline*). Wind is the driving force

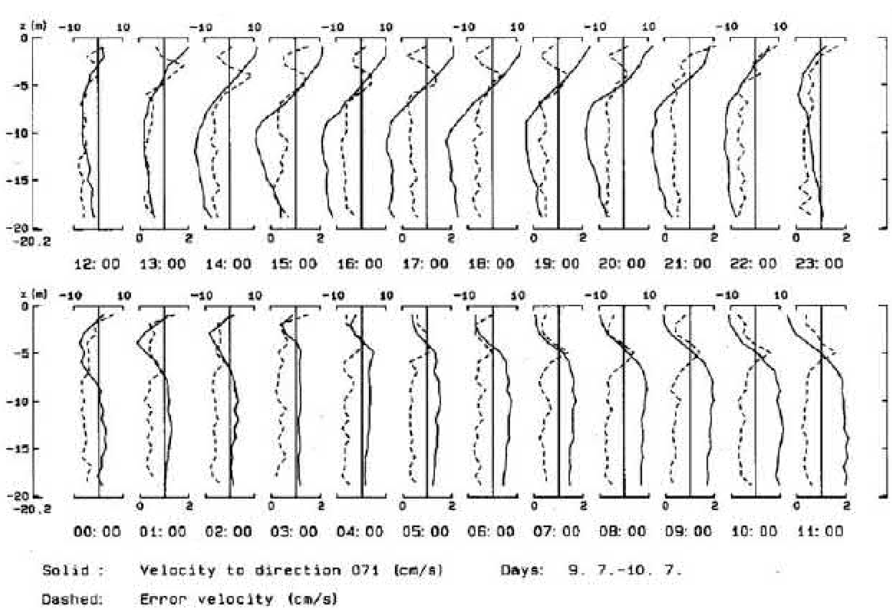


Figure 5.13: *Hourly mean values of current profiles (solid line, scale from -10 to 10 cm s^{-1}), measured with an acoustic doppler current profiler (ADCP) on 9. – 10. of July 1989 on Lake Päijänne. Dashed lines are error estimates (scale from 0 to 2 cm s^{-1}). Values below individual plots are times. Source: Pulkkinen (1989).*

for deep and water waves, but due to the shallow depth of lakes, wind-driven wave motion almost always reaches all the way to the bottom. With lakes, a theory combining both shallow and deep water waves is required, which is quite difficult. Seiche waves, standing waves oscillating at a basin specific characteristic period, are pure shallow water waves.

As a precondition for a seiche is tilting of the water level, which can be brought upon a lake by wind or difference in atmospheric pressure on the opposite sides of the lake. When the force holding the tilt ceases, the whole mass of water starts to oscillate back and forth over its equilibrium state, at a characteristic period. These kinds of waves became known at Lake Geneva in the 18th century, but the Aztecs might have observed this phenomenon on the Lake Texoco in Mexico before 1519. Seiche[4] is a standing wave, where the water level can be represented as a sum of the sine waves, propagating in opposite directions. The basin has nodes, where the water level does not change at all, and antinodes, where the water level changes the most, corresponding to the amplitude A of the wave.

The period of a seiche wave is dependent on the basin length. It can be thought of as the time it takes for a wave to move from one end of the basin to another and back. The period of a uninodal seiche in a rectangular basin is

$$T = \frac{2L}{\sqrt{gH}}, \tag{5.22}$$

[4]The word 'seiche' originates from Swiss French, and refers to a swing or oscillation

where L is the basin length. In the middle of the basin there is a single node, and the edges of the lake have the antinodes. This equation is called the *Merian's formula*, dating back to 1828. It has been noticed to give rather realistic results for natural lakes, with variation in results originating from the irregular shape of lakes. There can be more nodes than one, in which case the period of the wave is $\frac{1}{n}T$, where n is the number of nodes.

Example 5.8
If the depth of a lake basin is $H = 10$ m and the length is $L = 10$ km, the seiche wave propagates at a speed of $V = 9.90$ m s^{-1}, and the period of a single node wave is $T = 33.6$ min. If, on the other hand, $H = 4$ m and $L = 1$ km, then $V = 6.3$ m s^{-1} and $T = 5.3$ min.

When the water column is stratified, *internal waves* can form on interfaces between different layers. As the restoring force of these waves is the net gravity, which is the difference between gravity and buoyancy, and which is proportional to the density gradient between the layers. Internal waves do not require such a large forcing due to the small density gradients inside of the water column. This results in internal waves having amplitudes that are an order of magnitude larger than those of seiches or surface waves.

An internal seiche can form in the thermocline of a stratified lake. If density abruptly changes in the thermocline, the speed of a wave progressing there can be calculated as

$$V_i = \sqrt{g\frac{\rho_2 - \rho_1}{\rho_2}\left(\frac{1}{d_1} + \frac{1}{d_2}\right)^{-1}}, \tag{5.23}$$

where d_1 and d_2 are thicknesses of the epilimnion and hypolimnion, and ρ_1 and ρ_2 are their respective densities. The thermocline oscillation period is obtained similarly to the surface oscillation period: $T_i = 2LV^{-1}$. This oscillation can be observed as changes in isotherm depths. Because the period of internal seiches is relatively long, in the order of hours to days, wind changes its direction in between. For this reason pure wind driven internal seiches are rarely observed in Finnish conditions. Temperature oscillations attributed to internal seiches can also be in the hypolimnion. As an example, figure 5.14 has the development of temperature drawn for several depths in Lake Pääjärvi (Lammi, Finland). It can be noted that temperature fluctuates at various depths at the same time. By comparing the different curves in the plot it can be seen, that vertical shifts of about 6 m are connected to the temperature fluctuations.

If in stably stratified lake a particle of water is displaced from its location and then set free, the particle starts to oscillate at a frequency of

$$N = \sqrt{\frac{g}{\rho}\frac{\partial \rho}{\partial z}} \tag{5.24}$$

where z is the depth. This frequency is called the *Brunt-Väisälä frequency*, named after Welsh meteorologist David Brunt (1886 – 1965) and Finnish meteorologist Vilho Väisälä (1889 – 1969). The corresponding period $T = 2\pi/N$ is the shortest period, that a stratified water column can oscillate with. During summer stratification, the period is 1 – 10 minutes. This oscillation is often observed in the thermocline.

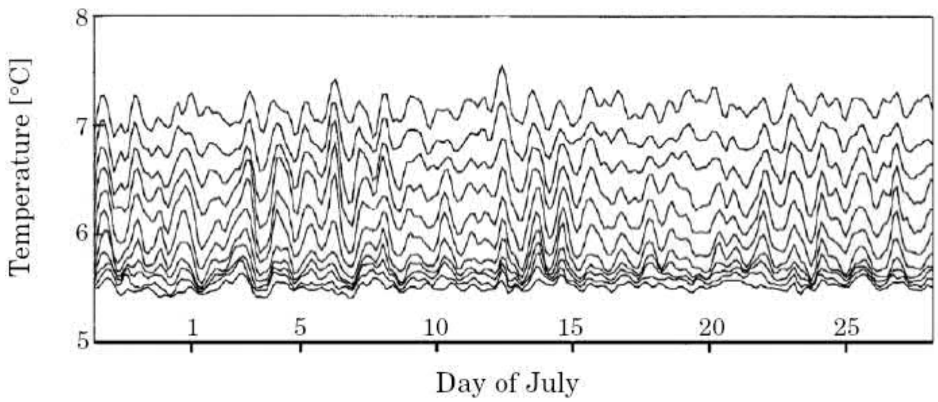


Figure 5.14: *Temperature of water at depths 22.9 m, 24.9 m, 26.9 m,... and 42.9 m in Lake Pääjärvi in Lammi in July 1986.*

5.5 Light conditions

5.5.1 Penetration of sunlight into lake water

Sunlight contains a wide band of the spectrum of electromagnetic radiation, but in practice water only lets visible light (380 – 760 nm) through. The band 400 – 700 nm is usually considered to be usable for photosynthesis. The penetration depth of ultraviolet and infrared radiation is in the order of one meter. Free passage of radiation is hindered by its absorption into water and subsequent transformation into heat and chemical energy, and scattering, i.e. the dispersion of light into different directions. Scattering is the result of rays of radiation hitting particles suspended in water and fluctuations in water density. Optical properties of natural waters are affected by water itself and optically active substances in it.

Upon arriving to a surface of water, light partly penetrates the surface and partly reflects back (Fig. 5.15). The outgoing part is αQ_s, where α is the albedo of the surface. From a still water surface, reflection follows Fresnel's law of reflection, according to which the angle of reflection is the same as is the angle of incidence. Specular reflection (also known as regular reflection) is indifferent to the wavelength of the incoming light, but it does depend on the angle of incidence. Only 2 % of light coming directly from above gets reflected. When the Sun is more than 20° above the horizon, less than 5 % of the light is reflected. At lower elevations, reflection of direct sunlight increases rapidly, but on the other hand then there is more diffuse light, which has the total reflection of 6.6 %. Rough surface of water causes *diffuse* reflection, which is reflection to many different directions at the same time, contrary to specular reflections, where reflection only shines in one direction.

When a ray of light enters the water, the ray is refracted according to Snell's law:

$$\frac{\sin\theta_i}{\sin\theta_j} = n_{ij}, \tag{5.25}$$

where $n_{ij} = 1.33$ is the refractive index of the air/water interface, θ_i is the angle of incidence, and θ_j is the angle of the refracted light from the normal of the surface. In water, sunlight gets absorbed and scattered. Some of the light is backscattered, and headed back to the water/air interface, where penetration and reflection happens again. When coming from water to air, the refractive index of the interface is $n_{ji} = 1/n_{ij} = 0.752$. In order for the light to penetrate this interface, the angle of incidence has to be less that 48.5°. Otherwise, the light gets completely reflected, for which reason the reflectivity of the backscattered light at the water/air interface is about 50 %.

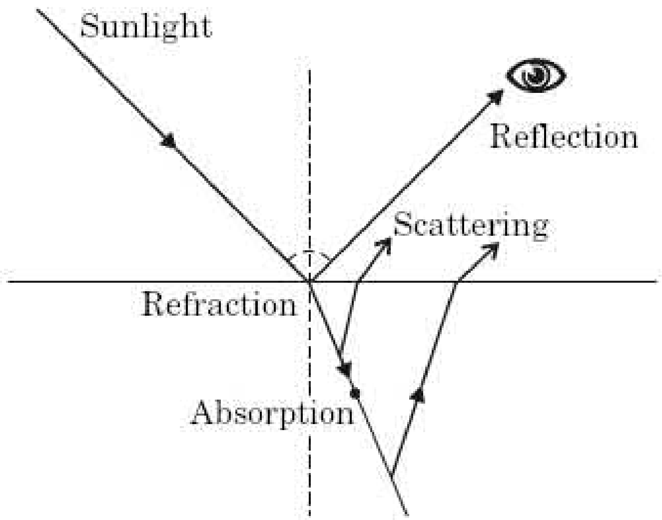


Figure 5.15: *Reflection, refraction, scattering and absorption of sunlight on the surface of water.*

Backscattering stemming from water into air is in the order of 10 % of the surface reflection.. The layer that is the source of this backscattering is about half of Secchi depth. The spectrum of backscattered light depends on the properties of optically active substances in the water. As a result, a satellite, or a human eye, observing the lake from above will see different colors in different lakes. Blue scatters the most, which is why very clean natural waters like the central basins of oceans, will seem blue when looking them from above.

5.5.2 Optically active substances in natural waters

The most important optically active substances, and the wavelength dependencies of absorption and scattering are presented in the table below

	Absorption	**Scattering**
Pure water	Strong > 600 nm	Low
Yellow substance	Strong < 550 nm	No scattering
Suspended matter	Low dependency	Depends on the particle size
Chlorophyll *a*	Absorption bands	Low

Pure water lets light pass so that the blue wavelengths of light penetrate water with the least effort and can still be observed at a depth of 100 m. Yellow substance is also called *coloured dissolved organic matter*, or CDOM for short. It specifically absorbs short wavelengths below 550 nm. For example, in very humic lakes short wavelengths get effectively absorbed and the lighting environment becomes brownish (Fig. 5.16). As light moves deeper, its spectrum gets altered by the optically active substances (Fig. 5.17). Absorption coefficient $a = a(\lambda)$ describes how fast light gets absorbed in the water, and scattering coefficient $b = b(\lambda)$ in turn describes how much of the radiation gets scattered on the way.

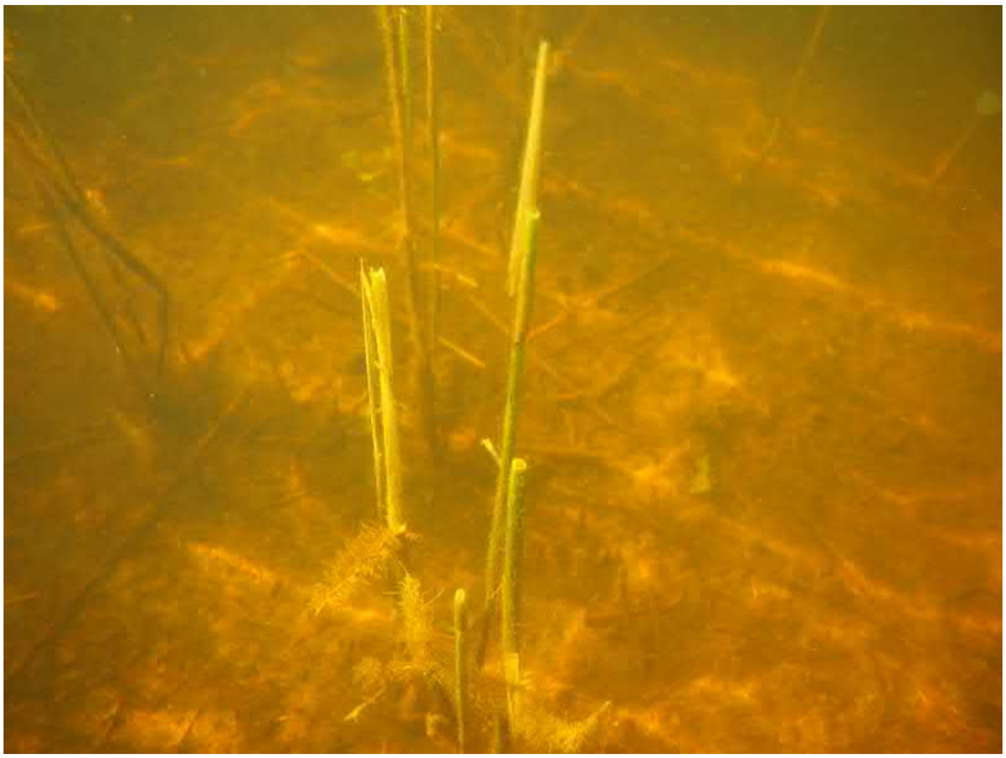


Figure 5.16: *Light conditions in humic lakes have a brown look into it. Photo: Matti Leppäranta.*

Coloured dissolved organic matter consists of various organic substances, that especially absorb short wavelengths. As a consequence, the peak of the spectrum is moved towards longer wavelengths and color of the water gets yellower. This is the namesake of yellow substance. Its absorption spectrum is modeled with an exponential law:

$$a_y(\lambda) = a_y(\lambda_0)\mathrm{e}^{-S(\lambda-\lambda_0)}, \tag{5.26}$$

where a_y is the absorption coefficient of CDOM, $S \approx 0.01$ nm^{-1} is a model parameter, λ is the wavelength and $\lambda_0 \approx 400$ nm is the reference wavelength. Absorption decreases by a factor $e^{-1} \approx 0.37$ for every $S^{-1} = 100$ nm increase in wavelength. Humic substances are a part of CDOM, and they give the brown color to many lakes. Figure 5.18 presents the absorption spectra of filtered and unfiltered samples from Lake Inari and Lake Vanajavesi.

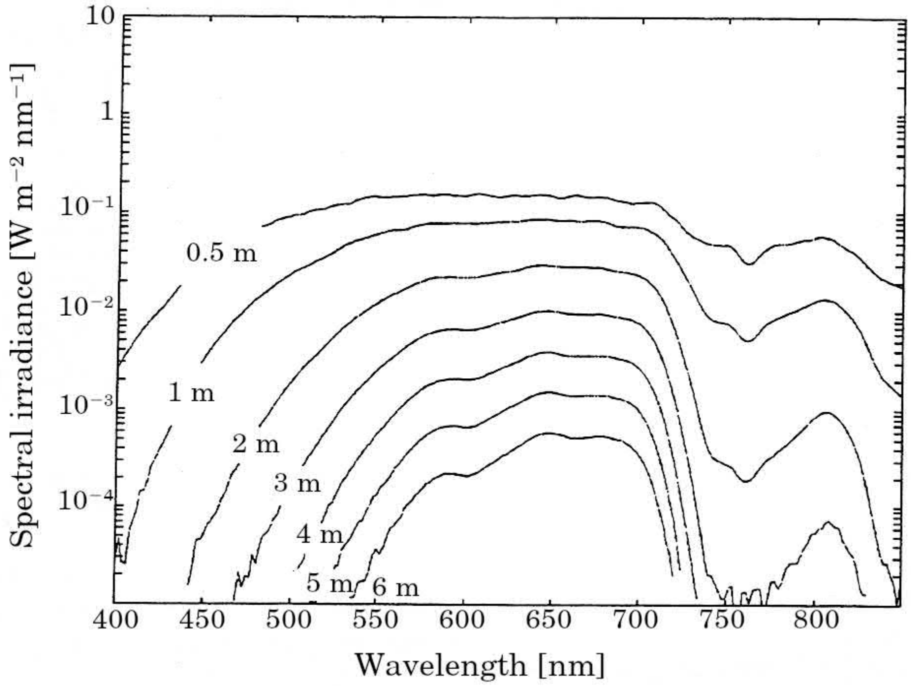


Figure 5.17: *Spectrum of sunlight in Lake Pääjärvi at various depths. Source: Blanco (1994).*

Filtering removes the effect of suspended matter. From this figure one can see, that CDOM absorbs short wavelengths and pure water long ones. The attenuating effect of suspended matter takes place mostly via scattering.

Suspended matter absorbs more in the short wavelengths, but the wavelength dependency is generally weak. The fraction of scattering is significant, and it is influenced by the particle size distribution. Particles that are less than a micrometer in size scatter short wavelengths the strongest, scattering from larger particles is only weakly dependent on wavelength. Suspended matter can have individual properties as well, like the color of a pollutant, which makes it stand out from its environment. Chlorophyll a is very selective in its absorption, with strong absorption bands at 430 – 440 nm and 660 – 690 nm.

Absorption and scattering cause attenuation of radiation with depth. In Finnish lakes, light is able to penetrate 0.5 – 10 m, depending on the quality of water. Attenuation of light in water is described by *Beer's law*

$$\frac{dE}{dz} = -c(\lambda;\, z)E. \tag{5.27}$$

Here, $E = E(z,\, \lambda)$ is irrandiance, the radiation originating from the upper half-space falling upon a unit of area, z is depth and c is the attenuation coefficient which is the sum of absorption and scattering coefficients: $c = a + b$. If c is not depth dependent, the solution becomes the exponential attenuation law

$$E(z, \lambda) = E(0, \lambda)\mathrm{e}^{-c(\lambda)z}. \tag{5.28}$$

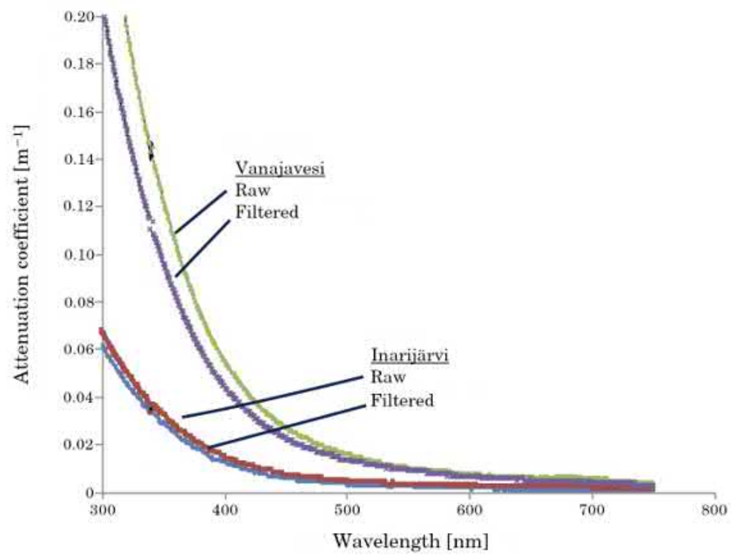


Figure 5.18: *Absorption spectra from Lake Inarijärvi and Lake Vanajavesi from summer 2014. Absorption due to pure water has been removed. Filtered samples have CDOM still in it, while the raw samples have chlorophyll-a and suspended matter in addition to CDOM. Source: Lammi Biological Station.*

Radiation attenuates so that at a depth of $z = c^{-1}$, $\mathrm{e}^{-1} \approx 0.37$ of the original radiation is left.

The radiation spectrum can be can be integrated with respect to wavelength for total radiation, and then the transmission of energy into the water is obtained. For the total radiation $\overline{E} = \overline{E}(z)$, an effective attenuation coefficient C is used. Then,

$$\overline{E}(z) = \overline{E}(0)\mathrm{e}^{-Cz}. \tag{5.29}$$

Two important empirical quantities in aquatic optics are the *Secchi depth* z_D and the euphotic depth z_e. The first one is defined as the depth down to which an observer can still see the Secchi disk. Euphotic depth is the depth down to which photosynthesis is still possible. It is defined as the depth where 1 % of the surface net radiation is still present. According to the exponential attenuation law $z_e = 4.6 \cdot C^{-1}$. Based on empirical data Secchi depth and euphotic depth are estimated as $z_D \approx 2 \cdot C^{-1}$ and $z_e \approx 2.5 \cdot z_D$.

In hydroecological research, even more important than knowing the energy transmitted is to know the number of light quanta, upon which the photosynthesis is dependent. This can be obtained from the formula

$$\overline{E}_q(z) = \int_{\lambda_1}^{\lambda_2} \frac{hc_0}{\lambda} E(\lambda; z) d\lambda, \tag{5.30}$$

where $\lambda_1 = 400$ nm and $\lambda_2 = 700$ nm, h is the Planck's constant and c_0 is the speed of light in vacuum. In this equation, the number of light quanta is integrated over the PAR[5] band. The flow on light quanta on the surface of Earth is in the order of 1000 – 5000 μmol m^{-2} s^{-1}, and in order to have any meaningful primary production 25 μmol m^{-2} s^{-1} is required.

On Earth's surface, radiation in the PAR band is very even, thus $\overline{E}_q \approx a \cdot \overline{E}$, where $a \approx 4.6$ μmol J^{-1}. In water, the spectrum changes and a is between 4.5 and 5.5, depending on water quality.

In this chapter we have taken a look at the physics of lakes and its significance for the hydrological cycle. Lakes have had an important part in Finnish lifestyle and culture. Transport over water is easier in the sheltered inland waters than on the sea, which is why boats meant for lakes are sleek and light to handle. *Savolaisvene* boat (Fig. 5.19) had taken its shape already in the Middle Ages. In the next chapter, we will take a look into the world of rivers and streams.

Figure 5.19: *Savolaisvene, a traditional boat used on Finnish lakes. Painting by Eero Järnefelt (1888). ©Hämeenlinna Art Museum.*

[5]Photosynthetically Active Radiation

Chapter 6

Rivers

6.1 Characterizing rivers

Watercourse refers to a natural or artificial channel by which water flows, pushed by gravity. These include rivers, streams and channels (Fig. 6.1). This chapter takes a look into their morphology, flow and the erosion they cause. The forcing by gravity is constant, from which follows that flow in watercourses is regular. This is in contrast to lakes, where the principal forcing is from wind, and gravity only acts as a restoring and balancing force. Rivers transport water from lake to lake and onwards to sea, and the key questions regarding the hydrology of rivers is the transport of water and whatever it carries with itself, and the erosion caused by the river flow.

Figure 6.1: *Kiutaköngäs in river Oulankajoki, which discharges into the White Sea (left) and the Mälkiä lock in the Saimaa Canal (right). Photo: Matti Leppäranta.*

The transport of water in rivers is given by discharge, which has L^3 T^{-1} as its dimension. Water is collected to a river as runoff from the river's catchment area. If the flow velocity is in the order of 1 m s^{-1}, and the length of the river is 500 km, water moves through in about six days. In the World's greatest river, the Amazon River, the transport of water is 209 000 m^3 s^{-1}. In the Finland's greatest river, Kemijoki, it is 560 m^3 s^{-1} (Table 6.1). In Finland's lesser rivers, like Rivers Aura and Vantaa, the discharge is in the order of 10 m^3

s^{-1}. The accumulation of water in the river catchment areas per unit area is 2.6 mm day^{-1} for Amazon, 0.9 mm day^{-1} for Kemijoki and 0.1 mm day^{-1} for Nile.

Rivers are characterized by the surface area of its cross section A and by length L (Fig. 6.2). *Wet perimeter* is the distance of the cross section where water and soil are in contact. These two quantities form the *hydraulic radius* of the watercourse

$$R = \frac{A}{P}. \tag{6.1}$$

For every unit length of dx, gravity affects the whole water body Adx, but friction only affects the surface Pdx. Thus, the greater the hydraulic radius, the smaller is the relative effect of friction. Friction affects all the way along the watercourse, as a result of which the length of the watercourse affects the flow velocity. The height h of the watercourse defines its potential energy. As water flows, this potential energy is depleted by friction and transformed into kinetic energy.

Example 6.1
If the cross section of a river is rectangular with a width of 50 m and a depth of 5 m, the area of the cross section is 250 m^2 and the wet perimeter is 60 m. In this case, the hydraulic radius is 250 m^2/60 m = 4.17 m. In general, the hydraulic radius for a rectangular watercourse that has a depth of H is $R = \frac{H^2}{\delta}\frac{1}{2H+H\delta^{-1}} = \frac{H}{1+2\delta}$, where δ is the ratio between the river's depth and width. If the cross section is half circle with a radius of H, hydraulic radius becomes $R = {}^1\!/_2 H$.

The length of a river is much larger than its depth or width, and therefore rivers have the shape of a line. Water flows downstream approximately in its lengthwise direction. This allows us to consider any watercourse one-dimensionally, where the x-axis runs downstream along the middle of the course. The flow is dominated by the x-directional velocity $u = u(x, y, z; t) > 0$, velocity components perpendicular to the u-component, v (velocity across the river) and w (vertical velocity), are very small: $u \gg v, w$. Temperature, and thus also density, are considered to be constants. Turbulence takes care of the mixing, as rivers and streams are relatively shallow and have a strong current.

Discharge is the water cross section area A multiplied by the mean flow velocity U

$$Q = UA. \tag{6.2}$$

Discharge describes the total transport of water. For a typical Finnish river $U \sim 1$ m s^{-1}, $A \sim 250$ m^2, and hence $Q \sim 250$ m^3 s^{-1}. For a small ditch, $U \sim 0.1$ m s^{-1}, $A \sim 10$ m^2, and $Q \sim 1$ m^3 s^{-1}. The continuity equation and the equation of motion (see section 3.3), the basic equations of flow dynamics, can be applied to rivers. Defining the discharge is one of the prime questions of the research on watercourses. Flow is regular, because gravity is the driving force. Besides discharge, the transport of matter with the flow and the soil erosion in the catchment area and the riverbanks themselves are important questions.

Table 6.1: *Examples of length, discharge and catchment areas of the great rivers of the world and some Finnish rivers.*

	Length	**Discharge**	**Catchment area**
River	**km**	$\mathbf{m^3\ s^{-1}}$	$\mathbf{10^3\ km^2}$
Amazon	7 000	209 000	7 050
Nile	6 900	2 800	3 250
Yangtze	6 300	31 900	1 800
Mississippi-Missouri	6 280	16 200	2 980
Yenisei	5 540	19 600	2 580
Yellow River	5 460	2 100	745
Paraná River	4 880	18 000	2 580
Congo River	4 700	41 800	3 680
Lena	4 400	17 100	2 490
Niger	4 200	9 570	2 090
Volga	3 650	8 080	1 380
Eufrat	3 600	856	884
Rio Grande	3 060	82	570
Tonava	2 890	7 130	817
Ganges	2 620	12 000	907
Kemijoki	550	560	51
Tornionjoki	520	370	40
Kymijoki	204	280	37
Vuoksi	162	690	68
Kokemäenjoki	121	240	27
Vantaa	101	15	2
Aurajoki	70	7	1

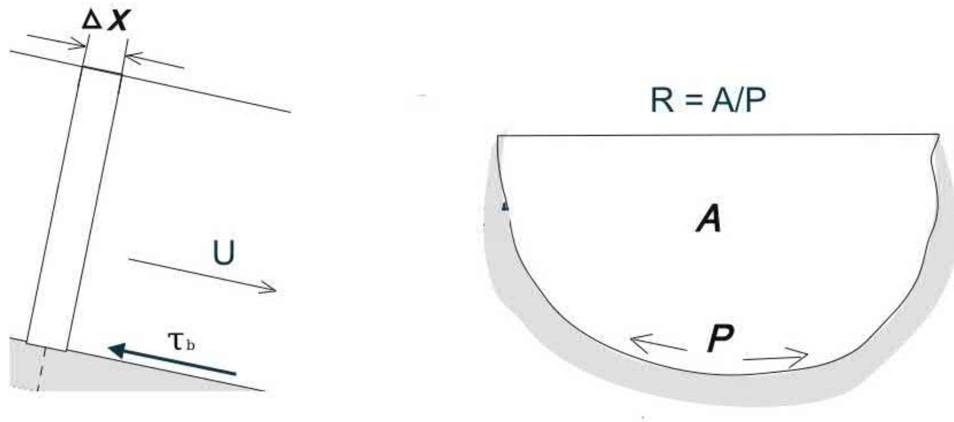


Figure 6.2: *Longitudinal section and cross section of a channel and the related quantities. Δx is a step in the direction of the channel, τ_b is the friction against the bottom over a unit of surface area, U is the flow velocity, A is the cross section area, P is the wet perimeter and L is the channel length.*

If the flow is stable and there are no tributaries (i.e. smaller streams joining the main flow), discharge has to be equal in all cross sections, $UA = constant$. This very central result for the research of rivers could be called the 'Leonardo da Vinci's equation', in honor of its developer. In an unstable situation the continuity equation can be applied, according to which the cross section of the river has to change if the discharge changes in the direction of the flow:

$$\frac{\partial A}{\partial t} = \frac{\partial Q}{\partial x} \tag{6.3}$$

where t is time. In other words, the water surface level in the river changes.

Turbulent flow mixes the water column effectively. If c is a mass concentration per unit volume, the mass transport is

$$M_c = \int_A cu\, dA. \tag{6.4}$$

In a well-mixed water column matter is spread evenly along the whole cross section, and mass transport can be estimated with the discharge. Then, mass transport becomes $M_c = cQ$.

The importance of rivers for mankind has manifested itself throughout history in many ways: through water supply, sanitation, fishing, transport, travel and recreation. As technology advanced, rivers began to be used for power production, from which conflicts arose between the usage of rivers and their protection and conservation (Fig. 6.3). In Finland, great rivers define landscapes and scenery, and Northern Bothnia is referred to as the *Land of the ten rivers* in the song of A. V. Koskimies:

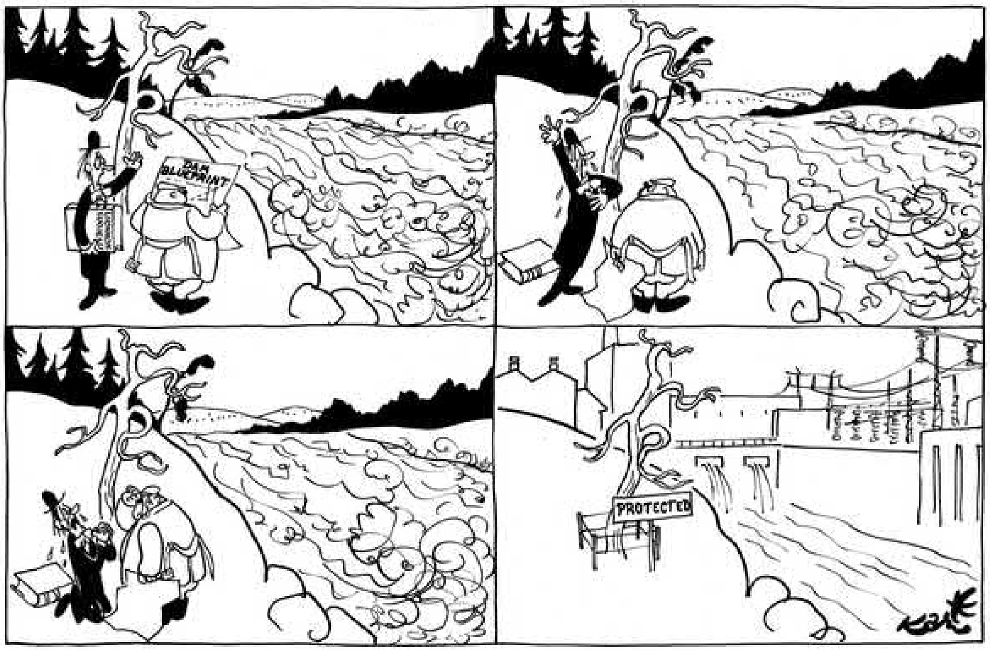


Figure 6.3: *'Concession', a strip by Kari Suomalainen, 19th May 1961. Translation: Joonatan Ala-Könni ©Kari Suomalaisen perikunta*

Kemi, Tornio, Ounas, Oulu ja Ii,	*Kemi, Tornio, Ounas, Oulu and Ii,*
olen nähnyt uomanne aavat.	*I have witnessed your wide plains.*
Ja mieleni laajeten lainehtii	*And my mind it expands, billows*
ja suoneni tarmoa saavat.	*and my veins, flowing with vigour.*
Jokes uljahat syöntäni suurentaa,	*Your rivers so proud they embiggen my heart.*
oi Kymmenen virran maa.	*O' the Land of the ten rivers.*

6.2 Flow dynamics in watercourses

6.2.1 Bernoulli's equation

The Navier-Stokes equation (section 3.3.1) is simplified greatly in the one-dimensional case, when the u and w components can be neglected as small. Then, the equation of motion can be written as

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - g\frac{\partial h}{\partial x} - \frac{1}{\rho}\frac{\partial p}{\partial x} + N\nabla^2 u. \tag{6.5}$$

On the left side there is the local change in flow velocity. On the right side there are the advection, the component of gravity that is along the inclined river, pressure gradient and internal friction. Hydrostatic balance prevails in cross sections that are perpendicular to the direction of the flow. The flow in rivers is turbulent, and the Reynolds number $Re = UL/\nu$ is greater than 10^4. Here, the scale of velocity is the flow velocity and length scale is the depth of the river. If, for example, $U = 1$ cm s^{-1} and $L = 100$ cm, then $Re = 7\,700$ and the flow is already nearly turbulent. Flow in natural rivers is faster, and the stream is typically larger, so the prevailing state of the flow is turbulent.

Let's start by considering a steady and frictionless flow, $\partial u/\partial t = 0$ and $N = 0$. Frictionless flow is far from the real state of affairs, but it still constitutes a useful theoretical reference. Equation (6.5) can then be written as

$$0 = \frac{\partial}{\partial x}\left(\frac{u}{2} + gh + \frac{p}{\rho}\right). \tag{6.6}$$

From this, the *Bernoulli's equation* can be derived (Daniel Bernoulli, 1700 – 1782):

$$\frac{u^2}{2g} + h + \frac{p - p_0}{\rho g} = constant, \tag{6.7}$$

where p_0 is the air pressure on the surface. Bernoulli's equation can be thought of as a balance of mechanical energy, which in equation (6.7) has been written as energy per unit of mass, also called the energy height. The terms on the left side of the equation describe the kinetic energy height, potential energy height (which is the same as the geometric height) and pressure height, respectively. On the surface of the stream $p = p_0$, in which case equation (6.7) describes the balance between kinetic and potential energies

$$\frac{u^2}{2g} + h = constant. \tag{6.8}$$

If $u = 0$, equation (6.7) results in the basic equation of hydrostatics, $p = p_0 + \rho gh$.

A second interpretation of equation (6.7) is acquired when it is multiplied by ρg. Then, the dimension of the terms will turn into pressure, and the resulting term $\frac{1}{2}\rho u^2$ is called the dynamic pressure. If $u = 1$ m s^{-1}, dynamic pressure is 0.5 kPa, which corresponds to the hydrostatic pressure created by a 5 cm layer of water. When the flow velocity becomes higher than this, dynamic pressure becomes relevant in the scale of the total pressure.

Example 6.2

Torricelli's (1608 – 1647) equation. Let us consider a wide vessel of water, that has a closed tube attached on the bottom of it. The tube is closed at a depth of h. When the tube is opened, water begins to flow out of it and the vessel empties. On the surface $u =\sim 0$, so the velocity of the water flowing out is $u = \sqrt{2gh}$ (Eq. 6.8). Torricelli's equation is a special case of the Bernoulli's equation.

Similarly, it can be shown that perpendicularly to the flow (y-axis) and in vertical direction (z-axis, or depth) the following holds:

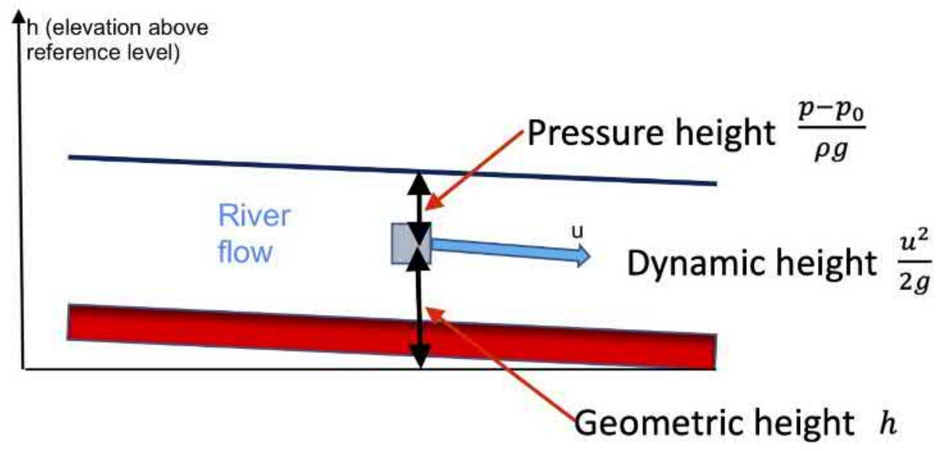


Figure 6.4: *Schematic representation of the Bernoulli Equation. Figure: Matti Leppäranta.*

$$0 = \frac{\partial}{\partial y}\left(\frac{p}{\rho}\right) \text{ and} \tag{6.9}$$

$$0 = \frac{\partial}{\partial z}\left(gh + \frac{p}{\rho}\right). \tag{6.10}$$

Across the stream *depth* = *constant*, and hence *pressure* = *constant*. Hydrostatic balance holds in the vertical direction.

Let us take a look at the steady state flow. We shall multiply all terms of Bernoulli's equation (Eq. 6.7) by $u\,dA$ and integrate over the cross section. This results in:

$$\int_A \frac{u^3}{2g} dA + \int_A u\left(h + \frac{p - p_0}{\rho g}\right) dA = \int_A u \cdot constant \cdot dA. \tag{6.11}$$

Flow velocity u along the stream is not quite constant due to boundaries, and the integration of its cube is done by using a correction factor α: $\int_A u^3 dA \approx \alpha A U^3$, where $\alpha \geq 1$ is dependent on the cross section of the flow. Based on empirical evidence, $\alpha \approx 1.3 - 1.4$ in natural rivers and streams. If the flow is constant throughout the cross section, $\alpha = 1$, which is the value often used if no other information is available. The second integral of the equation can be written as

$$\int_A u\left(h + \frac{p - p_0}{\rho g}\right) dA = \left(h + \frac{p - p_0}{\rho g}\right) \int_A u\, dA,$$

since the term inside of the brackets is a constant within the cross section. Because at the surface $p = p_0$, this constant is h. According to the continuity equation, the integral of the total transport of water $u\,dA$ over the cross section in a stable flow is UA, which is a constant. When the equation (6.11) is now divided by the discharge, we acquire Bernoulli's equation of flow in a watercourse:

$$\alpha\frac{U^2}{2g} + h = constant \tag{6.12}$$

Bottom friction is significant in the natural flow of a river, and for this reason Bernoulli's equation falls short downstream. This deficiency corresponds to a energy height equal to the work done by the friction.

Example 6.3
Let us have a look at a river leaving a lake. Water level in the lake is h_0, and flow velocity on the surface is $U_0 \approx 0$. Downstream the river $h < h_0$ and $U > 0$, and according to Bernoulli's (or Torricelli's) equation $U = \sqrt{2g(h_0 - h)}$. Flow velocity thus increases downstream. Because there is no friction, flow velocity is only dependent on the change in height, and not on the total length of the river. If $h_0 - h = 5$ m, then $U = 10$ m s^{-1}, which is way too big for a flow in a natural river. Typically, a 5 m drop in height corresponds to a ~ 5 km distance horizontally, which means a lot of energy is required in working against the friction.

6.2.2 Friction in the flow of rivers

Friction in turbulent flow, τ, is proportional to the square of the flow velocity. Hydrodynamic friction was scrutinized in section 3.3.1 (Fig. 3.11). Bottom friction of a river is here defined as $\tau = \rho C_b U^2$, where C_b is the resistance coefficient dependent on the bottom roughness. Work done by friction over a distance dx is

$$dW = \rho C_b U^2 P dx, \tag{6.13}$$

where P is the wet perimeter (Fig. 6.2). Friction loss is this work divided by the surface area A, and this in turn is transformed into loss in energy height h_f by dividing it further by ρg:

$$dh_f = \frac{C_b U^2}{Rg} dx. \tag{6.14}$$

We can add this to Bernoulli's equation, and in this way take friction into account

$$\frac{\alpha}{2g} U^2 + h - x\frac{C_b}{Rg} U^2 = constant, \tag{6.15}$$

where x is the distance covered by the flow. If conditions are steady, friction uses up the energy that is gained from the drop in height.

In a steady state $U = constant$. Then, equation (6.15) gives the classical formula of flow in a watercourse, *Chezy's equation*, from 1775:

$$U = \sqrt{\frac{hRg}{xC_b}} = C\sqrt{RS}, \tag{6.16}$$

where $S = h/x$ is the geometric slope of the river and $C = \sqrt{g/C_b}$ is Chezy's coefficient. Antoine Chézy (1718 – 1798) tried to estimate C for the River Seine during canal construction, and deducted that it is a constant. Later, experimental work around the world has revealed that it depends on the parameters of the river in question. In 1889, this research resulted in a form of Chezy's coefficient that carries the name of Robert Manning (1816 – 1897)

$$C = MR^{1/6} \tag{6.17}$$

where M is the Manning's coefficient. Table 6.2 has values of Manning's coefficient for various types of riverbeds.

Table 6.2: Manning's coefficients (M).

Riverbed	**M [m$^{1/3}$ s^{-1}]**
Smooth, solid and clean watercourse	40
Rocky bottom	25 – 28
Smooth ice	85 – 100
Smooth concrete	100

Example 6.4
Let us have a look at the effect of friction on flow velocity. Hydraulic radius is $R = 5$ m, slope is $S = 10^{-3}$ and Manning's coefficient $M = 30$ m$^{1/3}$ s^{-1}. In this case, $U = 2.8$ m s^{-1} and kinetic energy height is $\frac{\alpha}{2g}U^2 = 0.40$ m, when $\alpha = 1$ and it is assumed to remain constant in a steady flow. As the geometric height decreases, friction losses grow the same amount. At the beginning of this 100 km long hypothetical river, geometric height is $SL = 100$ m, and friction loss is zero. Along the way to the river mouth, work against friction is done for 99.6 m of energy height, and 0.4 m of it is transformed into kinetic energy. Without friction, flow velocity at the river mouth would be 44.3 m s^{-1}.

More advanced methods, like a friction coefficient based on Reynolds number and turbulence theory have not brought any meaningful improvements on calculating flow velocity in natural watercourses. Attempts have been made to find better methods for the estimation of Manning's coefficient, like the analysis of channel morphometry, but so far it has turned out, that it needs to be estimated on a case by case basis from measurements.

6.2.3 Waves in the river water column

Shallow water waves (see section 5.4.2) propagate along rivers at a speed of $v = \sqrt{gH}$. If $H = 2.5$ m, then $v = 5$ m s^{-1}, which is in the same range as is the flow velocity. Wave motions transport disruptions, and the ratio of the wave speed and flow speed tells whether these disruptions move upstream or not. For this reason shallow ater waves are important in river dynamics.

The ratio between flow speed and shallow water wave speed is defined as the *Froude number*, $Fr = U/\sqrt{gH}$ (see section 2.3.1). At the same time Froude number also describes the relationship between inertia and gravity. A flow is categorized into three distinct classes:

$$
\begin{aligned}
Fr &> 1 \quad \text{Supercritical flow} \\
Fr &= 1 \quad \text{Critical flow} \\
Fr &< 1 \quad \text{Subcritical flow}
\end{aligned}
$$

In supercritical flow waves are not capable of travelling upstream, since water flows faster than waves can move. In a subcritical situation waves are faster than the flow, and hydrodynamics become more difficult to handle.

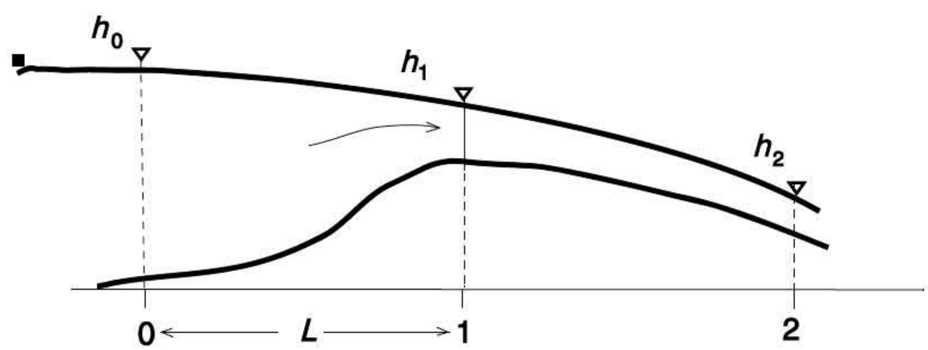


Figure 6.5: *Longitudinal profile of a river. Water level at the lake where the river starts is* h_0.

In an outlet river flowing from a lake (Fig. 6.5), an initial cross section can often be identified from where the river starts. It also defines the relation between discharge and water level. The highest point of the riverbed is usually on this spot, and it is called a sill. Quantities measured in the lake, at the initial cross section and downstream are marked with indices 0, 1 and 2 respectively. Because in the lake $U_0 = 0$, from the equation of energy height (Eq. 6.17) we get

$$U_1 = \sqrt{(h_0 - h_1 - h_f)\frac{2g}{\alpha_1}}. \tag{6.18}$$

Depending on the circumstances, two different cases for the height h_1 can be identified:

1. Height h_1 is only dependent on the height h_0. This is the situation when the flow below h_1 is supercritical, and disturbances formed below h_1 can not propagate upstream. This is the situation also, if the structures and basins affecting the flow are far enough downstream. Then, $h_1 = f_0(h_0)$.

2. Height h_1 is also affected by the water level at the point 2, in which case $h_1 = f_2(h_0, h_2)$.

Formation of supercritical flow requires a change in the watercourse. This change could, for example, be the sill described in figure 6.5. Flow speed in a critical flow is $U_c = \sqrt{gH_1}$, where H_1 is the depth of the watercourse at the sill. Because the distance between the lake and the sill is short, the effect from friction can usually be neglected. When we select the water level at sill as the reference level, equation (6.18) becomes

$$U_c = f_c\sqrt{\frac{2}{3}gh_0}, \tag{6.19}$$

where f_c is a coefficient dependent on friction and the shape of cross section.

Measurements of water level are preferably arranged in such a way that the assumptions of case 1 are applicable. Function f_0 can then be translated into a *stage-discharge relation*. It describes the relation between the lake water level and discharge rate (Fig. 6.6). It is defined by performing simultaneous measurements of discharge and water level. Constructing a single parameter stage-discharge relation requires that the watercourse is steady state and that it can not have changing conditions, like ice or vegetation. If the water level changes so fast that Bernoulli's equation for steady flow is not applicable, stage-discharge relation might become like the curve b in figure 6.6.

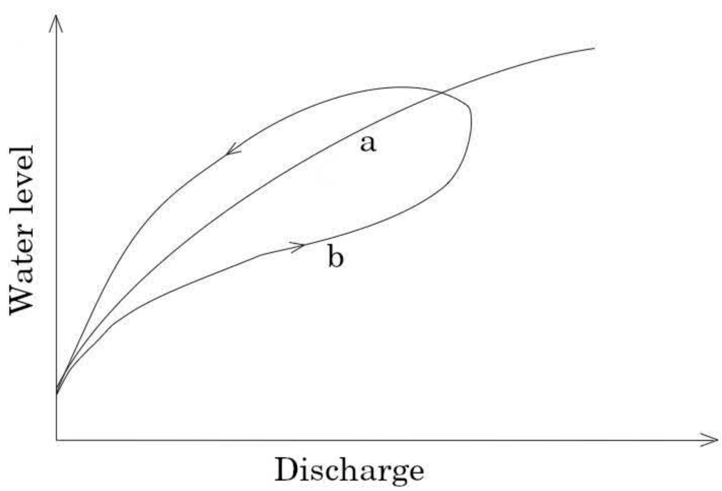


Figure 6.6: *Stage-discharge relation for a supercritical flow in a stable situation (a) and in an unstable situation (b).*

In Finland, the discharge is usually estimated with a single parameter stage-discharge relation. Two parameter relation (dependent on water level above and below the sill) is required, when the difference in water level between two consecutive lakes is small. As examples of stage-discharge relations are two figures: figure 6.7, which is a single-parameter relation composed by the Hydrological Office in Finland, and figure 6.8 which is a two-parameter relation, where the discharge is dependent on water levels above and below the sill. Time series of discharge are usually devised from water level and stage-discharge relation.

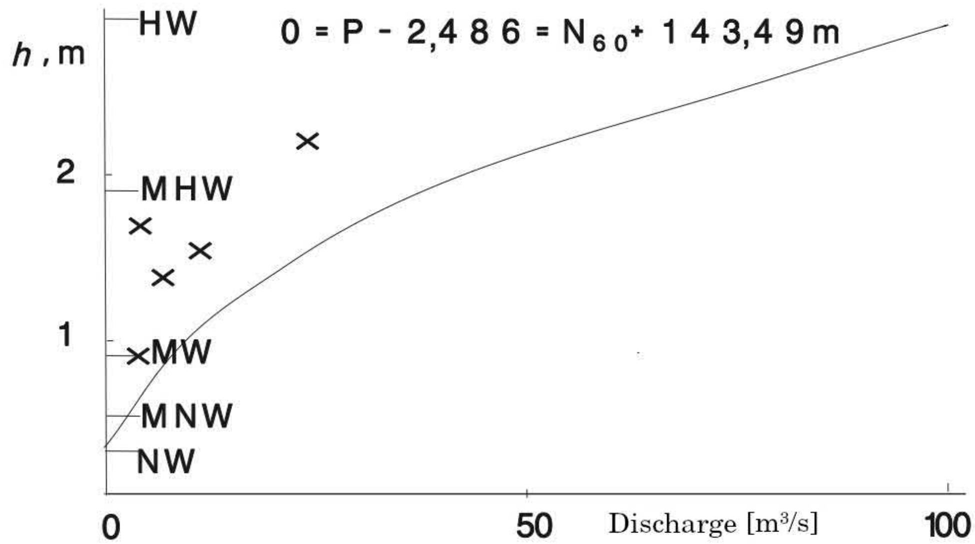


Figure 6.7: *Stage-discharge relation for the Kouheroisenkoski rapids in the Saarijärvi route. Text at the top of the figure describes the baseline water level (m) in relation to the reference point (P) and to the elevation standard of 1960 (N60). Winter measurements are marked with x. HW = high water, MHW = mean high water, MW = mean water, MNW = mean low water, NW = low water.*

6.3 Measuring discharge and water level

Discharge has been measured with current meters. The classical instrument is a propeller shaped device, the rotation rate of which is dependent on flow velocity. In vertical direction, 3 – 7 measurement depths are selected, and in horizontal direction this vertical flow profile is measured at 3 – 5 locations. Discharge is acquired by integrating over the whole cross section, which is the definition of discharge. Nowadays, mechanical current meters have been mostly replaced by acoustic meters based on the Doppler shift, which are able to measure the flow velocity in the whole water column simultaneously (Fig. 6.9). These direct measurements are used in the construction of stage-discharge relations.

In small streams, *measuring weirs* (Fig. 6.10) of different shapes and sizes can be utilized. Weirs allow the theoretical calculation of a stage-discharge relation. Figure 6.11 has examples of various measuring weirs. Figure 6.11a represents a triangular weir with 90° notch. In Finland, weirs are shaped like in figure 6.11b. This shape ensures that discharge can be accurately measured during both high and low water levels without the height difference becoming too large (Mustonen, 1965). The lower part of the weir is also easy to protect from freezing during the winter. Figure 6.11c describes a rectangular weir, and figure 6.11d depicts how water moves over the sharp edge of the weir.

Stage-discharge relation can be constructed for the rectangular weir (Fig. 6.11c) when the height h is measured from the lower edge of the weir as positive in the upwards direction. Width of the weir is B. Dashed lines denote streamlines (which can be interpreted as a tube), whose energy height in the upper basin ($U = 0$) is $h = h_0$. The target of this calculation is the cross section where the bottom of the stream ejecting through the weir is at its

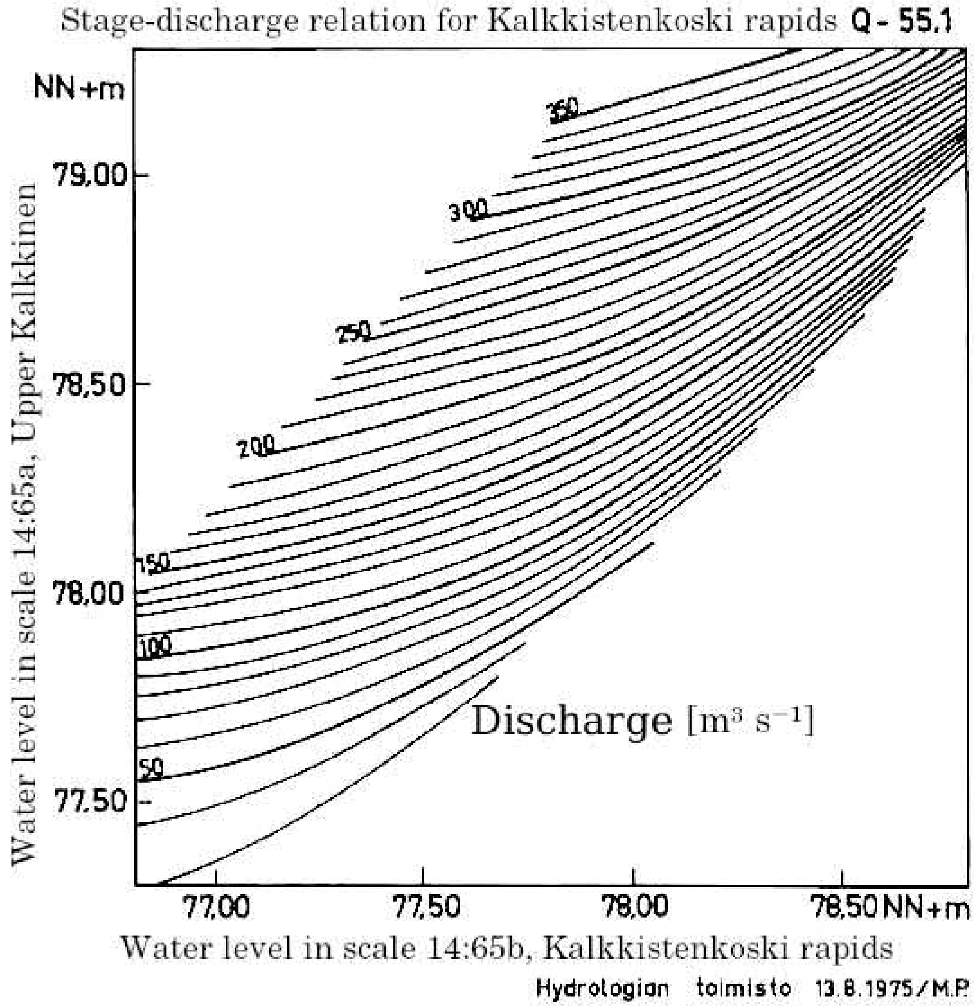


Figure 6.8: *Stage-discharge relation for the Kalkkistenkoski rapids in Päijänne (Mustonen, 1986). Vertical axis has the water level in lake Päijänne and horizontal axis has the water level at the Kalkkistenkoski rapids. Water level is given in relation to the elevation system NN.*

highest. There, the direction of the flow is mostly horizontal, and the vertical acceleration is zero. Additionally we assume, that the flow is frictionless for the short distance between the upper basin and the weir notch. According to Bernoulli's equation the stream speed is $u = \sqrt{2g(h_0 - h)}$. discharge Q is now acquired by integrating

$$Q = \int_{\Delta_1}^{h-\Delta_2} \sqrt{2gy} B dy = \frac{2}{3}\sqrt{2g} B h_0^{3/2} \left[\left(1 - \frac{\Delta_2}{h_0}\right)^{3/2} - \left(\frac{\Delta_1}{h_0}\right)^{3/2} \right], \tag{6.20}$$

where $y = h_0 - h$. Lengths Δ_1 and Δ_2 are difficult to measure, but they are small compared to h_0. The terms inside of the square brackets can thus be condensed into a single term, the discharge coefficient m. This can be written as the *dam equation*:

Figure 6.9: *The most significant innovation in measuring flow velocity since the propeller current meter has been the Acoustic Doppler Current Profiler (ADCP). In Finland this method was adopted in 1994, and it has made the measurement of large discharge values significantly easier. Photo: SYKE (Kuusisto, 2008).*

$$Q = \frac{2}{3} m \sqrt{2g} B h_0^{3/2}. \tag{6.21}$$

The discharge coefficient has to be determined by calibration, but typically it ranges between 0.6 – 0.7. Because Δ_1 and Δ_2 are dependent on h_0, m is also dependent on it.

Example 6.5
Discharge is easy to determine from the power output, efficiency and elevation drop of hydroelectric power stations. Power output is $M = \gamma \rho g (h_0 - h_1) Q$, where γ is the efficiency coefficient, and h_0 and h_1 are the respective heights of the upper and lower basins. If the height difference is 10 m and discharge is 100 m^3 s^{-1}, the power output with a hypothetical 100 % efficiency would be 10 MW.

Figure 6.10: *Melt water flowing through the Kotioja measurement weir in Ranua. The site was founded in 1976 for the investigation on the effects of forestry. Photo: SYKE (Kuusisto, 2008).*

Thus, determining discharge can often be made just by measuring the water level, which is much simpler. There are three ways of measuring water level (Fig. 6.12):

- With a fixed scale, drawn to a rock, bedrock or a pillar of a bridge
- With a pole fixed to the bottom, or with a portable stake with a scale
- With a recording device, such as a *limnigraph*.

The most common type of a limnigraph is one where changes in water level are drawn via a float and a thin wire to the memory of the device, digital or analog.

Discharge can also be derived by using an artificial tracer, that moves along with the flow and whose concentration is easy to determine. Various salts, dyes and radioactive substances have been used as tracers. Tracer can be added into a river in a solution, as a singular dose or at a constant feed rate q.

When a constant feed rate is employed, the discharge Q can be determined. After the adjustment time, the tracer concentration downstream has reached a steady state of c_2, and the tracer flow is c_2Q. If the tracer concentration in solution that is fed into the water is c_1, then by the conservation of matter it can be stated that $c_1q = c_2Q$.

Discharge can be determined also by releasing the whole tracer dose C into the river at once. Downstream from the release point, where the tracer has completely mixed into the water, concentration c of the tracer is measured as a function of time (Fig. 6.14). Because the tracer has been mixed into the whole cross section, the instantaneous flow of matter is

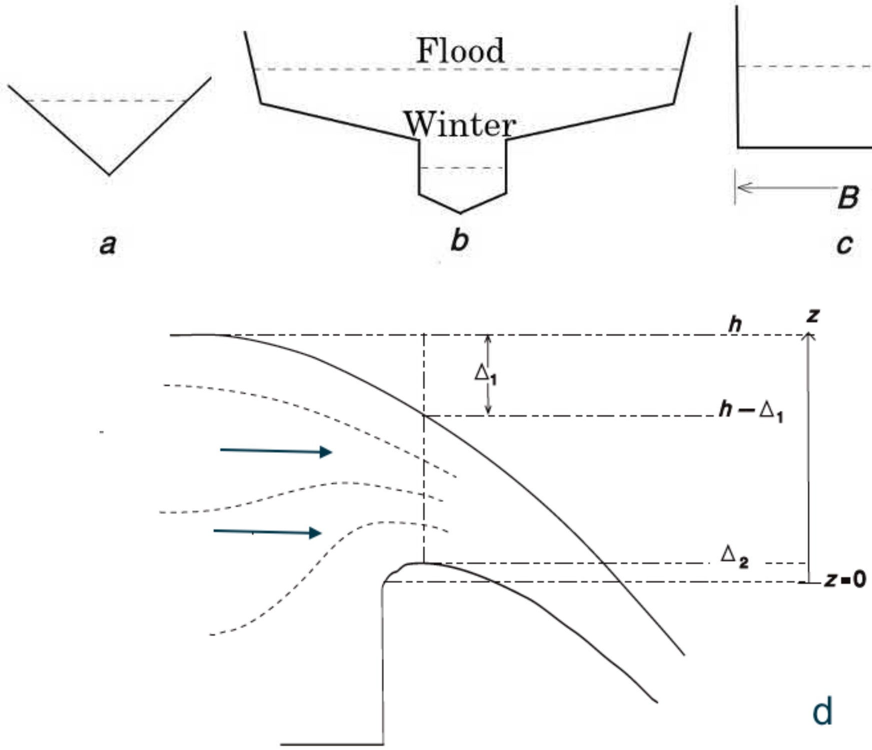


Figure 6.11: *Various weir types: a) triangular weir with a 90° angle, b) the most common weir in Finland, c) rectangular weir and d) flow of water past a weir.*

$M = cQ$, and the total amount of tracer that goes through the cross section can be acquired by integrating

$$C = \int_0^\infty M dt = \int_0^\infty cQ dt = Q \int_0^\infty c dt. \tag{6.22}$$

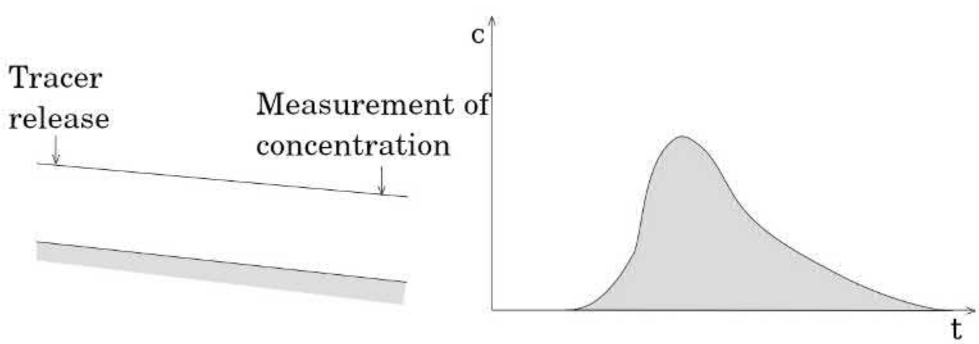


Figure 6.14: *Measuring discharge with a tracer. Tracer input is instantaneous.*

Figure 6.12: *Stamp commemorating the Pyhäkoski hydro power station (1963). The power station is located in river Oulujoki, and is one of the biggest in Finland. It was opened in 1951. Its output of electricity is 122 MW, and the fall height is 32.4 m. The stamp was designed by Olavi Vepsäläinen.*

6.4 Winter conditions

6.4.1 Freezing of rivers

In Finnish conditions, river and canals freeze in winter (Fig. 6.15). Ice conditions differ significantly from those found in lakes. The most important, and also the most visible, differences are the occurrence *frazil ice* and *ice jams*. Development of the ice conditions is intimately connected to the flow, and as a consequence river ice receives a much more layered structure than lake ice.

Especially in the autumn, turbulent flow keeps the water in a river well mixed, since density differences in cold water are small. Thus, the temperature of the water column can be assumed constant. Flowing and cooling water will eventually supercool[1], but only for a few hundredths of a degree. Ice cover begins to form from the shores. Places of intense turbulence have plenty of particles suspended in the water, which function as freezing nuclei. Supercooled water will nucleate onto these. In addition to suspended matter, snow flakes falling onto the surface of water can act as the freezing nuclei. Frazil ice, which consists of small and freely floating crystals of ice, is formed in this manner. As the small crystals freely move with the flow, they can stick onto the bottom sediment and form anchor ice, or onto the underside of the ice cover. From the river bottom, frazil ice can lift itself to surface due to its buoyancy.

In springtime, the ice cover will warm up and become soft due to sunlight. Eventually, the ice cover breaks into floes, which begin to drift downstream with the flow. Occasionally, they pile up forming ice jams.

[1]Supercooling of a liquid refers to a situation, where the temperature of the liquid falls below the freezing point, but still stays liquid.

Figure 6.13: *Water level scale and an observer in 1920s. Photo: SYKE (Kuusisto, 2008).*

The formation of frazil ice collects suspended matter from the water and brings that to the ice cover. Therefore, river ice contains impurities, unlike lake ice, which usually is very clean. Frazil ice that has first become anchor ice can even lift clumps of sediment to the ice cover.

6.4.2 Frazil ice and ice cover

Heat exchange between the river and the atmosphere happens as presented in chapter 4. As long as the surface of a river producing frazil ice is open, the water surface is at the freezing point. If the air is cold, turbulent heat exchange removes heat from the water into the atmosphere very efficiently. During frazil ice formation, the heat balance equation can be written as

$$\rho_i L_f \eta = -Q_n \tag{6.23}$$

where ρ_i is the density of ice, L_f is the latent heat of freezing, η is the volume of the produced ice per unit of area and time and Q_n is the surface energy balance. Crystals of frazil ice are small, between 0.1 mm and 1 mm.

The surface energy balance was addressed in section 4.1. During frazil ice production the following can be said about energy balance. Long wave radiation, of both the atmosphere and the water surface, and turbulent heat exchange are the dominating factors. The surface of the water can be considered to be at the freezing temperature. A linear model of the heat balance can be applied:

Figure 6.15: *A river in winter. The surface is partially covered in ice and partially open. Photo: Matti Leppäranta.*

$$Q_n \approx K_0 + K_1(T_a - T_0), \tag{6.24}$$

where K_0 and K_1 are model parameters dependent on time. To some extent they can be considered as constants. K_0 is determined by the radiation balance and K_1 mostly by the turbulent heat exchange. Under winter conditions $K_0 \approx -30$ W m^{-2} and $K_1 \approx 10$ W m^{-2} °C^{-1}. When frazil ice is forming $T_0 \approx 0$ °C and $Q_n < 0$, equations (6.23) and (6.24) give a value of $\eta = 6.5$ cm day^{-1} °C^{-1} for the ice growth rate, if the air temperature is -20 °C. Ice growth rate is thus larger for frazil ice production than for congelation ice growth, where the ice grows to the bottom of the ice cover and the ice slows down its own growth by insulation.

Solid ice cover begins to form from the shores towards the mid-stream, similar to congelation ice formation on lakes (see section 5.3.2). Due to frazil ice, river ice has more layers than lake ice. In the dead of winter, most rivers in Finland are covered by a solid ice cover, but places of rapid flow have permanent open water with frazil ice production. Frazil ice can form thick masses of slush, which in turn can cause floods and issues to power stations as they stick onto their machinery.

When a solid ice cover has formed, its thickness h increases when heat is transported from the water into the atmosphere ($Q_n < 0$). Turbulent flow brings heat onto the bottom surface of the ice, which slows down or prevents the formation on new ice. Temperature of the ice bottom surface is $T_h = 0$ °C in fresh water. Let's approximate the transport of heat with heat conduction through the ice. Then,

$$\rho_i L_f \frac{dh}{dt} + Q_w = k_i \frac{T_h - T_a}{h} \text{ and} \tag{6.25}$$

$$Q_w = \rho c C_H (T_w - T_h) U, \tag{6.26}$$

where Q_w is the heat flux from the water into the ice bottom, c is the specific heat capacity of water, $C_H \sim 10^{-3}$ is the heat exchange coefficient and T_w is the temperature of the water. The left side of equation (6.25) is the latent heat released by the formation of ice and the transport of heat from water to ice. The right side is the conduction of heat from the ice into the atmosphere. Heat originating from the water can be written as $Q_w = K_2(T_w - T_h)$, where $K_2 \approx 500$ W m^{-2} °C^{-1}.

Equations (6.25) and (6.26) do not have a general solution, but two special cases shed light on the growth of ice. If there is no heat coming from the water (Eq. 6.27), or if the ice thickness is constant and the heat going into the ice equals with the heat conducted through it (Eq. 6.28), we get, respectively,

$$h = a\sqrt{S},\ a = \sqrt{\frac{2k_i}{\rho_i L_f}},\ S = -\int_0^t T_0(t')dt' \text{ and} \tag{6.27}$$

$$h = \frac{k_i(-T_0)}{K_2 T_w},\ T_0 < 0\,°\text{C},\ T_w > 0\,°\text{C}, \tag{6.28}$$

where S is the sum of negative degree days and T_0 is the ice surface temperature. Often it is assumed that $T_0 \approx T_a$, but when the ice cover is thin this is not a very good approximation. The equilibrium thickness given by equation (6.28) is applicable, when the temperature of the water is at least 0.1 °C, otherwise reaching the equilibrium takes so long that it is not feasible in the real world.

In winter the discharge is usually rather small, since most of the precipitation is stored into the snow cover. Still, floods during warmer winters is something not unheard of. Ice cover slows the flow down due to friction, since ice makes the wet perimeter of the river longer and reduces the hydraulic radius, from which follows that the flow speed drops (see the Chezy-Manning formula, Eqs. 6.16 & 6.17). Thickness of the ice cover is in the same range as in lakes. Turbulence of the river brings heat from the water column, which slows down the growth of ice. On the other hand, places with open water produce extra frazil ice that can make the ice cover thicker downstream.

As frazil ice crystallizes on solid particles, it effectively collects solid matter from the flow. For this reason, layers formed by frazil ice contain plenty of particles (Fig. 6.16). Table 6.3 contains comparisons between the amounts of impurities in lake and river ice. These numbers contain also the fallout of particles from the atmosphere.

6.4.3 Ice break-up

Ice break-up in rivers is also a dynamic phenomenon. In spring, the ice cover becomes soft due to warm air and solar radiation. Eventually, it breaks and the now freely floating ice

Table 6.3: *Impurities accumulated in the ice cover of river Porvoonjoki and two lakes during 1997 – 1999 (Leppäranta et al., 2003). Values are averages from three winters. Ice thickness was 40 – 50 cm.*

	Dissolved matter [mg/L]		**Suspended matter [mg/L]**	
Site	**Water**	**Ice**	**Water**	**Ice**
Lake Vesijärvi	52.3	12.7	1.1	2
Lake Päijänne	31.3	11.3	1.4	2.1
River Porvoonjoki (site 1)	127.3	24	16	190
River Porvoonjoki (site 2)	135	24	16	190

floes travel along the flow. The movement of these floes is affected by the friction between them and the friction on water and shore. As groups of floes converge together, their internal friction increases and the floes can jam up. Ice jam formation is dependent on the morphology of the river, ice floe size and composition and the flow velocity. Critical flow velocity for dam formation is

$$U_c = K\sqrt{2g\frac{\rho - \rho_i}{\rho}h}, \tag{6.29}$$

where ρ is the density of water, $K \approx 1$ is a coefficient dependent of the shape of the floes and h is the thickness of the floes. If $U > U_c$, then the floes begin to pile on top of each other (Fig. 6.17).

ice jams have internal cohesion, and they will only break under a sufficient load. The rise of water level and accumulation of additional ice will increase the pressure exerted on the jam, while the ice floes becoming soft weakens the jam. Eventually, the jam will break and the floes will move forward with the stream, possibly to form a new jam downstream. Nevertheless, every movement brings the ice closer to the mouth of the river. Ice break-up on rivers can be described as a series of quasi-steady states, where ice jams are formed and freed up again as the masses of ice drift and stand still in a marching order dictated by the dynamics of the system of jams.

ice jams can cause floods and to prevent them, jams are sometimes blown up or booms are installed on the river to control the flow of ice. Especially in the rivers of Southern Lapland, cuts are sawn into the ice before it breaks up with specialized tractor driven circle saw, so that the break-up can be more controlled.

6.5 Water as an eroding and transporting medium

6.5.1 Modes of transport

Hydrological cycle transports not only water, but heat, dissolved and solid matter. In rivers, the transport of mechanical energy is also important. The upcoming section discusses the

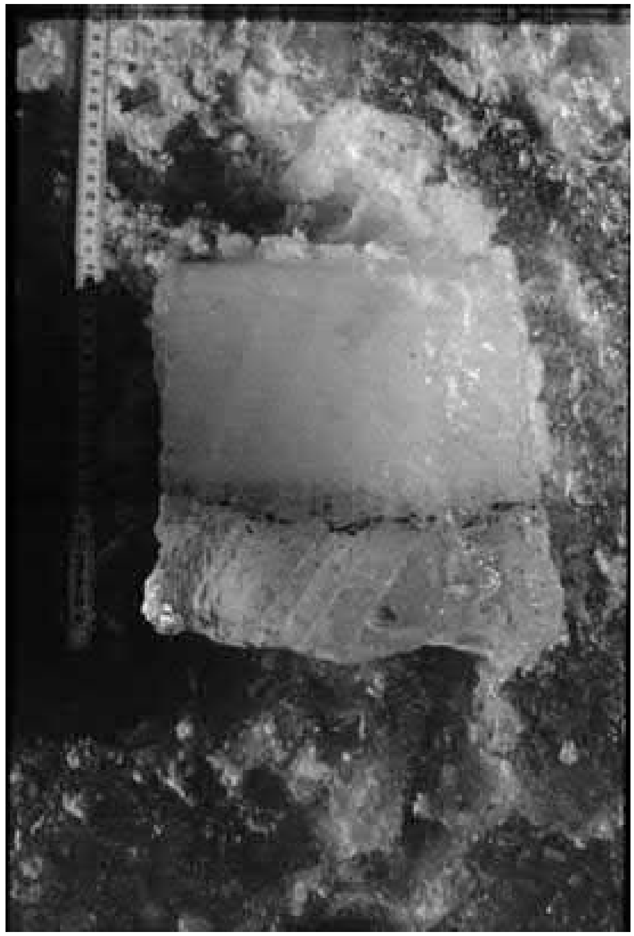


Figure 6.16: *Vertical structure of ice in River Porvoonjoki at Pukkila, March 1997. Photo: Matti Leppäranta.*

transport of solid matter along with the channel flow, i.e. sediment transport. Suspended matter can enter a river with runoff or carried by wind.

Solid (particulate) matter transported by the flow of rivers is divided into two groups:

- *Suspended matter*, which stays in the liquid due to the mixing involved with the turbulent flow
- *Bed-load transport*, which is composed of particles that roll on the bottom, or that get lifted occasionally from the bottom to slowly settle back there a little later. The size of these particles is so large that they will not stay in the water body.

The amount of suspended matter and bed-load transport is dependent on the flow velocity, density and viscosity of the liquid and the density and shape of the particles. According to Stokes' law (Eq. (3.4)) the sink velocity of a particle is dependent on the particle. Transport of particles happens in three steps: *detachment or erosion – transport – settling or accumulation.*

Based on their origin, particles are divided into three groups:

Figure 6.17: *An ice jam has lifted the water level of river Kalajoki 24th of April 1912. Local people have always gathered to witness the ice break-up. Photo: SYKE (Kuusisto, 2008).*

- *Suspended particles* are eluviated from the ground surface along with surface runoff.
- *Bed-load transport* is mostly composed of matter from the bottom of the channel.
- *Industrial and household waste* is usually organic and thus has a smaller density than minerals. Typically, they have a fiber like consistency.

6.5.2 Effects of sediment discharge on ground

The most visible effects of the transport of solid particles is the slow lowering of high ground forms, especially mountains, and the filling of low-laying ground. Discharge of sediments from the continents into the sea is continuous, causing a lowering of the continental mean height. It has been estimated that the average rate of erosion is 13.6 km^3 per year, which would add up to a mean rate of 3 cm per millennium for the lowering of continents. In addition, as a direct consequence of sediment particle transport, rivers change their shape. Either continually in the same direction, or periodically changing the direction of erosion (Fig. 6.18).

Water, changes in temperature and chemical processes cause the weathering of solid bedrock into small particles. When the particles formed in weathering get small enough, they begin to move with the flow of wind and water. Local variations in the rate of erosion are large. Erosion is at its most intense in areas where changes in temperature and precipitation are great. the greatest erosion happens in the Monsoon areas of Southeast Asia and the least erosion occurs in the deserts of Africa and Asia. Table 6.4 lists rivers that have a large rate of solid particle transport.

Figure 6.18: *Erosion is very strong along the Yellow River, and the shape of the river changes on the wide plains. The larger photo shows river plains. The smaller one shows erosion at a shore. Photo: Matti Leppäranta.*

In some smaller rivers of Indochina the solid matter transport is over 7 000 tons per square kilometer per year. In the northern zones, erosion and transport is small. In Northern Europe, vegetation protects the soil from erosion. According to measurements done in Sweden, the sediment discharge rate per surface area of River Torniojoki is 4 tons per year, and the River Ångermanjoki discharges 2 tons per year into the Sea of Bothnia.

In addition to changing the ground surface, sediment discharge also changes the river itself (Fig. 6.19). The geometry and morphology of rivers change over time. For example the Yellow River changes continuosly as it meanders its way on the wide plains. Changes in rivers can be categorized into a few types:

- *Meandering* is the occasional erosion and accumulation along the channel, which creates steep curves in the river. When the curving has proceeded far enough, the channel will straighten and the meandering starts over.
- *Silting* is a process associated with changes in the discharge. As the discharge increases and the flooding starts, the flow velocity is so large that it suspends solid particles from the bottom and transports them. This makes the channel deeper. When the flooding ends, the flow is not strong enough to suspend particles any more and they settle onto the bottom again.
- *Filling of reservoirs* happens when a river discharges into a reservoir where the flow velocity decreases and the particles settle to the bottom. This process fills river pools, lakes and artificial reservoirs, and is also the process behind the formation of river deltas.

Table 6.4: *Sediment discharge of selected rivers.*

River	**Surface area of the catchment** [10^3 km^2]	**Sediment discharge** [10^9 kg/a]	**Sediment discharge per unit of area** [10^3 kg/a/km^2]
The Yellow River	673	1 900	2 800
Ganges	956	1 450	1 500
Brahmaputra	670	730	1 100
Yangtze	1 900	500	260
Indus	970	440	450

6.5.3 Particle transport in a river

Water flow causes a shear stress over the bottom of the river, which resuspends particles from the bottom. As per the turbulent friction law, this force is proportional to square of the flow velocity. Gravity and cohesion forces between the particles try to hold the particle on the river bottom.

In figure 6.21 the various zones of particle transport are presented in a (u, d) coordinate system, where u is the flow velocity and d is the particle diameter. This type of a plot is called Hjulström's diagram (Graf, 1971). When the flow velocity is low and particle diameter is relatively small ('transport' zone) there is no suspension or settling. Sufficiently high flow velocity begins to resuspend particles from the bottom and into the flow ('erosion' zone). On the other hand, when the particle size exceeds a certain value particles begin to settle onto the river bottom ('deposition' zone). Particles with a diameter of 0.2 – 0.3 mm suspend and detach with the most ease. This is due to the fact that as the particle size drops the cohesion forces between the particles increase and with bigger particles gravity begins to dominate. In the aforementioned range of diameters the combined effect of gravity and cohesion forces are the smallest.

Because the suspension of a particle from the channel bottom is mostly a function of the flow velocity, relationships between flow velocity, discharge and sediment discharge has been under considerable scrutiny. If the sediment originates from the channel, this dependency can be rather strong. If on the other hand most of the sediment particles originate from the surface runoff of the catchment area, the dependency is weaker. Then, the time of maximum discharge and maximum sediment discharge might not match. The lag between these peaks can vary from hours to months.

This chapter has been about the flow, morphology and erosion on rivers, streams and canals. Rivers form an effective way of transport between water systems. Under winter conditions ice is formed, which has an effect on the flow and causes frazil ice formation and ice jams. In the next chapter, properties of ground water are taken a look at.

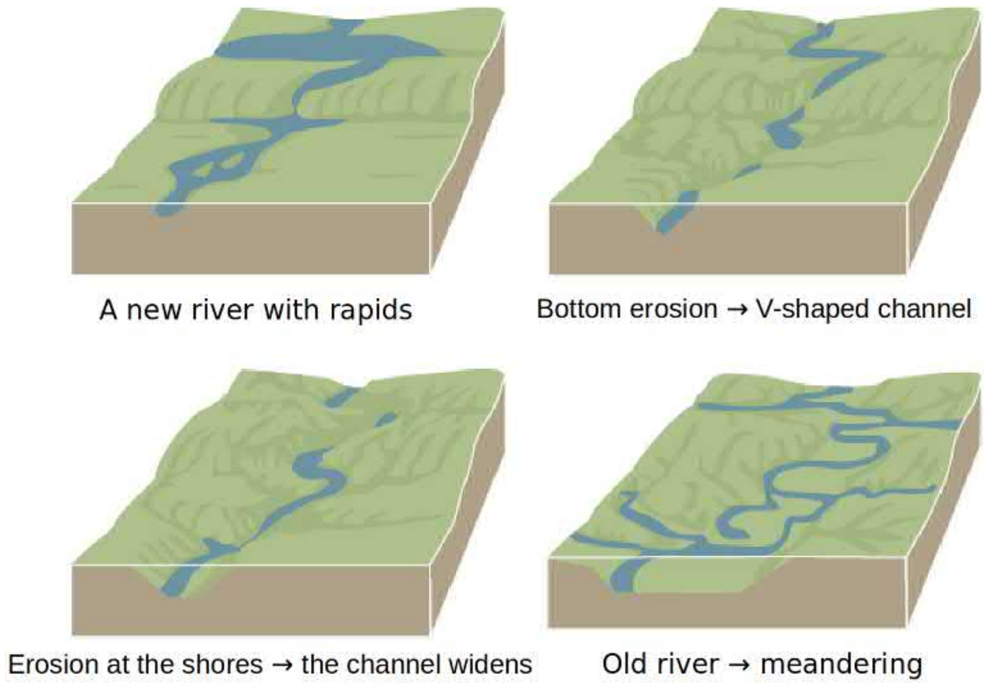


Figure 6.19: *Development of the geometry and morphology of a river over time.*

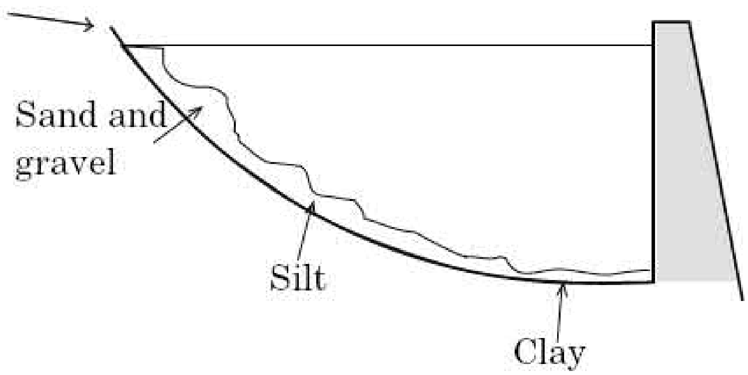


Figure 6.20: *Filling of a basin with sediment. Coarser grains settle near the river mouth and finer particles get transported further.*

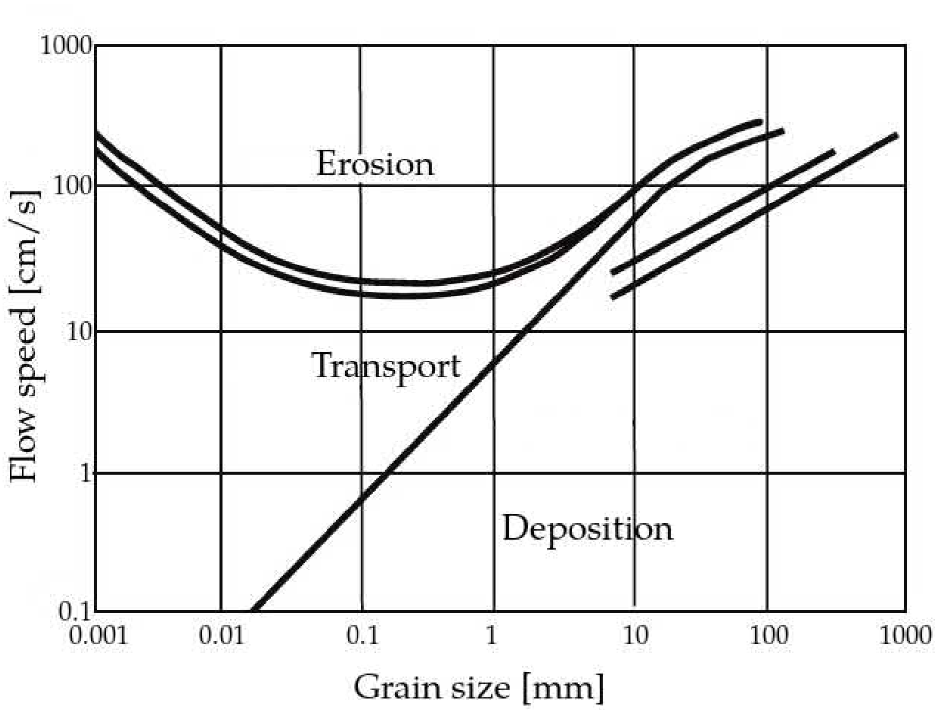


Figure 6.21: *Regions of particle transport in rivers according to Hjulström. Source: Graf & Altinakar (1971), modified.*

Chapter 7

Geohydrology

This chapter will be revolving around ground and its water resources. These resources are split into two types: soil water and groundwater. Groundwater forms the greatest storage of liquid fresh water on Earth. Soil water moves downwards pulled by gravity while groundwater is pushed onto move by pressure. In cold climate zones, water in ground freezes and forms frost. There exists both seasonal frost and permafrost. At the end of this chapter, glacial hydrology is briefly discussed.

7.1 Soil water

7.1.1 Soil type and water retention capacity

From the eyes of the mankind, Earth's lithosphere contains the most important fresh water resource of the planet (Fig. 7.1). Physical behaviour of this water is dependent on the and type. In Finnish climate, surface layer water freezes in the winter, which has an effect on the annual hydrological cycle. In climate zones colder than Finland, permafrost exists.

Water content in soil is dependent on the season, with highest values reached during the autumn rains. During winter, the soil water storage hardly changes at all, and in the spring the melting of snow increases the amount of water in the soil. Plentiful evaporation during the summer temporarily decreases the water storage in soil, with minimum being reached in August – September.

Underground water resources are divided into *soil water* and *groundwater* (Fig. 7.2). Soil water resides in the ground surface layer, where it is bound by the soil particles and forms an unsaturated zone. Plants take water from this layer, which has plenty of oxygen and where biological activity is strong. Flow of water is downwards.

Soil water can be discerned into three zones: *root zone, intermediate zone* and *capillary zone*[1]. Precipitation cast down on Earth partially flows as surface runoff, and partially infiltrates into the soil. Of this infiltrated water, a part stays in the root zone and gets transpired away. A part of the infiltrated water gets all the way into the capillary zone, which lays just above the groundwater. This slowly descending water is called gravitational water or percolating water.

[1]Sometimes also called the *capillary fringe*.

Figure 7.1: ***a)*** *Wet bog in Kainuu, Northern Finland,* ***b)*** *Arvo Koho maintaining a ground frost tube in 2004. Photo: a) Matti Leppäranta, b) SYKE (Kuusisto, 2008).*

Retention and movement of soil water is fundamentally dependent on the soil type. Soil types are classified into mineral soils, organic soils and chemical sediments. Soil separate is a separable fraction of mineral soil, that has a specific grain size. Soil types are classified by their grain size. This classification is very coarse and it varies a bit, depending on the application.

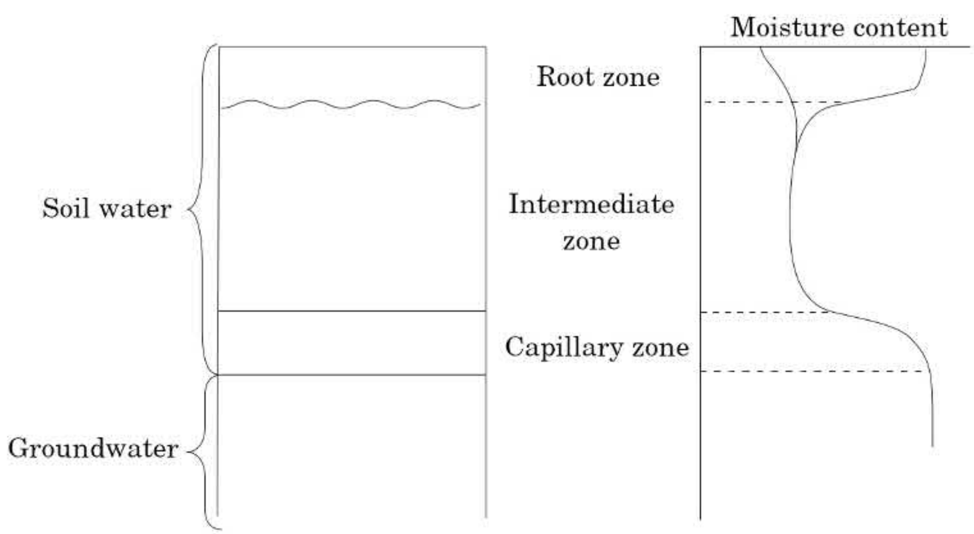


Figure 7.2: *Water zones in the ground and the corresponding profile of soil moisture content.*

Table 7.1: *Soil classes and their respective grain size distributions. Source: National Board of Waters (1976).*

Soil separate	**Grain size [mm]**
Clay	< 0.002
Silt	0.002 – 0.06
Sand	0.06 – 2
Gravel	2 – 20
Rocks	20 – 600
Boulders	> 600

Soil type gets its name (silt, sand, gravel) based on the median of the grain size distribution. Figure 7.3 represents some examples of grain size distributions. Clay stands for soil that has over 30 % of its mass as clay class grains. Moraine is a soil type formed in glacial processes containing unsorted grains. Moraine soil types are classified into silt, sand and gravel moraines. Peat and mud are organic soil types. Humus content of mud is over 6 % of its mass. During dry periods soil shrinks, which is affected by the soil type, and humus and clay content. Dry shrinking of soil increases when the fraction of organic matter in soil increases. It is 40 – 70 % when the fraction of organic matter is 6 – 20 %.

Example 7.1
Small grains are usually present in much greater numbers than bigger grains. If the probability density of soil grains is $f(R)$, where R is the grain radius, then the volume required by all the grains smaller than R is

$$V = \frac{4}{3}\pi \int_0^R r^3 f(r)\, dr$$

According to this formula if $f(R) \propto R^{-3}$, then volume is evenly distributed among all grain sizes. The exponent -3 in the number distribution describes the fact, that when the grain size drops to $1/10$, they grow thousandfold in numbers.

A molecule of water is a dipole, or in other words, it has an asymmetrical electrical charge (see section 3.1 & figure 3.1). For this reason it is in electric interaction with other molecules. The surface of soil particle is usually negatively charged, and due to the dipole forces molecules of water *adsorb*[2] to the surface of soil grains. Dipole forces dominate within 0.5 μm from the surface of the grain. Adsorbed layers are very thin. The smaller the grains are, the more they have surface area in relation to their mass and volume and the water retention capacity of the soil increases. Fine grained soils (clay and silt) can hold much more

[2]Adsorption is a physical process, where a fluid forms a thin layer on the surface of a solid. Porous, solid surfaces adsorb the strongest due to their large surface area.

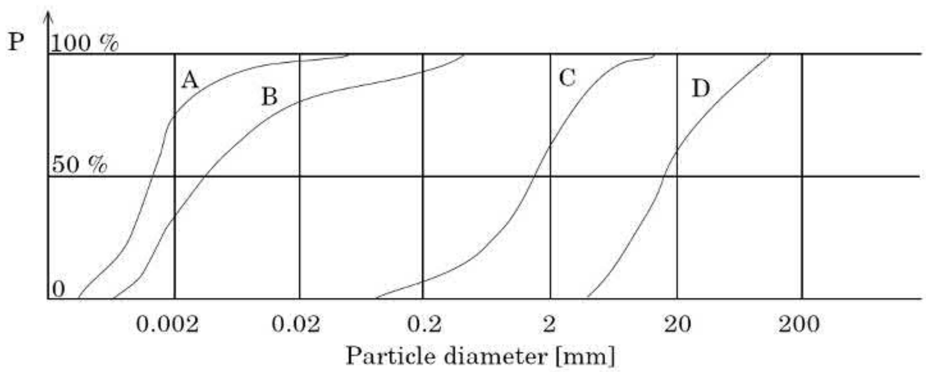


Figure 7.3: *Examples of the particle size distribution of various soil types: A and B are clay soils, C is sand and D is gravel. P is the fraction of the total mass that is in smaller grains. The 50 % level corresponds the median of the distributions.*

water than more coarse soil, like sand.

In addition to surface forces on soil particles, surface tension of water affects on the air/water boundaries in soil. Surface tension tries to decrease the volume of a water. Capillary rise, h', depends on the radius of the pore tube r. Approximately it can be said that $h' = 150\ \mathrm{mm}^2\ \mathrm{r}^{-1}$. Capillary zone exists due to surface tension. Retention of water depends on the dominating force, which gives rise to the various ways of water attachement in soil (Fig. 7.4).

Let us take a look at the soil water retention by starting from the Bernoulli equation (Eq. 6.7). Because flow velocities are very small (less than 1 mm s^{-1}), the kinetic energy term can be left out, and thus the energy height of soil water can be written as

$$H = h + \frac{p - p_0}{\rho g}. \tag{7.1}$$

Heights H and h are measured from a fixed reference height, positive upwards. The last term, $h_c = \frac{p-p_0}{\rho g}$, is in an unsaturated situation called the *capillary potential*, which represents the energy required to release water from a soil particle per unit of mass. Energy height H describes energy per unit of mass, and h represents the geometric height. Energy height is called potential, and in various applications it is called the piezometric head or hydraulic head, because it is the level that water would take if a vertical tube would be placed on the point of interest. Cohesion forces in the soil are included in the capillary potential.

Pressure of the soil water can be measured with a tube placed on the point of interest. Because in an unsaturated situation water is under suction, and the pressure head is negative, the measuring tube needs to be pointing down (Fig. **??**). Pressure is lower than the atmospheric pressure. The instrument described in figure **??** is called a tensiometer, and it can be applied to measure magnitude of the suction, when the soil water pressure is lower than the atmospheric pressure.

Capillary potential includes, in addition to capillary forces, all other forces affecting the suction. These include temperature gradient in soil, which mostly affects the movement of vapor in soil, and osmosis, which heads the flow towards higher concentration of dissolved

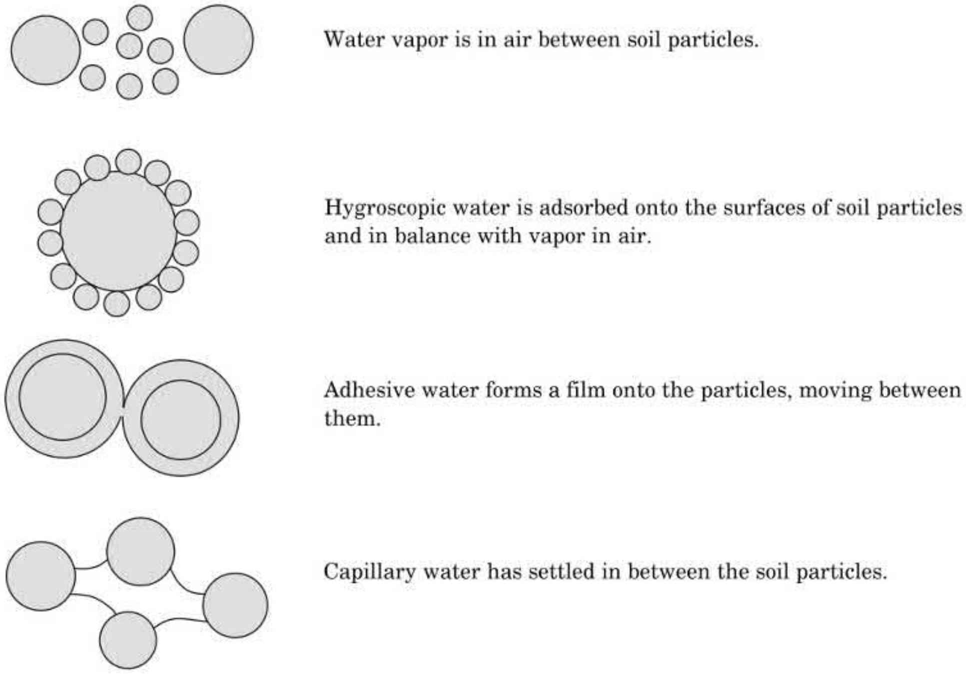


Figure 7.4: *Binding of water in ground.*

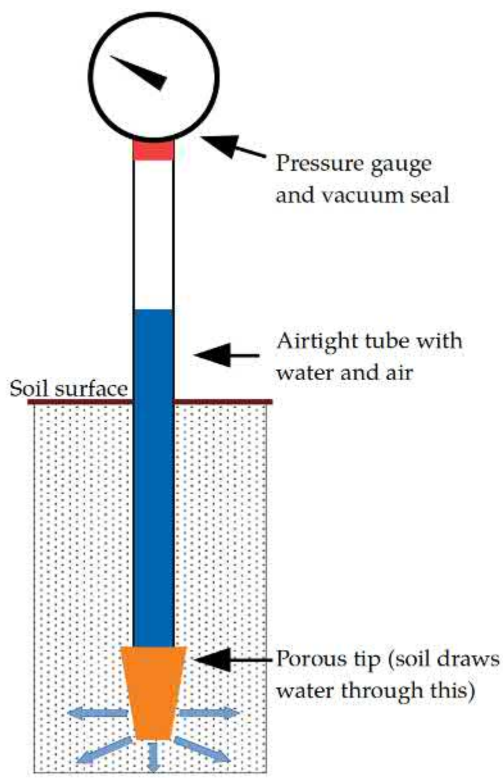


Figure 7.5: *Measuring suction in soil. The pressure gauge measures the difference in pressure between soil water and atmosphere ($p_0 - p$). Figure: Joonatan Ala-Könni.*

substances. Osmosis is important when water enters the root of a plant. Capillary potential is the energy required to free water from the binds of cohesion forces of the soil. Older literature uses *pF-value* to describe the capillary potential. It is the base-10 logarithm of the additive inverse of the capillary potential, $pF = \log_{10}(-h_c[\text{cm}])$, and has a unit of cm. Additive inverse of the capillary head is known as the *water retention capacity*, and the relationship of it with the soil moisture content is called the *water retention curve* (Fig. 7.6).

Let us have a look at a vertical cross section of soil. In an equilibrium where there is no flow, the various water zones under the soil surface can be resolved (Fig. 7.2). Flow of water in the root and intermediate zones is slow. Mostly, it happens as the movement of vapor between pores and as the flow of adhesive water. If the water content of the zone temporarily increases due to precipitation, the equilibrium is disrupted and the fraction of capillary water increases momentarily. This water moves downwards due to gravity. In the capillary zone, smaller pores are connected to the groundwater, and larger pores can retain air pockets, completely surrounded by water.

As was previously stated, in the root and intermediate zones capillary head depends on the moisture content. In the capillary zone and in the groundwater zone, respectively

$$p - p_0 = -\gamma(z - z_0) < 0 \quad \text{and} \tag{7.2}$$

$$p - p_0 = -\gamma(z - z_0) > 0, \tag{7.3}$$

where z is depth, z_0 is the groundwater surface depth and the product of gravity and density ρg is symbolized with γ.

Field capacity is the amount of water that is left in the soil after gravity has pulled all of the available water from saturated soil. Table 7.2 has values of capillary head that are required to release water. *Available water capacity* is defined as the difference between field capacity and wilting point. Typical values of available water capacity are presented in table 7.4.

7.1.2 Measuring soil water content

Soil constitution is described with *porosity* n and *water content* m. The least dense way to pack material consisting of uniformly shaped spheres is $\pi/6 \approx 0.52$ and the densest way is $\pi/3\sqrt{2} \approx 0.74$, which can be used as reference values. Grain size distribution affects the true porosity greatly. Wide distribution reduces porosity, since small grains can fill the gaps between larger grains. For this reason the porosity of moraine can be rather small. Often the concept of effective porosity is applied, which includes the most tightly absorbed water in the volume of the grains.

A connection can be made between the volume V, volume of water V_W and volume of air V_A

$$V_w = mV \quad \text{and} \tag{7.4}$$

$$V_a = (n - m)V. \tag{7.5}$$

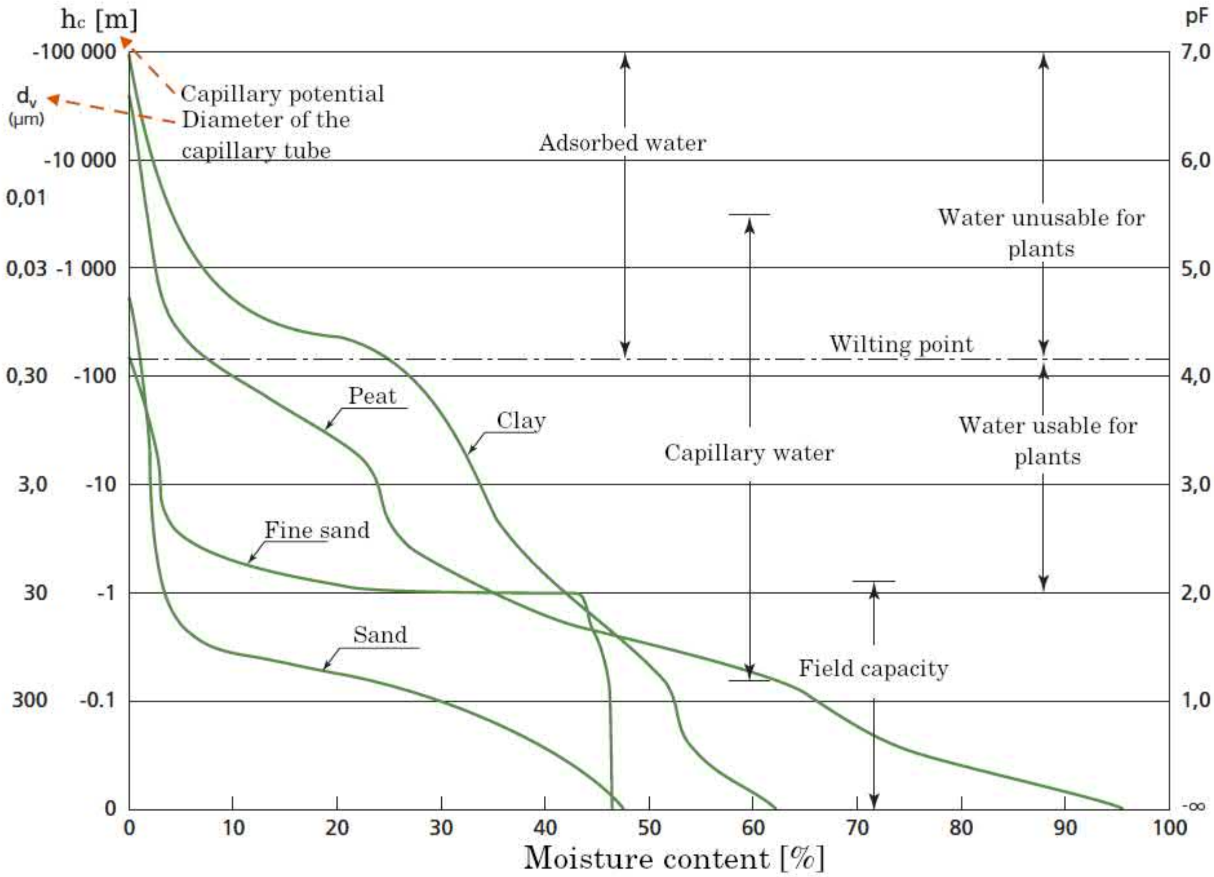


Figure 7.6: *Water retention curves of some soil types. The horizontal axis has the water content, and vertical axis has the water retention capacity represented as the diameter of a capillary tube d_v, capillary potential h_c and as pF-number,* $pF = \log_{10}(-h_c[\text{cm}])$

Knowing the soil water content is very important for many agricultural applications and in hydrological applications as a factor in runoff. In hydrology, water content is usually expressed as a volume fraction, but it can also be expressed in relation to the solid matter mass or total mass M. Water content expressed in relation to

- volume: $m_W = \frac{V_W}{V}, \quad 0 \leq m_V \leq 1$

- solid matter mass: $m_S = \frac{M_W}{M_S}, \quad 0 \leq m_S \leq \infty$

- total mass: $m_{SW} = \frac{M_W}{M_{SW}}, \quad 0 \leq m_{SW} \leq 1.$

Here, subscript W refers to water, S to solid matter and SW to the combination of water solid matter. Soil and water densities are

- wet bulk density: $\rho_M = \frac{M_W}{V}$

- soil density, or solid matter density: $\rho_S = \frac{M_S}{V_S}$

- dry density: $\rho_D = \frac{M_S}{V}$.

Example 7.2
Consider a soil sample. It has a volume of $V = 1$ m^3, solid matter mass of $M_S = 4\ 000$ kg and water mass of $M_W = 200$ kg. Then, by definition, water content relative to the total volume is $m_W = \frac{V_W}{V} = \frac{0.200\,\mathrm{m}^3}{1\,\mathrm{m}^3} = 20$ %, 5 % relative to the solid matter mass and 4.8 % relative to the total mass. Wet bulk density is 200 kg m^{-3}, soil density is 5 000 kg m^{-3} and dry density is 4 000 kg m^{-3}.

Table 7.2: *Retention of water and capillary head.*

Level of water adsorption	**Pressure [bar]**	**Capillary head [cm]**	**pF**	**Pressure [kPa]**
Hygroscopical limit. Below this, water only moves as vapor.	-30	-30 000	4.5	-3 000
Wilting point. Below this plants can not use soil water anymore.	-15	-15 000	4.2	-1 500
Field capacity. Above this, free water will drain down to lower layers.	-0.3	-300	2.5	-30

Table 7.3: Available water capacity for various soil types. Available water is the wilting point subtracted from the field capacity.

Soil type	**Clay**	**Fine silt – mid-fine silt**	**Coarse silt – fine sand**	**Sand**
Grain size [mm]	< 0.002	0.002 - 0.02	0.02 - 0.2	0.2 - 2
Available water capacity	14 %	16 %	10 %	7 %

Measuring the soil water content for hydrological applications is difficult. Water content would need to be followed in the same spot from year to year with a sufficient frequency so that the measurements would be representative of the mean values of the whole area in question. This sets requirements for the field capabilities of the instruments and for the repeatability of the measurement.

Oldest and perhaps the most widely used method is to take a soil sample of a known volume, weight it, then drain it at a temperature of 105 °C. The difference between wet and dry weight is the mass of water in the sample. The disadvantages of this method are that it is very laborious and it will eventually disrupt the study area.

Previously, an instrument based on the slowing down of fast neutrons, which depends on the soil moisture content, was rather widely used. This was a hydrological application of a method developed for the prospecting of oil in the 1960s. Basic components of this instrument are neutron emitting radioactive substance (Am-Be or Ra-Be) and a detector, which is a proportional counter or a crystal. The detector is capable of seeing only neutrons that have been slowed down.

Measurements of electric resistance are based on the fact that the soil resistance is inversely proportional to the soil moisture content. Water content can also be measured based on dielectricity, because relative permittivity (dielectric constant) is a function of soil water content. A more recent technique is based on measuring the soil dielectricity by ground penetrating radar and reflectometer (Time Domain Reflectometer, TDR).

7.2 Groundwater

7.2.1 Groundwater resources

Groundwater contains 30 % of the planet's total fresh water resources, and 99 % of the liquid fresh water resources. Half of it is shallow groundwater, meaning that it lays at no more than 800 m below surface, and the other half is located deeper. In mid-1970s, groundwater represented about 30 % of the water used by municipal water services in Finland. Nowadays it is over 60 % of total consumption, or about 35 m^3 s^{-1}. It has been estimated that the groundwater resources of Finland would be sufficient for a consumption of up to 45 m^3 s^{-1}. Based on this it would seem that the groundwater resources of Finland are sufficient. Exploiting them to the fullest has its own problems, since individual groundwater deposits are rather small, and quite often they are far from the place where they would be consumed (Mälkki 1999).

The mean groundwater retention time[3] is 240 years, but varies significantly depending on the groundwater storage in question. The deepest groundwater is in very limited interaction with the atmosphere and other shallow groundwater. Some groundwater storages have been filled thousands of years ago. For example Northern Europe has groundwater storages that were formed during the last ice age, and Sahara desert has some storages that are older still and have retained themselves through several glaciations.

Groundwater with a free surface forms a layer saturated with water where the hydrostatic pressure is higher than that of the atmosphere. At the top of the groundwater layer $p = p_0$, and the surface elevation depends on how much water has been infiltrated there from the soil water zone and the environment.

Based on the water retention curve (Fig. 7.6) it can be said, that for a pF-number corresponding the field capacity, water content of clay is 40 % and water content of sand is 10 % of total volume. Both soil types have a saturation water content of about 50 %. The difference between saturation and field capacity, 40 percentage points for sand and 10 for clay, is called the free water. Free water is moved by gravity, and electrical cohesion forces are not able to hold it.

From the human perspective groundwater resources containing plenty of free water are important. Resources of this kind are found from areas with sand and gravel formations, while in finer soil types the cohesion forces are so strong that moving of water is difficult. Storages which contain plenty of free groundwater are called *aquifers*. The most important aquifers of Finland have formed in eskers that resulted from the retreating phase of the Fennoscandian Ice Sheet. Usable groundwater is also present in bedrock cracks. In everyday language groundwater refers to this usable water.

Groundwater is special due to its purity. In most cases it is usable almost as is. As it infiltrates through soil layers, bacteria and other impurities harmful to human health are filtered out. Instead, useful dissolved salts are added into the water. Groundwater is also partly protected from pollutants falling from the sky, because most of the impurities are filtered in the soil. Groundwater is threatened more by slow acidification and increase in salinity due to the practise of spreading salt on roads during winter to keep them from freezing. Groundwater is in danger also when a pit dug for gravel extraction reaches the groundwater surface.

Aquifers are formed in a layer of coarse soil type, located over a layer that poorly conducts water. There are three basic types of aquifers:

- *Unlimited* or *free aquifer*
- *Limited* or *artesian aquifer*, where the surface of it is limited by an impermeable layer,
- *Perched groundwater*, which is a separated saturated layer over the actual groundwater surface.

The surface of a free aquifer is formed by the free surface of the groundwater, where pressure equals the external pressure, i.e. atmospheric pressure ($p = p_0$). Artesian aquifer is covered by a poorly conducting layer and throughout the aquifer pressure is higher than the

[3]Mean retention time is the storage volume divided by the rate of renewal.

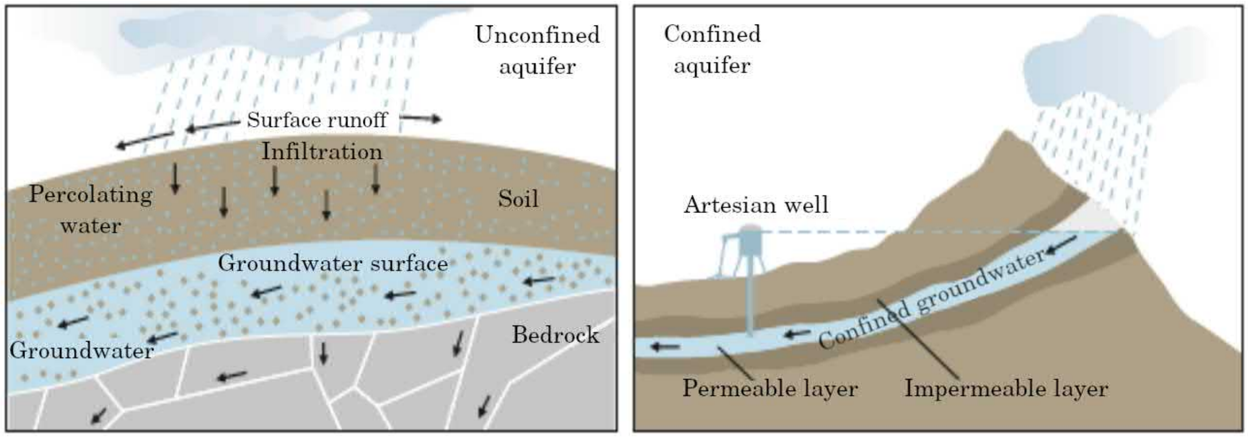


Figure 7.7: *In an unconfined aquifer (left) rainwater infiltrates into the soil and sinks all the way to the groundwater surface level. After that, it will flow following the slope of the surface. In a confined aquifer (right) there is an insulating layer above, and the pressure in the aquifer is determined by the height of the free surface.*

external pressure ($p > p_0$). If a tube is inserted into an artesian aquifer, water level in the tube will rise above the surface of the aquifer, potentially even above the ground surface. This level in our hypothetical tube is called the piezometric head or hydraulic head. Water in an artesian aquifer might originate from hundreds of kilometers away, from a more rainy area that lies higher up. Above the impermeable layer a second aquifer might form, which is called a perched aquifer.

Groundwater deposits get replenished by precipitation, rivers, lakes and other aquifers. Aquifers can also be artificially replenished, which might be necessary if due to increased use there is a possibility to deplete the aquifer. Many cities in Finland (for example: Turku, Jyväskylä, Kuopio and Lappeenranta) are reliant on artificial groundwater. Aquifers leak into the sea, other water systems or other aquifers, or they can be depleted through excessive exploitation.

7.2.2 Darcy's law and the flow of groundwater

Movement of groundwater is fluid flow through a porous medium. Gravity acts as the external forcing, which tries to set the groundwater level horizontal. Movement is hampered by friction F between water and the surrounding medium. Pressure, geometric height h and friction form an equilibrium, from which the flow of groundwater can be solved. The general form of this equation is

$$-\frac{1}{\rho g}\nabla p + \nabla h + \frac{1}{\rho g}F = 0. \tag{7.6}$$

The sum of pressure head and geometric height is the hydraulic potential H, and its gradient (∇H) drives the flow. Friction is proportional[4] to the flow velocity

[4] In laminar flow, this relation is linearly proportional to the flow velocity.

$$F = -\rho C U \tag{7.7}$$

where C is the friction coefficient.

From this, we obtain the *Darcy's law* (named after Henry Philibert Gaspard Darcy, 1803 – 1858), which estimates the flow rate $Q = UA$ through a soil filled tube (Fig. 7.8) over a cross section A, or the mean flow velocity:

$$U = \frac{Q}{A} = K\frac{H_1 - H_2}{L}, \tag{7.8}$$

where $K = C^{-1}$ is the hydraulic conductivity, H_1 & H_2 are the potentials at points 1 and 2 (Fig. 7.8) and L is the length of the hypothetical tube. The quantity $\frac{H_1-H_2}{L}$ is the hydraulic gradient, and the hydraulic conductivity or permeability describes the soil's ability to conduct water through it. Hydraulic conductivity has the dimension of speed, for example $K = 1$ cm s^{-1} gives a theoretical mean flow velocity of $U = 10^{-4}$ m s^{-1} when the hydraulic gradient is 1 %.

Notice that in equation (7.8) the potential difference $H_1 - H_2$ is divided by the length of the tube L, not by horizontal distance x, and thus

$$\frac{H_1 - H_2}{L} \to \frac{\partial H}{\partial s}, \text{ when } L \to 0 \tag{7.9}$$

where s is the distance in the direction of the tube. Approximation $\frac{\partial H}{\partial s} \approx \frac{\partial H}{\partial x}$ is applicable if the slope of the tube is small.

The variation in hydraulic conductivity is large. In addition to porosity, it is affected by the soil type (especially pore shape) and the size distribution of soil grains. Table 7.4 lists some soil types and their hydraulic conductivities. The table also lists conductivity classes and classification of associated aquifers (Bear et al., 1968). True flow velocity between the soil pores is

$$\widetilde{U}\frac{1}{n_e}U, \; n_e = \frac{V_{we}}{V}, \tag{7.10}$$

where $n_e \sim 10 - 30$ % is the effective porosity, and V_{we} is the volume of the water in the flow. Adsorbed water does not flow. Groundwater flow is usually laminar, which is affected by the temperature dependent viscosity, dissolved gases and the concentration of suspended particles in water. Due to the passages between soil particles forming a complex network, the flow of water follows a random path leading to rapid dispersion.

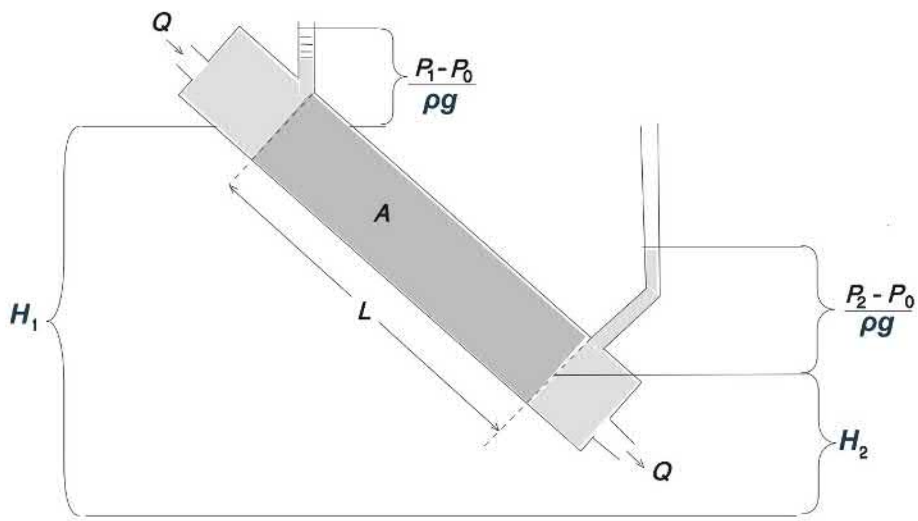


Figure 7.8: *Darcy's experiment to determine the hydraulic conductivity K. The tilt of the tube multiplied by K results in the flow rate across the tube's cross section.*

Table 7.4: Hydraulic conductivity and the usability of an aquifer based on it.

Soil type	**Hydraulic conductivity *K* [cm s^{-1}]**	**Groundwater usability**	**Conductivity class**
Gravel	$10^2 - 1$	Good	Permeable
Sand	$1 - 10^{-3}$	Good	Permeable $(1 - 10^{-2})$ – Semipermeable $(10^{-2} - 10^{-3})$
Silt & clay	$10^{-3} - 10^{-7}$	Bad	Semipermeable $(10^{-3} - 10^{-6})$ – Insulator $(10^{-6} - 10^{-7})$
Clay	$< 10^{-7}$	Negligible	Insulator

Example 7.3
Surface flow on a free aquifer. At the surface, the potential height equals the geometric height, $H = h$. Hydraulic gradient is 5 %. If the hydraulic conductivity of the soil is $K = 1$ cm s^{-1}, then the flow velocity of water is $U = K\frac{H_1 - H_2}{L} = 0.05$ cm s$^{-1} \approx 50$ m day^{-1}. This value of K would be realistic for a high quality aquifer. If we set $K = 10^{-7}$ cm s^{-1}, which represents clay and the slope is kept the same, then the annual movement of water is 2 mm.

Above, Darcy's law was applied to a sample of soil, but the law can be applied to flow field.

Water flows always to the direction where the change in potential is the highest, i.e. to the direction of the potential gradient. Therefore, the direction of flow is always perpendicular to the potential contours ($H = constant$).

Darcy's law has its limitations. Firstly, it requires a sufficiently small Reynolds number (laminar flow). If the flow is turbulent (very coarse soil, high potential gradient), Darcy's law can not be applied. In that case the friction term should also have term with square relation to the flow velocity: $F = -(\rho CU + \rho C_2 U^2)$. Secondly, in clay and decomposed peat, which have high cohesion, there is a threshold. In order for the flow to initiate, the potential gradient must exceed the threshold gradient.

Third limitation is the assumption of homogeneous soil. Usually soil has been formed as a consequence of sedimentation periods, that has resulted in a non-isotropic soil. For example, sea and lake bottoms have different sedimentation rates in different seasons. Little by little, due to land uplift or a decrease in sea level, the basin bottom has transformed to soil, which still retains its old, layered structure. In this kind of soil, water tends to flow along the layers and not across them. This can lead to a situation where the groundwater flow does not follow the potential gradient. Because these layers are typically almost horizontal, hydraulic conductivity has higher value horizontally than vertically.

Groundwater flow models built upon Darcy's law with proper boundary conditions define explicitly the flow. Soil type has to be known or assumed for every grid cell. The most important boundary conditions for the model are the following:

1. Impermeable layer: Flow close to this boundary must go along this layer, i.e. the potential surfaces are perpendicular to the impermeable layer.

2. On the boundary between an aquifer and a water system the flow is perpendicular to this surface. This boundary also is a potential surface, where the value of the potential is the same as the free level of water.

3. On a free surface of a groundwater aquifer the pressure has to be constant and the same as the atmospheric pressure at that level. Value of the potential on this surface is the same as the groundwater surface height on that spot.

Usually in groundwater flow models the flow field is solved numerically from the differential equation of flow (Eq. (7.6)) and the continuity equation. There are analytic solutions for some simple cases. One of these solutions is presented in the following section.

7.2.3 Dupuit assumption

Modeling unlimited flow leads to differential equations that can not be solved analytically, apart from some specific cases. In order to understand groundwater flow, these examples are enlightening.

Jules Dupuit (1804 – 1866) derived an equation for wells, that was based on two simplifying assumptions:

1. The slope of the flow is small. In this case, the derivative of the hydraulic potential in the direction of the flow can be approximated via horizontal distance x: $\frac{\partial H}{\partial s} \approx \frac{\partial H}{\partial x}$.

2. Flow velocity U is not dependent on the vertical coordinate z. Then, the flow velocity is everywhere the same as it would be on a free surface, $U = K\frac{\partial H}{\partial x}$.

Let us have a look at an unlimited aquifer, which has a horizontal bottom and initially no flow. A tube is inserted into the aquifer for pumping. When the pumping has been going on long enough, a steady state forms (Fig. 7.9). This case is called the Dupuit well equation.

The solution is constructed as follows. The level of the free surface before the pumping is H_0, the radius of the pumping tube is r_0 and r is the distance from the tube. When the pumping has stabilized, water level in the tube is H_1, and the flow velocity is $u = u(r)$, which is positive towards the pumping tube. In a steady state, every cylinder surface having a radius of $r > r_0$, has a constant flow Q passing through. According to Dupuit's assumption, flow velocity is not dependent on the vertical coordinate.

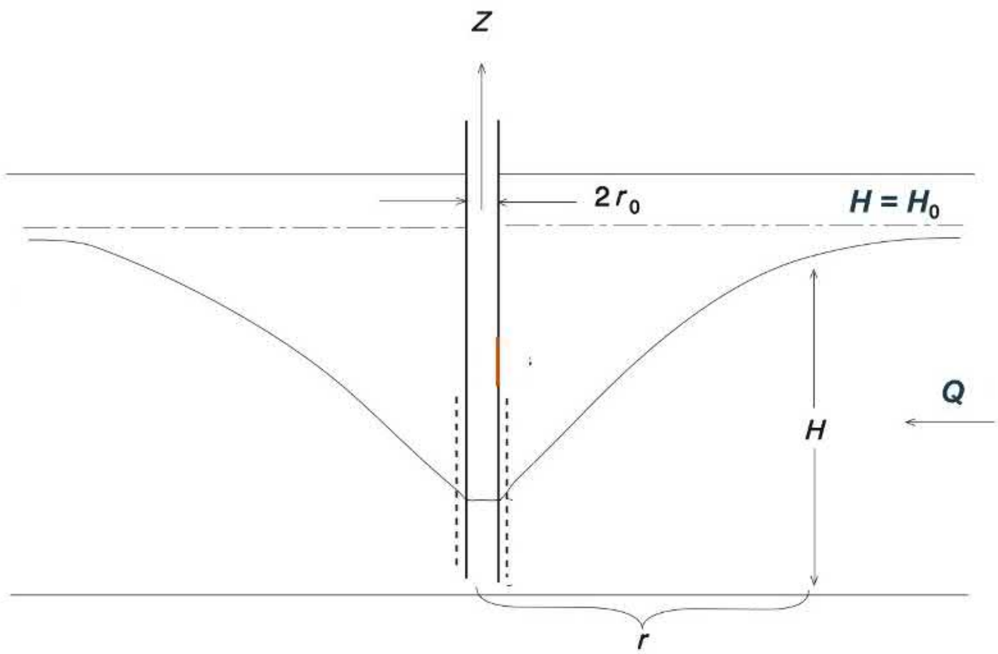


Figure 7.9: *Pumping an unconfined aquifer. This is the analytical solution from a stable state, when the boundary conditions are set as $H = H_0$ and $r = R$. r_0 is the diameter of the well.*

The flow through any cylinder surface is

$$Q = 2\pi r H u, \tag{7.11}$$

where H is the level of the free surface, which is dependent on the distance r from the tube. In a steady state Q is not dependent on the distance r. Because H is the potential height, Darcy's equation states that the radial flow velocity towards the tube is

$$u = K\frac{dH}{dr}. \tag{7.12}$$

By placing this velocity (Eq. 7.12) into equation (7.11) we acquire a differential equation for the potential height H. This separable differential equation can be integrated:

$$H^2 = \frac{Q}{\pi K}\ln r + B, \tag{7.13}$$

where B is a constant of integration. The form of this equation immediately reveals some limitations. When r increases, the potential height approaches infinity ($H \to \infty$), but the level must be equal or less than the original level ($H \leq H_0$). This problem stems from the fact that a continuous flow is assumed. But as the pumping continues, the effect of it just keeps reaching further out and no stable situation is reached until the aquifer is empty or unless the aquifer would be continuously replenished. For practical applications, equation (7.13) usually works for weeks to months from the start of the pumping, since changes in aquifers are slow.

A radius R, where the pumping still has a noticeable effect on the aquifer water level, can be defined. To solve the constant of integration B, condition $H = H_0$ when $r = R$ is set. Then, the equation for the level of the free surface becomes

$$H_0^2 - H^2 = \frac{Q}{\pi K}\ln\frac{R}{r}, \text{ where } H_0 = H(r_0) \text{ and } r_0 \leq r \leq R. \tag{7.14}$$

Figure 7.10 shows solutions of this well equation with varying values of hydraulic conductivity. As the hydraulic conductivity goes down, the profile of the well in an equilibrium gets deeper, and the edges (R) should be widened in order to have a more realistic solution. Equation (7.14) can be made dimensionless:

$$1 - \eta^2 = \Omega\ln\frac{1}{\xi}, \ \xi_0 \leq \xi \leq 1, \tag{7.15}$$

$$\xi_0 = \frac{r_0}{R}, \ \Omega = \frac{Q}{\pi K H_0^2}, \ \eta = \frac{H}{H_0}, \tag{7.16}$$

where Ω is the dimensionless flow rate, $\eta = \eta(\xi)$ is the dimensionless free surface level and ξ is the dimensionless distance. It can be seen, that by keeping the ratio of pumping and hydraulic conductivity the same, the same free surface level solution is reached. Additionally, the situation can be balanced with boundary conditions. The time scale of the solution is acquired with the hydraulic conductivity, $T \sim LK^{-1}$. By choosing the length scale as $L \sim R$, time scale is 1 hour – 1 day, when $K = 0.1 - 1$ cm s^{-1}.

The pumping can be carried out to establish the hydraulic conductivity K or to extract water. In the first case the flow rate Q is measured and the water level H is determined at several distances r. From these, the value of K can be solved. In the extraction of water a condition can be set, that the difference $H_0 - H$ can not exceed a certain value. The pumped outflow Q corresponding to this free surface level can be estimated with equation (7.14).

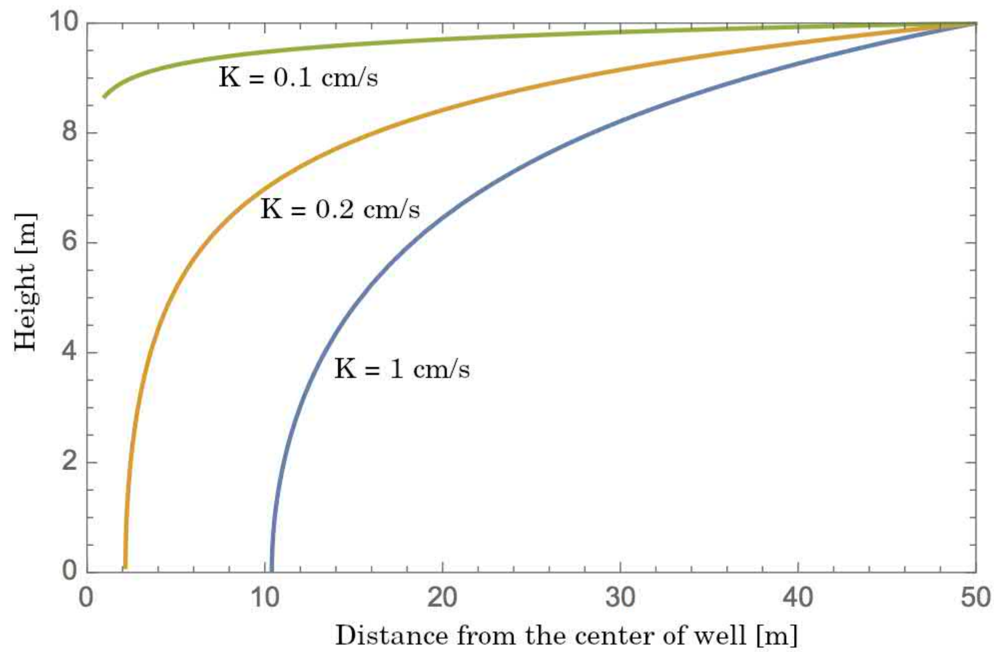


Figure 7.10: *Free surface of groundwater around a well. Here, the pumping rate is 0.2* m^3 s^{-1}*, and boundary conditions are 10 m free surface level at 50 m distance from the well. Hydraulic conductivity is 0.1 cm* s^{-1}*, 0.2 cm* s^{-1} *and 1 cm* s^{-1}*. Zero level of the height is over 10 m below ground.*

7.2.4 Measurements and pumping tests

Field studies of groundwater are done with groundwater tubes. They are drilled into representative spots over the aquifer. Water level is measured from the tube with a conductivity meter or a pressure sensor. The data can be recorded automatically. Other measurements and samplings are performed on these tubes as well. Flow rate from a spring can be measured with a weir.

Seismic sounding is based on the observation of the propagation of seismic waves in the ground. It reveals details about the depth of the overburden[5] and the bedrock. Low-frequency (VLF) radio waves are utilized to detect electromagnetic fields induced around, for example, veins of water in the ground. Measurements of gravity can be used to see anomalies in the density structure of the ground, and a ground penetrating radar can be used to detect layers in the ground.

Long-term test pumping, in the time scale of days to weeks, aims to establish the natural replenishment and the effects of the pumping on the environment. Pump tests also include drilling holes to measure the water level around the well.

There are several mathematical models to estimate the effects of short term pumping on the aquifer water level. These models include parameters for the hydraulic conductivity, thickness, storage capacity, boundary quality and input of water from the environment. Pumping is always done simultaneously with measurements of groundwater level in the

[5] A layer comprising of loose rocks and gravel on top of the bedrock.

surrounding area. By fitting the results from the field measurements into the model results, the parameters can be fine-tuned.

The amount and level of groundwater is also dependent on local factors. Groundwater discharges either via springs or directly to surface water bodies through their bottom. Exchange of water between groundwater and a river can be separated into four basic types: (i) river receives water from a local, intermediary or regional groundwater flow, (ii) the river is above the groundwater and gives water to the groundwater storage, (iii) the river is on the same level with the groundwater, sometimes receiving and sometimes delivering water and (iv) a throughflow system. The situation can change between these states depending on the water level of the river.

On coastal areas a boundary zone between saline and fresh water will form in the groundwater storage (Fig. 7.11). For the depth z of this boundary there is a solution, based on hydrostatic equilibrium. This so-called Ghyben-Herzberg relation is written as

$$z = \frac{\rho}{\rho_s - \rho} h \tag{7.17}$$

where $\rho_s = 1\ 025$ kg m^{-3} is the density of seawater ($S = 35$ ‰) and h is the groundwater level relative to the sea level. The equation does not take into account the flow of fresh groundwater into the sea. Thus, in reality in the upper parts of the groundwater zone the boundary extends further towards the sea.

In Finland, the quality of groundwater is good and the volume is sufficient for the population. For some locations, artificial groundwater is required. The most important research questions about Finnish groundwater are the uneven distribution, concentration of iron, manganese and fluoride, effects of the salting of roads in winter, and increase in nitrate concentration. Due to local pollution sources there are contaminated land areas. Toxic heavy metals are rarely found in excessive levels, and even then the levels are not very high. As an example in figure 7.12 is a groundwater station in Puolanka, Central Finland.

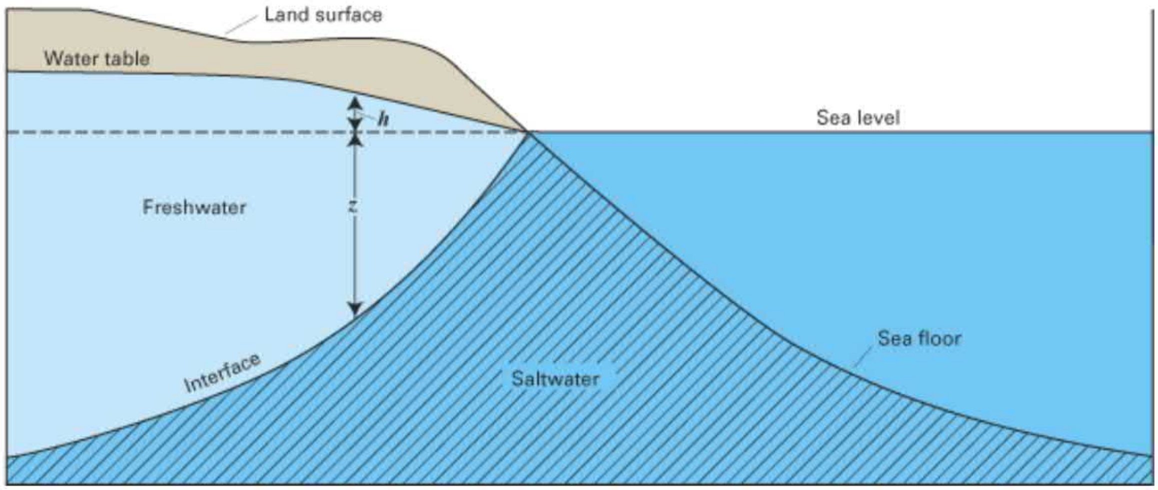


Figure 7.11: *The interface between fresh and saline groundwater near a coastline as described by the Ghyben-Herzberg relation. Source: Barlow (2003): Groundwater in freshwater-saltwater environments of the Atlantic coast, Fig. B-1. http://pubs.usgs.gov/circ/2003/circ1262/pdf/circ1262.pdf*

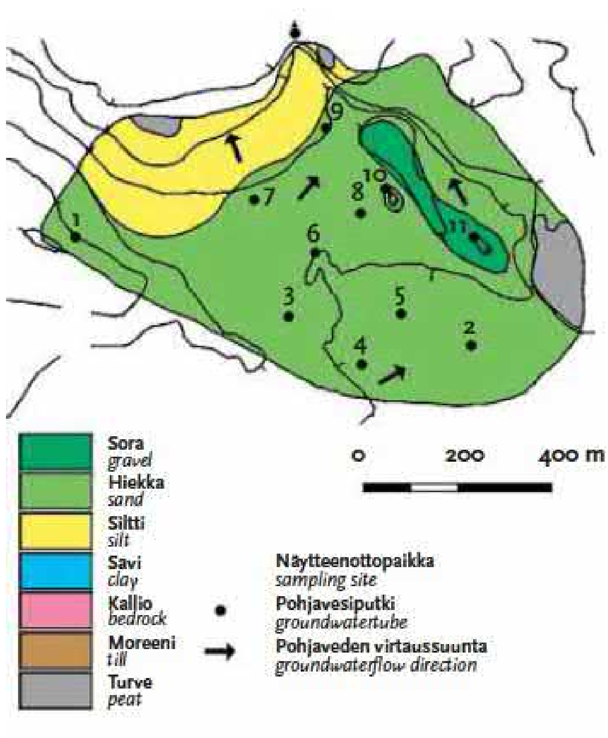


Figure 7.12: *Alakangas groundwater station in Puolanka. The area around the station is part of the Kuhmo–Paltamo–Haukiputaa esker section. Soil type is mostly sand. The general direction of the groundwater flow is from SW to NE. Source: The Finnish Environment Institute (Kuusisto 2008)*

7.3 Soil temperature

7.3.1 Equation of thermal conduction

Temperature of the soil has an effect on the hydrological cycle through several mechanisms. Evaporation depends on the temperature of the water on the surface, temperature gradient in the soil affects the movement of vapor and the temperature of plant roots affects transpiration. Flux of heat into the ground is one of the terms in the surface energy balance, which is a function the soil temperature. Soil freezing and frost is the result of temperatures below freezing point in the soil. Heat exchange between the soil surface and the atmosphere are affected by differences in the conductivity of heat in the soil, which in turn is affected by differences in the soil moisture content.

Figure 7.13 represents variations of temperature in Hyrylä (Tuusula, Southern Finland) in a hydraulically well conducting, sandy area (Lemmelä et al., 1981). The annual variation of temperature is dependent on the soil type and moisture content, and so the result in the figure is not to be held as universal. In this case, the annual temperature variations reached a depth of 6 – 7 meters, and in the surface layer the variations were about 20 °C. Dry soil experiences more variation than wet soil. For example the surface temperature of dry peat can reach over 50 °C. When the moisture content is high, evaporation and thermal conduction to deeper layers decrease the top layer temperature, and the minimum temperature increases due to the greater heat capacity and thermal conduction of wet soil.

In a porous medium, thermal energy can be transported via several methods (Fig. 7.14). These mechanisms are:

- Thermal conduction in water and solid particles
- Phase changes of water
- Diffusion of vapor
- Convection in water
- Convection in air
- Thermal radiation

Convection in this context refers to free convection, driven by density differences in the fluid in question. In addition to these mechanisms heat is *advected*, or in other words moved from one place to another along with the flow of soil water. In coarser soil types this term is significant when the vertical infiltration of water is strong. Figure 7.14 presents the significance of each heat transport term in different soil types at various levels of water content. One can notice, that transport via conduction is usually strong. For this reason, we will now take a closer look at conduction.

Thermal conduction in one dimension (the vertical z coordinate) is written as:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right), \tag{7.18}$$

where T is temperature, ρ is density, c is the specific heat capacity and k is the thermal conductivity. The left side of the equation describes the change in thermal energy. The right side is the diffusion of heat. Heat capacity ρc can be estimated as the weighted mean of the heat capacities of soil components (air, water, organic and inorganic matter).

Thermal conductivity depends on how the soil particles are positioned relative to each other and how water fills the pores in between them. Wet soil conducts heat much better than dry, and in wet soil evaporation takes up heat, which decreases the temperature. Minimum temperature in wet soil is greater than in dry soil due to the high heat capacity of water.

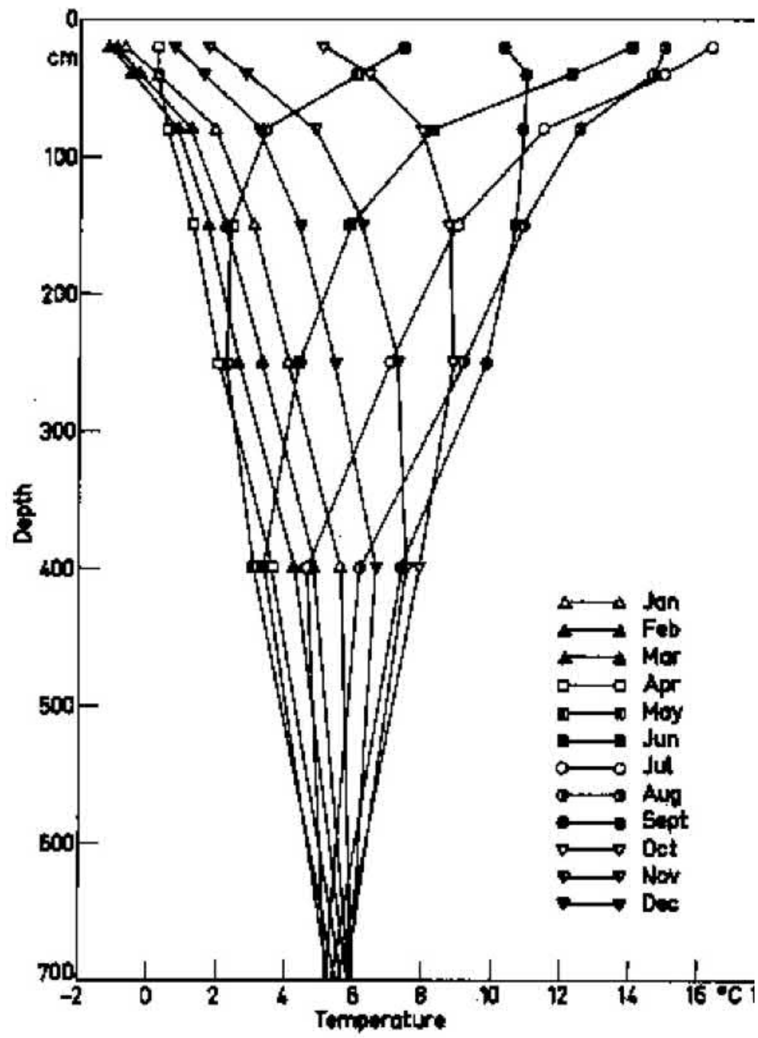


Figure 7.13: *Mean soil temperature during 1969 – 1973 in a sandy area in Hyrylä, Southern Finland. Source: Lemmelä et al. (1981).*

Let us now assume that heat capacity and thermal conductivity are constants. When equation (7.18) is divided by the heat capacity, the right side of the equation is left with the diffusion coefficient of heat $D = \frac{k}{\rho c}$, which has the dimension of L^2T^{-1}. The diffusion coefficient can be determined from temperature observations by studying the movement of the heat wave in soil. Table 7.5 lists values associated with the thermal properties of soil components. Figure 7.15 has an example of how the thermal conductivity of clay and sand is a function of moisture content and temperature. The diffusion coefficient describes the propagation of temperature signal, $L^2 \sim DT$. If $D = 10^{-7}$ m^2 s^{-1}, then during one month the signal propagates 0.5 m. This lag is well visible in figure 7.13.

Equation (7.18) can be analytically solved, if the boundary conditions consist of a sine wave temperature variation at the surface and a constant temperature $\overline{T}$ deep in the ground:

$$T(z,\, t) = \overline{T} + \Delta T e^{-\lambda z} \sin(2\pi\omega t - \lambda z + \phi),\ \lambda = \sqrt{\frac{\pi\omega}{D}}, \tag{7.19}$$

$$z = 0: \quad T(0,t) = \overline{T} + \Delta T \sin(2\pi\omega t + \phi), \tag{7.20}$$

$$z \to \infty: \ T \to \overline{T} \tag{7.21}$$

where $\overline{T}$ is the temperature deep in the ground, ΔT is the amplitude of the surface temperature, ω is the angular frequency of the temperature wave (corresponding period is $2\pi\omega^{-1}$) and ϕ is the phase. λ describes the exponential attenuation of the temperature amplitude and the phase shift progresses with depth. If the diffusion coefficient is 10^{-7} m^2 s^{-1}, in the case of the yearly temperature wave $\lambda^{-1} = 1.0$ m. Penetration of the temperature signal Z can be defined as $3\lambda^{-1}$, and then $\frac{\Delta T(Z)}{\Delta T(0)} = e^{-\lambda Z} \approx 0.050$. From this it can be seen, that the yearly variations in temperature are still visible at the depth of about 3 m.

Table 7.5: *Thermal properties of soil components at a temperature of 20 °C. (x) means, that the value corresponds to a mean. Source: Farouki (1981).*

	Density	**Specific heat**	**Thermal conductivity**	**Diffusion coefficient**
Medium	**kg m^{-3}**	**J kg^{-1} °C^{-1}**	**W m^{-1} °C^{-1}**	**m^2 s^{-1}**
Mineral soil (x)	2 650	730	2.9	$15 \cdot 10^{-7}$
Organic soil (x)	1 300	1 900	0.25	$1.5 \cdot 10^{-7}$
Water	1 000	4 200	0.6	$1.4 \cdot 10^{-7}$
Air	1 000	1 004	0.026	$0.21 \cdot 10^{-7}$

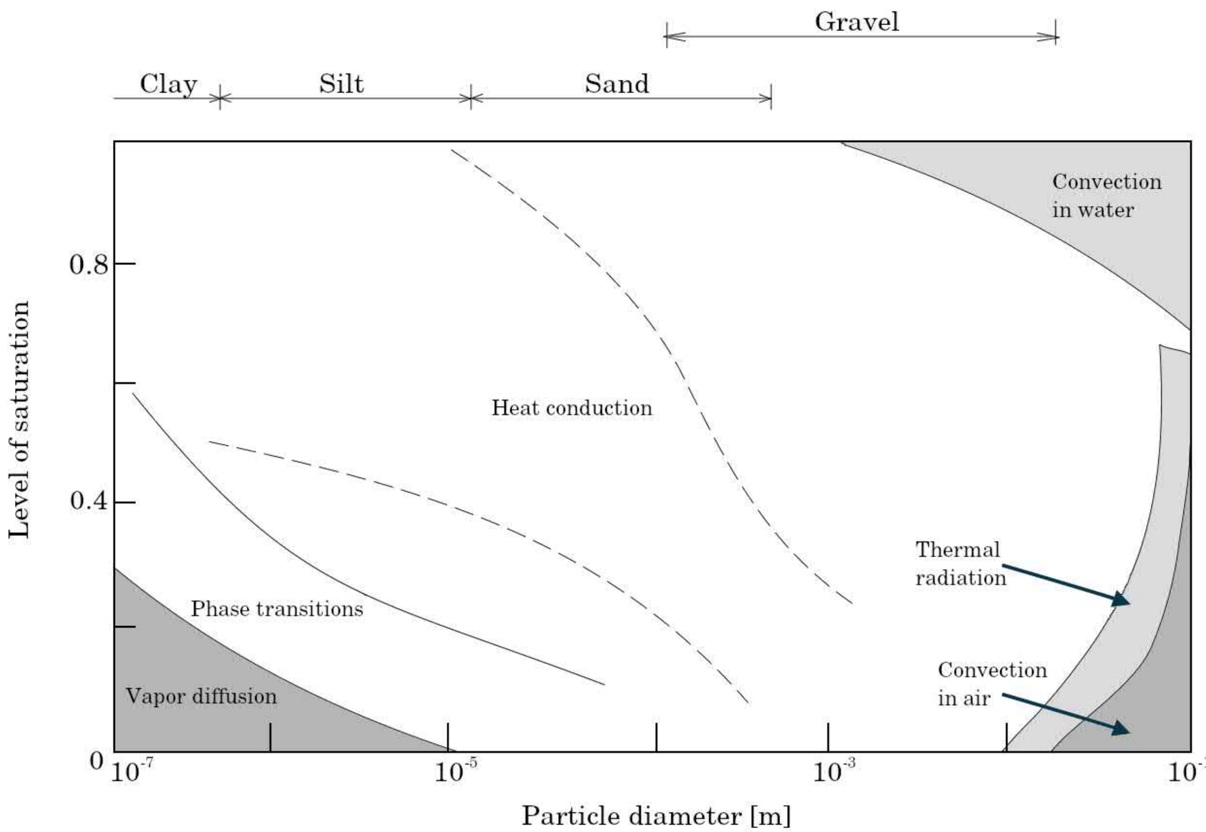


Figure 7.14: *Dominating mechanisms of heat transfer for different soil types and moisture contents. Water content is here relative to the saturation water content. Source: Farouki (1981).*

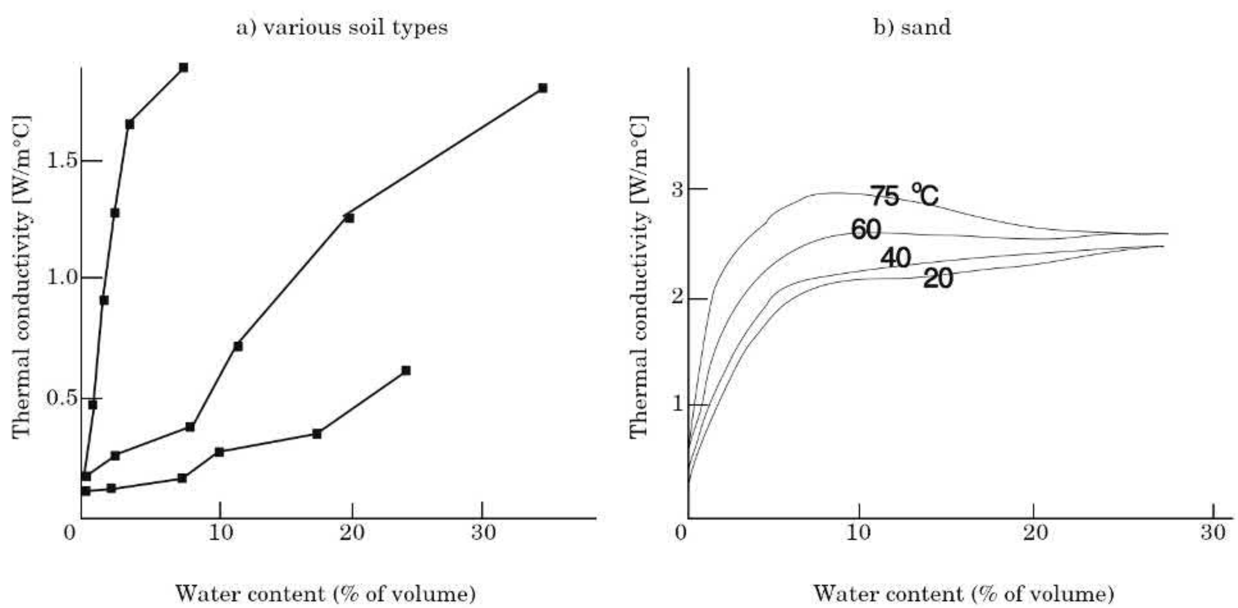


Figure 7.15: *Dependency of thermal conduction on the water content, for a) various soil types and for b) sand at different temperatures. Source: Farouki (1981), modified.*

7.3.2 Frozen ground

Water in surface layers of soil freezes when its temperature drops below 0 °C. This frozen layer is called *ground frost*, or just *frost*. *Permafrost* refers to a layer of frozen ground that has survived at least one summer without melting. Frost that has been formed in the winter and that melts in the spring is called seasonal frost. Formation of frost affects the hydrological cycle by binding soil water into a solid form and preventing liquid water from infiltrating deeper into the ground. Ground frost is problematic for human activities. During its melting the soil becomes soft, and moving along on roads and in the field becomes difficult. This season has a Russian language term *rasputitsa*[6] (Fig. 7.16). The formation and melting of frost deforms the soil through changes in the volume of water, which is called *frost heaving*.

Permafrost is prevalent in Northern Russia, Siberia, North America and in mountain ranges. In Siberia, permafrost layer can extend hundreds of meters into the ground. In Finland permafrost is only found in small patches in Northern Lapland. The depth of seasonal ground frost in Finland is mostly between 0 – 100 cm (Fig. 7.17), in Finland values over 1 m are only found in Northern Lapland. The depth of the frost layer is a function of air temperature and the thickness of insulating snow cover. There are different types of ground frost. *Cavity frost* forms in crumbly soil, like a field. Water is frozen onto the walls of cavities to form needle like structures which resembles rust. *Massive frost* is formed in soil containing sand and/or gravel with low water content. It is uniform and almost invisible, and rarely forms thick layers. *Layered frost* has a sheet like horizontal structure. The layers can be connected to each other or have some distance between them, depending on the soil moisture content.

One of the more well known ground frost formations is the *palsa*, a mound containing ice that rises on a bog. A palsa is born when a layer of seasonal frost does not melt completely during the summer and can grow further in size during the next winter. With this additional mass and thickness it is able to survive the upcoming summers even better. Palsas are not eternal, and due to their thermodynamics they melt away in a few decades.

The physics of the growth and melting of ground frost are similar to the freezing of lakes. The basic equation of ground frost growth is built from the continuity of heat transport from the soil through the snow and into the atmosphere:

$$\rho m L_f \frac{dh}{dt} = k_r \frac{T_f - T_0}{h} = k_s \frac{T_0 - T_s}{h_s} = Q_n, \tag{7.22}$$

where m is the soil moisture content, h is the thickness of the frozen layer, k_r and k_s are the respective thermal conductivities of ground frost and snow, T_f, T_0 and T_s are the temperatures of freezing, ground surface and snow surface, h_s is the snow depth and Q_n is heat transport from the atmosphere. If $T_0 = T_0(t)$ is known, the first equation becomes

$$h = a_m \sqrt{S},\ a_m = \sqrt{\frac{2k_r}{\rho m L_f}} \tag{7.23}$$

[6]This also is the etymology for the Finnish term *rospuutto*. A synonym for it is *kelirikko*. *Keli* means the weather conditions in regards to traveling, and *rikko* means 'broken'.

Figure 7.16: *"For my whole life I have lived in this village, and never have I wandered outside of it. And every single year during the rasputitsa, I wonder how will the world live... I mean... because the road to my house is unusable..." Kari Suomalainen 28.5.1957. ©Kari Suomalaisen perikunta.*

where S is the sum of freezing degree days. Compared to lake ice, the thickness of ground frost is greater by a factor of $m^{-1/2}$. This result requires that the ground is bare.

Snow is an excellent insulator, and it slows down the growth of ground frost. In the case of lake ice, snow turns into snow ice, but snow over ground frost only insulates. If the surface temperature of snow is known, equation (7.22) can be written as

$$\rho m L_f \frac{dh}{dt} = k_r \frac{T_f - T_s}{h + \gamma h_s}, \quad \gamma = \frac{k_r}{k_s}. \tag{7.24}$$

A good reference value for the thermal conductivity of snow is 0.15 W m^{-1} °C^{-1}, which corresponds a snow density value of 250 kg m^{-3} (see section 4.4.2). Thermal conductivity of ground frost is an order of magnitude larger, depending on the soil type and water content, so $\gamma \sim 10$, and thus according to equation (7.24), snow insulates about ten times better than frozen ground.

Example 7.4
Let us have a look at an idealized situation, where ground frost depth and snow cover thickness correlate as winter progresses, $h_s = \beta h$. In that case, equation (7.24) can be solved analytically

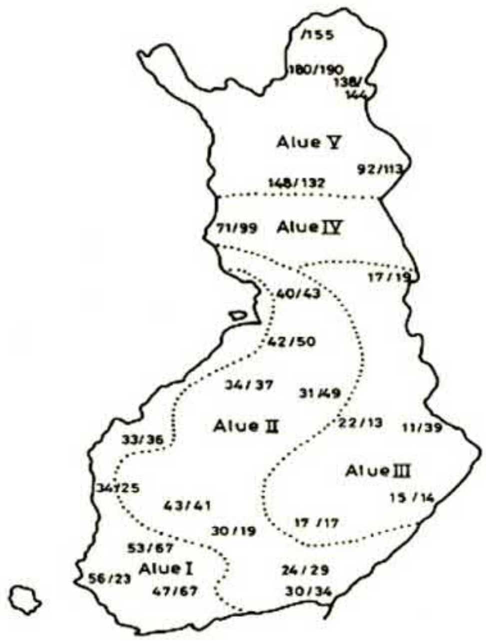


Figure 7.17: *The five different frost zones of Finland and the mean maximum frost thickness in open (first value) and in forest (second value) during 1968 – 1985. Source: Mustonen (1986).*

$$h = \frac{a_m}{\sqrt{1+\gamma\beta}}\sqrt{S}.$$

The growth of ground frost is slower in comparison to the case with bare ground: if $\gamma \sim 10$ and $\beta \sim 1$, the reduction factor is 0.3. If the snow depth is constant, the thickness of the ground frost will be

$$h = \sqrt{a_m S + (\gamma h_s)^2} - \gamma h_s.$$

The ratio between the ground frost thicknesses in the barren case and in the snow covered case is $\sqrt{1+\eta^2} - \eta$, where $\eta = \frac{\gamma h_s}{\sqrt{a_m S}}$. If the ground frost has a thickness of 100 cm in the bare ground case, it will be 24 cm under a 20 cm snow cover.

Example 7.5

The Hydrological Office has been using an empirical formula for the determination of ground frost depth (Soveri & Varjo, 1977)

$$h = \frac{42\text{ cm}}{h_s} \cdot \mu b \sqrt{S - 15\frac{h_s}{\text{cm}}\,{}^\circ\text{C}\cdot\text{day}},$$

where μ is the soil type coefficient and b is the field coefficient. For an open field, $b = 1$, and for a forested terrain $b = 0.9$. μ varies between $0.9 - 1.2$, so that lower values correspond to greater water content. If the snow thickness is 20 cm and the sum of negative degree days is $S = 1\,000$ °C·day, the depth of ground frost will be 45 – 67 cm.

These examples shed light onto the sensitivity of ground frost development to the snow accumulation. In Finland, both the air temperature and the snow accumulation reflect themselves strongly in the ground frost depth. Eastern Finland has more snow than Western Finland, and therefore in the east ground frost does not reach as great depths as it does in the west.

Melting of ground frost begins from the top, when all snow has melted and the rays of Sun begin to heat the top layer of the soil. As the melt progresses, the surface of the ground frost retreats deeper into the ground, and the additional heat required has to be conducted from the ground surface. The ground frost surface depth can be solved from the continuity of heat transport:

$$\rho m L_f \frac{dh_b}{dt} = -k_b \frac{T_f - T_0}{h_b} = -Q_n. \tag{7.25}$$

If the surface temperature is known, this equation can simply be integrated. It results in $h_b = a_b\sqrt{P}$, where $a_b = \sqrt{\frac{2k_b}{\rho m L_f}}$ is the melting-degree-day coefficient and $P = \int_0^t \min(0,\ T_f - T_0(t'))dt'$ is the sum of positive-degree-days. Ground frost melt is thus similarly dependent on temperature as is ground freezing. The coefficients of melting and freezing are very close to each other: $a_r \approx a_b$.

Heat can also be conducted to ground frost from below. This slows down growth and accelerates the melting of ground frost. If at the beginning of winter snow accumulation is large, heat coming from the ground might be enough to prevent any ground frost formation.

Equations (7.24) and (7.25) also give the possibility to take a look into the formation of permafrost. Then the thickness of frost in the winter must be greater than melting in summer. When there is only a little snow, permafrost is formed when $h > h_b$. This is the situation, if $S > P$, i.e. the sum of freezing-degree-days is greater than the sum of positive-degree-days. Snow accumulation and melting do not affect ground frost symmetrically. The more there is snow, the more there will also be insulation to decrease growth and, on the other hand, the snow will also protect ground frost from melting. In Northwestern Lapland the condition $S > P$ is fulfilled in most years, and spots of permafrost can be found.

In hydrology the amount of water bound into ground frost is also of significance. Water content of ground frost is dependent on the amount of precipitation in the previous autumn and on the capillary properties of the freezing soil. The last-mentioned factor causes water to be sucked up into the ground frost layer. A thick layer of frozen ground with a high water content accelerates the runoff of melt water in spring. On sunny, south sloping hills ground frost thickness is smaller than on shady areas.

Ground frost tends to form in a layer where changes in the soil moisture content are the highest. Below the soil moisture content decreases due to liquid water percolating down into groundwater and because frost absorbs moisture. Ground frost moisture content is at its highest at the onset of melting. Temperature of groundwater is always slightly higher than its surroundings and it releases heat. Variations in groundwater temperature are smaller than variations in the surrounding ground. When the groundwater level is close to the ground surface, the ground frost will remain thin.

Development of ground frost layer is also dependent on the local vegetation, which acts as insulation. On open fields there are herbaceous plants, and in forests there are trees. Due to this additional insulation, ground frost is 11 % thinner in forests than it is in open ground. On the other hand, very dense spruce forests thicken the ground frost because they intercept nearly all of the snowfall, and thus leave less insulation over the ground. Interception in deciduous and pine forests is significantly less, and the ground frost is shallower.

Ground frost melts both from above and below, and the ground temperature will follow the air temperature with a lag. In spring, melt water will quickly run off from an area that has a thick, moist and dense layer of ground frost. In coarser soil types water can infiltrate through frozen ground and fill groundwater aquifers.

Figure 7.18: *Damage due to frost heave near Jyväskylä, Central Finland in 1981. Photo: Risto Virpimaa (Photo collection of the Finnish Transport Agency/Mobilia museum, Kangasala).*

The development of ground frost can be studied with frost tubes. The tubes contain methylene blue, which is blue as a liquid, but turns clear upon freezing. Soil temperature measurements can be done by placing thermistor chains protected by tubing into the ground. Also, thickness of a ground frost layer can be measured with a ground-penetrating radar (GPR). The bottom of the layer usually gives off a strong reflection.

Increase in the ground frost moisture content causes frost heaving, which is the increase of soil volume upon freezing. It manifests itself as a rise in ground level, surface damage and in movement of structures (Fig. 7.18), which is intensified by the formation of ice lenses into the bases of pillars and rocks. When they grow, they will lift any structures above them. The increase in moisture content is a result of the capillary action, which typically is at strongest in silty soil. Capillary rise in clay is significant, but the flow speed of water is so slow that the moisture content of ground frost does not have enough time to increase. In turn, the capillary rise in sand is so low that the layer of ground frost does not have the capacity to absorb water from deeper levels. In Finland, all soil types freeze, but frost heaving is not present in all of them.

Structures susceptible to frost must be supported deeper than the maximum expected ground frost depth. This is because artificial structures conduct heat better than the surrounding combination of soil and snow. Freezing progresses deeper around structures. Capillary potential has an effect on how water is absorbed from lower layers. The lift of ground is due to the capillary absorption of water into the frozen layer from the soil below it.

7.4 Glaciers

Land ice accumulations form in areas where the snow accumulation in winter does not melt away during summer. Snow that has survived over at least one summer is called firn. New snow cover falls always over the old snow and can accumulate to significant thickness over the years. Firn is slowly transformed into ice through a melt-freeze metamorphosis or by compression alone. The latter case requires a pressure of 50 – 100 m of snow. In the snow–ice transformation, air channels between the crystals of ice are closed and sealed bubbles are formed. This also drops the attenuation of light inside the ice, which gives it a clear appearance that resembles ice more than snow. Its density then is 830 kg m^{-3} and air bubbles make up for 9.5 % of its volume. Deeper into the glacier, pressure compresses air bubbles and the ice becomes denser.

Land ice forms are divided into two groups according to their size: *ice sheets* or *continental glaciers* and *glaciers*. In the current climate there are two ice sheets on this planet: the Antarctic ice sheet and the Greenland ice sheet. These are c. 4 km thick at their thickest and they contain about $^2/_3$ of the planet's freshwater resources. They remain mostly unused, since they are in solid form and reside far from any significant population centres. Glaciers can be found in all latitudes, all the way to the Equator in the Andes mountains. Among the greatest glaciers are *Vatnajökull* in Iceland, *Nordaustlandet* in Svalbard and the *Southern Patagonian ice field* in the Andes mountain range. Vatnajökull is the largest glacier of the Northern Hemisphere. It is 400 – 700 m thick and has a surface area of 8 200 km^2.

There are no glaciers in Finland. During the last ice age, Finland was completely covered by the Fennoscandian Ice Sheet. It retreated and melted away about 5 000 years ago. Glaciers nearest to Finland can be found in Kebnekaise, Sweden (*Storglaciären*) and Skibotn, Norway (*Steindalsbreen*, Fig. 7.19). In the fells of Enontekiö, Northwestern Lapland there are occasional spots of firn, but they are so sparse and irregular that glaciers do not begin to grow. Cooling of the climate or an increase in the annual snow accumulation could change the current state of affairs in Enontekiö. Finnish researchers have mostly been involved in glaciology in Antarctica and Svalbard. In turn, geomorphology resulting from the last ice age has been studied in Finland for a long time in the fields of geology and geography.

Ice sheets and glaciers store the water in them for a long time, from hundreds of years up to a million years. Glacial storages of water are usually large, and in regards to hydrology it is important to know how much melt water they will produce during the summer.

Ice sheets and glaciers can be divided into two zones: accumulation and ablation zones (Fig. 7.20), separated by an equilibrium line. In the accumulation zone the net accumulation of snow is positive and in the ablation zone it is negative. Accumulation zone is covered in snow almost all the way to the equilibrium line. Surface at the equilibrium line consists of refrozen slush. Ice can be visible elsewhere in the accumulation zone as well, due to the movement of the glacier or the wind drift of snow, spots called *blue ice*. Surface of the ablation zone is ice. Central areas of ice sheets are accumulation zone, and melting happens at the edges. In glaciers, the areas high up have dry snow, lower down they have areas of bare, melting ice.

The mass balance of glaciers is written as

$$\frac{1}{\rho_i}\frac{dM}{dt} = P - E - R + Y \tag{7.26}$$

where M is the mass per unit of surface area, P is precipitation, E is evaporation, R is runoff and Y is the net accumulation of drifting snow. All terms are given as rate of change in liquid water equivalent.

In an equilibrium, a glacier gives up a water through evaporation and runoff the same amount that it receives through precipitation. Drifting snow causes local changes in the yearly mass balance of glaciers. Runoff happens also in solid form, when a glacier terminating in the sea calves icebergs. Annual mass accumulation is $P - E$, and when the effect from the drifting snow is assumed small, $R \sim P - E$. The amount of water received during the melting season is then $(P - E)A$, where A is the surface area of the glacier. Hydrological methods have been used to obtain direct measurements of the mass balance. Mass balance of the Antarctic ice sheet is dominated by precipitation and roughly equal amount of iceberg calving. Melting, evaporation and liquid runoff are small.

Example 7.6

Surface are of the Vatnajökull glacier is $A = 8\ 200\ \mathrm{km}^2$, and the annual precipitation is approximately 1 000 mm. If evaporation and the accumulation of drifting snow can be assumed small, then the scale of the net accumulation will be $P_n = 500$ mm. As the net accumulation discharges from one river into the sea, mean flow rate will become $P_nA/\mathrm{year} = 130\ \mathrm{m}^3\ \mathrm{s}^{-1}$. This is comparable to the discharge of river Kymijoki, which is 240 $\mathrm{m}^3\ \mathrm{s}^{-1}$ (see table 6.1). Melting season of a glacier is short. If it were four months, the mean discharge would be 390 $\mathrm{m}^3\ \mathrm{s}^{-1}$.

Figure 7.19: *Steindalsbree glacier, Skibotn, Norway. Glacial flow originates from the top, narrows down and creates a wider formation further down. Photo: Matti Leppäranta.*

Energy balance of a glacier is similar to the energy balance of any ground surface. Albedo is determined by the surface type, whether it is snow or ice. Heat is conducted from the surface down. Melting can be in the order of 1 meter during the melting season. In the Antarctic, there is very little melting, and almost all runoff happens as iceberg calving (Fig. 7.21).

Glaciers do flow, but ever so slowly. Glacial ice is viscous material, which flows under the force of gravity. In mountain glaciers, flow is on inclined surfaces, but ice sheets creep, or deform like a sticky liquid, under their own weight. This movement is driven by gravity, and the flow velocity is $10 - 1\ 000$ meters per year, or in the order of meter per day. Viscosity of glaciers is very high (10^{15} kg m^{-1} s^{-1}). Mountain glaciers flow downhill so, that melt from below and grow at the top.

Movement of glaciers can be divided into two types: basal sliding and internal deformation. Basal sliding can be anything between $0 - 100$ % of the total movement of a glacier. Internal deformation is viscose creep. Glaciers do not slide down like a solid object, but they deform during sliding. Ice sheets spread out under their own weight on a level surface. The maximum thickness of the Antarctic ice sheet is c. 5 km, which corresponds to the maximum structural strength of ice.

Land ice has liquid water stored on and around in *proglacial*[7] lakes. During the melting season liquid runoff travels to outlet rivers and it can form *epiglacial* lakes near the edge of the glacier. For example, 10 000 years ago in place of the Baltic Sea there was Baltic Ice Lake that had accumulated from the runoff of the Fennoscandian Ice Sheet. On areas

[7]The term *glacial lake* refers to lakes that have been formed due to glacial processes. Most of the lakes of Finland are glacial lakes. Proglacial lakes are on and around current glaciers.

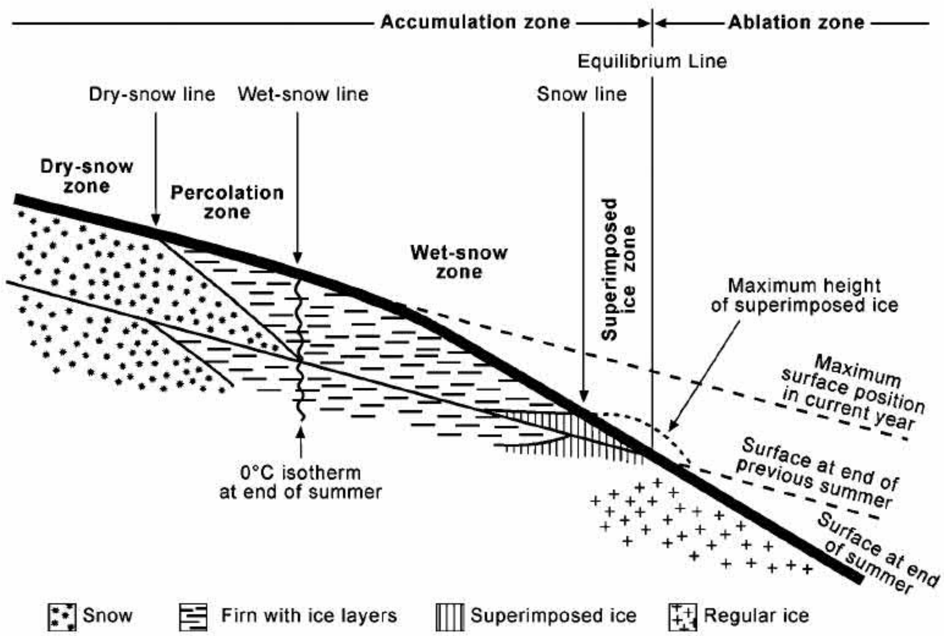


Figure 7.20: *Glacier zones. Source: Cuffey & Paterson (2010).*

of blue ice, solar radiation can melt ice and form *supraglacial* lakes. They can be open or covered by a thin layer of ice. Seasonal supraglacial lakes form during the summer and freeze completely during the winter. They have a depth of around one meter. Supraglacial lakes that survive over several years grow over time and eventually discharge due into the ice their own pressure. These discharges can be extremely severe and cause cracking of ice and flooding.

The most exotic of all lakes exist under the Antarctic ice sheet. When a balance between geothermal heat and the heat conduction through the ice sheet persists, water can remain liquid is *subglacial* lakes. The largest of them is Lake Vostok, which has a surface area of 15 000 km^2 and a depth of 150 m. High pressure persists under the ice sheet, for which reason the melting point of ice is at -3 °C.

Example 7.7
In the Icelandic language a surge of glacial floodwater is called a *jökulhaup*. One of the most famous cases was the Grimsvötn jökulhaup in 1934. The peak flow rate was $5 \cdot 10^4$ m^3 s^{-1} ($^1/_4$ of the flow rate of the Amazon river) and the extent of the flood was 100 km^2.

Ice sheets and glaciers form freshwater resource with a long retention time. Glacial melt water is very important for many local waterworks. They have been used for hydropower in Norway and as irrigation in the dry agricultural areas of Northern Italy.

This chapter has revolved around underground water resources and glaciers. Soil water is an unsaturated layer of water, where the movement of water is mostly vertical. Groundwater is a saturated layer of water below the soil water layer. Groundwater flows mostly

Figure 7.21: *An iceberg that has just calved from an ice shelf. Photo: Matti Leppäranta.*

horizontally, directed by pressure. Soil water balance is also influenced by temperature due to the dynamics of vapor and ground frost in the cold zones. As the Finnish poet Eino Leino wrote:

Suot ovat suuret Suomenmaassa	*Mires are wide in the Finnish lands*
rimmet sekä nevat,	*bogs and swamps,*
Suomen soilla hallan immet	*on the mires maidens of frost*
öisin laulevat	*singing in the night.*

After this chapter all of the storages of water – atmospheric water, lakes and underground water – have been covered. Between these storages transport happens via air, rivers and runoff both under and above ground. Runoff is the subject of the next chapter, which ends our examination on the hydrological cycle.

Chapter 8

Runoff

8.1 Principles of runoff

Runoff is one of the central events in hydrology (Fig. 8.1). World Meteorological Organization defines runoff as 'That part of the precipitation that flows towards the stream on the ground surface (surface runoff) or within the soil or bedrock (subsurface runoff)' (WMO, 1974). It transports rainwater back into the ocean and offers fresh water for the use of the nature and man. *Specific discharge* is runoff over a unit of time and surface area.

Figure 8.1: ***a)*** *Spring flood in Äkäslompolo (Lapland) and* ***b)*** *Kymijoki mouth, which discharges water from the Päijänne water system into the Gulf of Finland. Photos: Matti Leppäranta.*

A catchment area forms an entity into which water comes in as precipitation and leaves as runoff and evaporation, which produces the net change in the volume of water storage. Runoff can be solved from the water balance equation (Eq. 3.23)

$$R = P - E - \frac{1}{A}\frac{dV}{dt}, \tag{8.1}$$

where A is the surface area of the catchment. The storage of water can be in ground, water system, or in vegetation. Runoff, like flow rate, describes transport of water. Its dimension is length over time. Typical unit used is millimeters per day or year, the same units that are in use for precipitation and evaporation. Conversely, runoff as a volume over time is acquired by multiplying it with the catchment surface area. Dimension of discharge is volume over time, and it means the volume of water passing a cross section of a stream in unit time.

Example 8.1
The runoff into the Baltic Sea brings in annually a water equivalent of one meter spread over the whole sea. The catchment surface area 1.65 million km^2, and the surface area of the Baltic Sea is 0.393 million km^2. The mean precipitation over the catchment area is 500 mm per year. The catchment surface area is about four times as big as the surface area of the Baltic Sea, so the amount of precipitation is equivalent of 2.1 m layer of water over the sea. Approximately half of the precipitation is evaporated before it reaches the sea.

Runoff is caused by gravity. Cohesion forces in the soil and friction resist this movement. Vegetation influences runoff significantly. All in all, the formation and spatiotemporal development of runoff are complicated processes. There are ways to simplify it, which are considered in more detail in the upcoming sections. Runoff can be thought of as consisting of three parts: *surface runoff*, *soil water runoff* and *groundwater runoff*.

Figure 8.2 describes how a constant precipitation affects runoff and other components of the hydrological cycle as a function of time. The fraction of runoff, and especially of surface runoff, increases as the rain prolongs, as the fraction of the other components become constant when their corresponding storages fill up. Water flowing on ground surface ends up in small depressions and streams in a matter of minutes from the start of the rain. Soil water runoff or interflow develops in the following hours (Fig. 8.3). Groundwater runoff depends strongly on the soil water storage, and it joins watercourses after weeks, months or even years after the precipitation event. The effect of precipitation on groundwater flow can be detected rather quickly, but the flow itself is much slower. Generally, there has been consensus on the relation between old and new groundwater, where new water pushes old into the water system (Sklash & Farwolden 1979, Rodhe 1981, Laudon et al. 2007), although, lately this has been questioned. This process also has an effect on the transport of dissolved matter.

Dependencies between the different components of runoff are defined by topography, soil type and temporal development of precipitation and the melting of snow. Surface runoff is large, when the soil conducts water poorly. Poor seepage could stem from the fine grained nature of the soil, frozen soil or natural or man made soil compression. Pores in the soil can also be saturated with water due to past rain and cannot absorb any additional water.

Interflow, that is runoff happening in soil, is great if the soil conducts water well and below it there is an impermeable layer, which prevents any further infiltration into deeper layers. This is a likely scenario on agricultural fields, where nutrient rich humus on the top layer has dense subsoil below it. Groundwater runoff is large in areas where the soil absorbs water efficiently, and where the layers below conduct water well.

However, dividing runoff into these three categories is somewhat theoretical. Quite often surface runoff transforms into subsurface runoff and back to surface runoff before reaching a channel flow. During wet periods distinguishing between surface runoff and groundwater

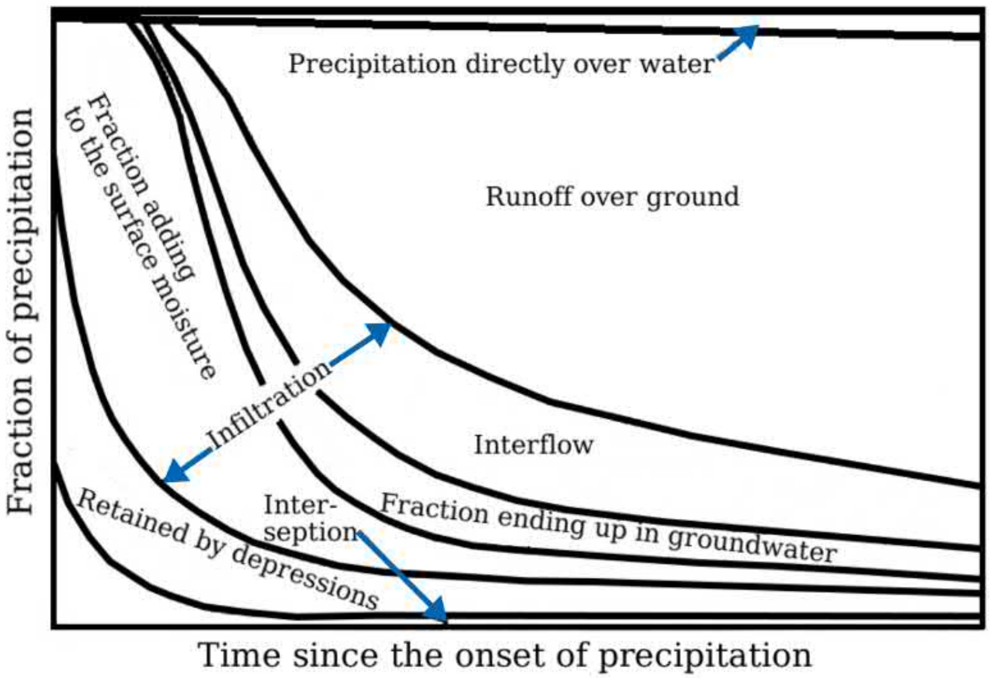


Figure 8.2: *Development of various components of runoff over time since the onset of precipitation. Source: Mustonen (1986), edited.*

runoff is difficult and inaccurate. On the other hand, after a dry period or during cold winters in lakeless areas, runoff can be considered to be almost exclusively groundwater runoff. Groundwater runoff at its simplest can be found in springs, and surface runoff on paved surfaces. Lake Sääksjärvi in Nurmijärvi and Lake Vesijärvi in Päijät-Häme get nearly all of their water from groundwater, and can thus be considered to be 'large springs'.

Relative sizes between different forms of runoff change depending on the season. During the melting of snow, fractions of surface and subsurface runoff are large. During a typical summer most of the runoff is groundwater runoff. Autumn rains increase the fraction of surface runoff, and they may cause more surface runoff if the soil pores are saturated with water. Runoff during the winter is almost predominantly groundwater runoff, apart from occasional short periods of above freezing temperatures.

Hydrograph is one of the key graphs in the field of hydrology. Figure 8.4 presents the shape of a typical short period discharge or runoff curve. Runoff is assumed to consist of two parts there: base flow and direct runoff. Base flow is the sum of groundwater runoff and slow interflow. Direct runoff forms from precipitation and the melting of snow, which quickly finds its way into rivers as surface runoff and interflow. It also includes the fraction of precipitation that is not retained by interception, depressions or filtration. Direct runoff is also called the effective precipitation or effective melting, and it can be discerned from a retention curve.

Three components can be seen in the hydrograph: rising limb, peak and draining or recession curve. The shape of the rising limb depends mainly on the type of the runoff event. If the effective precipitation or melting persist long enough for the runoff to reach the point of measurement from the most distant parts of the catchment area, then the rise time t_p

Figure 8.3: *Rain and melt water accumulation into a ditch. Photo: Matti Leppäranta.*

describes the runoff time of concentration typical for the catchment area. The turning point, which is found on the decreasing part of the hydrograph, is usually considered to be end point of surface runoff. After this, the hydrograph describes the draining of water storage developed on the catchment.

If the event behind the runoff reaches all over the catchment area, the recession curve after the turning point can be considered as a permanent property of the catchment area. Decrease of the runoff is assumed linear, and the basic equation and its solution are

$$\frac{dR}{dt} = k_r R \quad \text{and} \tag{8.2}$$

$$R(t + t_0) = R(t_0)e^{k_r t}, \tag{8.3}$$

where $t \geq 0$ is the time after the turning point t_0 and k_r is the recession constant. Time scale of this process is described by the inverse of the recession constant, and after a time period of $3k_r^{-1}$ the runoff has nearly ended, $R(t + t_0) = e^{-3}R(t_0)$, when $t = 3k_r^{-1}$.

There is no unambiguous way to separate base flow from the hydrograph. The first step is to find the moment, where runoff starts to grow. Defining the end point requires experience from inspection of several hydrographs from various rain events. An empirical formula can be used to help in this:

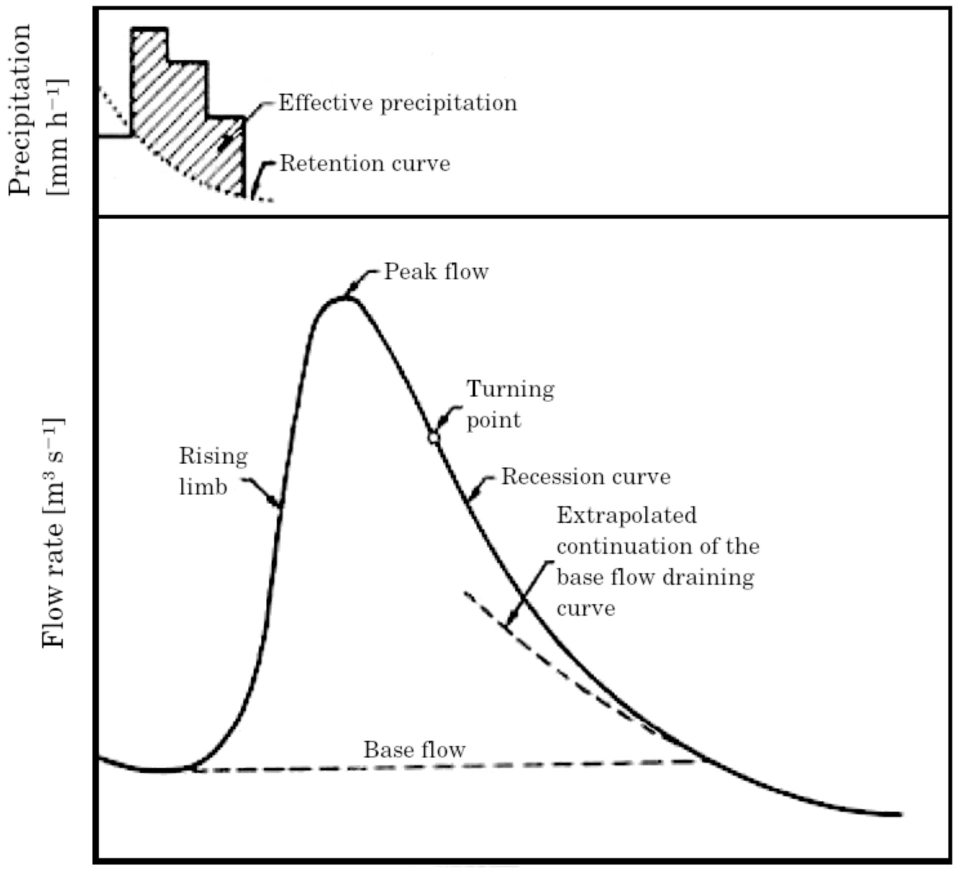


Figure 8.4: *Components of the hydrograph. Source: Mustonen, (1986), edited.*

$$\frac{t}{\text{day}} = 0.87\left(\frac{A}{\text{km}^2}\right)^{0.2}. \tag{8.4}$$

Separating the different components of the hydrograph is easiest in small, lakeless catchment areas where a dry season is followed by an intense rain event. The separation becomes more difficult in large catchment areas, due to lakes and the uneven distribution of precipitation.

In Finland, precipitation is much larger than evaporation, and thus runoff is always a significant part of the transport of water. This has also played part in creating a very diverse role of water in Finnish culture (Fig. 8.5).

8.2 Runoff models

8.2.1 General

Hydrological models aim to describe the whole hydrological cycle or just a part of it. Typically, a model has a central variable, for example runoff, soil moisture content, groundwater runoff or evaporation, against which the model is calibrated. Modeling is difficult especially due to surface runoff and interflow (subsurface flow). Input of water in runoff models comes from precipitation. The amount of runoff is determined by weather conditions and soil type.

Changes in discharge rate, water level and runoff of a water system are described by the following abbreviations:

High-	H		
Mean high-	MH	Q	Flow rate
Mean-	M	W	Water level
Mean minimum-	MN	q	Runoff
Minimum-	N		

Combining these we get quantities like MW = mean water level, HQ = high discharge and MNq = mean minimum runoff. The quantities refer to the observation period. For example, in a 20 year time series HQ_{20} is the maximum of the whole time series, MHQ_{20} is the mean of the annual maximums and MQ_{20} is the mean of the whole time series.

In many hydrological applications the amount of water that leaves the catchment area as runoff or river discharge has to be estimated from precipitation measurements. This problem is encountered for example when hydrological forecasts are made, or when statistical parameters of discharge have to estimated from a sparse data set. Advanced hydrological runoff models are also necessary when quantifying the effects of environmental changes into the hydrological cycle and when estimating the distribution of dissolved pollutants in a given catchment area.

Statistical methods were used in early runoff models. Older models were based on the empirical dependencies between precipitation, runoff and water level. The unit hydrograph method was developed in 1930s. It was used mostly to estimate the runoff after major rain events. Use of numerical models began to grow strongly in 1960s. One of the first models was the Stanford runoff model, developed in 1961 in the U.S.

Conceptual models followed numerical models. They are formed by making simplified approaches to the components of the hydrological cycle. As computational resources have much advanced, it has become possible to solve the formation of runoff directly from the differential equations of flow dynamics. Then, simplifying assumptions have to be made, because otherwise very detailed information about the catchment area would be required. A physical basis for the models is built out of the flow dynamics in basins, channels and through porous media, and thermodynamics, which includes phase changes. The greatest difficulties spawn from rain falling down on ground, because what happens then is strongly dependent on the properties of the surface. Water level in ground and in lakes, river discharge, evaporation and the transport of water out of catchment areas are the output of runoff models. This information is required to make flood forecasts, to maintain hydroelectric power plants, and to protect nature and environment.

One of the most widely used conceptual models is the HBV-model (Hydrologiska Byråns Vattenbalansavdelning, Bergström & Forsman, 1973) developed by Sten Bergström. In this

Figure 8.5: *'Passage', by Pekka Halonen (1892). Photo: Finnish National Gallery/Matti Janas, ©Finnish National Gallery/Ateneum*

model, the catchment area is divided into smaller sub-catchments, each of which has its own water balance calculated. Runoff results from overflows of storages of water. The Finnish Environment Institute SYKE operates a country-wide watershed model, WSFS (Watershed Simulation and Forecasting System). It is based on refined HBV-model (Vehviläinen, (1982, 1992, 1994)).

8.2.2 Effective precipitation and unit hydrograph

Rain falling on ground is divided into three parts: direct runoff, water seeping into soil and groundwater, and direct evaporation. Water that ends up in groundwater and water that evaporates are losses. Effective precipitation is the difference between precipitation and losses, and it is denoted by P_e.

The concepts of effective precipitation and unit hydrograph were created by L. K. Sherman in 1930s. More effective models had already partially superseded the unit hydrograph method, but it still contained properties that aided in understanding the progression of the process of runoff. Runoff due to the effective precipitation is called the direct runoff R_d. Figure 8.6 presents the division of precipitation into the effective precipitation and losses, and also the division of runoff into direct runoff and groundwater runoff.

The amounts of effective precipitation and direct runoff have to be equal. These totals can be acquired from the equation

$$\wp_e = \int_0^T P_e dt \quad \text{and} \tag{8.5}$$

$$R_d = \int_0^\infty R_d dt, \tag{8.6}$$

where T is the duration of the rain event. On the basis of the conservation law $\wp_e = R_e$. The total amount of effective precipitation is affected by the amount and duration of the precipitation event, season and the storage ability of the soil, which is called the antecedent precipitation index. It describes the effect of past precipitation on the soil moisture content and the soil's ability to receive new rain. Antecedent precipitation index is acquired from the recursive formula:

$$\Gamma_i = r\Gamma_{i-1} + P_{i-1} \tag{8.7}$$

where Γ_i is the antecedent precipitation index of day i, P_i is the precipitation on day i and r is a constant or a seasonal parameter. Through experimentation it is possible to determine the dependency between the effective precipitation and the aforementioned quantities. Antecedent precipitation index is dependent on the soil moisture content and the amount of water that the soil is still able to receive.

Example 8.2
To understand the antecedent precipitation index, let us have a look at two extreme cases:
a) $P_i = 10$ mm day^{-1} ($i = 0$) or 0 ($i > 0$) and $r = 0.2$. Then, $\Gamma_1 = 10$ mm day^{-1}, $\Gamma_2 = 2$ mm

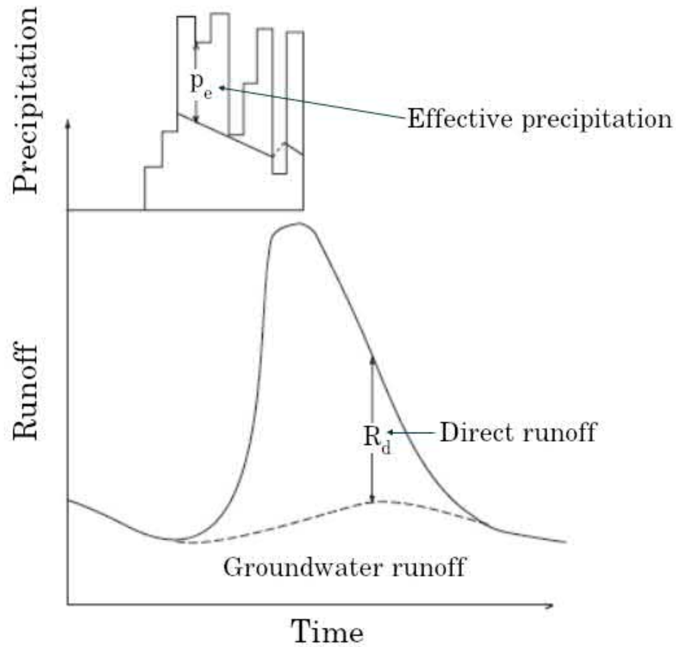


Figure 8.6: *Formation of runoff during and after rain. Differentiating effective precipitation and direct runoff.*

day^{-1}, $\Gamma_3 = 0.4$ mm day^{-1} and so forth. The antecedent precipitation index gets smaller, as time from the rain event passes.
b) $P_i = 5$ mm day^{-1} = constant and $r = 0.2$. Now, $\Gamma_1 = 5$ mm day^{-1}, $\Gamma_2 = 6$ mm day^{-1}, $\Gamma_3 = 6.2$ mm day^{-1} and so forth. The antecedent precipitation index grows as the soil fills up with water.
c) The limits for Γ_i are $\Gamma_i = P_{i-1}$, when $r = 0$, and $\Gamma_i = \Sigma_{k=0}^{i-1} P_k$, when $r = 1$. In the first case, soil always draws the rainwater deeper, and in the second case rainwater always remains in the surface layer.

Now that the determination of effective precipitation from the total precipitation has been explained, we can move on to see how the direct runoff is distributed over time. Let us assume, that the 'effective precipitation – direct runoff' system is linear, which means that rain events happening at different times can be summed together. In reality, this linear assumption is not always valid, because progression of the effect of precipitation in a water system is dependent on runoff.

From the linear assumption it follows, that the runoff at time t, which is due to effective precipitation $P_e(s)ds$ at time s, is proportional to this effective precipitation (Fig. 8.7). It can be written as

$$dR_d(t) = u(t-s)P_e(s)ds, \tag{8.8}$$

where u is the *instantaneous unit hydrograph* (IUH) caused by instantaneous precipitation. Based on the linear assumption IUH is an intrinsic property of the water system. It is not dependent on, for example, effective precipitation or runoff.

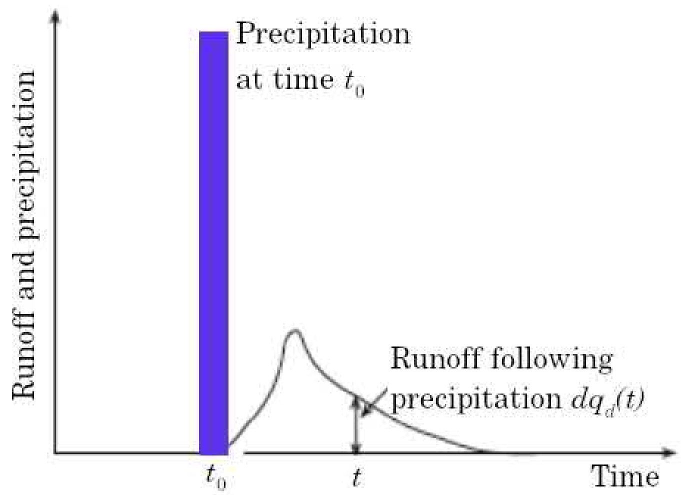


Figure 8.7: *Direct runoff $q(t)$ as a result of effective precipitation at t_0.*

On the basis of the linear assumption, runoff $R_d(t)$ at time t is acquired by integrating equation (8.8) over time:

$$R_d(t) = \int_0^t u(t-s)P_e(s)ds. \tag{8.9}$$

The time t is zero at the moment the rain event begins.

In practice, runoff is not determined as a continuous function of time, but rather it is calculated for short timesteps. The time from the beginning of the rain is divided to timesteps with a length of Δt. Mean values of runoff R_d and effective precipitation P_e at periods $[0, \Delta t]$, $[\Delta t, 2\Delta t]$ are marked by subscripts 1, 2, ... Now, equation (8.9) can be written as

$$\begin{aligned} R_{d1} &= P_{e1}U_1, \\ R_{d2} &= P_{e2}U_1 + P_{e1}U_2, \\ &\dots \end{aligned} \tag{8.10}$$

Coefficient U_1 describes the fraction of effective precipitation that is drained as direct runoff during that timestep, U_2 describes the same fraction for the next timestep and so forth. Coefficient U_i is a dimensionless quantity. The dimension of the function $u(s)$ on the other hand is 1/time. Because the amounts of effective precipitation and direct runoff are the same, it follows that

$$\sum_i U_i = 1, \int_0^\infty u(s)ds = 1. \tag{8.11}$$

Example 8.3
Let us take a look at a case, where $u = \lambda e^{-\lambda t}$.
a) If P_e = constant, then $R_d = P_e(1 - e^{-\lambda t})$.
b) If there is a sudden shower $P_e = \delta(s)$, then $R_d = P_e e^{-\lambda t}$. Here, δ is the Kronecker delta, which is defined as

$$\delta(s) = 0,\ s \neq 0 \text{ and } \int_{-\infty}^{\infty} \delta(s) f(s) ds = f(0).$$

c) If P_e = constant, then the coefficients in equation (8.10) become $U_1 = 1 - e^{-\Delta t}$, $U_2 = e^{-\Delta t} - e^{-2\Delta t} = e^{-\Delta t}(1 - e^{-\Delta t})$, and so forth.

Instantaneous unit hydrograph is determined by investigating the effect of many separate short rain events on the runoff. The use of IUH is hampered by many factors:

1. The assumption that the water system functions linearly is a source of errors.
2. In reality the shape of the function $u(s)$ is determined by the amount of effective precipitation. Its 'length' increases as P_e increases.
3. This method does not allow for the calculation of total runoff, it is only suitable for the determination of direct runoff.
4. The concepts of *direct runoff* and *base flow* are inaccurate, and differentiating them is difficult.
5. In Finnish conditions, runoff is often the result of many small precipitation events, and there usually is no sufficient number of clearly separate events to calibrate the method.

8.2.3 WSFS watershed model

The Finnish Environment Institute (SYKE) operates the WSFS (Watershed Simulation and Forecasting System) runoff model. The input data for this model are precipitation, mean temperature and potential evaporation. Potential evaporation can be solved via meteorological observations. WSFS models several processes, all of which have their own process model: precipitation model, snow model, depression storage model, soil water model and interflow and groundwater runoff models (Fig. 8.8). The effect of elevation is also included in the runoff model.

The fill levels of storages are described by variables, whose dimension is length and unit is usually millimeter. Each storage has their own limits. Filling and draining of these storages are controlled by various tuning parameters. The storage maximums and other parameter values are acquired through experience and by calibrating the model with observations of water level and discharge.

The precipitation model calculates the precipitation falling onto a catchment area. Its phase is also determined at the same time. After this, the measurement errors of precipitation are compensated and the effect of elevation on the rain is taken into account. Finally, the areal precipitation is calculated. Weather radar observations are used to estimate the areal precipitation of the last five days. Values older than that are acquired from weather station data.

The snow model calculates the snow accumulation, sublimation, melting and the extent of snow cover. Melting is estimated based on the daily mean temperature, separately for open fields and forests. Meltwater is first stored in the snow cover, depressions on the ground and bogs (which is a depression storage in the model). As the melting progresses the meltwater quickly drains into rivers and streams as surface runoff and interflow. Part of the water is retained in the soil water storage and filtered down to groundwater, from where the discharge is slow.

Snow melts faster in open fields, bogs and clear-cut areas due to solar radiation and the wind driven turbulent heat exchange. Forest protects the snow cover from the Sun and wind, which causes the snow to melt slower. This time difference is typically about two weeks. If the melt is interrupted, the snow that has been retained in the forests can cause sudden floods during times when all of the snow from fields, roads and population centers has already melted a while ago. Part of the meltwater is frozen into ditches and depressions, and the elevation and the direction of the slope affect melting as well.

The soil water model solves the storage of water bound in the soil between 0.5 m to 1.5 m of depth. It is replenished by precipitation and meltwater, and drained by evaporation and percolation into groundwater. Part of the incoming water is stored into the soil layer, some of it is evaporated and the rest form runoff via soil water and groundwater storage. During summer, the size of soil water storage is between 50 – 100 mm. With precipitation, this fill level determines the amount of runoff and the development of floods. During spring and late autumn this storage is usually full and the accuracy of the flood forecast is dependent on the accuracy of the areal snow water equivalent and/or the reliability of temperature and precipitation forecasts. Evaporation from soil is based on potential evaporation and the fill level of the soil water storage. From the soil water storage water is transported into interflow storage (intermediate storage), from where a quickly discharging runoff will form the flood peak. Further, the water will be carried into the groundwater storage, from where the water will discharge over a longer period of time. With these storage models, values for precipitation losses, runoff lag and water storage can be solved.

Other submodels included in the WSFS are the river model, the flood area model and the lake model. Several different types of models can be used for the modeling of river reaches: simple models solving the lag and dissipation of a flood wave or dynamic physical models based on momentum and conservation equations. The river model currently in use describes the river as a series of basins. It uses the information about the river length, width and slope. With this type of model, water level for the whole length of the river can be calculated. For important reaches of the river the the solution can be made more accurate with in-situ measurements.

The lake model solves the volume and water level of lakes from inflow, outflow, local

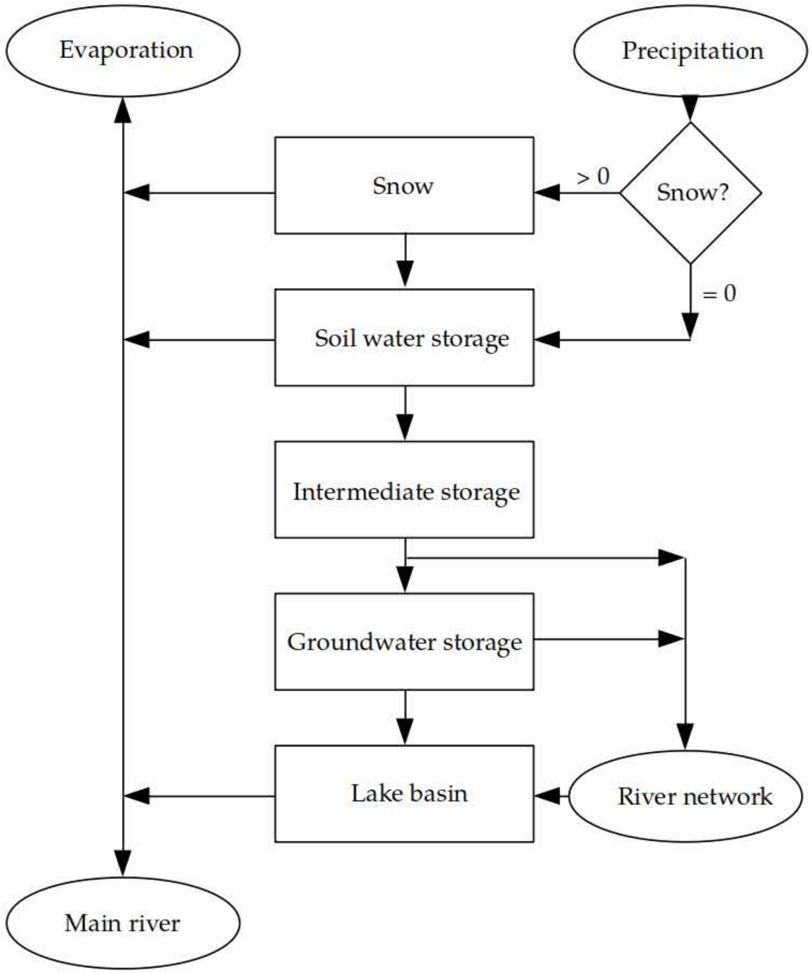


Figure 8.8: *Structure of the core of the WSFS water system model. The quantities are handled as daily values. The unit of all flux quantities is mm* day^{-1}*, snow, soil water, intermediate storage and groundwater have a unit of mm, the unit for the lake basin storage is* m^3 *and river discharge is in* m^3 s^{-1}.

runoff, evaporation, and precipitation. Evaporation is calculated from potential evaporation and lake surface temperature. Lakes warm up and cool down slowly, which decreases the evaporation in spring and increases it in autumn in comparison to the potential evaporation. Human activity has a significant effect on the water balance of many lakes, because they are regulated through damming and release of water. For each regulated lake there exists a regulation guideline, which is taken into account in the model.

Figure 8.9 presents a one year forecast for the water level in Lake Pielinen, produced by the WSFS model. The model takes into account the statistical variation in the water level. In addition to this, it also takes into account the variation in weather forecasts. This is achieved by running the model with over 50 different forecasts. Mostly, they are received from the European Centre for Medium-Range Weather Forecasts (ECMWF).

Lakes, as well as bogs, act as intermediate storage for runoff in nature and in the model,

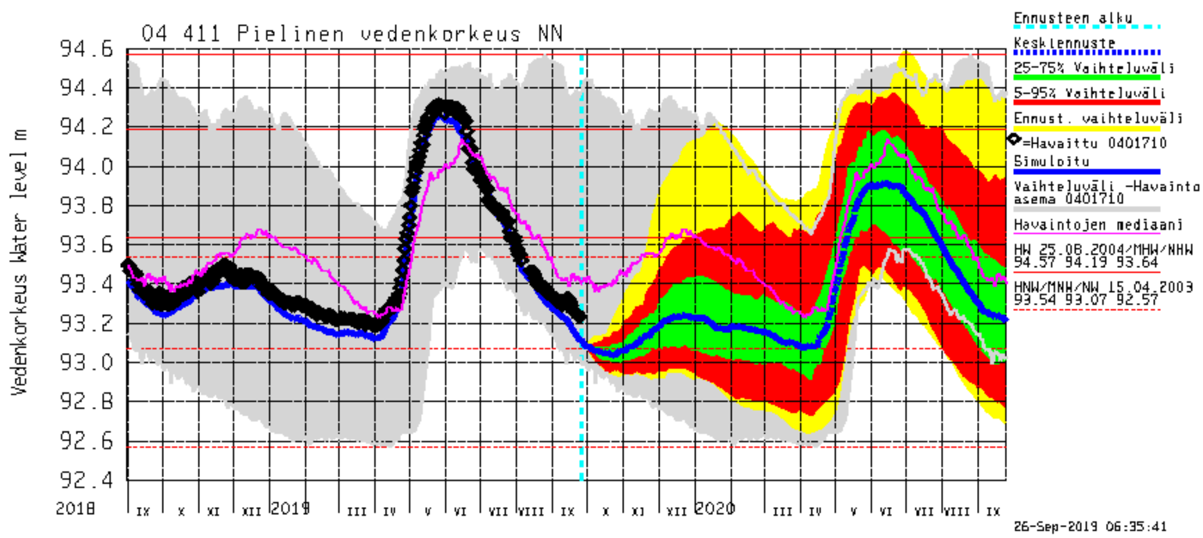


Figure 8.9: *Simulated water level in Lake Pielinen (26.9.2019), produced by the WSFS model. Vertical axis is water level in meters (coordinate system NN), horizontal axis is time. Translation of legend: Beginning of forecast, mean forecast, 25 – 75 % range, 5 – 95 % range, Total range of forecast, Observed, Simulated, Range of observations, Median of observations.*

delaying the runoff peak and smoothing it. Minimum flows are bigger in areas with plenty of bogs and lakes, than in an area with just a few of them.

The watershed model is assembled from runoff, river, flood area and lake models to describe a certain water system. The model is calibrated for each water system by using an optimization model to seek suitable parameters. Calibration is performed continuously, by applying automated measurements of water level and other quantities. In addition to automated algorithms, expert knowledge is also applied.

8.3 Water quality models

8.3.1 Models describing the quality of water in runoff

Various models have been developed to estimate the load of nutrients and other dissolved matter discharging with runoff into bodies of water. The key component in them is the calculation of runoff (Fig. 8.10). To compute a solution of the water quality, different approaches have been applied. In Finland, models from the INCA (Integrated Nitrogen in CAtchments) family and SWAT (Soil and Water Assessment Tool) models have been used. They are suitable to be ran on a personal computer. Water quality is also assessed in the WSFS model. It is called VEMALA (Huttunen et al., 2008), and it is actively being developed by SYKE. It has been used to estimate the nutrient load into the Baltic Sea, both currently and under the changing climate.

INCA-N is a mathematical, process based nitrogen model, originally developed in England (Wade et al., 2002). The model simulates the effects of the changing environment (climate change, land use change and fallout change) on the nitrogen load. It has been

Figure 8.10: *Small creatures that move with the runoff are being collected on field course at the Lammi biological station. Photo: Anna-Riikka Leppäranta.*

widely used for small catchments and large river systems. The INCA model takes into account the most significant sources of nitrogen, and simulates different sources of nitrogen and their transformation processes in local soil and watersheds. The model has a user interface and five components: land use, fallout, hydrology, soil and rivers.

The model has been further improved in SYKE for a cold climate by adding a snow model into it and by improving the computation of the soil temperature (Fig. 8.11). In applying the model to river Yläneenjoki, it was possible to assimilate real time nitrate measurements. In the last few years, comparative models have been developed for sediment transport (INCA-Sed), carbon transport (INCA-C) and a specialized model for pathogen transport (Rankinen et al., 2016). The second model usable on personal computers is SWAT, which is a dynamic, catchment area based model developed in the U.S. (Arnold et al., 1998).

8.3.2 Models describing the quality of lake water

There are several approaches to modeling the quality of water in lakes. At its simplest, this means that the concentration of a substance, for example phosphorus (c_P) at the lake outlet can be solved from the incoming phosphorus load (MP) by applying the continuously stirred tank (CSTR) method (Frisk, 1989). This method is based on the following assumptions:

- The lake basin is completely mixed, and the incoming load is immediately mixed into

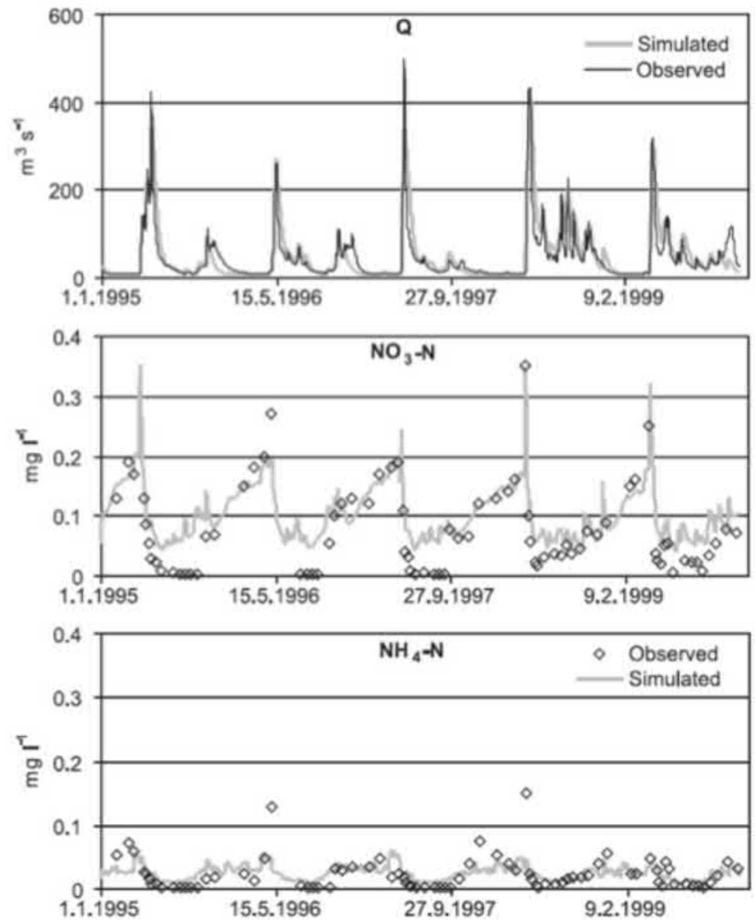


Figure 8.11: *Result given by the INCA-N model for the mouth of the River Simojoki. It simulates the amounts of nitrate ($NO_3 - N$) and ammonia ($NH_4 - N$). Source: Rankinen et al., (2004).*

the whole lake.

- From the previous assumption it follows, that the concentration at the lake outlet is the same as it is in rest of the lake.
- Hydraulic balance reigns over the lake, which means that the amount of incoming and outgoing water has to match.

Based on these assumptions, a general mass balance equation can be written for the basin:

$$V\frac{dc}{dt} = I_c - cQ + S_cV, \tag{8.12}$$

where V is the basin volume, c is the substance concentration, I_c is the total load of the substance into the basin, Q is the flow rate at the lake outlet, and S_c is the source/sink term, describing the effect of the basin's internal reaction on the substance concentration (like sedimentation, decomposing and release from the sediment). This equation can be solved by using the assumption for the balance $\frac{dc}{dt} = 0$, in which case the basin phosphorus concentration c_P will reduce to

$$c_P = \frac{I_P - S_P}{Q}, \tag{8.13}$$

where S_P is the retention of phosphorus in the lake (kg per year) and I_P is the total phosphorus load into the basin, which is usually directly proportional to the phosphorus load $I_P = R^* c_0 T$. The proportionality coefficient, or rather, the retention coefficient R^* receives basin specific values. There are many different formulas for its estimation. The following is the equation given by Frisk (1978)

$$R^* = 0.9 \frac{c_0 T}{B + c_0 T} \tag{8.14}$$

where $B = 280$ mg m^{-3} month is a parameter, c_0 is the load/volume (mg m^{-3}) and T is the retention time (in months). The retention coefficient is dimensionless, and it is dependent on the initial concentration and retention time. The theoretical value $T = V/Q$ for retention time is often used. Based on the assumption that the incoming substance is perfectly mixed into the whole basin, phosphorus concentration at the lake outlet can now be calculated as follows

$$c_P = (1 - R_P^*) \frac{I_P}{Q} = 1 - 0.9 \cdot \left(1 + B \frac{Q}{I_P T}\right)^{-1}. \tag{8.15}$$

Meriläinen & Veijola (1985) applied this method into soil water in an equilibrium. They had at their disposal observations of flow rate and concentration of streams. The weighted mean of concentration was 87 μg P L^{-1}. In their calculation, a value of 0.45 m^3 s was used for runoff, with which value they got a result of 1 200 kg per year of phosphorus load into soil water. They calculated the retention coefficient at two different mean depths (1.5 m and 2.0 m) and with corresponding retention times (20 months and 26 months), and got retention coefficients of $R^* = 0.80$ and $R^* = 0.78$. Mean values of phosphorus concentration at soil water outlets was thus 17 μg P L^{-1} and 20 μg P L^{-1}.

Solution for the concentration equation (Eq. (8.12)) can also be presented as dynamic (i.e., changing over time). Then, the equation becomes

$$c(t) = c_P - (c_P - c_0) \exp\left[- \left(\frac{Q}{V} + K \right) t \right], \tag{8.16}$$

where c_0 is the initial concentration in the basin, c_P is the concentration in a stable situation, when $t \to \infty$, and K is the sedimentation coefficient. Any other internal processes in the basin have been neglected. For the sedimentation coefficient of total phosphorus volume in Finnish lakes, a value of $K = 1 \cdot 10^{-6}$ mg P m^{-3} s^{-1} has been used. Malve (2007) has developed the statistical LLR/Lakestate model, which is based on the CSTR model. It presents the result as a probability distribution. The model can be used as a cost-effective method of evaluating the required reductions of total nutrient load on lakes and for probabilistic ecological classification (Fig. 8.12). CSTR hydraulics is useful for shallow lakes with no stratification, and for estimating long term means in lakes that undergo periodic overturns.

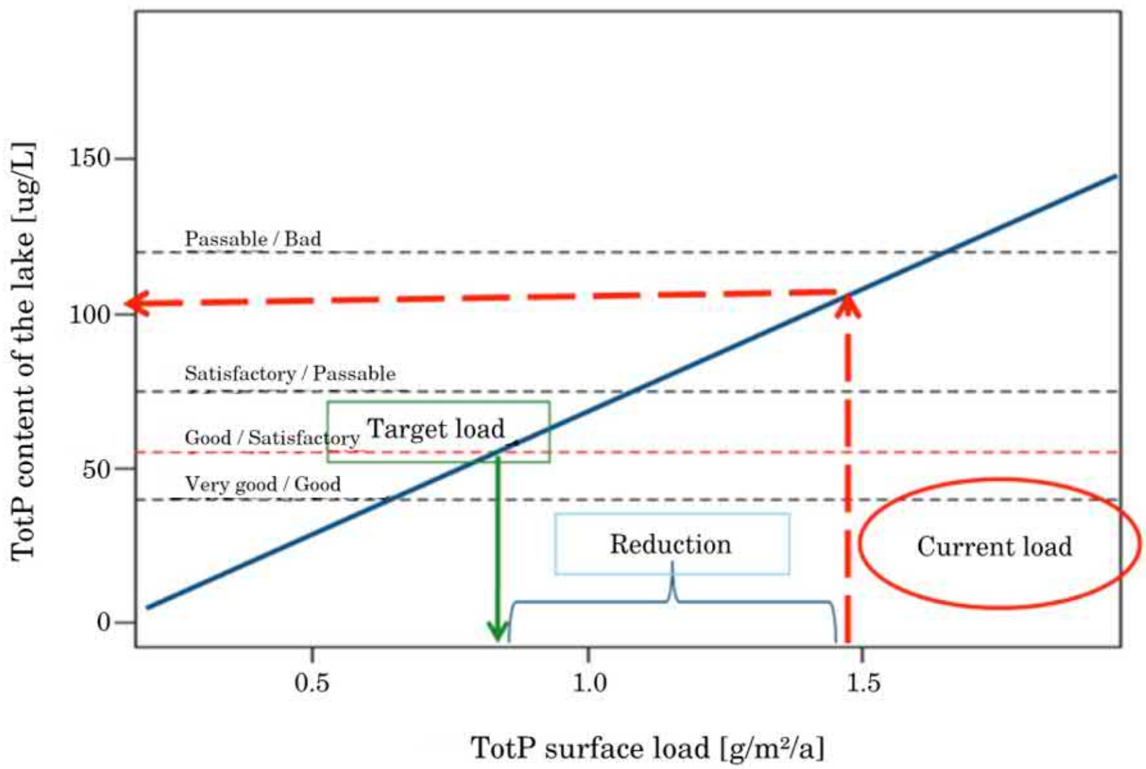


Figure 8.12: *An example of an LLR-model result for the reduction of total phosphorus load (TotP). The desired reduction is defined based ecological classifications (Good / Passable, in this case.)*

Plug Flow Reactor (PFR) is a complete opposite of the CSTR method. In PFR hydraulics, no mixing is assumed at all, and all transport is assumed to be with the flow. In it, the concentration equation is solved for distance (x), and solution becomes

$$c(x) = c_0 \exp\left(-\frac{kx}{\langle u \rangle}\right), \tag{8.17}$$

where k is the reduction constant out of the system (for example, decomposing), and $\langle u \rangle$ is the mean flow velocity in the channel. In the solution, first order kinetics have been applied in the reduction of the substance. In it, the reduction is directly proportional to its concentration. There are different values for k, which are based on field and laboratory measurements. PFR hydraulics is mostly applicable for rivers, but it has also been applied to elongated lakes to describe the flow of surface water under an ice cover.

CSTR or PFR methods are not sufficient, when studying variation of substance concentration in a complex lake or river system, or when values for short period variations are required. River systems are usually handled as one-dimensional, while lakes and seas are handled as two or three-dimensional. In such cases, advection-dispersion models are typically applied. In them, transport is considered to be due to advection and dispersion. Advection is the transport of matter via a flow, and diffusion is molecular of turbulent diffusion, which tries to equalize differences in concentration.

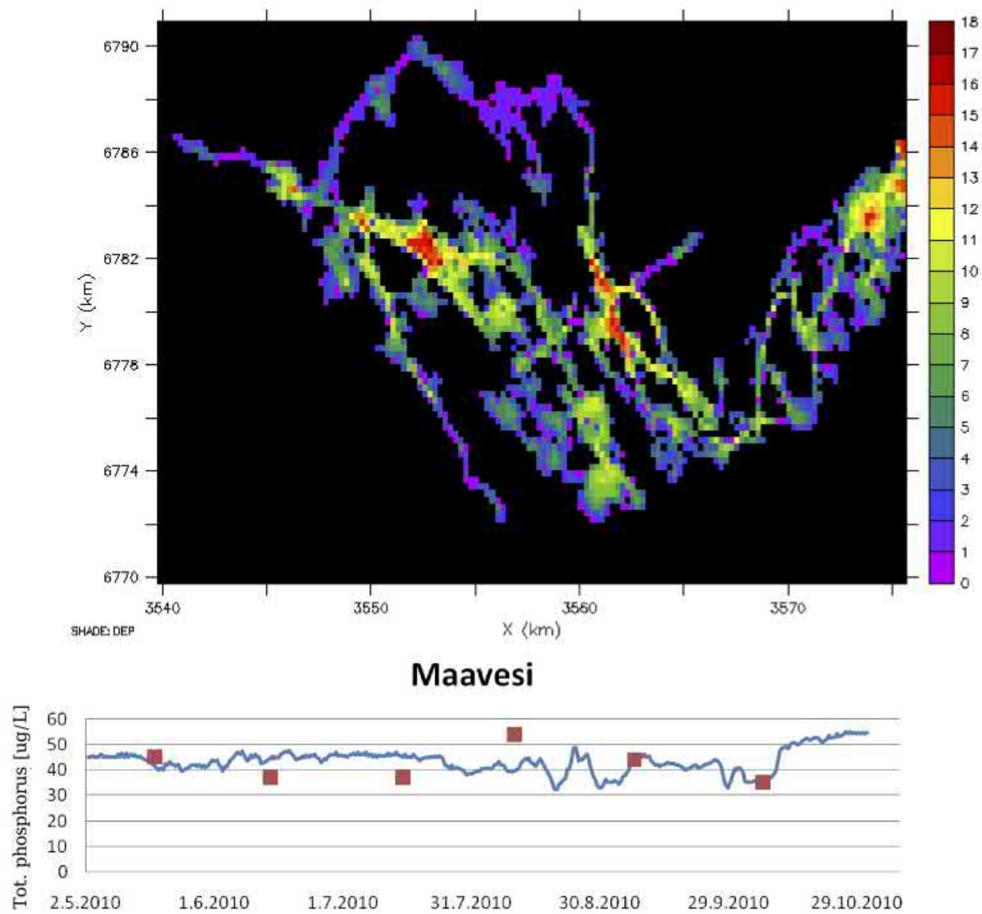


Figure 8.13: *Example on the usage of the 3-dimensional Coherens model. In the upper figure is the model grid, with colors indicating depth in meters. The lower figure shows a computed result (blue line) and in-situ observation (red squares) from northern part of the model area (Maavesi, South Saimaa, Finland). Source: Liukku & Huttula (2013).*

An advection-diffusion equation describing the concentration $c = c(\mathbf{x},\ t),\ \mathbf{x} = (x,\ y,\ z)$ of some substance in an area Γ can be added to a flow model

$$\frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = \nabla \cdot (D \nabla c) + \Psi(c, T, ...), \tag{8.18}$$

where $\mathbf{u} = (u,\ v,\ w)$ is the flow velocity, D is the diffusion coefficient, and Ψ includes biogeochemical transformations that are dependent on temperature T and other properties of water. On the left side of the equation (8.18) is local change in concentration and advection, and on the right side are the change in concentration due diffusion and the effects of biogeochemical processes. Vertical flow can result in deposition onto the bottom of the basin. Biogeochemical processes usually include decomposing, biological respiration, interaction with the sediment, aeration at the surface and processes due to anoxia (like the release of phosphorus from the sediment). At the open edges of the basin, the boundary condition of the equation describes the change in the substance concentration with the inflow and outflow:

$$\mathbf{x} \in \partial\Gamma : \frac{\partial c}{\partial t} = c \frac{\partial u}{\partial \mathbf{n}}, \tag{8.19}$$

where $\partial\Gamma$ is the edge of the area and $\mathbf{n}$ is its normal.

The most common quantities describing the quality of water in these models are oxygen concentration, nutrient concentration (nitrogen and phosphorus) and chlorophyll a (Fig. 8.13). Temperature correction is applied for biogeochemical reactions and Michaelis-Menten reaction kinetics for the production and decomposing of chlorophyll a. In it, the production rate of chlorophyll a is linearly proportional to the high nutrient concentrations until to specific upper threshold. The effect of light is described with a similar limiting threshold. The combined effect of light, nutrients and temperature is acquired by multiplying the limiting coefficients for each part. Respiration and other processes included in the model receive a similar treatment. The calibration and validation of the model requires a representative group of samples from different parts of the lake in order to confirm the spatial and temporal variation produced by the model.

Chapter 9

Future

This book has presented the basics of hydrology, with an emphasis on the key questions regarding hydrology in Finland. This has included the physical properties and quality of natural waters: hydrometeorology, hydrology of lakes, and rivers and geohydrology. The core concepts of the water cycle were presented first, details were added later on, and a more holistic picture was compiled in the chapter regarding runoff and runoff models. The goal has been to offer a textbook for a beginner in the field of hydrology.

Water on our planet undergoes an eternal cycle. It is evaporated from the surface into the atmosphere, transported by the winds and returned back to the surface as precipitation (Fig. 9.1). Water is purified as it cycles through the atmosphere, as evaporation only transports molecules of water. Upon reaching the ground, water is transported as runoff into groundwater and surface water, eventually ending either back into the atmosphere or the ocean. The total amount of water is conserved in the cycle. Small loss happens into the space around Earth, but some additional water is brought in by comets.

Water also moves through the biosphere and human hands. Foreign substances are being dissolved and particles mixed into water, and they are transported further along with the movement of water. The quality of water and its usability change over time. Environmental load brings harmful substances into water, and the changing climate affects the availability and temperature of water.

Clean, fresh water is a necessity for life. Distribution of clean water on Earth is not even, which has caused and will cause, massive economical and political issues. Finnish water resources are adequate, but unevenly distributed. For example, the Päijänne tunnel brings water for over 100 km to serve the needs of the capital and its surroundings. Water quality has become ever more important, and maintaining it requires more drastic measures.

The biggest human users of water are agriculture and industry. Improvements in the efficiency of agriculture has led to increased use of irrigation systems, which sometimes leads to an overuse of water, which in turn causes depletion of water resources somewhere else. Industry requires water especially for the production of energy and cooling. Canals have been used as a mean of transport for thousands of years, and many canal networks connecting multiple water systems have been built. Fishing has been exercised in lakes and rivers as well, although these resources are not as great as those in the ocean. During the last several decades, tourism and recreational activities in natural waters have become ever more popular users of water resources, which creates further requirements for the water quality (Fig. 9.2).

Figure 9.1: *Rivers function as important transporters of water. Ice off in a river is always an awe inspiring sight. Oldest Finnish hydrological time series is the ice off of River Tornionjoki, starting from year 1696. Pictured is River Kaanaanjoki (Mäntsälä, Southern Finland). Photo: Matti Leppäranta.*

Figure 9.2: *A summer's day on a sail boat on lake Päijänne. Recreational use of water systems is undergoing a strong increase. Photo: Matti Leppäranta.*

In addition to the problems of water shortage, there can also be problems with too much water. Heavy rain and large discharges in rivers are the biggest problems when assessing the human and economical damage. Heavy rain can also cause landslides as they change the physical properties of the soil. Snow and ice can also cause significant problems. Ice dams in rivers cause floods in Finland every year. If the climate warms up, floods caused by frazil ice will become more common especially in Southern Finland as rivers do not get a full ice cover anymore. Ice storms occurring in North America damage forests and buildings and cause disruptions to traffic. In these storms, cold air is advected from north with supercooled water droplets freezing onto any surface. In 1998, the city of Montreal was without electricity for five days due to an ice storm.

Harmful substances in surface water as well as in groundwater cause problems. These substances can be divided into nutrients and toxins. Toxins harm aquatic ecosystems and pose a threat to the local use of water resources. Widespread problems also arise from the spread of nutrients, both from point loads coming from population centers and diffuse load originating from agriculture. Eutrophication has become a key environmental question especially regarding lakes. Flushing of microbes is also an issue. In Finland, especially during early spring, diarrhea epidemics sometimes occur that have an unclear origin. Household waste water treatment and medical residue are also playing a role in the water quality.

Water engineering, regulation of lakes and rivers, and other use and conservation of water has given a reason to many disputes. Hydropower stations prevent salmon from swimming upstream to spawning areas, from which long battles in courthouses have followed, like the one concerning the River Kemijoki that has been ongoing since the 1950s. Special routes for fish have been built for example for River Oulujoki, and similar projects are ongoing on other rivers as well. Even demolition of some smaller power stations have been discussed to allow salmon to spawn, like at rivers Vantaanjoki and Hiittolanjoki. Water traffic has seen restrictions on some lakes and rivers, like the banning of motor boats and snow mobiles. The retention time of Finnish lakes is typically between a few months and a few years, but just changing the water is not always enough, as sediments can contain deposited harmful substances. The sediment will then slowly release these substances back into the water. One of the most problematic of these cases are the waters that lay below pulp and paper industry centers. Massive amounts of waste fiber (so-called *zero fiber*) has accumulated in the sediments, as the factories have not been able to filter them out of the waste water.

The state of groundwater is very important since it is the most suitable for household use. In addition to industrial waste, another major threat to its quality is the salting performed on roads during winter. For this reason salting has been limited in many areas. One of the greatest single accidents to have affected water resources in Finland was the massive gypsum pond leak at the Talvivaara nickel mine in 2012. $1.2 \cdot 10^6$ m^3 of waste water containing several hundreds of tons of metals was released into the environment.

As a result of citizen activism numerous trusts and societies have been founded to protect many of Finland's most important lakes and rivers. Among the oldest of them is the Pyhäjärvi Institute and Foundation (est. 1989 in Eura), Vesijärvi Foundation (in Lahti) and Vanajavesi Foundation (in Hämeenlinna). Rivers have also received this kind of attention, e.g. the Kymijoki Water and Environment Society. Keeping lakes and rivers in a good shape promotes the wellbeing of local population, fishing, tourism and the ecological balance of the local nature (Fig. 9.3).

Figure 9.3: *The usage of fisheries in Finnish lakes has remained regrettably low. Although vendance (Lat: Coregonus albula, Fin: muikku) still remains as a very popular traditional Finnish dish. Photo: Matti Leppäranta.*

The natural variation of our planet's climate is large and chaotic. Regular, so-called Milankovitch cycles[1] with periods of tens of thousands of years can be found, as well as irregular variations, like the North Atlantic oscillations (NAO for short) and El Niño - South Pacific oscillation (ENSO). NAO describes the westerly winds coming from the Atlantic, and it has a major role in the weather in Europe. Especially the winter temperature has a large natural variation: in Finland winter can be mild when westerly winds are blowing, and harsh when the cold, continental winds blowing from the east are dominating. The monthly mean temperature in Helsinki in January in the 20th century shows this well: -17 °C for the coldest January and +3 °C for the warmest.

Discerning the possible human effect from natural variation is difficult. The consensus of the Intergovernmental Panel on Climate Change (IPCC) is based on the views of the majority of researchers studying the field and is widely considered to be the best available forecast. It is necessary to point out though, that there are researchers who do not share this view and who consider the human effect on climate to be negligible.

There is a significant difference between the solar radiation coming to Earth and the thermal radiation leaving it. Solar radiation is short wave (0.3 – 3 μm), and the thermal radiation emitted by Earth is long wave (5 – 50 μm). Greenhouse gases in the atmosphere act like the glass roof of a greenhouse: they let the solar radiation fall onto the surface of Earth, but partially block the outgoing terrestrial thermal radiation. The *greenhouse effect* caused by these gases keeps the mean temperature on our planet suitable for life. The most

[1] Milankovitch cycles are the result of the periodic oscillations found in the tilt of the Earth's rotational axis and orbital parameters.

significant greenhouse gases are water vapor (responsible for 36 – 70 % of the greenhouse effect), carbon dioxide (9 – 26 %), methane (4 – 9 %) and ozone (3 – 7 %). Concentration of carbon dioxide in the atmosphere has gone up 31 % and methane concentration 149 % when compared to their pre-industrial levels in 1750. According to modern research doubling the concentration of carbon dioxide would result in 1.5 – 4.5 °C increase in mean surface temperature (IPCC, 2014).

During the years 1906 – 2005 change in the mean surface temperature was 0.74 °C per century. During the last 50 years the rate of change has been nearly double: 0.13 °C per decade. Estimates for the rate of increase in the future vary widely: 1.1 – 6.4 °C increase from the mean temperature of 1980 – 1999. This variation is due to many reasons. All aspects of the behavior of the climate system are not well known, and it is made more difficult by the unpredictability of human behavior. Scenarios concerning the amount of emissions in the future have wide variations due to the unknowns in the population growth and the development of the methods of energy production. A scenario based on low emissions results in an increase of 1.8 °C by the year 2100. In the case of the most pessimistic emission scenario this increase is 6.4 °C.

IPCC has also reported on the long term changes in precipitation and wind patterns. Precipitation increased significantly in Northern Europe in 1900 – 2005. According to the latest climate models the amount of precipitation is predicted to remain similar in Finland, but that seasonally it would change so that winters would get more rain and summers less.

If climate change occurs as IPCC has predicted, it would have a significant impact on the water cycle. An immediate effect is an increase in the temperature of surface waters, as can be seen from the change in the surface heat balance. This has an effect on the summer stratification of lakes and the oxygen content of the hypolimnion. In the high latitudes, like in Finland, there is a change also in the length of the ice covered period, especially making the ice-off date earlier. Increases in temperature also make algal blooms more prevalent and contribute to eutrophication (Fig. 9.4). Groundwater reacts slowly to increasing temperatures, but changes in flow patterns may be observed due to the changing viscosity.

If winters gets shorter spring floods might get smaller due to smaller accumulation of snow. On the other hand, floods might be experienced during the winter as melting periods can be seen more even in the middle of winter. Accumulation of snow and properties of snow cover affect heavily the development of ground frost and surface water. Plant and animal life might face adaptation challenges as snow conditions change, but these changes are difficult to predict. For example, the ability of reindeer to eat lichen from under the snow might be hindered.

Another immediate response to warming climate is the increase in evaporation, due to the fact that the saturation pressure increases heavily with the increased temperature of air. If a catchment area has plenty of surface water, increase in evaporation might decrease the storage of water. Increased evaporation might be seen in groundwater as well, but with some time lag. Change in evaporation is larger when the original temperature is higher. In a water system the effect of increased evaporation might be compensated by the increase in precipitation. Hydrological consequences of increased rains are simple in principle: more water results in larger storage of water. Evaporation compensates increases in storage, if precipitation has first increased the soil moisture content.

Figure 9.4: *Eutrophication of lakes has led to commonplace and strong algal blooms. Photo: Jouko Sarvala.*

Whether the mankind is able to reduce the emissions is one of the key questions regarding our future. In principle, any negative or positive effects of the possible climate change are local, but the catastrophe caused by a sudden increase in temperature would be felt globally. Regardless of climate change, sustaining and improving the quality and ecological state of natural waters is of utmost importance. A positive feedback loop can be observed in the upkeep of quality and recreational use of water systems (Fig. 9.5).

Hydrology can be expected to remain a central field of geophysics, due to water being such an integral part of our life. Development in observational and computational methods will continue on its path, resulting in ever improved methods of research, monitoring and mathematical models. The physics of the water cycle is rather well understood, but the environmental factors and forcing require plenty of more work. Increases in computational capabilities let us take into account the composition and structure of soil more accurately than ever before. Even more challenging is the modeling of dissolved matter and solid particle transport with the cycle of water.

Interdisciplinarity between neighbouring fields of science has been getting more common in hydrology recently. Especially in ecology water is a critical resource, and water resources and ecology are in mutual interaction. Hydrology also has connections to the human history, due to the fact that population centers have always needed water. Hydrological information has been used in archaeological research when studying ancient methods of water supply. In planetary geophysics the availability of water beyond Earth is one of the key questions of the near future.

Water resources that are so far unused reside in the continental ice sheets, but their distance from large population centers make their use impractical. Towing icebergs to areas suffering from lack of water was studied as an option in 1980s, when it was deemed as not economically feasible. Icebergs disintegrate as they arrive to warmer waters. Also the desalination of seawater is not economically feasible in large scale yet, although it is

Figure 9.5: *Recreational use of water systems has increased, that has a large impact on the local population and tourism. One of the more recent water related hobbies is SUP-boarding (SUP stands for stand-up paddle boarding). Photo: Petriina Köngäs.*

performed locally in many places.

Water resources, their controlled use and upkeep of their quality are some of the key challenges during this century. The total amount of water is conserved, but its quality may worsen and distribution change due to changes in the surrounding environment. Water is a biological necessity for people, but it has also signified longing, harmony and beauty. Being close to water has equaled peace of mind (Fig. 9.6).

Figure 9.6: *'Evening bells', Isaak Levitan (1860 – 1900). Levitan was a famous Lithuanian-Jewish painter, who was especially known for his naturalistic paintings. ©Tretyakov Gallery, Moscow.*

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Annex. Mathematics basics

Anna-Riikka Leppäranta

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1. INTRODUCTION

This annex introduces the basic mathematics that is needed to understand an introductory course in hydrology. The annex is designed especially for students with limited high school mathematics education. Derivations of formulae and proofs of mathematical statements have been partly omitted. For more information and basic mathematics, please, consult applicable web sites, e.g., look for GeoGebra for plotting and Wolframalpha for solving equations. In the University of Helsinki system, MatLab can be used by the students.

2. POWERS AND POLYNOMIALS

2.1. Power formulae

Rules for power calculations are needed, e.g., when manipulating dimensions and units, and powers of ten. They are based on the definition of power

$$a^n = a \cdot a \cdot \ldots \cdot a \quad (a \text{ taken } n \text{ times in the product})$$

Thus, powers can always be changed into multiplication for straightforward treatment. The following rules are useful.

(1) $a^m \cdot a^n = a^{m+n}$

(2) $\frac{a^m}{a^n} = a^{m-n}$

(3) $(ab)^n = a^n b^n$

(4) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

(5) $(a^m)^n = a^{mn}$

(6) $a^1 = a$

(7) $a^0 = a$, when $a \neq 0$ (0^0 is not defined)

(8) $a^{1/n} = \sqrt[n]{a}$, especially $a^{1/2} = \sqrt{a}$

(9) $(a+b)^2 = a^2 + 2ab + b^2$

(10) $(a-b)^2 = a^2 - 2ab + b^2$

(11) $(a+b)(a-b) = a^2 - b^2$

Negative exponents are often used in text instead of the division sign, for example m/s = m s^{-1}. In mathematical physics, the solution must have the correct unit that can be used in checking calculations.

Example 2.1. Units and dimensions. Pressure in a liquid body is given by $p = gh$, where g is the acceleration due to gravity and h is depth. The equation can be solved for the depth as $h = \frac{p}{g}$, and the unit of depth is meter, denoted as $[h] = \text{m}$. The unit can be checked as

$$[h] = \frac{[p]}{[g]} = \frac{\text{Pa}}{\text{kg/m}^3 \cdot \text{m/s}^2} = \frac{\frac{\text{kg} \cdot \text{m}/s^2}{\text{m}^2}}{\text{kg/m}^3 \cdot \text{m/s}^2} = \frac{\text{kg} \cdot \text{m}^{-1} \cdot s^{-2}}{\text{kg} \cdot \text{m}^{-2} \cdot \text{s}^{-2}} = \text{m}$$

Here, the pressure is force per area, force is mass multiplied by acceleration, and thus the unit of pressure is expressed with the basic units as Pa = kg·m·s⁻²·m⁻².

Einstein, Newton, and Pascal are playing hide-and-seek on the beach. Einstein closed his eyes and started counting. Pascal ran away to hide, but Newton just took a stick and draw a square on a sand around himself and stayed at his place. Einstein opened his eyes and saw Newton. Einstein: "You really suck at this game, Newton, I found you already!" Newton: "No you didn't, you found Pascal. See - one Newton over one square meter!"

2.2. Powers of ten

Powers of ten are used to express very small and large numbers. Calculators also employ this formulation.

Example 2.2. Application from Example 2.1.

(i) $10^3 = 10 \cdot 10 \cdot 10 = 1000$
(ii) $10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$
(iii) $2.5 \cdot 10^5 = 250\,000$
(iv) $3.45 \cdot 10^{-4} = 0.000345$
(v) It makes calculation easier to first work through the powers, e.g.,

$$\frac{10^{-3} \cdot 10^5}{10^{-7} \cdot 10^{22}} = \frac{10^2}{10^{15}} = 10^{2-15} = 10^{-13}$$

Note. Prefixes of units refer to powers of ten, for example

(a) kilo = k = thousand = $1000 = 10^3$
(b) milli = m = one-thousandth = $0.001 = 10^{-3}$
(c) mikro = μ = 10^{-6}
(d) nano = n= 10^{-9}

2.3. Polynomials

Polynomials are functions of an argument, noted here by x, consisting of terms with the argument in nonnegative powers and a coefficient. For example, $y = x^2, y = 3x^6 - 7,$ and $f(x) = \frac{1}{3}x$ are polynomials, and $y = \sqrt{x}$ and $f(x) = \frac{1}{x}$ are not. Polynomials are classified according to the degree or their highest power term.

(i) Zeroth order: constant functions $f(x) = a$, e.g., $f(x) = 5, y = 0$ (Fig. 1).
(ii) First order: $f(x) = ax + b$, e.g., $f(x) = 3x - 1,$ and $f(t) = 2t$ (Fig. 1).
(iii) Second order: $f(x) = ax^2 + bx + c$, e.g., $f(x) = x^2 - 2x + 10$ and $y = x^2$ (Fig. 2). The solutions of are obtained by the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $(a \neq 0)$ (Fig. 2).
(iv) Higher order polynomials, for example $f(x) = x^3$ (odd) and $f(x) = x^4$ (even) (Fig. 2).

Note. The second-order solution can be also used to estimate slope (see Example 3.2).

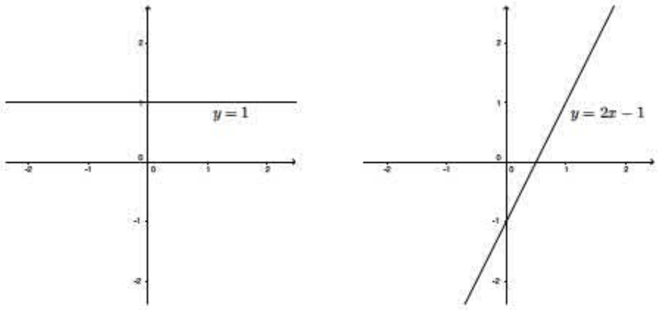


Figure 1. Graphs of constant and first-order polynomials.

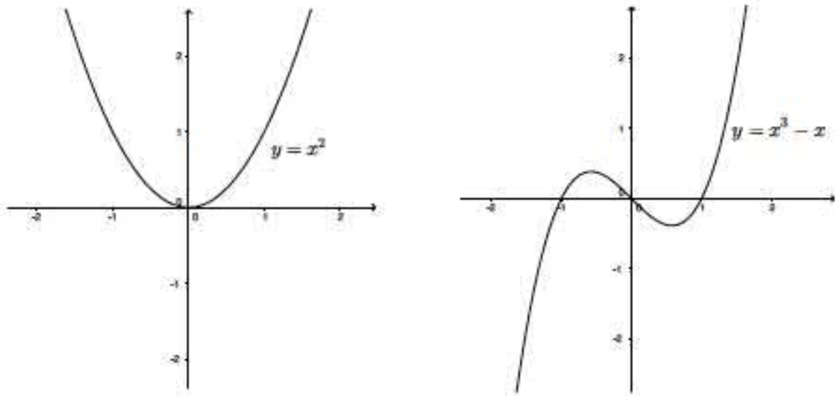


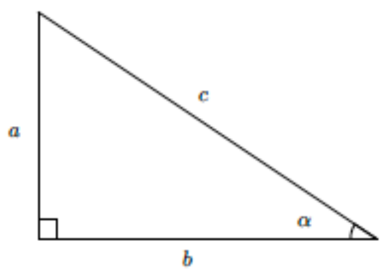
Figure 2. Graphs of second and third-order polynomials.

3. TRIGONOMETRIC FUNCTIONS

Trigonometric functions are associated with angles and periodic phenomena, and they are widely used in physics. Trigonometric functions can be defined with right triangles by the ratios of their sides (Fig. 3), tri = three, gono = angle, metric = measurement. There are six ratios between the sides, but normally only three of them are used,

$$\sin\alpha = \frac{a}{c}, \cos\alpha = \frac{b}{c}, \tan\alpha = \frac{a}{b} = \frac{\sin x}{\cos x}$$

Also, the Pythagoras theorem $a^2 + b^2 = c^2$ is needed.



Fig, 3. Right triangle.

Trigonometric functions and the Pythagoras theorem can be used to solve any side or angle of a right triangle, when two of these quantities are known. The angles can be expressed on degrees or radians, 360° = 2π rad = full circle.

Example 3.1. Components of a vector. Often a vector is decomposed into components along x- and y-axes. This goes easily with the sine and cosine functions (Fig. 4).

$$v_x = v \cos \alpha \,, v_y = v \sin \alpha$$

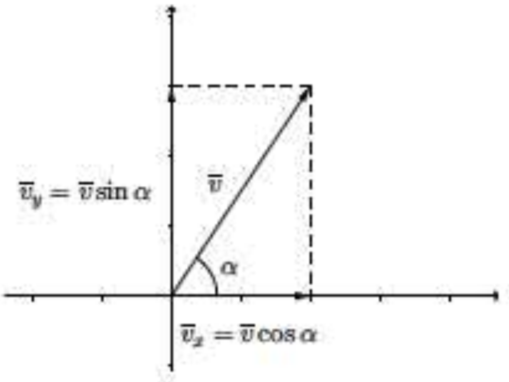


Fig. 4. Getting components of the vector $\vec{v}$.

To solve for an angle, inverse functions of the trigonometric functions or arcus functions are needed. These are denoted as $\arcsin x, \arccos x$ and $\arctan x$, in calculators they are often denoted by $\sin^{-1}, \cos^{-1}$, and $\tan^{-1}$.

Example 3.2. The equation $2\sin^2 x + 3\sin x - 3 = 0$ can be solved using the second-order polynomial case (see Section 2.3):

$$\sin x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 2 \cdot (-2)}}{2 \cdot 2} = \frac{.3 \pm 5}{4}$$

that gives $\sin x = \frac{1}{2}$ or $\sin x = -2$. The latter solution is not good since always $|\sin x| \leq 1$, and thus the former solution is valid and $\sin^{-1} = 30°$ (when looking for an acute angle).

Note. In many calculators and spreadsheet computation software it is assumed that angles are in radians. Own system can be checked by $\sin^{-1} = 30° = 0.52$ rad. The software has a function for the transformation.

Note. There are many formulas for transformation of trigonometric functions, e.g., formulas for double angles. Such can be found in handbooks. If the student knows complex numbers, the transformation formula can be derived from the Euler's formula $e^{i\phi} = \cos \phi + i \sin \phi$.

3.1. Unit circle

The unit circle is a circle with radius equal to 1 (Fig. 5). A right triangle can be drawn inside the circle so that the hypotenuse overlaps the x-axis, and then the sine and cosine can be read from the coordinate axes directly (this is the idea of the unit circle). This circle also presents the mathematical angle direction as counterclockwise from x-axis, while the geographic direction goes clockwise from y-axis.

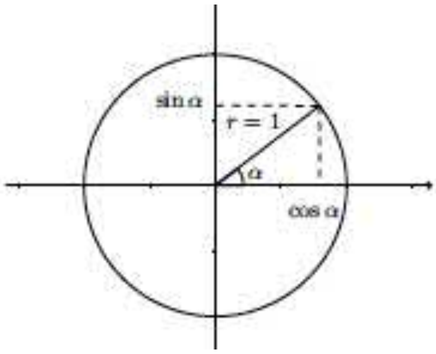


Fig. 5. Unit circle.

Graphs of the functions

The graphs of sine and cosine functions are waves (Fig. 6), because they are periodic with the same values repeating at 2π rad intervals of the argument (x). Many natural phenomena, such as surface waves in lakes and temperature variations in soil are periodic, and sine and cosine functions are employed in their analyses. The graphs show the specific features of sine and cosine:

$$\sin 0 = 0, \sin\frac{\pi}{2} = 1, |\sin x| \leq 1$$

$$\cos 0 = 0, \cos\frac{\pi}{2} = 1, |\cos x| \leq 1$$

$$\sin x = \cos\left(\frac{\pi}{2} - x\right)$$

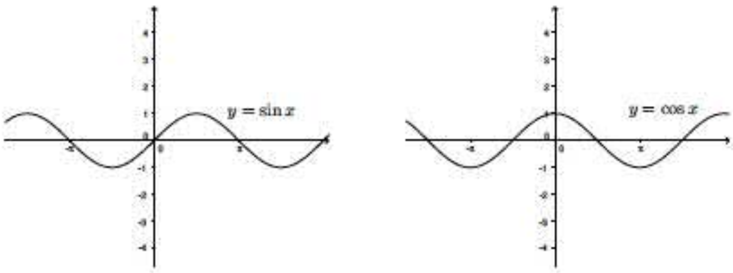


Fig. 6. Graphs of sine and cosine functions.

The tangent function looks fully different (Fig. 7). This function is not defined at $x = n\frac{\pi}{2}$, since then $\cos x = 0$ (and $\tan x = \infty$). Tangent is not limited like sine and cosine but can have any value.

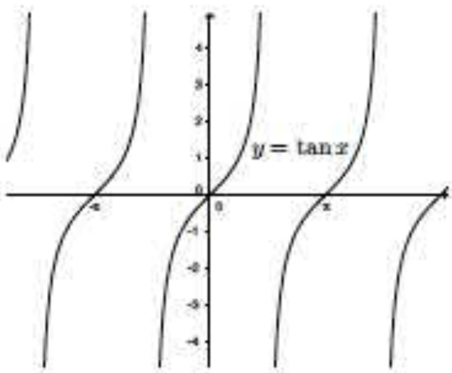


Fig. 7. Graph of tangent function.

4. Exponent and logarithm functions

4.1. Exponent function

Exponent functions have the argument in the exponent, for example $y = 2^x$ or $f(t) = 3te^t - 1$. The base number of exponential functions is often $e = 2.71828\ldots$, Neper's number. In this chapter the base of e is taken.

The exponent function e^x $(x \geq 0)$ grows fast with its argument, and correspondingly the function e^{-x} $(x \geq 0)$ decreases fast (Fig. 8). The terms "exponential growth" or "exponential decay" are often used as references. The exponential function e^x is defined for all values of x and its value is always positive. If the exponent is a complicated function. the notation $e^{f(x)} = \exp[f(x)]$ is employed. In treating exponential functions, the calculation rules with powers can be utilized (see Section 2.1), for example

$$e^x e^y = e^{x+y}, e^{-x} = \frac{1}{e^x}, e^0 = 1$$

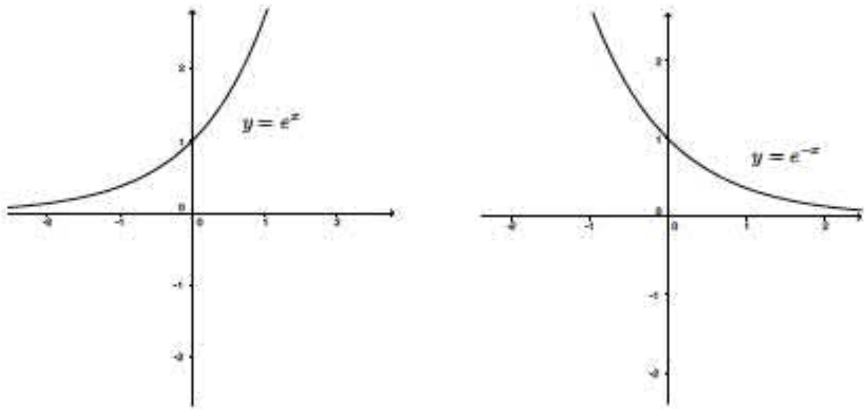


Fig. 8. Graph of exponential function.

Example 4.1. Exponential growth. The growth of bacterial population is exponential until a certain limit. The population grows by division of an individual into two bacteria and so on. After each division of individuals, the population has doubled. This can be described by the function $f(x) = 2^x$, where x is the number of duplications.

4.2. Logarithm function

Logarithm gives the exponent. For $x > 0$,

$$\log_k x = y, \text{when } k^y = x$$

This is read "the k-base logarithm of x is y". The common base logarithms are noted as

$$\log_e x = \ln x \text{, and } \log_{10} x = \lg x$$

In some calculators, the ten-base logarithm is noted as log. In this chapter, after the following example only e-based logarithms are used. The base can be easily changed.

Example 4.2. To calculate logarithm.

(1) $\log_2 8 = 3, \text{because } 2^3 = 8$

(2) $\lg 10{,}000 = 4$, because $10^4 = 10{,}000$
(3) $\lg \frac{1}{100} = -2$, because $10^{-2} = \frac{1}{100}$

The calculation rules of logarithm follow from the definition of the function

$$\ln xy = \ln x + \ln y$$

$$\ln \frac{x}{y} = \ln x - \ln y$$

$$\ln x^a = a \ln x$$

The definition also gives

$$\ln e^x = x, e^{\ln x} = x, \text{and} \ln 1 = 0$$

Logarithm function is the inverse of exponential function. It is defined only for positive argument values, and the increase by x decreases toward zero with increasing x (Fig. 9).

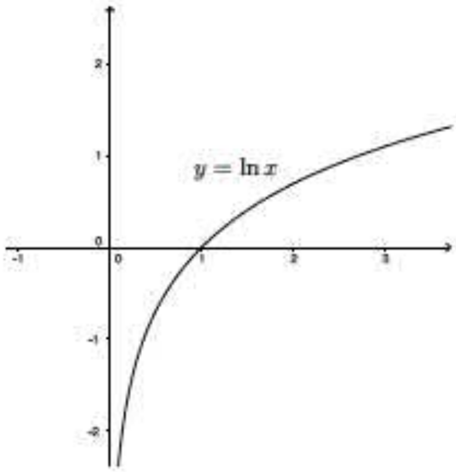


Fig. 9. Graph of logarithm function.

Solving for the exponential equation

Example 4.2. The exponential equation is solved by taking the logarithm (any base) of both sides of the equation as follows:

$$2^t = 20 \rightarrow \ln 2^t = \ln 20 \rightarrow t \ln 2 = \ln 20 \rightarrow t = \frac{\ln 20}{\ln 2} \approx 4.3$$

Plotting data

When looking into the dependence between two quantities, the data are plotted in (*xy*)-coordinate system. To understand the dependence, coordinate transformations can be performed to produce linear dependence or a straight line in the plot. Thus,

(1) If $y = ax + b$, (*x*,*y*)-coordinate system works as such,
(2) For exponential relation $y = ae^{bx}$, a straight line is obtained in $(x, \ln y)$-system.
(3) For a power law relation $y = ax^b$, a straight line is obtained in $(\ln x, \ln y)$-system.

The Neper's number base can be replaced by any other base.

5.1. Derivative

The derivative describes the rate of change. At a given point of a curve, the derivative equals the slope of the tangent of the curve (Fig. 10).

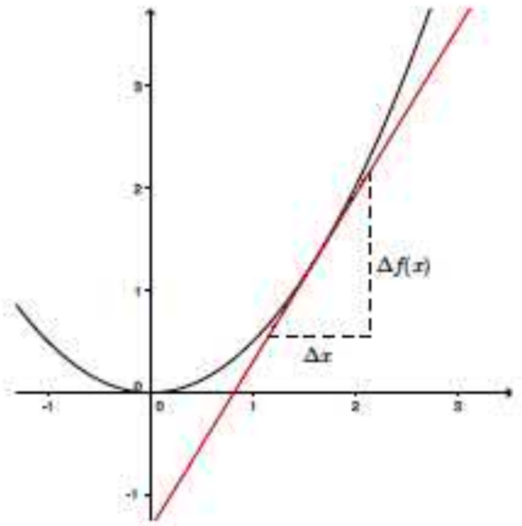


Fig. 10. Derivative equals the slope of the tangent.

Mathematically, the derivative is the limit of them slopes of secants approaching the tangent, notated by $f'(x)$:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{\Delta x \to 0} \frac{\Delta f(x)}{\Delta x}$$

The latter formulation is popular in physics, with Δ ("delta") standing for small change. "lim" stands for the limit, when h or Δx approaches zero. The derivative of a function $f(x)$ can be denoted by

$$f'(x), Df(x), \frac{df(x)}{dx}, \text{or } \frac{d}{dx}f(x)$$

A function can be derived also for several times. For example, the second time derivative of function $f(t)$ is denoted by $f''(t)$ *or* $\frac{d^2f(t)}{dt^2}$.

Note. All functions do not possess derivatives, even continuous functions. For example, the function $f(x) = |x|$ has a sharp corner at $x = 0$ where a tangent cannot be drawn.

Rate of change

The rate of change, i.e., derivative, can be examined from the form or graph of the function. Consider the tangents in Fig. 11.

(1) At point (1), the tangent points sharply up, so that the derivative is positive, and the function is growing.
(2) At point (2), the tangent is horizontal, and the rate of change is zero. The function has a local maximum here. The extremes –minima and maxima – of a function are found from the points where the derivative is zero (but not all points with zero derivative are extremes).
(3) At point (3), the function is decreasing slowly.

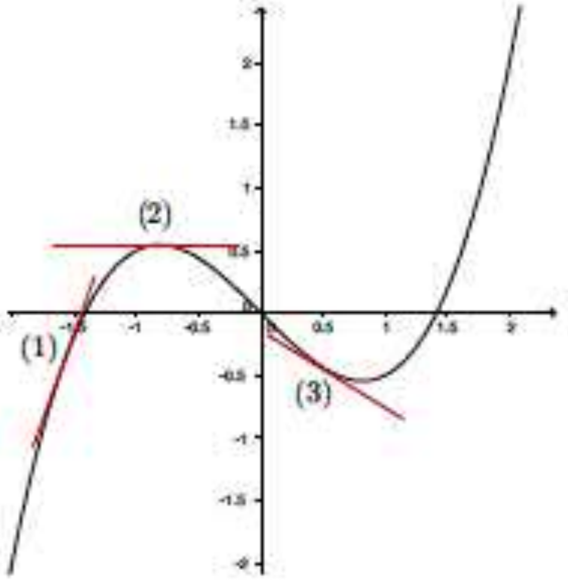


Fig. 11. Tangents on a curve.

Derivative in physics

Derivative is a key tool when examining changing quantities. Understanding derivatives helps to read physical formulae, for example $\frac{dT}{dt}$ stands for the rate of change of temperature T with time t. By taking derivatives, new quantities are obtained. The first derivative of distance s with respect to time is velocity v, and the first derivative of velocity with respect to time is acceleration a,

$$\frac{ds}{dt} = v, \frac{d^2s}{dt^2} = \frac{dv}{dt} = a$$

This can be also deduced from graph. Derivative is the slope of the tangent, i.e., $\frac{\Delta y}{\Delta x}$. The slope of distance is thus $\frac{\Delta s}{\Delta t}$ that is the dimension of velocity.

Derivation rules

Taking the derivative as the limit as defined above is laborious, and in practice derivation rules are utilized. Common rules are given below. Derivation is a linear operation, it can be performed term by term and constants can be directly moved out:

$$D[f(x) + g(x)] = Df(x) + Dg(x)$$

$$D[af(x)] = aDf(x)$$

and furthermore

(1) $Da = 0$ (the derivative of a constant is zero)
(2) $Dx = 1$
(3) $Dx^n = nx^{n-1}$
(4) $D\sin x = \cos x$
(5) $D\cos x = -\sin x$
(6) $D\tan x = \frac{1}{\cos^2 x}$
(7) $De^x = e^x$

(8) $D\ln x = \frac{1}{x}$

Example 5.2 We can get using the derivation rules:

(1) $D(x^3 + 5x - 1) = 3x^2 + 5$
(2) $D2\sin x = 2\cos x$

The derivation rules for products, quotients and combined functions are, respectively,

(1) $D[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$

(2) $D\frac{f(x)}{g(x)} = \frac{f'(x)g(x)-f(x)g'(x)}{[g(x)]^2}$

(3) $Df[g(x)] = f'[g(x)]g'(x)$

Example 5.3. Derivation of combined functions. The function $y = \sin x^2$ is combined of two functions, inner function $g(x) = x^2$ and outer function $f(x) = \sin x$. Thus, $f[g(x)] = \sin x^2$, and $f'[g(x)] = \cos x^2$, and $g'(x) = 2x$. The rule (3) above gives

$$D\sin x^2 = \cos x^2 \cdot 2x = 2x\cos x^2$$

5.2. Partial derivative

Partial derivatives are used with functions of several arguments. For example, the function $f(x, y)$ with two variables as the arguments has the first-order partial derivatives as

$$\frac{\partial f(x,y)}{\partial x} \text{ and } \frac{\partial f(x,y)}{\partial y}$$

Partial derivation is performed similarly to the normal derivation. If derivation is performed for the variable x, other variables are then kept fixed and treated as any constants.

Example 5.4. Consider the function $f(x, y) = x^2 - 2xy$. The partial derivatives are

$$\frac{\partial(x^2-2xy)}{\partial x} = 2x - 2y \text{ and } \frac{\partial(x^2-2xy)}{\partial y} = -2x$$

Example 5.5. In many practical calculations, partial derivatives can be replaced by finite difference ratios. For example, if the water surface is tilted by wind stress in an elongated lake of length L, the hydrostatic pressure difference across the lake is

$$\frac{\partial p}{\partial x} = \frac{\partial \rho g h}{\partial x} = \frac{\rho g \partial h}{\partial x} = \frac{\rho g \Delta h}{L}$$

The first equation uses the definition of hydrostatic pressure (ρ and g are water density and acceleration due to gravity), in the second equation ρ and g are assumed constants, and in the third equation the partial derivative is approximated by the ratio of water level difference Δh to the lake length L.

6. INTEGRATION

6.1. Integral function

Integration is inverse operation of derivation. When looking for an integral function of a given function, one may consider what is the function to be derived. Thus,

$$\int f(x)dx = F(x), \text{if } F'(x) = f(x)$$

Here dx is the integration factor, which will be discussed more in Section 6.2.

Example 6.1. Let us integrate function $2x$. This is the derivative of function x^2, and thus

$$\int 2xdx = x^2$$

Checking: $Dx^2 = 2x$, i.e., the correct integration function has been found. On the other hand, also $D(x^2 + 5) = 2x$ and $D(x^2 - 100) = 2x$, because the derivative of a constant is zero. Therefore, any constant can be added to the integral function. The full answer to the integral function includes an undetermined constant C:

$$\int 2xdx = x^2 + C$$

Integration formulae

Integration formulae follow directly from the derivation formulae. Common ones are listed below. Integration is a linear operation, and therefore it can be performed term by term and constant coefficient can be moved in front.

$$\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$$

$$\int af(x)dx = a\int f(x)dx$$

Furthermore,

(1) $\int adx = ax + C$
(2) $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad (x \neq -1)$
(3) $\int \sin x\, dx = -\cos x + C$
(4) $\int \cos x\, dx = \sin x + C$
(5) $\int e^x dx = e^x + C$
(6) $\int \frac{1}{x}dx = \ln|x| + C$

Example 6.2.

$$\int \left(x^3 - \frac{5}{x}\right) dx = \int \left(x^3 - 5 \cdot \frac{1}{x}\right) dx = \frac{1}{4}x^4 - 5\ln|x| + C$$

More guides for integration are presented in Section 6.3.

6.2. Definite integral

The definite integral gives the area between the x-axis and the function curve in the x-axis interval $[a, b]$, denoted by

$$\int_a^b f(x)dx = /_a^b F(x) = F(b) - F(a)$$

Integration can also be performed over a given area A, denoted as $A = \int_A f dA$.

Area

The area between the x-axis and the curve of the function in the x-axis interval $[a, b]$ is easy if the function is a constant or of a first-degree (Fig. 12). For example, for the function $f(x) = x^2$, this area does not come out with the basic geometric formulae.

In general, area calculations are based on summing rectangles, which cover the object as closely as possible (Fig. 13). The narrower the rectangles are, the more accurate they give the area in question. The height of a rectangle approximately equals the value of the function f at that point, and the base of the rectangle is a small step in the x-axis, dx, so that the rectangle area is $A(x) = f(x) \cdot dx$. In numerical integration, the areas of all these rectangles are added together for an approximation of the total area. The integral sign is $\int$ is in fact a stretched "S" that refers to sum.

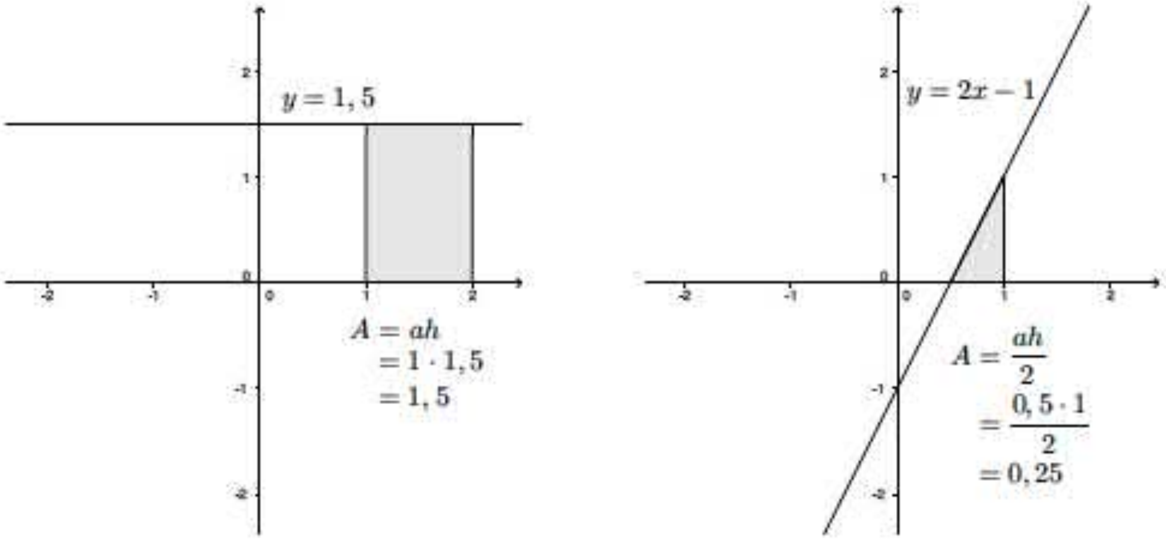


Fig. 12. Calculation of areas.

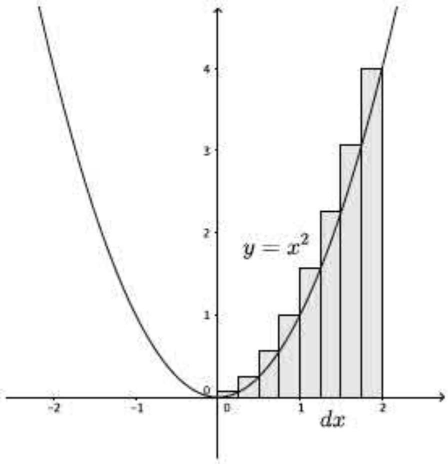


Fig. 13. Calculation of the surface area using rectangles.

Example 6.3. Calculation of definite integrals.

(1) Second-order function $\int_0^1 x^2 dx = /_0^1 \frac{1}{3}x^3 = \frac{1}{3}1^3 - \frac{1}{3}0^3 = \frac{1}{3}$
(2) Exponential function $\int_0^2 e^x dx = /_0^2 e^x = e^2 - e^0 = e^2 - 1 \approx 6.4$

Integration in physics

The definite integral gives area or accumulation. Integration produces new quantities. For example, integration of velocity with respect to time gives distance (cf. Derivation in physics section). The quality of the new quantity can be seen from the graph: integral is the sum of the rectangles $x \cdot y$, thus the integration of velocity with respect to time gives the unit as s·m/s = m, i.e., the unit of distance.

6.3. Integration methods

Mathematically integration is easier than derivation since most functions can be integrated. But in practice, explicit integration can be challenging, or even impossible. For example, a simple looking function $f(x) = e^{x^2}$ cannot be expressed in closed form. However, the definite integral $\int_a^b e^{x^2} dx$ can be numerically approximated. Here methods are presented to determine integral functions. If integration formulae are not available but derivation rules are known, integration formulae can be derived. In addition, it needs to be understood whether integral functions are sought, or definite integrals are calculated. often a difficult integration problem can be solved in many ways. First, one's favorite should be applied, and if that does not work, then take the next method,

Chain rule

The chain rule follows from the derivation rule of combined functions:

$$\int f'[g(x)]g'(x)dx = f[g(x)]$$

This formula can be used, if the combined function and the integral of the inner function are known.

Example 6.4. Consider the integral $\int 2x(x^2+1)^3 dx$. Here we can take $g(x) = x^2 + 1$, which indeed appears like being inside, and then $g'^{(x)} = 2x$. The chain rule gives now

$$\int 2x(x^2+1)^3 dx = \frac{1}{4}(x^2+1)^4 + C$$

The result can be checked by taking the derivative.

Substitution method

The substitution method is based on the chain rule, and it makes integral functions easy to obtain. In this method, a part of the original function is replaced by a simpler form. When this is done, also dx and integration limits in the definite integral must be changed.

Example 6.5. Consider again the integral $\int 2x(x^2+1)^3 dx$. Try a substitution $t = x^2 + 1$. When the derivative is taken for both sides, we have $dt = 2xdx$. After rearrangement, we have

$$\int 2x(x^2+1)^3 dx = \int (x^2+1)^3 2xdx = \int t^3 dt = \frac{1}{4}t^4 + C = \frac{1}{4}(x^2+1)^4 + C$$

where at the end the substitution $t = x^2 + 1$ was returned.

Example 6.6 Definite integral. The definite integral $\int_0^1 e^{3x} dx$ is calculated using the substitution method. For $t = 3x$, we have $dt = 3dx$ or $dx = \frac{1}{3}dt$. The integration limits need to be transformed: at $x = 0$ we have $t = 0$, and at $x = 1$ we have $t = 3$. with these substitutions, the integral is obtained as

$$\int_0^1 e^{3x} dx = \int_0^3 e^t \frac{1}{3} dt = \frac{1}{3}\int_0^3 e^t dt = \frac{1}{3} /_0^3 e^t = \frac{1}{3}(e^3 - 1) \approx 6.4$$

Partial integration

The formula for partial integration is obtained by integrating the derivation rule of a product and rearranging the terms. Thus (the argument x is omitted for clarity):

$$\int_a^b D(fg)dx = \int_a^b (f'g + fg')dx \rightarrow \int_a^b f'gdx = /_a^b fg - \int_a^b fg'dx$$

The idea is to identify the function to be integrated as a product of two functions, one factor at least easy to integrate and one easy to derivate. When these functions are applied in the formula, an integral remains to be solved but that is easier than the oirginal.

Example 6.7. The integral $\int_0^1 x \sin x$ needs to be calculated. A good choice is $g(x) = x$, with $g'(x) = 1$, and $f'(x) = \sin x$, with $f(x) = -\cos x$. The partial integration formula gives then

$$\int_0^1 x \sin x \, dx = /_0^1 - \cos x \cdot x - \int_0^1 - \cos x \cdot 1 \, dx$$
$$= /_0^1 - \cos x \cdot x + \sin x = -\cos 1 + \sin 1 \approx 0.30$$

7. Differential equations

Example 7.1. Examples of differential equations.

(1) First-order simple differential equation: $\frac{dy}{dx} = 2x$
(2) First-order differential equation with function E and its first derivative:
$$\frac{dE}{dz} = -\lambda E$$
(3) Second-order differential equation with function h and its second time derivative:
$$m\frac{d^2h}{dt^2} = -mg$$

Equation (1) can be directly integrated for the solution $y = x^2 + C$. Also, equation (3) can be solved by integration twice, because the equation includes the unknown function only in the second derivative. Normally the direct integration does not apply. In this chapter, the methods to solve first-order separable and linear differential equations are treated.

7.1. Separable differential equations

Separable equations are of the form

$$\frac{dy}{dx} = p(x)q(y)$$

where y is a function of x, $y = y(x)$. Separable equations thus have on one side the derivative of y and on the other side a product of functions of x and y, $p(x)$ and $q(y)$.

Example 7.2. Examples of separable equations.

(1) Equation depending only on y:

$$\frac{dy}{dx} = ky, \text{i.e.}, p(x) = k, q(y) = y$$

(2) Equation depending on x and y:

$$\frac{dy}{dx} = \frac{y^2}{x}, \text{i.e.}, p(x) = \frac{1}{k}, q(y) = y^2$$

For example, equation $\frac{dy}{dx} = xy - 1$ is not separable. To solve a separable equation, it is first assumed that $q(x) \neq 0$. The equation is transformed by moving $q(x)$ to the left side and then integrating both sides:

$$\frac{dy}{dx} = p(x)q(y) \rightarrow \frac{dy}{dx} \cdot \frac{1}{q(y)} = p(x) \rightarrow \int \frac{dy}{dx} \cdot \frac{1}{q(y)} dx = \int p(x)dx$$

$$\int \frac{1}{q(y)} dy = \int p(x)dx$$

The last equation is solved then by direct integration.

Example 7.3. Solve the equation $y\frac{dy}{dx} - 2x = 0$. First, it is changed into the form $\frac{dy}{dx} = \frac{2x}{y}$, and we have $p(x) = 2x, q(y) = \frac{1}{y}$ so that the equation is separable. Thus, we have

$$\int ydy = \int 2xdx \rightarrow \frac{1}{2}y^2 + C_1 = x^2 + C_2$$

We can combine the constants, and the solution is $\frac{1}{2}y^2 - x^2 = C$.

Example 7.4. The equation of the attenuation of sunlight in surface waters $\frac{dE}{dz} = -\mu\lambda$ is solved. Here we can take $E = y, z = x$, and find that the equation is separable with $p(x) = -\lambda, q(E) = E$, and thus

$$\int \frac{1}{E} dE = -\int \lambda dz \rightarrow \ln|E| + C_1 = -\lambda z + C_2 \rightarrow e^{\ln|E|+C_1} = e^{-\lambda z + C_2} \rightarrow$$

$$e^{\ln|E|} = e^{-\lambda z + C} \rightarrow |E| = e^{-\lambda z}e^{C} \rightarrow E = Ae^{-\lambda z}$$

Here the equation was integrated, and the constant were combined into one (that is allowed since they are arbitrary). In the last step, the absolute value signs in $|E|$ could be removed since it must be positive, and the constant e^C was replaced by A. The solution shows that sunlight attenuates exponentially in water. The constant A can be determined from a boundary condition, e.g., $E(0) = A$.

7.2. First-order linear differential equations

The first-order linear differential equations have the standard form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

They depend linearly on the function y. For example, the equation $\frac{dy}{dx} + xy^2 + 1 = 0$ is not linear since y is in the second power. The general solution of the standard form is

$$y = e^{-\int P(x)dx} \int e^{\int P(x)dx} Q(x)dx$$

Example 7.4. Solution of the equation $\frac{dy}{dx} - \frac{y}{x} = 2x$. Here $P(x) = -\frac{1}{x}, Q(x) = 2x$, and $\int P(x)dx = -\int \frac{1}{x}dx == -\ln x$. Inserting them into the general solution gives

$$y = e^{lnx} \int e^{-\ln x} \cdot 2xdx = x \int e^{\ln x^{-1}} \cdot 2xdx = x \int 2dx = x(2x + C)$$

In the initial value problem, the constant C is determined from a given initial condition.

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Appendix 2: Study problems

Chapter 1 Introduction

1-1 What is the role of the hydrosphere in climate?

1-2 How much precipitation falls in Finland per person in one year?

1-3 The population of Helsinki uses the water from Lake Päijänne for the household water. How much has this impact on the water level of the lake?

Chapter 2 Methods

2-1 The spring high water level H of a non-regulated lake follows the exponential distribution $p(H)=\lambda e^{-\lambda H}$, where $\lambda=1$ m^{-1}. What is the 10-year maximum water level at the probability 0.9?

2-2 Based on observations of random variables $X \wedge Y$, linear regression equations $Y=a+bX \wedge X=a^{'}+b'Y$ have been determined. Does the equation $b^{'}=b^{-1}$ hold in general? Justify your answer.

2-3 Show that for Gaussian distributions we have $F(x;\mu,\sigma^2)=F\left(\frac{x-\mu}{\sigma};0.1\right)$, where $F(x;\mu,\sigma^2)$ is the cumulative distribution function with the parameters μ (mean) and σ^2 (variance).

2-4 In mechanics, the dimensions of all quantities P can be expressed using the three fundamental quantities M – mass, L – length, and T – time: $dim\,P=M^a L^b T^c$. Show the expressions of the dimensions of evaporation, acceleration. density, force, pressure, and energy.

Chapter 3 Water and circulation of water

3-1 Determine the 2nd order polynomial of the density of fresh water with maximum at 4 °C and exact values at 0 °C and 20 °C.

3-2 In the Antarctic continent, subglacial lakes exist beneath 4 km thick ice sheet. Estimate the lake temperature, assuming that it is at the freezing point.

3-3 In spring, the *in situ* electric conductivity in a boreal lake was measured as 65 µS cm^{-1}. The temperature of water was 8 °C. What was the concentration of dissolved substances?

3-4 How the water salinity and concentration of dissolved substances are related to each other? What is the concentration in ocean water that has the salinity of 35 ‰?

3-5 In meromictic lakes full turnover is prohibited by the high salinity of the deep water. How much the deep-water salinity must be in the climate of Finland to reach the meromictic condition, assuming that the surface water is fresh?

Chapter 4 Hydrometeorology

4-1 The temperature of moist soil is 10 °C. How much are the specific humidity and water vapor pressure at the surface?

4-2 Estimate how much evaporation can account for from moist soil during a summer day in Finland? Justify your response.

4-3 Determine the theoretical mean surface temperature of the Earth based on the radiation balance when the atmosphere is ignored. The emissivity of the Earth is 0.9, the albedo is 0.34, and the incident solar radiation is 1.37 kW m^{-2}. Compare the result with the outcome of observations of 14 °C and explain why they are different.

4-4 Estimate using the radiation balance whether "snowball Earth" is a possible equilibrium.

4-5 For a snow cover, the thickness is 50 cm, the density is 300 kg m^{-3}, and the mean temperature is –10 °C. How much energy is needed to a) raise the snow cover temperature to 0 °C, and b) to melt the snow thereafter?

4-6 In a spring day in southern Finland the weather was as follows: rainy (precipitation 20 mm d^{-1}), air temperature 6 °C, and wind speed 5 m s^{-1}. Snow cover seemed to be decreasing. What were the principal factors behind snow melting? Justify your answer.

Chapter 5 Physics of lakes

5-1 The water level of Lake Suur-Saimaa was NN + 76.00 m on July 1st, in a given year. In July, the mean inflow was 640 $m^3\,s^{-1}$, the mean outflow was 420 $m^3\,s^{-1}$, and monthly precipitation was 54 mm. Calculate the July evaporation, when the water level was NN+76.08 m at the end of July. The surface area of Lake Suur-Saimaa is 4 467 km^2.

5-2 The water balance of lakes is expressed as $\frac{dV}{dt} = P - E + I - R$, where V is the water storage, t is time, P is precipitation, E is evaporation, I is inflow, and R is outflow. How these quantities must be defined to provide dimensional consistency.

5-3 The surface area of a lake is 113 km^2. a) In September, the mean inflow was 17.0 $m^3\,s^{-1}$ and the mean outflow was 20.5 $m^3\,s^{-1}$. During September the water level sank by 9.3 cm and the precipitation was 70 mm. How much was the evaporation in September? b) The mean depth of the lake is 7.7 m. The annual mean inflow is $7.74 \cdot 10^8$ m^3 and annual precipitation is 683 mm. What is the flushing time of the lake.

5-4 Net heat input into a lake is 50 W m^{-2}. Assuming that the heating is distributed evenly into a 5 m thick surface layer, estimate how much the temperature increases in this layer in two days.

5-5 Assume that in the beginning of October the temperatures of lake surface and air are both 8 °C. Then the air temperature starts to decrease by 1 °C d^{-1}. How much the temperatures of the lake surface and air are after 10 days?

5-6 The mean depth of a lake is 10 m and the wind speed is 5 m s^{-1}. Estimate the speed of the resulting wind-driven surface current.

5.7 Determine the period of longitudinal Seiche in a) Lake Tuusulanjärvi (mean depth 2 m and length 9 km), b) Lake Pääjärvi, Lammi (mean depth 15 m and length 9 km), and c) Lake Geneve (depth 150 m and length 75 km).

5.8 What is the period of internal Seiche in a lake with length 10 km, depth 10 m (constant), and thermocline depth 5 m? The temperature is 20 °C in the upper layer and 10 °C in the lower layer.

5.9 Attenuation of sunlight in lake water is modelled using the exponential attenuation law $E(z)=E(0)e^{-\kappa z}$,where E is irradiance, z is depth, and κ is the attenuation coefficient. The euphotic depth of lakes is defined as the depth where the irradiance has reduced to 1 % of the level at the surface. Determine the euphotic depth for a clear water lake ($\kappa=0.2$ m^{-1}) and turbid lake ($\kappa=2$ m^{-1}).

Chapter 6 Rivers

6-1 Estimate how long it takes for a water particle to flow through a river in southern Finland to the Gulf of Finland. The river length is 200 km, the cross-section is rectangular with 50 m width and 3 m depth, and the river altitude decreases evenly from 100 m above the sea level to the Gulf of Finland.

6-2 The mean discharge of a river is 250 m^3 s^{-1}, the mean speed is 1 m s^{-1}, the river width is 100 m, and the cross-section is rectangular. How much are the river depth and slope?

6-3 Estimate how much are the kinematic height, pressure height, and geometric height (i.e., the terms of the Bernoulli equation in height dimension) in the upper and lower River Vantaanjoki. How much is the frictional loss?

6-4 The mean monthly precipitation P (in mm) and the corresponding correction coefficients k over the River Kyrönjoki drainage basin in 1961 – 1970 are the following:

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
P	29	25	25	26	44	38	59	88	67	54	46	37
k	1.43	1.45	1.42	1.24	1.17	1.04	1.04	1.07	1.08	1.14	1.27	1.31

Calculate the uncorrected and corrected annual precipitation. What is the correction coefficient of the annual precipitation.

6-5 The discharge curve gives the outflow Q from a lake as a function of the water level elevation $h:Q=f(h)$.This can be written as $Q=c(h)A$, where c is a function of h and A is the surface area of the lake. What are the dimension and interpretation of the function c.

6-6 The river length is 100 km, the cross-section is rectangular with depth 10 m and width 50 m, and the elevation change through the river down to the mouth is 50 m. What is the flow speed at the mouth? Use Chezy formula with the Manning coefficient $M \approx 50$ m$^{1/3}$ s^{-1} and approximate the slope of frictional loss by the river slope. How the flow speed changes, if a) depth, b) width, c) length or d) slope is doubled?

Chapter 7 Geohydrology

7-1 One cubic meter soil sample contains 20 % water and 10 % air by volume. The apparent density of the sample is 2 500 kg m^{-3}. Determine the water content relative to the sample mass, bulk density, and dry density.

7-2 A cylindrical well is located in the yard of a private house. The well diameter is 1 m. How much the well can provide water, if the hydraulic conductivity of the soil is 0.5 mm s^{-1}?

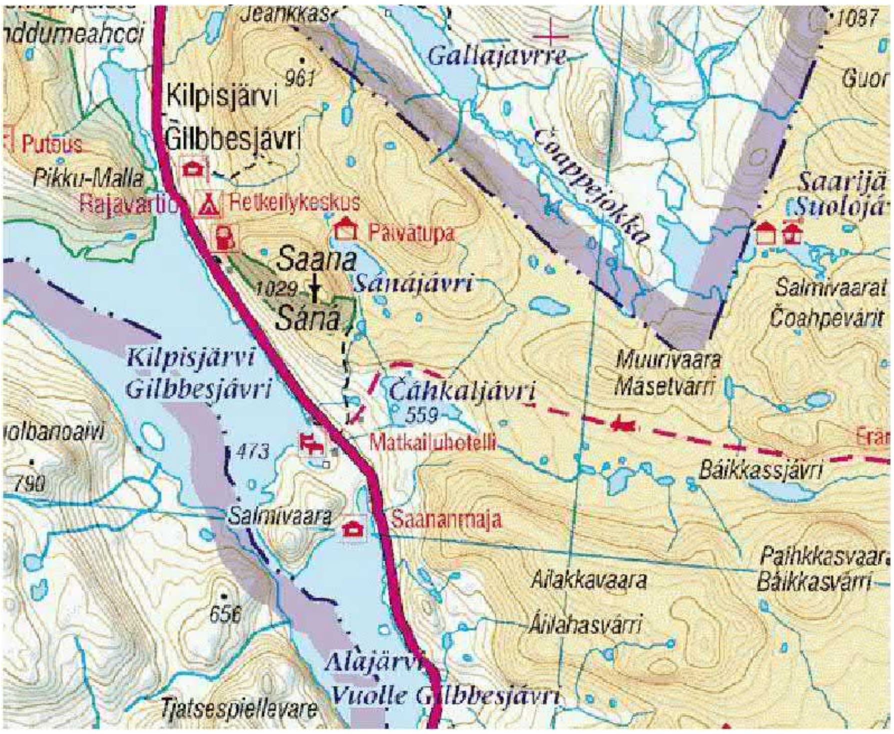
7.3 The surface area of a lake is 10 km^2. The area of the drainage basin is 100 km^2 and the slope is 2 %. How much the hydraulic conductivity of snow needs to be to allow all meltwater to flow down to the lake?

7-4 Is it preferable to build a root cellar in a mineral soil or organic soil? Justify your response.

7-5 A formula for the depth of the frozen ground reads for open land as $d_{fg} = \frac{1}{d_{snow}} \cdot 42\, C \sqrt{S - 15\, d_{snow}}$, where C = 0.9–1.1 depending on the soil quality and S is the sum of freezing-degree-days. Compare this with the Stefan's formula for lake ice thickness, $h = a\sqrt{S}$, where a = 2–3 depending on the snow accumulation. When frozen ground is thicker than lake ice?

Chapter 8 Runoff

8-1 Draw the boundary of Lake Cáhkáljávri drainage basin in the chart below.



8-2 The mean corrected annual precipitation is 520 mm in Vuotos drainage basin of River Kemijoki. The basin area is 9 406 km^2. A plan was made to build there a reservoir with maximum surface area of 236 km^2. Estimate how much the reservoir would have change the annual runoff, if the annual evaporation from the lake surface were 300 mm? The mean runoff in Finland is 10 L s^{-1} km^{-2}.

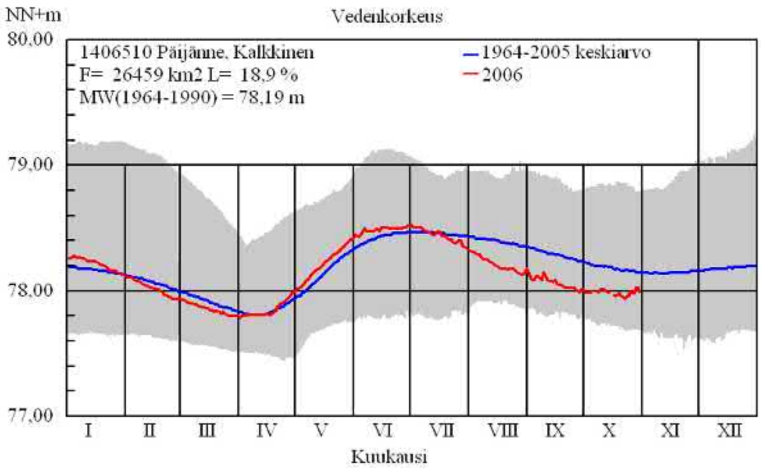
8-3 Consider the hydrology of Lake Päijänne in summer 2006 based on the time series data below (www.ymparisto.fi). In fall 2006, the following results were obtained for the water level and discharge of the lake. Find out:

a. How much lower was the water level of Lake Päijänne at Kalkkinen on September 30, 2006, compared with the long-term average?

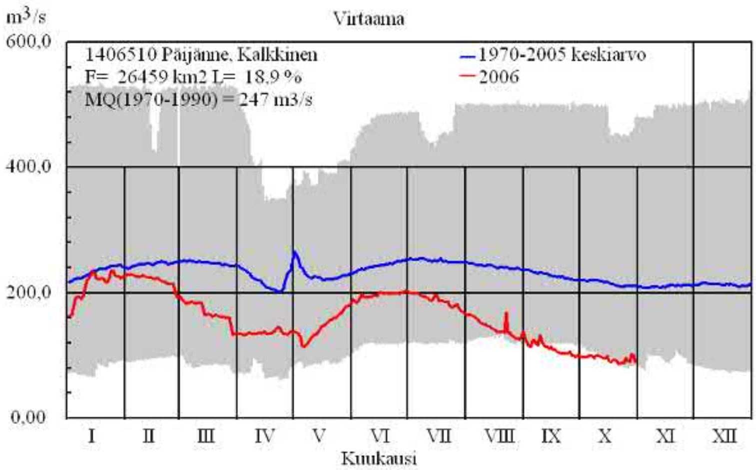
b. Calculate how much water would have been needed to raise the water level to normal? (The surface area of the lake is 1 116 km^2 at the normal water level). How many loads of tank trucks that would account for? Estimate yourself the tank size.

c. How much was the mean outflow of Lake Päijänne in July–September 2006?

d. How many loads of tank trucks would be needed to transport the outflow water in part c?



Water level (blue = mean, red = 2006).



Discharge (blue = mean, red = 2006).

8-4 The Leppävesi drainage basin of Lake Päijänne (#14.311) has an area of 17 684 km^2 with outflow at Vaajakoski. In 1995 the mean outflow was 146 m^3 s^{-1}. The Tourujoki drainage basin has an area of 334 km^2 with outflow at Lohikoski. Estimate the 1995 discharge at Lohikoski, and compare the result with the observed mean value of 1.6 $m^3 s^{-1}$. Is there a difference and why?

Chapter 9 Hydrology and future

9-1 How lake physics has impacts on lake ecology and vice versa?

9-2 Give your interpretation on the following Finnish folklore knowledge:
- carried water does not stay in well
- summer dries what it moistens
- lowlands possess risk for nightfrost
- frozen ground brings a pig home