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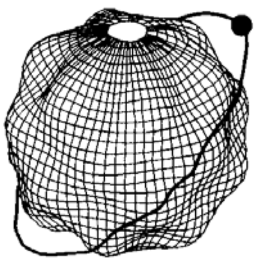

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IGeS Bulletin N. 12
Summary

I section: "Reviewed Papers"

II Section : "Communications and News"

Developing and testing the ellipsoidal gravity model manipulator ELGRAM

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Abstract

The commonly used representation of potential as a truncated series of spherical harmonics leads to global models that consist of set of coefficients c_{nm}, s_{nm} (OSU91, EGM96), that can be successively used by a spherical harmonics manipulator to compute values of the potential and its functionals (geoid undulation, gravity anomaly, deflection of the vertical) at any point of given coordinates (ϕ, λ, h).
The approximations involved both in the computations of the coefficients, from satellite and terrestrial measurements, and in their use in the synthesis of potential functionals make desirable the development of different computational techniques.
In view of the actual requirements of more and more precise potential representation it is useful to develop a manipulator working with series of ellipsoidal harmonic functions.
To this aim, appropriate equations relating geoid undulation, gravity anomaly, deflections of the vertical to anomalous potential T expanded in ellipsoidal series have been deduced, and a source code for their computation have been written and tested, comparing results with 'classical' spherical synthesis.
In the paper the main steps of analytical computations are shown, the structure and use of the new software is illustrated and some results of the comparisons are reported.

1 Ellipsoidal harmonic expansions

For many geodetic computations the anomalous gravitational potential of the earth is usually expanded in series of spherical harmonic functions:

$$T = \sum_{n=0}^{\infty} \sum_{m=0}^{n} v^s_{nm} \left(\frac{R}{r}\right)^{n+1} Y_{nm}(\theta, \lambda) \qquad (1)$$

and usually represented as a truncated series, up to $n = N_{max}$

It is however possible to expand T using ellipsoidal harmonic functions (Heiskanen, Moritz (1990))

$$T = \sum_{n=0}^{N_{max}} \sum_{m=0}^{n} v^e_{nm} \frac{Q_{nm}\left(i\frac{u}{E}\right)}{Q_{nm}\left(i\frac{b}{E}\right)} Y_{nm}(\theta_e, \lambda) \qquad (2)$$

The presence of Legendre function of second kind $Q_{nm}\left(i\frac{u}{E}\right)$ has been an obstacle up to now for the computation of such a series, the main difficulties in using truncated series of ellipsoidal harmonics coming from :

- The dependence on both order n and degree m of the Q_{nm}, that causes longer computational time for the terms containing the ellipsoidal "height" u (compared with spherical series (1) that can be computed separating the angular part Y_{nm} from the simple radial terms $\left(\frac{R}{r}\right)^{n+1}$, independent of m).

- The computation of the Q_{nm} functions cannot be done using the simple recursive relations that hold identically for Q_{nm} and for P_{nm}, because in this case the recursive relations show an unstable behavior due to their imaginary argument (Sona (1995)).

The first problem cannot be avoided, being intrinsic in the ellipsoidal representation, (we can only simplify computations with some interpolations), but with the more and more powerful processors now available it is today a minor problem.
The second and more important obstacle can be overcome computing the Q_{nm} with hypergeometric functions, as suggested in Thong and Grafarend (1989) or in Sona (1995).
The difficulties arising in such a direct computation of Q_{nm} functions however (for each n,m and each value of 'height' u) are better reduced by introducing the 'boundary layer approach' , that means by putting :

$$1 < \frac{u}{b} < 1 + 10^{-3} \qquad (\text{h<6700 m}),$$

approximation that holds for most of the earth surface. This leads to the direct approximation of the terms

$$\overline{Q}_{nm}(u) = \frac{Q_{nm}\left(i\frac{u}{E}\right)}{Q_{nm}\left(i\frac{b}{E}\right)}$$

in (2) as

$$\overline{Q}_{nm}(u) = \left(\frac{b}{u}\right)^{n+1}\left(\frac{u}{b}\right)^{e^2\frac{(n+1)(n+2)+m^2}{2n+1}} \qquad (3)$$

This expression has been proved to be very close to $\overline{Q}_{nm}$, up to a very high order and degree (n<1.5 10^5) and in the same time does not display any computational irregularity (singularity, instability) (Sona (1995)).

The 'limit layer' approximation is therefore a good solution to simplify and speed up the computation of $\overline{Q}_{nm}$ making thus sufficiently manageable a computer program working directly in ellipsoidal harmonics.

2 Spherical and ellipsoidal model coefficients

In order to develop a computer program that uses ellipsoidal harmonic truncated series to synthesize anomalous potential T and its functionals, it is necessary to transform the relationship exploited by the worldwide used classical 'spherical' software (for instance f477 or f388 by R.Rapp, (1982)) into the corresponding ellipsoidal ones, and, of course, it is necessary to use an ellipsoidal global model, that is, a set of ellipsoidal coefficients $v^e_{n'm}$ (eq.2).
This is not available, at the moment, and all the global geopotential models available to IGeS consist in the well known sets of spherical harmonic coefficients (e.g. OSU91A, EGM96, by Rapp,or Wenzel, (1999))

So the only way to have an ellipsoidal global model, with the purpose of checking results obtained with a new ellipsoidal model manipulator , (comparing values of ζ , Δg, ξ, η at several point ϕ_i, λ_i, h_i) is to translate a spherical model into the corresponding ellipsoidal one, that means, to transform the set of spherical coefficients v^s_{nm}, into the corresponding set of ellipsoidal coefficients $v^e_{n'm}$, through the well known exact relations existing between the two sets (Jekeli (1988)).
Therefore we created a fictitious set of ellipsoidal coefficients by transforming the coefficients of the spherical global model EGM96 (Nmax=360, Lemoine et al.1998) ; the values of $v^e_{n'm}$ can be computed as linear combination of v^s_{nm} , with $n = n'$, $n' - 2, n' - 4, n' - 2k$..... :

$$v^e_{n'm} = S_{nm}\left(\tfrac{b}{E}\right)\sum_{k=0}^{W}\lambda_{knm}v^s_{n-2k,m} \qquad (4)$$

where

$$\lambda_{knm} = \frac{(2n-2k)!\,n!}{(2n)!\,k!\,(n-k)!}\sqrt{\frac{(2n-4k+1)(n-m)!\,(n+m)!}{(2n+1)(n-2k-m)!\,(n-2k+m)!}}\left(\tfrac{E}{R}\right)^2$$

$$\bar{S}_{nm}\left(\tfrac{b}{E}\right) = \frac{\left(\tfrac{R}{E}\right)i^{n+1}(2n)!}{2^n\,n!}\sqrt{\frac{\varepsilon_m(2n+1)}{(n-m)!\,(n+m)!}}Q_{nm}\left(i\tfrac{b}{E}\right) =$$

$$= \left((i\tfrac{b}{E})^2 - 1\right)\left(i\frac{R}{b}\right)^{n+1}\left(i\frac{E}{b}\right)^m F\left(\tfrac{n+m+2}{2}, \tfrac{n+m+1}{2}; n+\tfrac{3}{2}; -\left(\tfrac{E}{b}\right)^2\right)$$

and F is the hypergeometric Gauss function (Jekeli (1988), Petrovskaya (2000)).
As it can be seen in eq. (4) each spherical coefficient v^s_{nm} is involved in the computation of all $v^e_{n'm}$ of the same order m and $n' \geq n$, $n'=n, n+2, n+4,.....,n+2k$. Therefore a global model, described by spherical coefficients up to order and degree 360, yields through (4) ellipsoidal coefficients that are significantly different from zero also for n'>360.

3 Ellipsoidal expansion of geodetic functionals of T

For the purpose to make the ellipsoidal computations comparable with the spherical one (up to spherical degree max Nmax=360), the (fictitious) ellipsoidal model EGM96ell has been built up to degree N^e_{max}=400 (of course, coefficients with m>360 are missing in this model).

Starting from the usual relations :

$$\zeta = \frac{T}{\gamma} \qquad (5)$$

$$\Delta g = \frac{\gamma}{\gamma}\cdot\nabla T + \left(\frac{\partial\gamma}{\partial h}\right)\frac{T}{\gamma} \qquad (6)$$

$$\varepsilon = -\frac{dN}{ds} = -\frac{1}{\gamma}\frac{dT}{ds} \qquad (7)$$

expansion of main geodetic functionals in ellipsoidal harmonics can be deduced, in order to compute from a set of ellipsoidal coefficients the anomalous potential T and its functionals height anomaly ζ , gravity anomaly Δg and deflections of the vertical ξ and η.
Being simply related with the anomalous potential T, the height anomalies ζ can be easily computed inserting the coefficients c^e_{nm}, s^e_{nm}, in (2):

$$T = \sum_{n=0}^{N_{max}} \sum_{m=0}^{n} \bar{Q}_{nm}(u) P_{nm}(\theta_e)\left(c^e_{nm} \cos m\lambda + s^e_{nm} \sin m\lambda\right) \ldots(8)$$

On the contrary, for some terms in equations (6) and (7) more analytical computations are needed. In (6), the projection of the gradient of T on the normal gravity direction in ellipsoidal coordinates can be computed as:

$$\frac{\underline{\gamma}}{\gamma}\cdot\nabla T = \frac{\gamma_u}{\gamma}\frac{\partial T}{\partial s_u} + \frac{\gamma_\beta}{\gamma}\frac{\partial T}{\partial s_\beta} \quad (9)$$

where γ_u and γ_β can be derived from the closed formula of normal potential U (Heiskanen and Moritz (1995))

$$\gamma_u = \frac{1}{w}\left[\omega^2 u \cos^2\beta - \frac{kM}{u^2+E^2} + \frac{\omega^2 a^2}{2q(b)}\left(\sin^2\beta - \tfrac{1}{3}\right)\frac{dq(u)}{du}\right]$$

$$\gamma_\beta = \frac{\omega^2 \cos\beta \sin\beta}{w\sqrt{u^2+E^2}}\left[a^2\frac{q(u)}{q(b)} - \left(u^2+E^2\right)\right]$$

where $w = \sqrt{\dfrac{u^2+E^2\sin^2\beta}{u^2+E^2}}$ and $q(u) = \dfrac{1}{2}\left[\left(1+3\dfrac{u^2}{E^2}\right)\operatorname{artg}\dfrac{E}{u} - 3\dfrac{u}{E}\right]$.

The ellipsoidal derivatives of the anomalous potential T:

$$\frac{\partial T}{\partial s_u} = \frac{1}{w}\frac{\partial T}{\partial u} \quad (10)$$

$$\frac{\partial T}{\partial s_\beta} = \frac{1}{w\sqrt{u^2+E^2}}\frac{\partial T}{\partial \beta} = \frac{-1}{w\sqrt{u^2+E^2}}\frac{\partial T}{\partial \theta_e} \quad (11)$$

($\theta_e=\pi/2-\beta$) can be derived by writing the anomalous potential T with the expansion (8) and remembering that the line element in ellipsoidal coordinates is given by

$$ds^2 = w^2 du^2 + w^2\left(u^2+E^2\right)d\beta^2 + \left(u^2+E^2\right)\cos^2\beta\, d\lambda^2 \ . \ (12)$$

Inserting the approximate expression (3) for the $\overline{Q}_{nm}$ one gets

$$\frac{\partial T}{\partial u} = \sum_{n=0}^{N_{max}} \sum_{m=0}^{n} \frac{K_{nm}}{u} \overline{Q}_{nm}(u) P_{nm}(\theta_e)\left(c^e_{nm} \cos m\lambda + s^e_{nm} \sin m\lambda\right) \quad (13)$$

with $$K_{nm} = e^2 \frac{(n+1)(n+2)+m^2}{2n+1} - (n+1)\ ,$$

and

$$\frac{\partial T}{\partial \theta_e} = \sum_{n=0}^{N_{max}} \sum_{m=0}^{n} \overline{Q}_{nm}(u) \frac{\partial P_{nm}(\theta_e)}{\partial \theta_e}\left(c^e_{nm} \cos m\lambda + s^e_{nm} \sin m\lambda\right) \quad (14)$$

As verified by testing one step at a time, the different approximations used in the two manipulators yield Δg values which cannot be compared because the gradient is taken along significantly different directions (while height anomalies results very close in values to the spherical ones). An additional check has therefore been planned, by comparing the values of $|\nabla T|^2$ in spherical and ellipsoidal coordinates, as this quantity is coordinate independent, thus testing whether the partial derivatives of T are computed properly or not:

$$|\nabla T|^2 = \left(\frac{\partial T}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial T}{\partial \theta}\right)^2 + \left(\frac{1}{r \sin\theta}\frac{\partial T}{\partial \lambda}\right)^2$$

$$|\nabla T|^2 = \left(\frac{1}{w}\frac{\partial T}{\partial u}\right)^2 + \left(\frac{1}{w\sqrt{u^2+E^2}}\frac{\partial T}{\partial \beta}\right)^2 + \left(\frac{1}{\sqrt{u^2+E^2}\cos\beta}\frac{\partial T}{\partial \lambda}\right)^2$$

All the terms needed to compute $|\nabla T|$ are already present in f477 as part of computation of ζ, Δg, ξ, η, therefore only few changes in that source code have been added.

The horizontal derivatives of T in ellipsoidal coordinates are also needed, to compute the two components of vertical deflection $\underline{\varepsilon}$:

$$\eta = -\frac{dN}{ds_\lambda} = -\frac{1}{\gamma}\frac{dT}{ds_\lambda} \qquad \xi = -\frac{dN}{ds_\varphi} = -\frac{1}{\gamma}\frac{dT}{ds_\varphi}$$

The η component is easy to compute and to compare with corresponding spherical value, because both coordinate systems use the same definition for λ.
From eq. (12) we get the simple relation :

$$\frac{dT}{ds_\lambda} = \frac{1}{\sqrt{u^2+E^2}\cos\beta}\frac{dT}{d\lambda}$$

On the contrary, ξ component needs more analytical computations, in fact we can write

$$\frac{dT}{ds_\varphi} = \frac{1}{f(u,\beta)}\frac{dT}{d\varphi}$$

where we have to compute $\frac{dT}{d\varphi}$ as

$$\frac{dT}{d\varphi} = \frac{\partial T}{\partial u}\frac{\partial u}{\partial \varphi} + \frac{\partial T}{\partial \beta}\frac{d\beta}{d\varphi} \quad ,$$

and $f(u,\beta)$ is deduced from relation (12) , with dλ = 0 and written as :

$$ds_\varphi{}^2 = \frac{w^2 du^2 + w^2 \left(u^2 + E^2\right) d\beta^2}{d\varphi^2} \cdot d\varphi^2$$

that gives :

$$ds_\varphi = w\sqrt{\left(\frac{du}{d\varphi}\right)^2 + \left(u^2 + E^2\right)\left(\frac{d\beta}{d\varphi}\right)^2} \cdot d\varphi \quad .$$

The derivatives of T with respect to u and β expanded in ellipsoidal harmonic series have already been computed for Δg determination (eq (13) and (14)).
Derivatives of u and β with respect to ϕ have to be computed from the relations used to transform geodetic into ellipsoidal coordinates :

$$\begin{aligned} (N + h)\cos\phi &= \sqrt{u^2 + E^2}\cos\beta \\ \left(\tfrac{b^2}{a^2} N + h\right)\sin\phi &= u \cdot \sin\beta \end{aligned} \qquad (15)$$

$$\frac{du}{d\varphi} = (M + h)\frac{\sqrt{u^2 + E^2}}{u^2 + E^2 \sin^2\beta}\left[\sqrt{u^2 + E^2}\sin\beta\cos\varphi - u\sin\varphi\cos\beta\right]$$

$$\frac{d\beta}{d\varphi} = (M + h)\frac{1}{u^2 + E^2 \sin^2\beta}\left[\sqrt{u^2 + E^2}\sin\beta\sin\varphi + u\cos\varphi\cos\beta\right]$$

(M and N are the principal radii of curvature)

Finally, calling

$$S_{(0)} = \sum_{n=0}^{N_{max}} \sum_{m=0}^{n} \bar{Q}_{nm}(u) P_{nm}(\theta_e)\left(c^e_{nm} \cos m\lambda + s^e_{nm} \sin m\lambda\right)$$

$$S_{(1)} = \sum_{n=0}^{N_{max}} \sum_{m=0}^{n} \frac{K_{nm}}{u} \bar{Q}_{nm}(u) P_{nm}(\theta_e)\left(c^e_{nm} \cos m\lambda + s^e_{nm} \sin m\lambda\right) \qquad (16)$$

$$S_{(2)} = \sum_{n=0}^{N_{max}} \sum_{m=0}^{n} \bar{Q}_{nm}(u) \frac{\partial P_{nm}(\theta_e)}{\partial \theta_e}\left(c^e_{nm} \cos m\lambda + s^e_{nm} \sin m\lambda\right)$$

$$S_{(3)} = \sum_{n=0}^{N_{max}} \sum_{m=0}^{n} \bar{Q}_{nm}(u) P_{nm}(\theta_e)\left(c^e_{nm} \sin m\lambda + s^e_{nm} \cos m\lambda\right)$$

ζ, Δg, ξ, η can be written as

$$\zeta = \frac{1}{\gamma} S_{(0)}$$

$$\Delta g = \frac{1}{\gamma}\left[F_1 \cdot S_{(1)} + F_2 \cdot S_{(2)} + F_3 \cdot S_{(0)}\right]$$

$$\xi = -\frac{1}{\gamma} F_4 \left[F_5 S_{(1)} + F_6 S_{(2)}\right] \qquad (17)$$

$$\eta = -\frac{1}{\gamma} F_7 S_{(3)}$$

These relations (similar to those used in spherical manipulators) link values of ζ and Δg to ellipsoidal coefficients c^e_{nm}, s^e_{nm} .

4 ELGRAM : an ellipsoidal harmonic manipulator

The program developed up to now computes ζ , Δg and ξ,η at sparse points (ϕ_i,λ_i,h_i) or on a regular (ϕ,λ) grid with constant height h, from a set of ellipsoidal coefficients and it is detailed in the following.
The main computational steps are:
1) geodetic coordinates (ϕ_i,λ_i,h_i) are transformed into the ellipsoidal coordinates (β_i,λ_i,u_i) solving the relations (15), for u and cosβ.
Note that, as both u and β depend on both h and ϕ, the surfaces h=constant and ϕ=constant are not transformed into surfaces u=const or β=const, that means: a geodetic grid with $\Delta\lambda$=const, $\Delta\phi$ =const (typically used in global computations) is not transformed into a regular ellipsoidal grid with constant $\Delta\beta$= and $\Delta\lambda$ (only the spacing $\Delta\lambda$=const is preserved, as λ is the same in the two coordinate systems)

2) the $\overline{Q}_{nm}$ functions are computed using (3). It has been verified that a quadratic interpolation between three fixed heights gives $\overline{Q}_{nm}$ with sufficient approximation. Thus the computation of them for each value of u can be avoided: a subroutine computes the $\overline{Q}_{nm}$ functions at two constant values u_1=1000m, u_2=4000m (the third value is fixed: for u=b $\overline{Q}_{nm}=1$ $\forall$ n,m) and the coefficients to be used in the interpolation. Another subroutine then interpolates at the point height u_i the proper values of $\overline{Q}_{nm}$ for each n,m.

3) as in spherical harmonic manipulators, the normalized Legendre functions $P_{nm}(\theta_e)$ and their derivatives $\frac{\partial P_{nm}(\theta_e)}{\partial\theta_e}$ are computed for the ellipsoidal value of $\theta_e=\pi/2-\beta$ using recursive relations.

4) the normal gravity $\gamma=\gamma(\phi,h)$ and its derivative are approximated by the relations (Heiskanen Moritz (1995)):

$$\gamma = 9.78032677\frac{\left(1+0.00193185135\sin^2\phi\right)}{\sqrt{1-e^2\sin^2\phi}} - h\cdot 0.3086\cdot 10^{-5}$$

$$\frac{\partial\gamma}{\partial h} = -\frac{2\gamma_a}{a}\left(1+f_2\sin^2\phi+f_4\sin^4\phi\right)\cdot\left(1+f+m-2f\sin^2\phi\right)$$

5) finally, the values of ζ , Δg and ξ, η are computed through (16) and (17)

5 The source code

ELGRAM source code has been developed in FORTRAN77 language , compiled and tested on a UNIX HP Workstation. It is organized in a main program (ELGRAM) and 8 subroutines; the main program defines (as usual) parameters values, file names,variables, vectors dimensions, reads input data, calls subroutines for the different computation steps and writes on files the output required.

The parameter nmax represents maximum order and degree achievable in the expansions, and it is used to define 9 matrix (nmax+1)x(nmax+1) and 5 vectors, with dimension that can be seen in Table 1, and it is now set at 400 correspondingly to the coefficients file EGM96ee (derived as said in par.1 from global model EGM96); if desired, it can be easily raised, when new higher degree ellipsoidal model will be available.

ce(n,m), se(n,m)	ellipsoidal coefficients	(0:nmax,0:nmax)
qnm1(n,m), qnm2(n,m),qnmu(n,m) anm(n,m),bnm(n,m)	values of Q_{nm} at three fixed heighs and interpolation coefficients	
pol(n,m),der(n,m) pnn(0:nmax), dnn(0:nmax), radq(n)	values of Legendre polinomials and derivatives	(2*nmax+1)
sinml(n),cosml(n)	recursive values of sin(mλ) cos(mλ)	(nmax)

Table 1

(We remember that a max ellipsoidal degree N^e_{max}=400 corresponds to a spherical global model of max degree N_{max}=360, as explained in par.1)

The same values of the fundamental constants a, e^2, ω^2, kM (Tab.2) as adopted by f477 have been used in ELGRAM to compare values of potential functionals computed at the same test points :

1/f = 298.257222101
KM = 3.986005 10^{14}
ω^2 = 7.292115 10^{-5}
a = 6378137.0

Table 2

As seen in fig.1 the program starts asking interactively for:

1) Name of global model coefficients file (the only one available at the moment is EGM96ee)
2) Choice between computation on sparse (0) or gridded (1) points
3) a) name of file containing n, ϕ, λ, h, of sparse points
   b) number of points
   or
   a) constant value of h
   b) limits and steps of the grid
   c) type of output
4) maximum degree of the computation (always lower than nmax)
5) name of output file.

```
global model (coefficients file) ?
sparse point (0) or grid (1) ?

sparse points  ==>  Points file (n,φ,λ,h)?
                    number of points ?

       grid    ==>  value of h (constant) ?
                    limits and step of the grid ?
                  (φmin φmax λmin λmax Δφ Δλ)
              output: n,φ,λ,values (0) or grid (1) ?
          if (1) ==> choose functional: 1) geoid undulation
                                        2)gravity anomalies
max degree ?
output file ?
```

Fig.1

The program then reads the model coefficients from coefficients file and, after that, for every point (ϕ,λ) read from the point file (or computed on knots of a regular grid), it performs the computational steps described in par.3 going through the following subroutines:

qnm012	computes Q_{nm} for each n,m up to maximum degree and order ngra
intqnm	computes the coefficients a_{nm} b_{nm} c_{nm} for the successive quadratical interpolation at given height u
geotoel	performs the coordinate transformation $(\phi,h)\Rightarrow(\beta,u)$
intqnm2	evaluates $Q_{nm}(u)$ at ellipsoidal height u
polleg dscml	compute surface harmonics $Y_{nm}(\beta,\lambda)$
Presint sintell	produce the final values of the functionals via (16) (17)

On the output file a header contains

1) Name of the model file
2) Max degree used
3) Name of the input point file and number of computed points,
   or ϕ_{min} ϕ_{max} λ_{min} λ_{max} $\Delta\phi$ $\Delta\lambda$ in4 the gridded case

then follow the output values:

a) n ϕ λ h β du ζ Δg ξ η in one line for each point,

or

b) gridded values of selected functional from north to south, from west to east; (organized in blocks at constant ϕ, variable λ)

6 Comparing ellipsoidal and spherical synthesis

As mentioned ELGRAM has been tested by comparing its results with 'spherical' results yielded by f477 (Rapp(1982)): height anomalies ζ, gravity anomaly Δg, anomalous potential gradient $|\nabla T|$, and vertical deflection components ξ,η, have been compared at several points.
In the following are reported some results of comparisons performed on regularly spaced points ($\Delta\lambda=\Delta\phi=0.5°$) along a quarter of the reference meridian $\lambda=0°$ and along a quarter of the parallel $\phi=45°$ (at variable heights).

The following plots display the satisfactory values obtained for both the absolute and relative differences of height anomalies, $|\zeta_s - \zeta_e|$ and $|\zeta_s - \zeta_e|/\zeta_s$

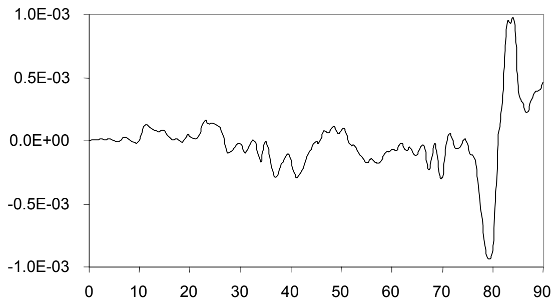

Fig.1 Height anomalies absolute differences (m) along a quarter of parallel $\phi=45^{\circ}$

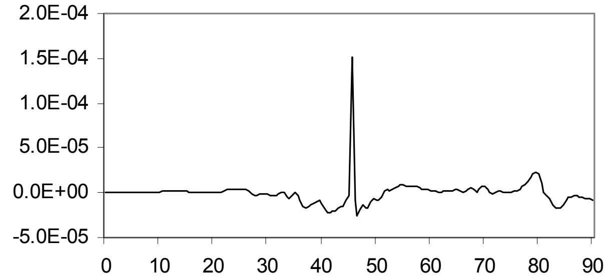

Fig.2 Height anomalies relative differences along a quarter of parallel $\phi=45^{\circ}$

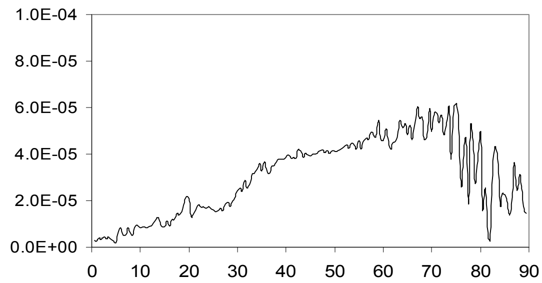

Fig.3 Height anomalies absolute differences (m) along a quarter of reference meridian $\lambda=0^{o}$

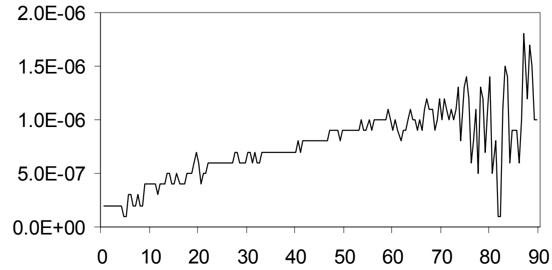

Fig.4 Height anomalies relative differences along a quarter of reference meridian $\lambda=0^{o}$

The spike of relative errors at $\phi=45^{o}$, $\lambda=0^{o}$ in Fig.2 is explained by the very small value of ζ at that point.

On the contrary, when we came to Δg we didn't get small errors, and this is easily understood remembering the different approximations used in the two manipulators, that lead to Δg values which are not comparable. However, as said in par.1, we compared also $|\nabla T|$ values, and we obtained very satisfactory results, as can be deduced from the following plots:

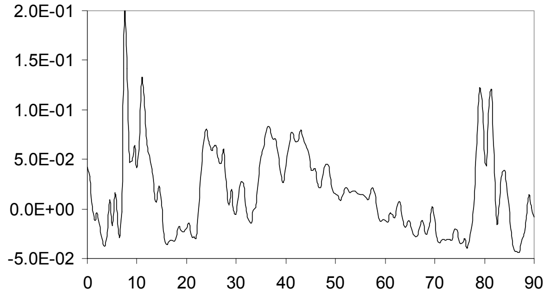

Fig.5 Absolute differences of $|\nabla T|$ (mGal) along a quarter of parallel $\phi=45^{o}$

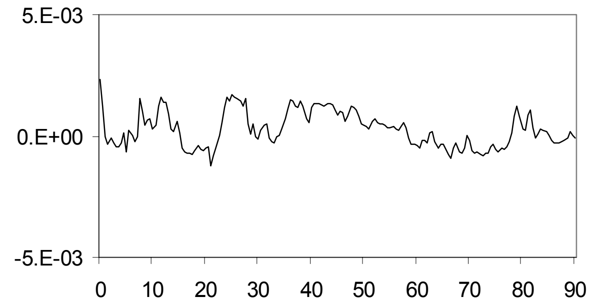

Fig.6 Relative differences of $|\nabla T|$ along a quarter of parallel $\phi=45^{\circ}$

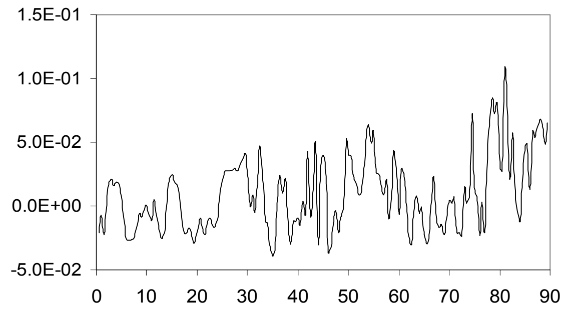

Fig.7 Absolute differences of $|\nabla T|$ (mGal) along a quarter of reference meridian $\lambda=0^{o}$

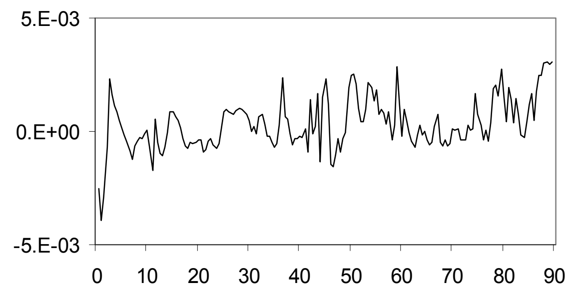

Fig.8 Relative differences of $|\nabla T|$ along a quarter of reference meridian $\lambda=0^{o}$

In this way we have the proof that horizontal derivatives of T in eqs (13) and (14) are properly computed.

The comparisons of ξ,η on the same test points are represented in Fig. 9 to 16 confirming the good agreement between ellipsoidal and spherical model manipulator, in the sense that the maximum error is of the order of the smallest noise in the measurements.

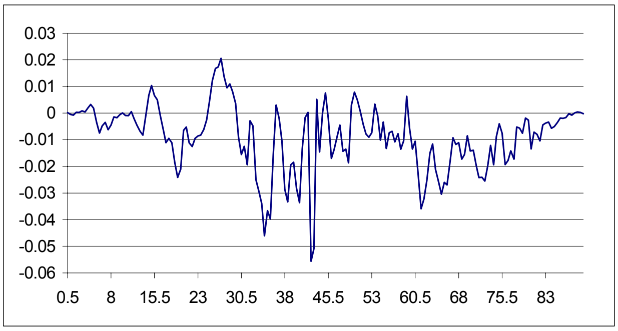

Fig.9 Absolute differences of ξ (sec) along a quarter of reference meridian λ=0^{o}

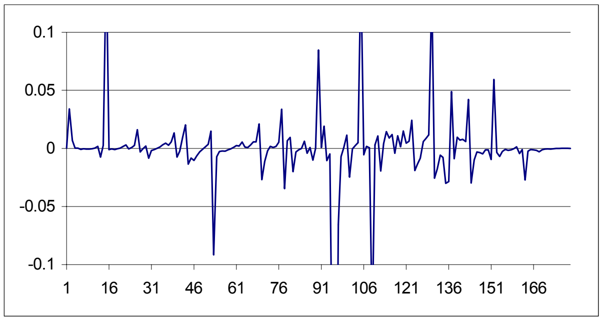

Fig.10 relative differences of ξ along a quarter of reference meridian λ=0^{o}

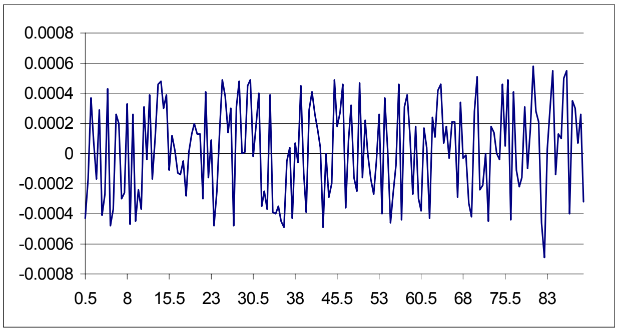

Fig.11 absolute differences of η (sec) along a quarter of reference meridian λ=0^{o}

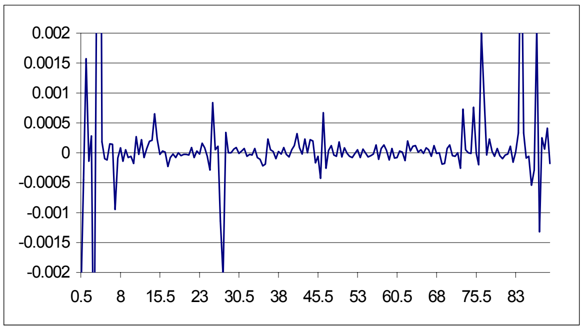

Fig.12 relative differences of η along a quarter of reference meridian λ=0^{o}

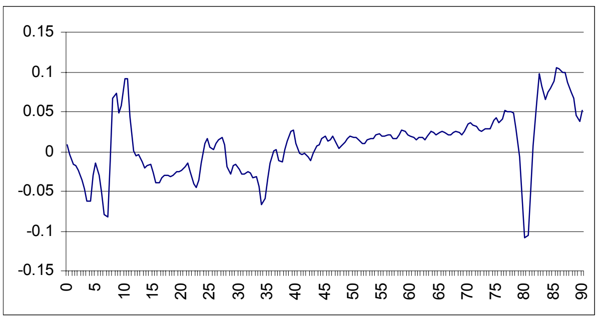

Fig.13 Absolute differences of ξ (sec) along a quarter of reference parallel φ=45^{o}

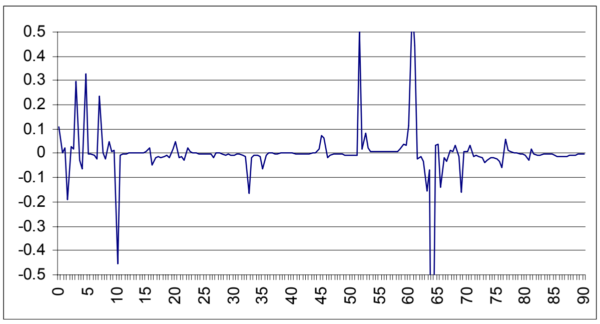

Fig.14 Relative differences of ξ along a quarter of reference parallel φ=45^{o}

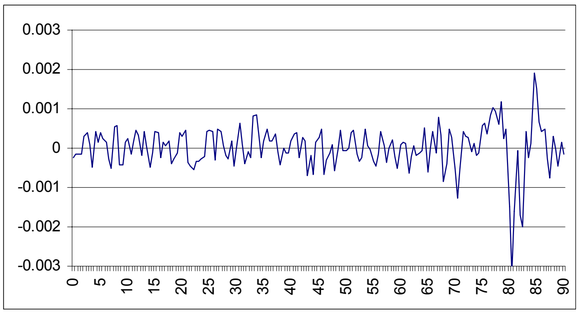

Fig.15 Absolute differences of η (sec) along a quarter of reference parallel φ=45°

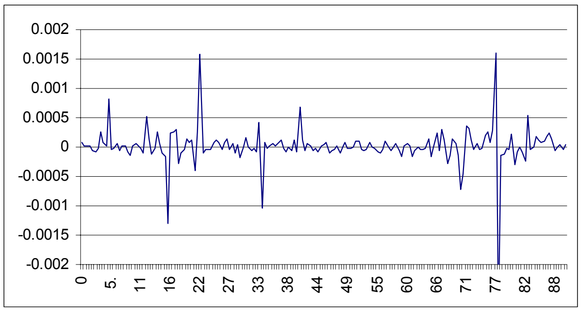

Fig.16 Relative differences of η along a quarter of reference parallel φ=45°

7 Conclusions and further developments

Tests performed on the new ellipsoidal gravity manipulator ELGRAM have got a very positive results; at present the software is able to compute at gridded or sparse points the basic functionals of the anomalous potential ζ, Δg, ξ,η.
We remember that this means that ELGRAM is able to perform the synthesis of ζ , Δg and ξ, η in the topographic layer (h<6700m) , given an ellipsoidal geopotential model.

The software is now freely available through the I.Ge.S. web, where one can find the source code ready to be compiled (main program and all subroutines), the input files for execution of some examples (including those reported in this paper), the point files used for testing the code along a quarter of parallel and of meridian and the corresponding output files for checking results and for training . The ellipsoidal coefficients file computed from EGM96 is also given to allow the use of ELGRAM software.

It is still necessary to test it up to order and degree 1440; this will require a careful optimization of routines and the introduction of FFT along parallels to save computing time.

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