NON-LINEAR TRANSIENT HEAT CONDUCTION ANALYSIS OF INSULATION WALL OF TANK FOR TRANSPORTATION OF LIQUID ALUMINUM

by

Miroslav M. ŽIVKOVIĆ[*], Aleksandar V. NIKOLIĆ, Radovan B. SLAVKOVIĆ, and Fatima T. ŽIVIĆ

Faculty of Mechanical Engineering, University of Kragujevac, Kragujevac, Serbia

This paper deals with transient non-linear heat conduction through the insulation wall of the tank for transportation of liquid aluminum. Tanks designed for this purpose must satisfy certain requirements regarding temperature of loading and unloading, during transport. Basic theoretical equations are presented, which describe the problem of heat conduction finite element analysis, starting from the differential equation of energy balance, taking into account the initial and boundary conditions of the problem. General 3-D problem for heat conduction is considered, from which solutions for two- and one-dimensional heat conduction can be obtained, as special cases. Forming of the finite element matrices using Galerkin method is briefly described. The procedure for solving equations of energy balance is discussed, by methods of resolving iterative processes of non-linear transient heat conduction. Solution of this problem illustrates possibilities of PAK-T software package, such as materials properties, given as tabular data, or analytical functions. Software also offers the possibility to solve non-linear and transient problems with incremental methods. Obtained results for different thicknesses of the tank wall insulation materials enable its comparison in regards to given conditions.

Key words: *heat conduction, finite element analysis, wall insulation, Galerkin method, PAK-T, optimization*

Introduction

Liquid metal transportation represents a complex task, primarily due to high temperature of the metal, rapid cooling, changes of the metal structure during the transport, time interval in which the liquid metal needs to be transferred from the foundry to the rolling mill, *etc*. During the last years, many researchers have studied problems associated with optimization of the thickness and composition of insulation wall of tanks and other

---

[*] Corresponding author; e-mail: zile@kg.ac.rs

engineering constructions, by application of numerical methods [1 – 6]. Tanks for liquid aluminum transportation must have a wall made of special materials in order to prevent cooling of aluminum below the melting point and solidification point, during its transfer from the foundry to the rolling mill.

Wall of tank for liquid aluminum transportation has a multilayered structure, made of materials which have low coefficient of conduction. Part of insulation wall that is in direct contact with liquid aluminum is made of concrete. External layer of the insulation wall is made of S235JRG2 steel. High insulating, heat resistant materials are used in between these internal concrete layer and external steel layer. The first layer is made of Insulfrax High Temperature Insulation Paper [7], which is easily die cut and fabricated for a variety of thermal applications including gaskets, aerospace heat shields, mold liners, refractory back-up insulation. The next layer is made of VATRAMIL® GM 22 [8], which is high temperature resistant mass with excellent fire protection.

The objective of this paper is to optimize and obtain the best solution for thickness of concrete and VATRAMIL® GM 22 layers within the insulation wall of the tank, by application of PAK-T [9] software package, which is based on finite element (FE) method and heat conduction laws [10-13]. Requested condition is to prevent liquid aluminum to cool down below 750 $^{\circ}$C after 8 h, as well as to keep the temperature of the outer layer made of S235JRG2 steel below 100 $^{\circ}$C. Coefficients of conduction of insulation materials are changing with the temperature, what is taken into account.

Software package PAK-T [9] is developed based on the theory of the heat conduction and theoretical foundation is given at the beginning of the paper. Application of PAK-T in optimization of the tank wall thickness is presented further in this paper.

Basic eguation of heat conduction

Differential eguation of energy balance

Differential equation of energy balance equation is based on fundamental principle of energy conservation. Namely, change of material internal energy per time unit, within elementary volume dV, is equal to the quantity of heat energy accumulated in that same volume per time unit. Therefore, the following equation can be stated:

$$\frac{dQ}{dt} = \frac{dU}{dt} \tag{1}$$

where, dQ and dU are changes of heat and internal energy within volume dV in elementary time interval dt. Change of internal energy can be expressed in the following form:

$$\frac{dU}{dt} = \rho c \frac{dT}{dt} dV \tag{2}$$

where ρ is material density, c – specific heat, and T – temperature. By using figure 1, expression $\mathrm{d}Q/\mathrm{d}t$ can be formulated as:

$$-\frac{\mathrm{d}Q}{\mathrm{d}t} = \left(q_x + \frac{\partial q_x}{\partial x}\mathrm{d}x - q_x \right)\mathrm{d}y\mathrm{d}z + \left(q_y + \frac{\partial q_y}{\partial y}\mathrm{d}y - q_y \right)\mathrm{d}x\mathrm{d}z$$
$$+ \left(q_z + \frac{\partial q_z}{\partial z}\mathrm{d}z - q_z \right)\mathrm{d}x\mathrm{d}y - q\mathrm{d}V \tag{3}$$

where q_x, q_y and q_z are heat flux components, as variables that represent the quantity of heat flow through the surface unit per time unit; q is the heat source intensity (heat quantity per time unit and volume unit). Flux component signs have been taken into account in eguation (3): plus sign corresponds to the plus flux projection onto the external normal on the surface. Also, minus flux through the surface, corresponds to heat energy accumulation in volume $\mathrm{d}V$. It can be considered that $q > 0$ if the heat source is in volume $\mathrm{d}V$ (at the point of material) and $q < 0$ in case of heat sink.
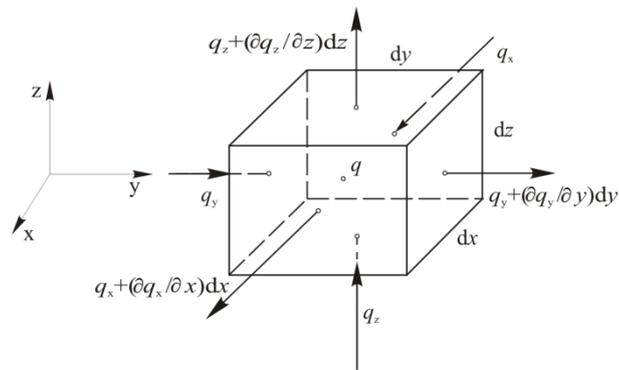

Figure 1. Elementary volume $\mathrm{d}V$ with heat flux components

Conduction of heat through solid bodies is defined by Fourier's law of heat:

$$q_i = -k_i \frac{\partial T}{\partial x_i} \qquad i = 1,2,3 \quad \text{no summation by i} \tag{4}$$

where k_i, namely k_x, k_y and k_z, are coefficients of heat conduction, in case of orthotropic material. In case of isotropic material, the following is valid:

$$k_x = k_y = k_z = k \tag{5}$$

By substituting (3) and (2) into energy balance equation (1) and using (4), differential equation is obtained in the following form:

$$-\rho c \frac{\mathrm{d}T}{\mathrm{d}t} + \frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right) + q = 0 \tag{6}$$

or by using index notation:

$$-\rho c \frac{\mathrm{d}T}{\mathrm{d}t} + \sum_{j=1}^{3} \frac{\partial}{\partial x_j}\left(k_j \frac{\partial T}{\partial x_j}\right) + q = 0 \tag{7}$$

Initial and boundary conditions

General solution of differential equation of heat conduction comprises indefinite functions and constants. Solution for temperature field $T(x, y, z, t)$ that satisfies given initial and boundary conditions is to be found, in scope of practical solving of the problem. There is a unique solution for given initial and boundary conditions. Initial conditions are given only for transient problems. They assume that temperature distribution is known at the initial moment $t = 0$, namely:

$$T(x, y, z, 0) = f_0(x, y, z) \tag{8}$$

where $f_0(x, y, z)$ is given function of material points coordinates.

In general, boundary conditions can be:

a) given temperature at S_1 part

$$T = T_s(x, y, z, t) \tag{9}$$

b) given flux at S_2 part

$$q_n = q_n(x, y, z, t) \tag{10}$$

c) given heat convection at S_3 part

$$q_h = h(T_0 - T_s) \tag{11}$$

d) given radiation at S_4 part

$$q_r = h_r(T_r - T_s) \tag{12}$$

where S_1, S_2, S_3 and S_4 are parts of the surface S, as symbolically represented in figure 2.
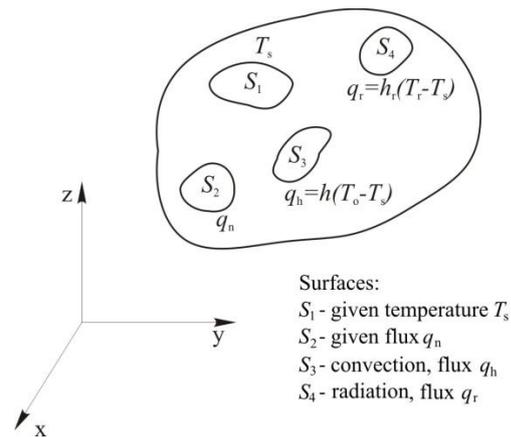
Temperature T_s is surface temperature, q_n, q_h and q_r are fluxes through the surface, T_0 is environment temperature, h is coefficient of convection, h_r is coefficient of radiation and T_r is the temperature of radiation source.

Equation that need to be satisfied for the boundary condition b) can be written as follows, based on Fourier's law of heat (4):

$$q_{\mathrm{n}} = k_{\mathrm{x}} \frac{\partial T}{\partial \mathrm{x}} n_{\mathrm{x}} + k_{\mathrm{y}} \frac{\partial T}{\partial \mathrm{y}} n_{\mathrm{y}} + k_{\mathrm{z}} \frac{\partial T}{\partial \mathrm{z}} n_{\mathrm{z}} = \sum_{i=1}^{3} k_{\mathrm{i}} \frac{\partial T}{\partial \mathrm{x}_{\mathrm{i}}} n_{\mathrm{i}} \qquad at \; S_2 \tag{13}$$

where $n_1 = n_{\mathrm{x}}, n_2 = n_{\mathrm{y}}, n_3 = n_{\mathrm{z}}$ are projections of unit vector of surface normal (S_2, S_3, and S_4). In case when material characteristics do not depend on temperature and radiation does not exist as a boundary condition, problem is linear, due to a fact that all differential equations that describe heat conduction law and boundary conditions, are linear dependent on temperature.

Figure 2. Boundary conditions for heat conduction through solid

Surfaces:
S_1 - given temperature T_{s}
S_2 - given flux q_{n}
S_3 - convection, flux q_{h}
S_4 - radiation, flux q_{r}

Incremental finite element equations and Galerkin method application

The derivation of the finite element balance equations is based on equations given in the previous text. Galerkin method is then applied, for derivation of FE equations. Based on differential equation of heat conduction (7), the following can be written:

$$-\int_{\mathrm{V}} \rho c h_{\mathrm{I}} \frac{\mathrm{d}T}{\mathrm{d}t} \mathrm{d}V + \int_{\mathrm{V}} \left[h_{\mathrm{I}} \sum_{j=1}^{3} \frac{\partial}{\partial \mathrm{x}_{\mathrm{j}}} \left(k_{\mathrm{j}} \frac{\partial T}{\partial x_{\mathrm{j}}} \right) \right] \mathrm{d}V + \int_{\mathrm{V}} h_{\mathrm{I}} q \mathrm{d}V = 0 \qquad \mathrm{I} = 1, 2, \ldots, \mathrm{N} \tag{14}$$

where h_{I} are interpolation functions and N is number of nodes per element, and V is finite element volume. Three-dimensional (*3-D*) isoparametric finite element is applied, as defined in [4]. Interpolation functions, geometry and number of nodes are also adopted. Temperature (T) in a point of element, defined in natural coordinates ξ, η, ζ is given as:

$$T\left(\xi, \eta, \zeta, t \right) = \sum_{\mathrm{I}=1}^{\mathrm{N}} h_{\mathrm{I}} T^{\mathrm{I}} \tag{15}$$

or in matrix form:

$$T = \mathrm{HT} \tag{16}$$

Where:

$$\mathrm{H} = \begin{bmatrix} h_1 & h_2 & \ldots & h_{\mathrm{N}} \end{bmatrix} \tag{17}$$

$$\mathrm{T^T} = \begin{bmatrix} T^1 & T^2 & \dots & T^N \end{bmatrix} \tag{18}$$

are row matrix of interpolation functions and column matrix of nodal temperatures, respectively. Eight-node 3-D finite element with temperature at the element point and nodal temperatures is shown in figure 3.
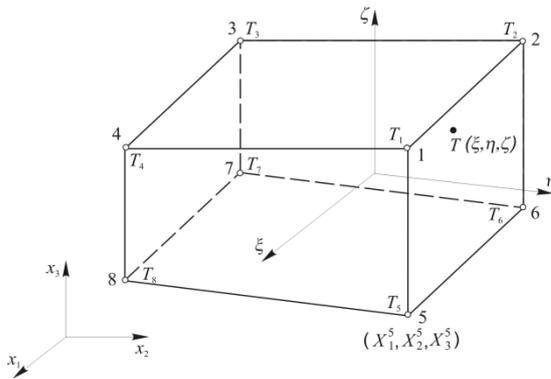

Figure 3. Three-dimensional (*3-D)* isoparametric finite element for interpolation of the temperature field

Applying the partial integration and Gauss theorem on second integral in (14), the following is obtained:

$$\int_V \left[h_I \sum_{j=1}^3 \frac{\partial}{\partial x_j} \left(k_j h_{J,j} \right) \right] dV T^J = -\int_V \left(\sum_{j=1}^3 k_j h_{I,j} h_{J,j} \right) dV T^J + \int_S \left[h_I^s \sum_{j=1}^3 k_j \frac{\partial T}{\partial x_j} n_j \right] dS \tag{19}$$

When element surface heat flux q_n is given and based on (13), the following can be written:

$$\int_S \left[h_I^s \sum_{j=1}^3 k_j \frac{\partial T}{\partial x_j} n_j \right] dS = \int_S h_I^s q_n dS = Q_I^{q_n} \tag{20}$$

where $Q_I^{q_n}$ are heat flux column matrix components and h_I^s are interpolation functions for nodes on surface S_2. If the heat convection is given for the surface S_3 expression is obtained as:

$$\int_S \left[h_I^s \sum_{j=1}^3 k_j \frac{\partial T}{\partial x_j} n_j \right] dS = \int_S h_I^s h \left(T_o - T_s \right) dS = -K_{IJ}^h T^J + Q_I^h \tag{21}$$

Summation over repeated index $J \left(J = 1, 2, \dots, N \right)$ is assumed in the previous equation. Convection matrix coefficients K_{IJ}^h and convection column matrix Q_I^h are expressed as follows:

$$K_{IJ}^h = \int_S h h_I^s h_J^s dS \tag{22}$$

$$Q_I^h = \int_S h h_I^s T_o \mathrm{d}S \tag{23}$$

For radiation boundary condition according to analogy with (21) – (23), the following expression can be written:

$$\int_S \left[h_I \sum_{j=1}^3 k_j \frac{\partial T}{\partial x_j} n_j \right] \mathrm{d}S = \int_S h_I^s h_r \left(T_r - T_s \right) \mathrm{d}S = -K_{IJ}^r T^J + Q_I^r \tag{24}$$

where the following relations are valid:

$$K_{IJ}^r = \int_S h_r h_I^s h_J^s \mathrm{d}S \tag{25}$$

$$Q_I^r = \int_S h_r h_I^s T_r \mathrm{d}S \tag{26}$$

Using (19) – (26), (15) and (14), the system of equations of the following form is obtained:

$$C\dot{T} + KT = Q \tag{27}$$

where matrix components for C and K and column matrix Q are determined as:

$$C_{IJ} = \int_V \rho c h_I h_J \mathrm{d}V \tag{28}$$

$$K_{IJ} = K_{IJ}^k + K_{IJ}^h + K_{IJ}^r \tag{29}$$

$$Q_I = Q_I^q + Q_I^{q_n} + Q_I^h + Q_I^r \tag{30}$$

Coefficients of heat conduction matrix K_{IJ}^k are given by:

$$K_{IJ}^k = \int_V \left(\sum_{j=1}^3 k_j h_{I,j} h_{J,j} \right) \mathrm{d}V = \int_V \left(k_x h_{I,x} h_{J,x} + k_y h_{I,y} h_{J,y} + k_z h_{I,z} h_{J,z} \right) \mathrm{d}V \tag{31}$$

where derivatives of interpolation functions are given by $h_{I,x} = \partial h_I / \partial x, \ldots, h_{J,z} = \partial h_J / \partial z$. In case of isotropic material, relations (5) are valid, and coefficients K_{IJ}^k can be expressed in following form:

$$K_{IJ}^k = \int_V k \left(h_{I,x} h_{J,x} + h_{I,y} h_{J,y} + h_{I,z} h_{J,z} \right) \mathrm{d}V \tag{32}$$

Convection matrix components K_{IJ}^h and radiation matrix components K_{IJ}^r are determined by (22) and (25). It should be noted that matrices C_{IJ} in (30) and K_{IJ} in (31) are symmetric. Column matrix Q_I^q in (32) can be written as:

$$Q_I^q = \int_V h_I q \mathrm{d}V \tag{33}$$

while $Q_I^{q_n}, Q_I^h$ and Q_I^r are determined by (20), (23) and (26).

By using interpolation matrix H, defined by row matrix according to (17), matrices in (27) can be written in a next form:

$$K^k = \int_V B^T k B dV \tag{34}$$

$$K^h = \int_S h H^{sT} H^s dS \tag{35}$$

$$K^r = \int_S h_r H^{sT} H^s dV \tag{36}$$

$$C = \int_V \rho c H^T H dV \tag{37}$$

$$Q^q = \int_V q H^T dV \tag{38}$$

$$Q^{q_n} = \int_S q_n H^{sT} dS \tag{39}$$

$$Q^h = \int_S h T_o H^{sT} dS \tag{40}$$

$$Q^r = \int_S h_r T_r H^{sT} dS \tag{41}$$

Row matrices H^s contain interpolation functions h_I^s for the surfaces. The matrix B in (34) has a next form like:

$$B = \begin{bmatrix} B^1 & B^2 & ... & B^N \end{bmatrix} \tag{42}$$

where, the submatrix for the node "I" consist of derivatives of interpolation functions with respect to coordinates x, y and z:

$$B^I = \begin{bmatrix} h_{I,x} \\ h_{I,y} \\ h_{I,z} \end{bmatrix} \tag{43}$$

Equation (27) represents energy balance equation for 3-D finite element transient heat conduction. Total number of equations is equal to number of nodes. One temperature value corresponds to each node. In case of 2-D, previous expressions remain unchanged, except that integrals over a volume V are practically reduced to integrals over surface S of the finite element, as shown in [10]. Interpolation functions h_I have appropriate forms for 2-D problem.

Solving of the balance equation of the structure

Matrix equation of the structure balance has the form (27) in case of transient problems.

We observe transient heat conduction in case when material constants c, k_j, h depends on the temperature or radiation boundary condition is used. The implicit iterative scheme for solution of system equations (29) is developed as follows. We set up a condition to satisfy (27) at the final step and consequently, this equation is written in the following form:

$$C^{(0) \ t+\Delta t}\dot{T}+K^{(0) \ t+\Delta t}T = \ ^{t+\Delta t}Q^{(0)} \tag{44}$$

where $C^{(0)}$ and $K^{(0)}$ are matrices that correspond to the known temperatures ^tT, at the beginning of the procedure, and $^{t+\Delta t}Q^{(0)}$ is the column matrix that corresponds to time "$t+\Delta t$" (at time functions) and known temperatures ^tT. Then the following approximation for rate of temperature change can be written:

$$^{t+\Delta t}\dot{T} = \frac{\Delta T^{(1)}}{\Delta t} \tag{45}$$

$$^{t+\Delta t}T = \ ^tT+\Delta T^{(1)} \tag{46}$$

where $\Delta T^{(1)}$ are increments of the temperature for the first iteration. Therefore, solution is obtained from (44) as:

$$\Delta T^{(1)}=\left(\hat{K}^{(0)}\right)^{-1 \ t+\Delta t}\hat{Q}^{(0)} \tag{47}$$

where:

$$\hat{K}^{(0)}=K^{(0)}+\frac{1}{\Delta t}C^{(0)} \tag{48}$$

and:

$$^{t+\Delta t}\hat{Q}^{(0)} = \ ^{t+\Delta t}Q^{(0)}-K^{(0) \ t}T \tag{49}$$

By substitution of temperatures:

$$^{t+\Delta t}T^{(1)} = \ ^tT+\Delta T^{(1)} \tag{50}$$

into (44), where instead of $C^{(0)}$, $K^{(0)}$ and $^{t+\Delta t}Q^{(0)}$ we put $C^{(1)}$, $K^{(1)}$ and $^{t+\Delta t}Q^{(1)}$, it may be, therefore, concluded that the left hand side is not equal to the right hand side. Equation (44) is then transformed into following form:

$$\frac{1}{\Delta t}C^{(1)}\left(^{t+\Delta t}T^{(1)}-\ ^tT+\Delta T^{(2)}\right)+K^{(1)}\left(^{t+\Delta t}T^{(1)}+\Delta T^{(2)}\right) = \ ^{t+\Delta t}Q^{(1)} \tag{51}$$

namely, for the i-th iteration:

$$\frac{1}{\Delta t}C^{(i-1)}\left(^{t+\Delta t}T^{(i-1)}-\ ^tT+\Delta T^{(i)}\right)+K^{(i-1)}\left(^{t+\Delta t}T^{(i-1)}+\Delta T^{(i)}\right) = \ ^{t+\Delta t}Q^{(i-1)} \tag{52}$$

where the following expression for temperature is used:

$$^{t+\Delta t}T^{(i-1)} = \ ^tT+\Delta T^{(1)}+\Delta T^{(2)}+...+\Delta T^{(i-1)} \tag{53}$$

which represents the known temperature from the last iteration. Obviously, the solution of (52) for temperature increment $\Delta T^{(i)}$ is:

Živković, M. M., *et.al.*: Non-linear Transient Heat Conduction Analysis of Insulation …

S308   THERMAL SCIENCE: Year 2010, Vol. 14, Suppl., pp. S299-S312

$$\Delta T^{(i)} = \left(\hat{K}^{(i-1)} \right)^{-1} {}^{t+\Delta t} \hat{Q}^{(i-1)} \tag{54}$$

where:

$$\hat{K}^{(i-1)} = K^{(i-1)} + \frac{1}{\Delta t} C^{(i-1)} \tag{55}$$

and:

$${}^{t+\Delta t} \hat{Q}^{(i-1)} = {}^{t+\Delta t} Q^{(i-1)} - K^{(i-1)} {}^{t+\Delta t} T^{(i-1)} - \frac{1}{\Delta t} C^{(i-1)} \left({}^{t+\Delta t} T^{(i-1)} - {}^{t} T \right) \tag{56}$$

Iterative procedure is continued until temperature increment at nodes is not sufficiently low, what can be expressed in the following form:

$$\left\| \Delta T^{(i)} \right\| \le \varepsilon_a \tag{57}$$

or

$$\frac{\left\| \Delta T^{(i)} \right\|}{\left\| \Delta T^{(1)} \right\|} \le \varepsilon_r \tag{58}$$

where ε_a and ε_r are selected absolute and relative tolerances and $\left\| \Delta T \right\|$ is temperature increment norm.

Analysis of tank for transportation of liquid aluminum

Tanks designed for liquid aluminum transportation must satisfy certain requirements regarding temperature of loading and unloading during transport. Liquid aluminum temperature during loading is 900 $^{\circ}$C. Requested condition is to prevent liquid aluminum to cool down below 750 $^{\circ}$C after 8 h, as well as to keep the temperature of the outer layer made of S235JRG2 steel below 100 $^{\circ}$C. Observed tank has multilayered insulation wall (figure 4.).
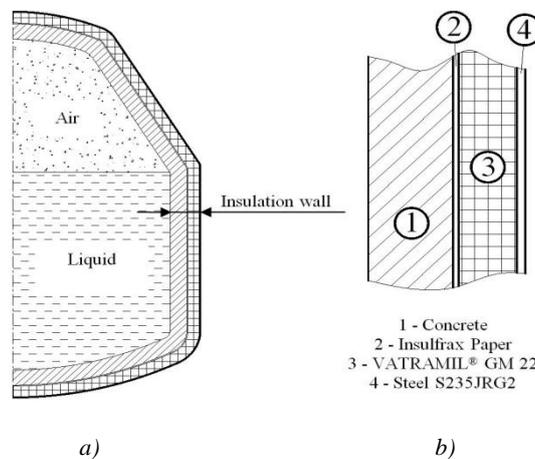

a)   *b)*

Figure 4. a) Schematic representation of the tank model, b) Multilayered insulation wall

Živković, M. M., *et.al.*: Non-linear Transient Heat Conduction Analysis of Insulation …
THERMAL SCIENCE: Year 2010, Vol. 14, Suppl., pp. S299-S312

S309

Problem is modeled with 2-D axisymmetrical four-node finite element, since the tank is axisymmetrical structure. The tank is not filled with aluminum up to the top.

Coefficients of conduction for used materials are temperature dependent, and they are presented in figure 5. Materials are separately given in two diagrams since VATRAMIL® GM 22 and Insulfrax Paper have very low conduction coefficients.
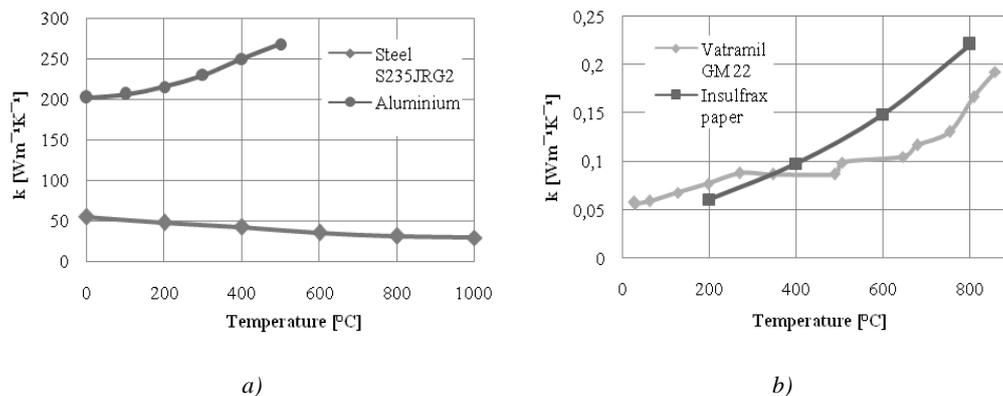

a)   *b)*

Figure 5. a) Coefficients of conduction of Steel S235JRG2 and Aluminum versus temperature, b) Coefficients of conduction of VATRAMIL® GM 22 and Insulfrax Paper versus temperature

Initial conditions for problem modeling are set up as follows: aluminum temperature at finite elements nodes 900 $^{\circ}$C and initial temperature of the insulation wall 20 $^{\circ}$C. Convection between the outer layer of the insulation wall and the environment is taken in account, with adopted convection coefficient 10 W/m²K and environment temperature 20 $^{\circ}$C.

Results and discussion

Solved problem is non-linear since material characteristics are temperature dependent. The solution is obtained in 48 time steps with time increment of 600 s. The problem of non-linear transient heat conduction is solved for three cases according to different layer thicknesses of VATRAMIL® GM 22 and Concrete as shown in Table 1.

Table 1. Thickness of the materials used for insulation wall

Material	Material thickness [mm]		
	Case 1	Case 2	Case 3
Concrete	80	75	70
Insulfrax paper	5	5	5
VATRAMIL® GM 22	55	60	65
Steel S235JRG2	8	8	8

Calculation in software PAK-T [9] was performed to obtain the optimized solution for wall thickness in order to satisfy prescribed conditions. Results obtained in Case 1 and Case 2 did not completely satisfied prescribed conditions, since the liquid aluminum temperature is below 750 ^{o}C after 8 hours, what can be clearly seen in figure 6.a. Prescribed condition for temperature on the outer layer of the insulation wall (below 100 ^{o}C), was satisfied in all three analyzed cases, what can be seen in figure 6.b. Temperature field for all three analyzed cases of the material layer thickness, after finished calculations, is presented in figure 7.
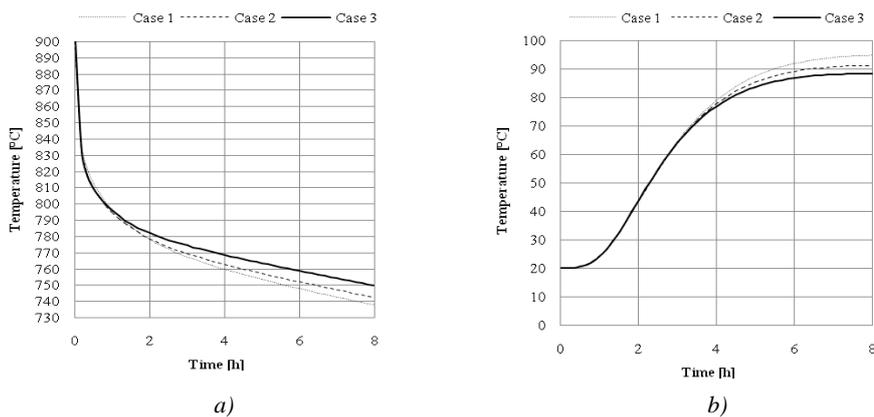

Figure 6. a) Change in temperature over time, at the inner part of the insulation wall, b) Change in temperature over time, at the outer part of the insulation wall
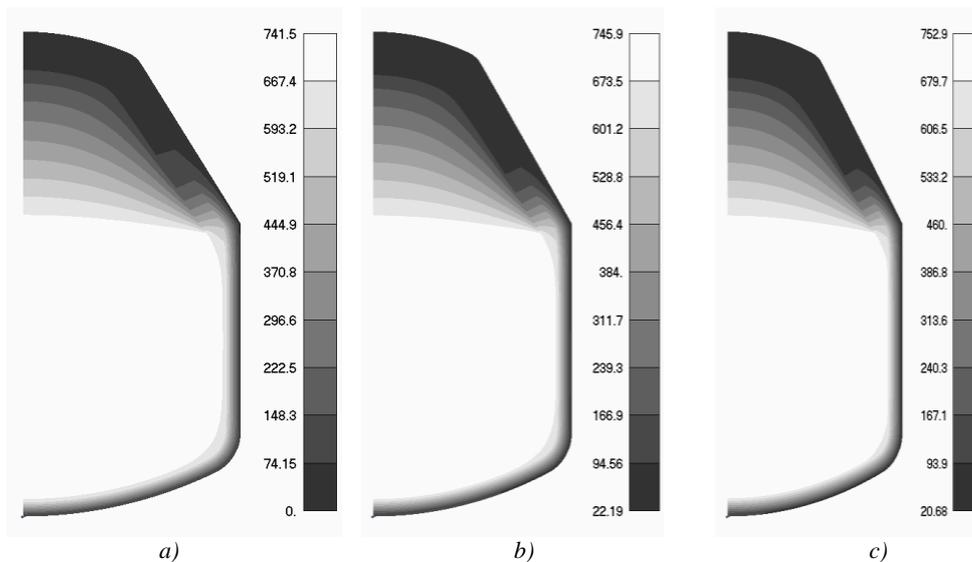

Figure 7. Temperature field of the tank model, a) Case 1, b) Case 2, c) Case 3

Temperature of the liquid aluminum in Case 3, is above 750 $^{\circ}$C after 8 hours, and the temperature of the outer part of the insulation wall is below 100 $^{\circ}$C.

Conclusions

This paper deals with possible optimization of real construction design of tank for liquid aluminum transport, by application of software package PAK-T. Complete software package is based on heat conduction theory, starting with balance equation and considering initial and boundary conditions. Forming of FE matrices and procedure for solving energy balance equations by implicit iterative methods were presented. The main objective was to found optimal solution for material layer thickness at the tank insulation wall. Obtained FE results shows that only case of analyzed material layer thicknesses used for insulation wall (70 mm Concrete, 5 mm Insulfrax paper, 65 mm VATRAMIL$^{®}$ GM 22 and 8 mm S235JRG2 steel) can be used for liquid aluminum transport.

Nomenclature

c – specific heat, [Jkg^{-1}K^{-1}]
h – coefficient of convection, [Wm^{-2}K^{-1}]
h_r – coefficient of radiation, [Wm^{-2}K^{-1}]
h_I – interpolation function, [–]
k – coefficient of heat conduction, [Wm^{-1}K^{-1}]
n – surface normal, [–]
Q – heat source intensity, [Wm^{-3}]
Q – heat energy, [J]
Q_h – convection flux, [Wm^{-2}]
Q_n – given flux, [Wm^{-2}]
Q_r – radiation flux, [Wm^{-2}]
Q_x – heat flux component, [Wm^{-2}]
S – surface, [m^2]
t – time, [s]
T – temperature, [K]
T_s – surface temperature, [K]
T_0 – environmental temperature, [K]
T_r – temperature of radiation source, [K]
ΔT – temperature increment, [K]
U – internal energy, [J]

V – volume, [m^3]

Greek letters

ε_a – absolute tolerance, [–]
ε_r – relative tolerance, [–]
ρ – material density, [kgm^{-3}]

Subscripts

x, y, z   – Cartesian co-ordinates
ξ, η, ζ – natural co-ordinates
N   – number of nodes per element

Superscripts

h   – convection
k   – conduction
r   – radiation
s   – surface
t   – time
Δt – time increment

Acknowledgment

S312

Živković, M. M., et.al.: Non-linear Transient Heat Conduction Analysis of Insulation …
THERMAL SCIENCE: Year 2010, Vol. 14, Suppl., pp. S299-S312

References

[1] Dombayci A., Golcu, M., Pancar, Y., Optimization of Insulation Thickness for External Walls Using Different Energy-Sources, *Applied Energy*, *83* (2006), 9, pp. 921-928

[2] Khemis, O., *et al*., Experimental Analysis of Heat Transfers in a Cryogenic Tank without Lateral Insulation, *Applied Thermal Engineering*, *23* (2003), 16, pp. 2107-2117

[3] Comakli, K., Yuksel, B., Optimum Insulation Thickness of External Walls for Energy Saving, *Applied Thermal Engineering*, 23 (2003), 4, pp. 473–479

[4] Arulanantham, M., Kaushika, N. D., Coupled Radiative and Conductive Thermal Transfers Across Transparent Honeycomb Insulation Materials, *Applied Thermal Engineering*, *16* (1996), 3, pp. 209-217

[5] Chen, Y. G., Parametric Evaluation of Refrigerated Air Curtains for Thermal Insulation, *International Journal of Thermal Sciences*, *48* (2009), 10, pp. 1988-1996

[6] Babac, G., Sisman, A., Cimen, T., Two-Dimensional Thermal Analysis of Liquid Hydrogen Tank Insulation, *International Journal of Hydrogen Energy*, *34* (2009), 15, pp. 6357-6363

[7] ***, Insulfrax High Temperature Insulation Paper, http://www.insulfrax.com/

[8] ***, VATRAMIL$^®$ GM 22 High Temperature Resistant Mass, http://www.vatramil.de/

[9] Kojić, M., *et al.*, PAK-T – Program for FE Heat Transfer Analysis, Faculty of Mechanical Engineering, University of Kragujevac, Kragujevac, Serbia, 2003

[10] Kojić, M., *et al.*, Finite Element Method I – Linear Analysis (in Serbian), Faculty of Mechanical Engineering, University of Kragujevac, Kragujevac, Serbia, 1998

[11] Bathe, K. J., Finite Element Procedures in Engineering Analysis, Prentice-Hall, Englewood Cliffs, N. J., USA, 1996

[12] Lewis, R. W. *et al.*, The Finite Element Method in Heat Transfer Analysis, John Wiley & Sons Ltd, Chichester, West Sussex, England, 1996

[13] Lazić, V., *et al.*, Theoretical-Experimental Determining of Cooling Time ($t_{8/5}$) in Hard Facing of Steels for Forging Dies, *Thermal Science, 14* (2010), 1, pp. 235-246