

## Scalable six bar linkage mechanism for re-orienting and aligning objects: Design methodology

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### ABSTRACT

This paper summarizes the work done in the context of an industrial research project in collaboration between the Italian Institute of Technology and the company Fameccanica Data Spa. This paper introduces the design of a six bar linkage mechanism, actuated by a single motor, for re-orienting and aligning objects. The main application consists of grasping empty bottles lying flat on an input conveyor, rotating, aligning and placing them in upright position on another conveyor. The breakthrough aspect of the general and scalable design introduced lays in a system of kinematic equations whose solutions correspond to the linkage lengths. Such lengths define a six-bar-linkage able to re-orient and align vertically a generic object. Finally, this paper presents the mechanism prototype created for such industrial application for performing tests in an operational environment at high speed.

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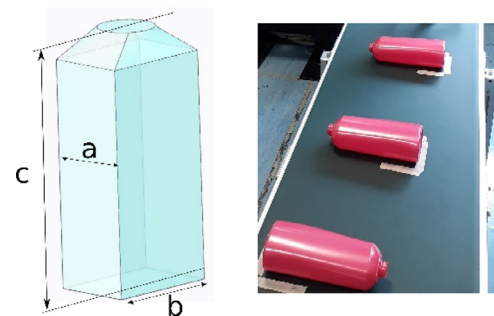
### 1. Problem statement

In the bottles filling industry, at a certain point of the production line, bottles have to be oriented to a vertical upright position in order to be filled.

In general, bottles have to be picked from a conveyor where they lay flat, after the unscrambling process, erected and moved to another conveyor leading them to the filling station.

Fameccanica Data Spa unscrambling process leaves the bottles flat and orderly spaced among each other in an input conveyor. The space in between bottles depends on bottle sizes, which can vary greatly. Even if the shape can also vary, all bottles can be inscribed in a cuboid of dimensions  $c \times b \times a$ . While all bottles are flat, laying on one of their widest surfaces, their orientation is not set by the unscrambling system, so that they can lay either in one or in the other direction presenting their neck toward one or the other side of the input conveyor (see, for instance, Fig. 1).

Since the velocity of the input conveyor varies in the range of 0.4 – 0.7m/s and its length is approximately six meters, a bottle goes from the beginning to the end of the input conveyor in approximately ten seconds. This means that a large number of bot-



**Fig. 1.** Schematic of bottles with dimensions and Example of bottles random orientation on conveyor.

tlers are processed in a small period of time and, thus, the process is considered a high speed process.

Therefore, the problem is to pick bottles and re-orient them.

Because bottles vary greatly, the design has to be independent of bottle shape and scalable. Since the orientation of bottles on the input conveyor is not set, the design should also be independent of their original neck position on the input conveyor. Since time is key to successful industrial processes, the design has to be modular and several systems should be able to work in parallel.

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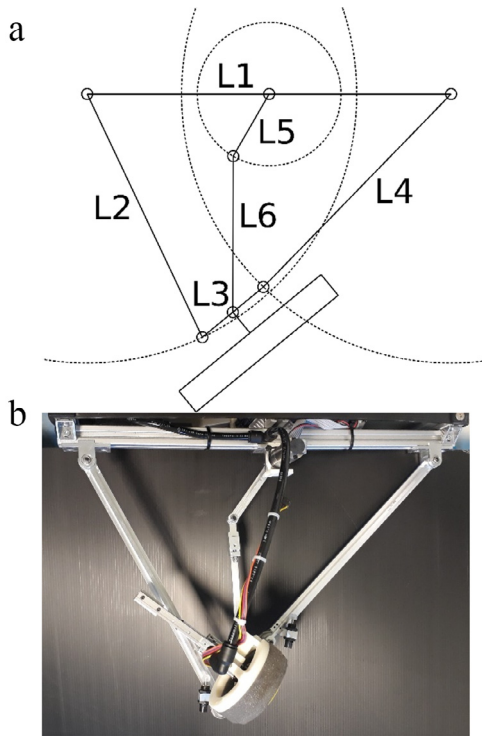


Fig. 2. Scheme of scalable six bar linkage mechanism (a) and last prototype used during tests (b).

## 2. Design

Several re-orienting systems for the bottle filling industry have been patented (Pugh et al., 1975; Gomez, 1989; 刘君基, 2010). All of them orient bottles while they are transferred from one conveyor to another, placed at lower height with respect to the first; they rely on a combination of fixed supports and guides to rotate the bottles during the passage from the upper to the lower conveyor. The design presented here, also patent pending, instead, consists of a gripper that can be mounted on any type of robot. The gripper presents several advantages over the traps and guides fixed on the conveyors. It is independent of bottle shape, because a vacuum cup is used to pick the bottles, independent of position of the bottle, because it can rotate the bottle of  $\pm 90^\circ$  to have the bottle neck up, and scalable, both in terms of size of bottles and in terms of number of bottles to be manipulated at the same time. The gripper is the outcome of several design iterations that led to a lean, versatile, scalable system consisting, essentially, of a planar mechanism actuated by a single motor.

Its schematic is depicted in Fig. 2, showing also the prototype realized for the last tests.

It is a six bar mechanism with links L2 and L4 of equal length and L1 serving as frame. The only motor that is needed to rotate the bottle is mounted between L1 and L5, in the central upper joint. An additional linear motor is mounted on L3 allowing adjusting the distance between the vacuum cup fixed rigidly to L3 and L3. This is represented by a line in the scheme in Fig. 2.a. The possibility to adjust the distance between L1 and the bottle is necessary to allow the positioning of bottles with different thicknesses in vertical upright position with their center of gravity aligned to the motor. This is necessary to be able to manipulate in the same way bottles with necks oriented oppositely, i.e. to place in the same position bottles that are rotated  $+90^\circ$  and  $-90^\circ$ . This linear motor, anyhow, is not necessary for the orientation movement; it's only used once, when the system is set to pick and rotate another

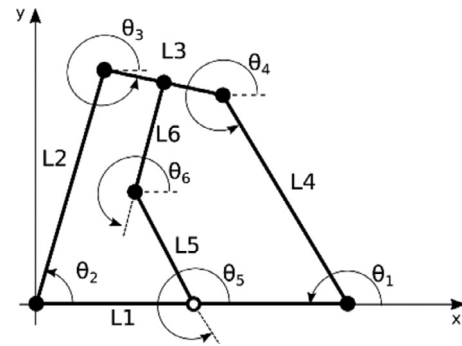


Fig. 3. Scheme of scalable six bar linkage mechanism used for writing the equations.

type of bottle. The unscrambling system, in fact, only processes a certain type of bottle at time.

The mechanism needs to be able to rotate bottles of  $90^\circ$  in both directions. Depending on where the bottle neck is, in fact, a rotation of  $\pm 90^\circ$  allows the bottles to be placed vertically in upright position with the neck on top.

## 3. Equations of the scalable mechanism

After having conceived the mechanism, its synthesis was generalized by finding a single equation that would allow the lengths of single links to be determined upon the definition of the trajectory and size of the object to be picked and rotated. Although the equation does not have analytical solution, it can be solved numerically, after having defined a desired trajectory through a set of values for the two independent variables, the input and output angles. The lengths of links obtained are then checked against geometrical constraints related to the bottle size and if necessary, re-calculated.

The scheme used to write the equation is shown in Fig. 3.

By considering the left side of the scheme, a first loop closure equation can be written, summing up the position vectors relative to half of L1, L2, half of L3, L6 and L5. The loop closure equation can be written in scalar form by considering its components in x and y axes (eq. (1) and eq. (2) respectively).

$$L_2 \cos \theta_2 + L_3/2 \cos \theta_3 + L_6 \cos \theta_6 + L_5 \cos \theta_5 + L_1/2 \cos \theta_1 = 0 \quad (1)$$

$$L_2 \sin \theta_2 + L_3/2 \sin \theta_3 + L_6 \sin \theta_6 + L_5 \sin \theta_5 + L_1/2 \sin \theta_1 = 0 \quad (2)$$

Since L1 lays on the frame, the angle  $\theta_1$  is constant and equal to  $\pi$ . By substituting it in the equations above and re-writing them isolating the terms containing the angle  $\theta_6$ , eq. (3) and eq. (4) are obtained from eq. (1) and 2 respectively:

$$-L_6 \cos \theta_6 = L_2 \cos \theta_2 + L_3/2 \cos \theta_3 + L_5 \cos \theta_5 - L_1/2 \quad (3)$$

$$-L_6 \sin \theta_6 = L_2 \sin \theta_2 + L_3/2 \sin \theta_3 + L_5 \sin \theta_5 \quad (4)$$

By squaring eq. (3) and eq. (4) and summing them up, a single equation (eq. (5)) can be obtained:

$$L_6^2 = L_2^2 + L_3^2/4 + L_5^2 + L_1^2/4 + L_2L_3(\cos \theta_2 \cos \theta_3 + \sin \theta_2 \sin \theta_3) + 2L_2L_5(\cos \theta_2 \cos \theta_5 + \sin \theta_2 \sin \theta_5) - L_1L_2 \cos \theta_2 + L_3L_5(\cos \theta_3 \cos \theta_5 + \sin \theta_3 \sin \theta_5) - L_1L_3/2 \cos \theta_3 - L_1L_5 \cos \theta_5 \quad (5)$$

Eq. (5) can be re-written in a short form considering special products of binomials (eq. (6)):

$$L_6^2 - L_2^2 - L_3^2/4 - L_5^2 - L_1^2/4 = L_2L_3 \cos(\theta_2 - \theta_3) + 2L_2L_5(\cos \theta_2 - \cos \theta_5) - L_1L_2 \cos \theta_2 + L_3L_5 \cos(\theta_3 - \theta_5) - L_1L_3/2 \cos \theta_3 - L_1L_5 \cos \theta_5 \quad (6)$$

Considering that the length of links 2 and 4 are the same, thus  $L_2=L_4$ , eq. (6) contains all variables of interest, all links lengths, and three angles:  $\theta_2$ ,  $\theta_3$  and  $\theta_5$ .

Out of the three angles included in the equation,  $\theta_2$  is not of interest, since, once the link lengths are established, its value depends on the other two angles:  $\theta_5$ , the angle defined by the motor, thus the input angle, and  $\theta_3$ , the angle defining the orientation of the bottle, thus the output angle.

To remove  $\theta_2$  from the equation, another loop closure equation was written, summing up the position vectors relative to the outer part of the whole scheme: L1, L2, L3 and L4. This loop closure equation can be written in scalar form by considering its components in x and y axes (eq. (7) and eq. (8) respectively).

$$L_2 \cos \theta_2 + L_3 \cos \theta_3 + L_4 \cos \theta_4 + L_1 \cos \theta_1 = 0 \quad (7)$$

$$L_2 \sin \theta_2 + L_3 \sin \theta_3 + L_4 \sin \theta_4 + L_1 \sin \theta_1 = 0 \quad (8)$$

As before, the angle  $\theta_1$  is constant and equal to  $\pi$ . By substituting it in the equations above and re-writing them isolating the terms containing the angle  $\theta_4$ , eq. (9) and eq. (10) are obtained from eq. (7) and 8 respectively:

$$-L_4 \cos \theta_4 = L_2 \cos \theta_2 + L_3 \cos \theta_3 - L_1 \quad (9)$$

$$-L_4 \sin \theta_4 = L_2 \sin \theta_2 + L_3 \sin \theta_3 \quad (10)$$

By squaring eq. (9) and eq. (10) and summing them up, a single equation (eq. (11)) can be obtained:

$$L_4^2 - L_2^2 - L_3^2 - L_1^2 = 2L_2L_3(\cos \theta_2 \cos \theta_3 + \sin \theta_2 \sin \theta_3) + 2L_1L_2 \cos \theta_2 - 2L_1L_3 \cos \theta_3 \quad (11)$$

Apart from links lengths, variables of interest, Eq. (11) contains two angles:  $\theta_2$  and  $\theta_3$ .

By solving Eq. (11) for  $\theta_2$  and by substituting the result in eq. (6), a single equation containing all interesting variables, all link lengths and only two angles, the input and output angles, can be obtained.

To solve Eq. (11) for  $\theta_2$  is necessary to introduce the tangent of  $\theta_2$  as variable. Given:

$$t = \tan \left( \frac{\theta_2}{2} \right) \quad (12)$$

$$\sin \theta_2 = \frac{2t}{1+t^2} \quad (13)$$

$$\cos \theta_2 = \frac{1-t^2}{1+t^2} \quad (14)$$

By substituting them in eq. (11), a second order equation in t is obtained (eq. (15)).

$$\begin{aligned} &(-L_1^2 - L_2^2 - L_3^2 + L_4^2 + 2L_3(L_1 + L_2)\cos\theta_3 - 2L_1L_2) \cdot t^2 + \\ &-4L_2L_3\sin\theta_3 \cdot t - L_1^2 - L_2^2 - L_3^2 + L_4^2 + 2L_3(-L_2 + L_1)\cos\theta_3 + \\ &+2L_1L_2 = 0 \end{aligned} \quad (15)$$

Eq. (15) can be rewritten grouping recurring terms in three additional variables, A, B and C. Given:

$$A = -L_1^2 - L_2^2 - L_3^2 + L_4^2 + 2L_1L_3 \cos \theta_3$$

$$B = 2L_2L_3 \cos \theta_3 - 2L_1L_2$$

Eq. (15) becomes eq. (16):

$$(A+B) \cdot t^2 - C \cdot t + (A-B) = 0 \quad (16)$$

**Table 1**

The four precision points used to generate five equations necessary to calculate all link lengths (values have been read from the actual CAD model).

	Output angle $\theta_3$ (rad)	Input angle $\theta_5$ (rad)
Position 1	5.7095	5.1487
Position 2	5.5944	5.2360
Position 3	5.4728	5.3233
Position 4	4.7070	5.7683

Eq. (16) has two solutions; by keeping the larger value for t, re-writing the equation in terms of the angle  $\theta_2$ , eq. (17) is obtained:

$$\theta_2 = 2 \arctan \left( \frac{C + \sqrt{-4A^2 + 4B^2 + C^2}}{2(A+B)} \right) \quad (17)$$

By substituting eq. (17) in eq. (6), eq. (18) is obtained. This is a single equation embedding only interesting variables: the six link lengths L1, L2, L3, L4, L5 and L6 and the two angles  $\theta_5$  and  $\theta_3$ , the input and output angles respectively.

$$\begin{aligned} &L_6^2 - L_2^2 - L_3^2/4 - L_5^2 - L_1^2/4 = \\ &L_2L_3 \cos \left( 2 \arctan \left( \frac{C + \sqrt{-4A^2 + 4B^2 + C^2}}{2A+2B} \right) - \theta_3 \right) + \\ &+ 2L_2L_5 \cos \left( 2 \arctan \left( \frac{C + \sqrt{-4A^2 + 4B^2 + C^2}}{2A+2B} \right) - \theta_5 \right) + \\ &- L_1L_2 \cos 2 \arctan \left( \frac{C + \sqrt{-4A^2 + 4B^2 + C^2}}{2A+2B} \right) + \\ &+ L_3L_5 \cos(\theta_3 - \theta_5) - L_1L_3/2 \cos \theta_3 - L_1L_5 \cos \theta_5 \end{aligned} \quad (18)$$

Next step consists of using Freudenstein's equations. Freudenstein's equations are a set of equation developed for the analysis and design of four-link mechanisms (F. Freudenstein, 1954; F. Freudenstein, 1954; Freudenstein, 1955). Initially, it consists of a single equation that becomes a system of multiple equations when pair of so-called input/output angles are given and substituted in such single equation.

Here, once the length of the frame link L1 has been defined, as with Freudenstein's equations, eq. (18) can be used to generate four different equations substituting the values for the input and output angles, corresponding to four precision points. Therefore, once the length of the frame link and the trajectory the object to re-orient have been defined, the latter by four points, i.e. four sets of two values for the input and output variables, by substituting these values in eq. (18), four equations are generated and can be solved numerically to calculate the lengths of the six links. Since two links have same length and one is set, the number of precision points is limited to four.

Analogously, by using the smaller value for t and re-writing eq. (15) in terms of  $\theta_2$ , another equation like eq. (18), could be written. This means that two sets of values for the lengths of the six links can be obtained. Actually, since the solution of the system of equations is not analytic and numerical solvers have to be used, the number of solutions is higher than two.

The solution should be chosen by checking the values of link lengths obtained against the requirements related to the size of the object to re-orient as explained in the next paragraph.

#### 4. Test and results

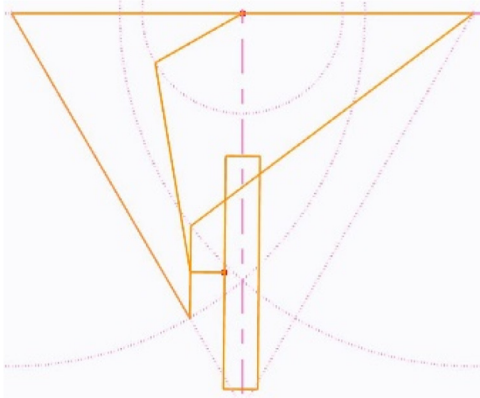
The system for synthesizing the six bar linkage mechanism was tested using a numerical algorithm based on Levenberg-Marquardt Newton's method available in Matlab software.

The input values for the trajectory, i.e. the four precision points used to generate four different equations, are summarized in Table 1.

The numerical solutions obtained during the tests, gave several meaningful sets of values for the six bar linkage link lengths, of

**Table 2**  
Six-bar linkage link lengths: actual CAD values and values generated numerically (L1 was set equal to 405 mm).

	CAD model length (mm)	Solution 1 generated length (mm)	Solution 2 generated length (mm)
L1	405	405	405
L2 = L4	300	219.41	330.98
L3	88	89.17	84.61
L5	80	86.61	71.40
L6	174	64.51	217.66



**Fig. 4.** Scheme of scalable six bar linkage mechanism with bottle for object size constraints.

which two sets are listed in Table 2 together with the original lengths of the CAD model.

The solution has to satisfy additional geometrical requirements related to the size of the object to be re-oriented.

In order to place the object in vertical upright position with its center of mass aligned with the motor, eq. (19), embedding object size  $a$ , has to be satisfied. In order not to have the object interfering with the motor, eq. (20), embedding object size  $c$ , has to be satisfied. And, finally, in order to be able to place the object on the outlet conveyor in a standing position, eq. (21), embedding object size  $c$ , has to be satisfied.

Considering a generic object of given dimensions  $a$ ,  $b$  and  $c$ , the geometrical constraints affecting the possibility to rotate it with the six bar linkage mechanism described in this paper are expressed in eq. (19), eq. (20) and eq. (21). Variables are clarified by Fig. 4. The value of angle  $\theta_2$  to consider is the one corresponding to the vertical position of the bottle, thus the vertical position of L3.

$$\frac{a}{2} + v \leq \left( \frac{L_1}{2} - L_2 \cos \theta_2 \right) \quad (19)$$

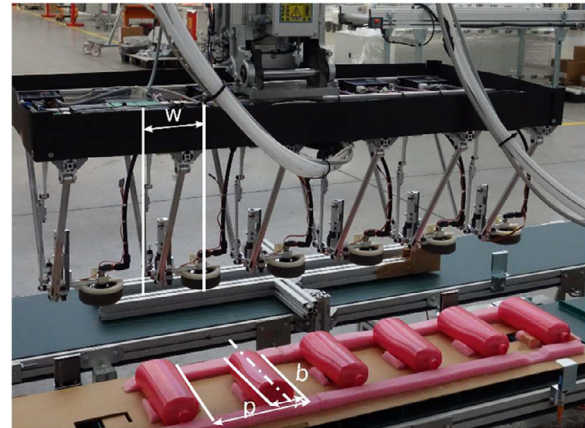
$$c \leq (L_2 \sin \theta_2 - L_3/2) \quad (20)$$

$$c \geq L_3 \quad (21)$$

While sizes  $a$  and  $c$  are directly connected to the links lengths, size  $b$  is affecting the possibility to manipulate different objects with several six bar linkage mechanisms mounted in parallel on the same gripper. If  $b$  is such that half of  $b$  plus the width of the six bar linkage,  $w$ , is smaller than the pitch between objects,  $p$ , (see Fig. 5) on the inlet conveyor, no interference between six bar linkages occurs. Otherwise, it is not possible to use more than one six bar linkage mechanisms in parallel.

In the case of the bottles used during the physical tests, with dimensions:

$$\begin{aligned} a &= 60 \text{ mm} \\ b &= 85 \text{ mm} \end{aligned}$$



**Fig. 5.** Gripper composed of 6 six bar linkage mechanisms used during the last tests and nomenclature relative to multiple objects manipulation.

$$c = 210 \text{ mm}$$

out of the two solutions listed in Table 2, only the second allows the correct manipulation of the object.

Since speed is a key factor in industry, it was deemed necessary to be able to scale the design also in terms of number of grippers to use together. Therefore, having as limit only the payload of the robotic arm supporting the gripper, in the physical tests, a gripper comprising six mechanisms has been successfully used. The orientation of bottles was determined before from the cameras installed into the unscrambling system and the direction of rotation of the motor, set by these input data.

## Conclusions

The paper presents a design methodology for synthesizing a six bar linkage mechanism using a single actuator to pick and rotate a generic object that can be inscribed in a cuboid of known dimensions  $a$ ,  $b$  and  $c$ . Such methodology consists of the definition of a set of equations (analogously to Freudenstein's equations for four-bar-linkages) that are, then, solved numerically. These solutions allow to, with extra geometrical constraints, define those values:  $a$ ,  $b$  and  $c$ . The mechanism has been originally designed for the bottle filling industry in the context of a joint research project between the company Fameccanica Data Spa and the research center IIT. The design is patent pending. As future work, a detailed kinematic and dynamic analysis of the new mechanism integrated in an industrial robotic system will be performed. In addition, the trajectories for the rotating process with this new system (six-bar-linkage together with an industrial robot) will be designed and optimized.

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