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Jedan postupak za izbor bočnih karakteristika pneumatika sa aspekta minimalnog vijuganja prikolice putničkog motornog vozila

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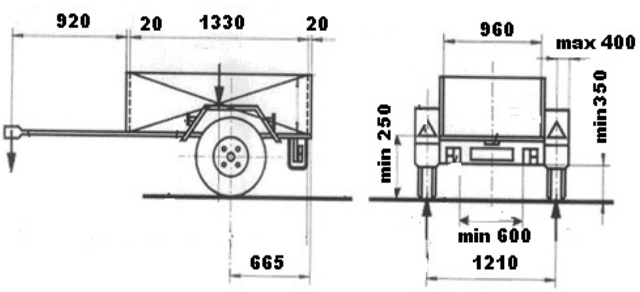
U početnoj fazi projektovanja motornih vozila, a time i njihovih prikolica, dimenzije i vrsta pneumatika se usvajaju prema zahtevanoj nosivosti, iz kataloga proizvođača. Fino doterivanje karakteristika, posebno bočnih karakteristika pneumatika se vrši u kasnijim fazama projektovanja, primenom modela ili eksperimentalnih metoda. U ovom radu, vijuganje prikolice, kao dela sistema putničko vozilo-prikolica, analizirana je uz korišćenje njenog jednostavnog dinamičkog modela. Jednačine kretanja su izvedene uz korišćenje Njutnovih zakona za konstantnu uzdužnu brzinu vozila. Pneumatici su opisani modelima zasnovanim na aproksimaciji polinomima višeg stepena. Optimalan izbor odgovarajućeg parametra bočnih karakteristika pneumatika je izvršen primenom Hooke-Jeevees-ove metode. Konstruktivna ograničenja parametara su uvedena primenom spoljašnjih kaznenih funkcija. Optimizacioni proces minimiziranja ugla vijuganja prikolice započet je sa tri grupe početnih vrednosti optimirućih parametara. Analize su pokazale da je dovoljno isti započeti sa sredinom intervala definisanosti parametara, a dobijeni podaci omogućavaju brz izbor bočnih karakteristika pneumatika prikolice u fazi njenog idejnog projektovanja.

Ključne reči: prikolica, vijuganje, optimizacija, pneumatici, bočne karakteristike

1. UVOD

U početnoj fazi projektovanja motornih vozila, a time i njihovih prikolica, dimenzije i vrsta pneumatika se usvajaju prema zahtevanoj nosivosti, iz kataloga proizvođača. Fino doterivanje karakteristika, posebno bočnih karakteristika pneumatika se vrši u kasnijim fazama projektovanja, primenom modela ili eksperimentalnih metoda.

Kontakt pneumatika i tla ima veliki značaj za ponašanje vozila na putu, uopšte, a time i prikolice kao dela vučnog voza. U literaturi postoji više modela pneumatika koji se koriste u analitičkim i/ili numeričkim istraživanjima sistema vozila [1-3].



Slika 1: Prikolica putničkog motornog vozila

Većina modela podrazumeva poznavanje deformacije pneumatika kako unutar tako i izvan kontaktne površine [2]. U ovom radu su korišćeni modeli pneumatika zasnovani na eksperimentalnim istraživanjima, aproksimirani polinomima višeg stepena [4], pri čemu je učinjen pokušaj da se na osnovu jednostavnog modela prikolice i minimiziranja njenog vijuganja definišu parametri koji će omogućiti lakši izbor bočnih karakteristika pneumatika u toku projektovanja prikolice.

U literaturi [5-8], postoje modeli prikolice različite strukture i složenosti, o čemu će kasnije biti više reči. Posmatrana prikolica [9], slika 1, čiji su geometrijski i inercijalni parametri dati u Tabeli 1.

Tabela 1: Parametri prikolice

Parametar	Opterećena	Neopterećena
Ukupna masa, m, kg	530	200
Rastojanje od uređaja za spajanje do osovine, L, m	1,625	1,625
Položaj težišta od uređaja za spajanje, a, m	1,395	1,015
Moment inercije za glavnu vertikalnu težišnu osu, I_z, kgm^2	194	73
Površina čeone projekcije prikolice, m^2 /Koeficijent otpora vazduha, Ns^2/m^4	1,2/0,4	1,2/0,4
Površina bočne projekcije prikolice, m^2/ Koeficijent otpora vazduha, Ns^2/m^4	0,63/0,4	0,63/0,4

U narednom tekstu će biti reči o modeliranju vijuganja prikolice.

2. MODEL PRIKOLICE

Analize su pokazale [6] da pri brzinama većim od 35 - 45 km/h, dolazi do pojave bočnih vibracija, usled vijuganja, što ima nezgodne posledice po kretanje vučnog vozila, jer se pogoršava upravljanje, povećava se potrošnja goriva i habanje pneumatika i zglobnih veza.

Pri modeliranju vozila, a samim tim i prikolice, mogu se koristiti modeli različitih struktura [10-19], ali se u praksi veoma često koristi princip da njihova složenost

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bude tolika da obuhvati samo one veličine koje se analiziraju [12].

Ovo se pravda činjenicom da složeniji modeli zahtevaju korišćenje većeg broja geometrijskih i inercijalnih parametara, koji, često, nisu poznati. Njihov orijentacioni izbor dovodi do većih grešaka u procesu dinamičke simulacije, pa je opravdano koristiti što jednostavnije modele.

Imajući to u vidu, u ovom radu je posmatrana jednoosovinska prikolica za putnička vozila uz sledeća ograničenja [5,6] :

- veza prikolice i vučnog vozila (uređaj za spajanje) je zglobna,
- vertikalne vibracije prikolice su zanemarene i
- kretanje zgloba je pravolinijsko.

Iz mehanike je poznato da tela koja vrše kretanje u ravni imaju tri stepena slobode, tj. za opisivanje njihovog kretanja je potrebno napisati tri diferencijalne jednačine. Međutim, kako mi u ovom slučaju posmatramo vučno vozilo koje se kreće pravolinijski, konstantnom brzinom, opravdano je da se posmatra samo vijuganje prikolice.

Na slici 2. prikazano je kretanje prikolice, pri čemu Ψ predstavlja njeno vijuganje u odnosu na pravolinijsko kretanje zgloba C (uređaja za spajanje).

Na slici 2. korišćene su sledeće oznake:

- F_{cx} i F_{cy} - komponente sile na zglobu,
- R_{vx},R_{vy} – otpor vazduha prikolice u pravcu kretanja i bočnom pravcu vučnog vozila respektivno.

U ovom radu su korišćene vrednosti posmatranih komponenata otpora vazduha:

$$R_{vx} = K_{vx} A_x v^2 \sin\psi \\ R_{vy} = K_{vy} A_y v^2 \cos\psi \qquad (1)$$

gde su:

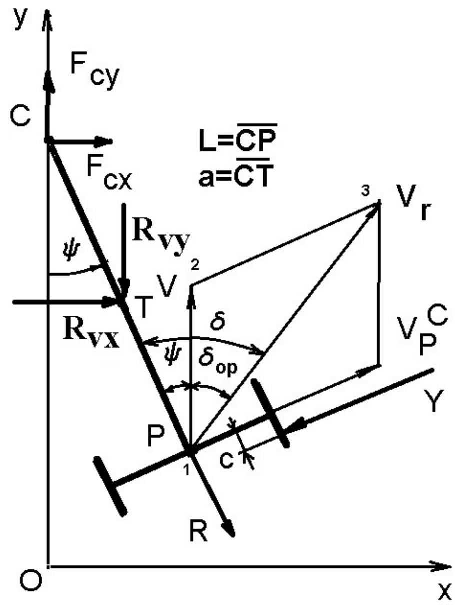
- K_{vx},K_{vy} – koeficijenti otpora vazduha prikolice u pravcu njenog kretanja i normalno na njega, respektivno, a
- v brzina kretanja vučnog vozila.
- R - rezultujuća sila otpora kretanja prikolice. U našem slučaju je korišćena relacija:

$$R = G\sin\alpha + Gf\cos\alpha + R_{vx}\cos\psi + R_{vy}\sin\psi + R_j \qquad (2)$$

- G - težina prikolice,
- α -ugao podužnog nagiba puta,
- R_j-otpor inercijalnih sila (za slučaj ravnomernog kretanja jednak nuli).
- Y - bočna reakcija tla (deluje za veličinu „c“ iza centra točka [5,6,10-19]),
- P - tačka na podu prikolice iznad osovine,
- $\dot{\psi}$ - ugaona brzina i
- v_p^C - relativna brzina tačke P u odnosu na tačku C.

Brzina v_P^C se može izraziti kao:

$$v_P^{\,C} = L\dot{\psi} \qquad (3)$$



Slika 2: Šema dejstva sila na prikolicu

Apsolutna brzina tačke P je data izrazom:

$$\overrightarrow{v_P} = \vec{v} + \overrightarrow{v_p^{\,C}} \qquad (4)$$

Ugao koji ova brzina gradi sa osom osovine predstavlja stvarni ugao povođenja pneumatika, tj.:

$$\delta = \psi + \delta_{op} \qquad (5)$$

Da bi smo izračunali veličinu ugla δ, posmatraćemo trougao sa temenima 123, i odgovarajućim uglovima $\delta_{op}, \gamma, \varepsilon$. Sa slike je očigledna relacija:

$$\psi + \delta_{op} + \varepsilon = 90^o \qquad (6)$$

Primenom sinusne teoreme na posmatrani trougao, imamo:

$$\frac{v}{\sin\varepsilon} = \frac{v_P^{\,C}}{sin\delta_{op}} \qquad (7)$$

Imajući u vidu izraz (7), imamo:

$$\frac{v}{\cos(\psi + \delta_{op})} = \frac{L\dot{\psi}}{\sin\delta_{op}} \qquad (8)$$

Primenom adicione teoreme, imamo:

$$\cos(\psi + \delta_{op}) = \cos\psi\cos\delta_{op} - \sin\psi\sin\delta_{op} \qquad (9)$$

Za male uglove se može napisati:

$$\sin\psi\sin\delta_{op} \approx 0$$

$$\cos\psi\cos\delta_{op} \approx 1$$

Imajući u vidu da je δ_{op} mala veličina, može se pretpostaviti da je ugao približno jednak sinusu ugla, pa se konačno može napisati izraz za povođenje pneumatika:

$$\delta = \psi + \frac{L\dot{\psi}}{v} \qquad (10)$$

Kako je već napomenuto, prikolica, kao kruto telo koja se kreće u ravni, ima tri stepena slobode kretanja. U ovom radu je analizirano samo vijuganje, pa diferencijalna jednačina koja opisuje to kretanje glasi:

$$\left(J_z + m_p a^2\right)\ddot{\psi} = -Y(L+c) + R_{vx} a\cos\psi - R_{vy} a\sin\psi \qquad (11)$$

gde zbir u zagradi predstavlja moment inercije poluprikolice u odnosu na tačku *C*, a

- M_s moment stabilizacije pneumatika [5,6,10-19].

Za male uglove, diferencijalna jednačina (11) se može napisati u obliku:

$$\left(J_z + m_p a^2\right)\ddot{\psi} = -YL - M_s + R_{vx}a - R_{vy}a\psi \tag{12}$$

U literaturi postoji više modela pneumatika [1-4]. Ocenjeno je celishodnim da se u ovom radu koriste modeli bočnih karakteristika pneumatika iz [4]:

$$\begin{aligned} Y &= (x_1 + x_2\delta + x_3\delta^2 + x_4\delta^3)(x_5 + x_6 Z) \\ M_s &= (x_7 + x_8\delta + x_9\delta^2 + x_{10}\delta^3)(x_{11} + x_{12} Z) \end{aligned} \tag{13}$$

gde su:

- x_i i=1-12 parametri modela pneumatika,
- δ - izračunati ugao povođenja pneumatika (izraz 8), a
- *Z*-dinamička reakcija tla (u konkretnom slučaju približno jednaka statičkoj reakciji tla).

Pri tome je uvedeno ograničenje u vidu granične bočne sile, koja je definisana silom prianjanja [5,6,10-19]:

$$Y_{\max} = \varphi Z \tag{14}$$

gde su:

- *Z* –normalna dinamička reakcija tla, a
- φ -koeficijent prianjanja.

Imajući u vidu karakter bočne sile i momenta stabilizacije (izraz 13), diferencijalna jednačina (12) je nelinearna, sa konstantnim parametrima, pa ju je moguće rešiti samo numerički.

Radi verifikacije (kalibrisanja) modela, izvršena je dinamička simulacija vijuganja prikolice sa pneumaticima 145SR13, čije su bočne karakteristike poznate [4], a njihovi parametri modela prikazanih izrazom (13) su dati u Tabeli 2.

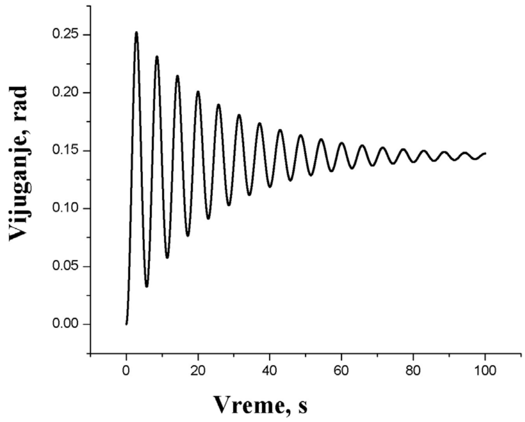
Tabela 2: Parametri test pneumatika 145SR13

x_1, -	-0,699
x_2, rad^{-1}	17,80
x_3, rad^{-2}	-1,17
x_4, rad^{-3}	202
x_5, N	23,00
x_6, -	0,0045
x_7, -	-0,23
x_8, rad^{-1}	1,40
x_9, rad^{-2}	-0,23
x_{10}, rad^{-3}	0,0101
x_{11}, Nm	-20
x_{12}, m	0,0141

Integracija diferencijalne jednačine (12) – koja je prethodno prevedena u sistem od dve diferencijalne jednačine prvog reda, je izvršena Metodom Runge Kuta, sa korakom 0,01 s, u 10000 tačaka, što obezbeđuje pouzdanost rezultata u oblasti 0,01 do 50 Hz, a što je prihvatljivo sa aspekta oscilatorne udobnosti i ponašanja vozila na putu [12,20].

Radi ilustracije, na slici 3. prikazan je vremenski zapis vijuganja opterećene prikolice pri brzini od 20 m/s. Analizom podataka sa slike 3, može se uočiti da parametri test pneumatika daju prihvatljive rezultate modela prikolice. Treba napomenuti, da su dinamičke pobude prikolice u bočnom pravcu, tokom simulacije, poticale od podužne i bočne komponente otpora vazduha. Testiranje je vršeno sa opterećenom i neopterećenom prikolicom, a ocenjeno je celishodnim da se izračunaju i efektivne vrednosti vijuganja koje su date u Tabeli 3.

Analizom podataka iz Tabele 3 se može utvrditi da na efektivnu vrednost ugla vijuganja prikolice utiču njeno opterećenje i brzina kretanja. Pri tome su vijuganja veća kod opterećenog stanja i veću brzinu kretanja, što ukazuje na potrebu da se za te eksploatacione uslove izvrši i izbor parametara bočnih karakteristika pneumatika, jer su za tu prikolicu, na osnovu nosivosti, predviđeni pneumatici 175/70R13 [9] čije bočne karakteristike nisu poznate.



Slika 3: Vijuganje opterećene prikolice sa pneumaticima 145 SR13, pri brzini 20 m/s

Tabela 3: Zavisnost vijuganja prikolice od opterećenja i brzine kretanja

Stanje opterećenja	Brzina kretanja, m/s	Efektivna vrednost ugla vijuganja, rad
Neopterećeno	10	0,06572
Neopterećeno	20	0,12957
Opterećeno	10	0,07544
Opterećeno	20	0,14679

3. METOD

U cilju optimalnog izbora parametara bočnih karakteristika pneumatika, korišćena je optimizaciona metoda Hooke-Jevesa, koja je detaljno objašnjena u [21]. Radi ilustracije, blok dijagram metode optimizacije je prikazan na slici 4. Kako je isti detaljno opisan u [12], ovde o njemu neće biti više reči. Granične vrednosti intervala prihvatljivih parametara pneumatika su definisane uvođenjem spoljašnjih kaznenih funkcija, definisanjem vrednosti funkcije cilja, izvan oblasti definisanosti parametara, 10^{20}. Program je realizovan u Paskalu, na personalnom računaru.

U ovom slučaju, funkcija cilja je definisana kao efektivna vrednost ugla vijuganja prikolice, tj.:

$$z = \frac{1}{T}\sqrt{\int_0^T \psi^2 dt} \tag{15}$$

gde su:

- *z* – funkcija cilja,
- ψ - ugao vijuganja prikolice,
- *t* – vreme, a
- *T*-period posmatranja vijuganja.

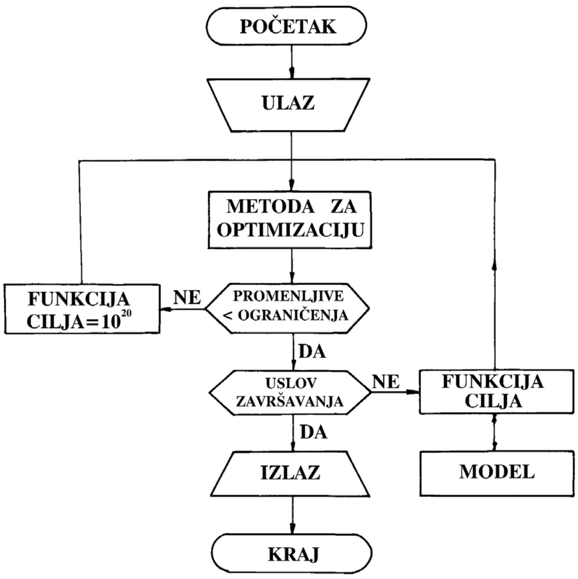
Program je realizovan sa uvedenim graničnim vrednostima parametara pneumatika:

$$-10 < x_i < 20,\ i = 1 \div 12$$

Kako sve metode nelinearnog programiranja konvergiraju ka najbližem minimumu funkcije cilja od polazne tačke, problem nalaženja globalnog minimuma je veoma težak [21]. Zbog toga je u ovom radu ocenjeno celishodnim da se optimizacija vrši sa tri grupe početnih vrednosti optimirućih parametara:

- sredina intervala oblasti definisanosti parametara,
- 0,8 od gornjih graničnih vrednosti i
- 1,3 od donjih graničnih vrednosti.

Pretraživanje funkcije cilja je vršeno sa korakom od 0,1; a iterativni proces je završavan kada je razlika dve susedne vrednosti funkcije cilja bila manja od 10^{-20}. Izračunavanje optimalnih vrednosti parametara je vršeno za opterećeno i neopterećeno stanje prikolice, a rezultati prikazani u Tabelama 4 i 5.



Slika 4: Blok dijagram korišćene metode za izbor parametara pneumatika

Tabela 4: Optimalni parametri pneumatika za neopterećenu prikolicu

Veličina	$0.5(x_d+x_g)$
x_1, -	1,051936315832336E-014
x_2, rad^{-1}	1,879999999999998E+001
x_3, rad^{-2}	9,500000000000218E+000
x_4, rad^{-3}	1,839999999999998E+001
x_5, N	1,789999999999998E+001
x_6, -	1,559999999999995E+001
x_7, -	3,999999999999895E-001
x_8, rad^{-1}	1,200000000000003E+000
x_9, rad^{-2}	1,859999999999996E+001
x_1, rad^{-3}	4,200000000000003E+000
$x_{11, Nm}$	5,799999999999989E+000
$x_{12,\ m}$	3,999679999993426E-001
F-ja cilja	2,163335731040705E-021
Br. iter.	7135

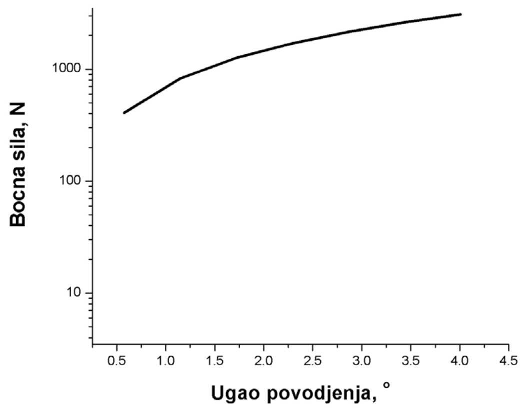
Tabela 5: Optimalni parametri za opterećenu prikolicu

Veličina	$0,5(x_d+x_g)$
x_1, -	1,051936315832336E-014
x_2, rad^{-1}	1,329999999999997E+001
x_3, rad^{-2}	1,999999999999999E+001
x_4, rad^{-3}	-9,999999999999984E+000
x_5, N	-9,900000000000075E+000
x_6, -	1,299999999999997E+001
x_7, -	-3,999999999999897E-001
x_8, rad^{-1}	1,999999999999999E+001
x_9, rad^{-2}	-9,999999999999973E+000
x_1, rad^{-3}	2,000000000000000E+001
$x_{11, Nm}$	1,990000000000003E+001
$x_{12,\ m}$	-1,339848440140420E-001
F-ja cilja	5,062664962514430E-008
Br. iter.	10285

4. ANALIZA PODATAKA I DISKUSIJA

Analizom podataka iz Tabela 4 i 5 može se utvrditi da korišćena metoda optimizacije konvergira samo u slučaju početnih vrednosti, na sredini intervala, oblasti definisanosti optimirućih parametara. U slučaju započinjanja procesa optimizacije sa početnim vrednostima koje su bile za 30% manje od gornjih graničnih, odnosno veće od donjih graničnih vrednosti, nisu dobijeni realni rezultati, pa te vrednosti nisu ni prikazane u Tabelama 4 i 5. Zbog toga se nameće zaključak da je opravdano iterativni proces optimizacije započeti sa sredinom intervala oblasti definisanosti parametara.

U konkretnom slučaju su vrednosti funkcije cilja za opterećenu prikolicu veće nego kod neopterećene prikolice, ali se ocenjuje opravdanim da se parametri bočnih karakteristika predviđenih pneumatika usvoje za slučaj opterećene prikolice. Zbog toga je za taj slučaj izračunata bočna sila, na osnovu koje treba izvršiti konačni izbor karakteristika pneumatika (pre svega koeficijenta bočne krutosti i pritiska vazduha jer su projektom definisane njihove dimenzije). Radi ilustracije, karakter bočne sile je prikazan na slici 5, sa koje se vidi karakter bočnih karakteristika koje moraju zadovoljiti pneumatici prikolice 175/70SR13. Napominje se da se pri izboru pneumatika treba rukovoditi činjenicom da krive stvarnih pneumatika budu što približnije izračunatim. Zbog toga se ne može govoriti o optimalnim, već prihvatljivim karakteristikama pneumatika.



Slika 5: Željene bočne karakteristike pneumatika, za predviđeno statičko opterećenje pneumatika

5. ZAKLJUČAK

Izbor vrste i dimenzija pneumatika prikolice se vrši na osnovu propisane nosivosti. Izbor parametara bočnih karakteristika se vrši u kasnijim fazama projektovanja, primenom modela ili eksperimentalnih metoda. Na osnovu izvršenih istraživanja može se zaključiti da se razvijena metoda može koristiti u praksi za izbor najprihvatljivijih bočnih karakteristika pneumatika u prethodnoj fazi projektovanja prikolice.

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A Procedure for Selecting the Lateral Characteristics of tires from the Aspect of Minimal Yaw of the Passenger Motor Vehicle's Trailer

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In the initial phase of motor vehicles design, and therefore their trailers, the dimensions and type of tires are adopted according to the required payload, from the manufacturer's catalogue. The fine-tuning of the characteristics, especially the lateral characteristics of the tires, is carried out in the later stages of design, using models or experimental methods. The yaw of the trailer, as part of the passenger vehicle-trailer system, using its simple dynamic model, is analysed in this paper. The equations of motion were derived using the Newton's laws for constant longitudinal velocity of a vehicle. Tires are described by models based on higher degree polynomial approximation. Optimal selection of the corresponding parameters of the lateral characteristics of the tires was made using the Hooke-Jevees method. Constructive parameter constraints have been introduced by the use of external penalty functions. The optimization process of minimizing the trailer's yaw angle was started with three sets of initial values of the optimizing parameters. Analyses have shown that it is sufficient to start with the midpoint of the parameter definition interval and the obtained data enable a quick selection of the lateral characteristics of the trailer's tires at the conceptual stage of its design.

Keywords: Trailer, Yaw, Optimization, Tires, Lateral characteristics

1. INTRODUCTION

In the initial phase of a design process of motor vehicles, and therefore, their trailers, the dimensions and type of tires are adopted according to the required payload, from the manufacturer's catalogue. The fine-tuning of the characteristics, especially the lateral characteristics of the tires, is carried out in the later stages of design, using models or experimental methods.

Tire and ground contact are of great importance for the behaviour of vehicles on the road, in general, and thus for trailers as part of a towing train. There are several tire models in the literature that are used in analytical and/or numerical studies of vehicle systems [1-3].

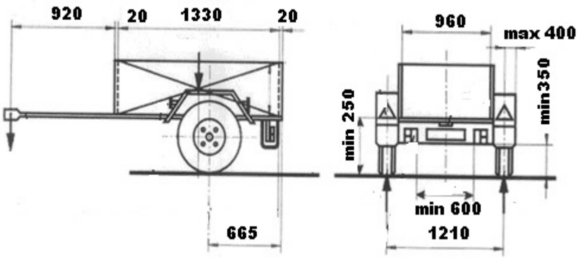

Figure 1: Trailer of the passenger motor vehicle

Most models involve knowledge of tire deformation both inside and outside the contact surface [2]. Tire models, based on experimental studies, approximated by higher-degree polynomials [4], have been used in this paper, and an attempt has been made to define, based on a simple trailer model and minimize its yawing, parameters that will allow easier selection of lateral tire characteristics during trailer design.

In the literature [5-8], there are trailer models of varying structure and complexity, which will be discussed later. Observed trailer [9] is shown in Figure 1, whose geometric and inertial parameters are given in Table 1.

Table 1: Trailer parameters

Parameter	Loaded	Unloaded
Total mass, m, kg	530	200
Distance from coupling device to axle, L, m	1.625	1625
Centre of gravity position of the coupling device, a, m	1.395	1.015
The moment of inertia for the main vertical centre of gravity axis, I_z, kgm^2	194	73
Trailer frontal projection surface, m^2 /Air drag coefficient, Ns^2/m^4	1.2/0.4	1.2/0.4
Trailer side projection surface, m^2/Air drag coefficient, Ns^2/m^4	0.63/0.4	0.63/0.4

The following section will discuss the modelling of trailer's yawing.

2. TRAILER MODEL

Analyses have shown [6] that at speeds exceeding 35 - 45 km/h, lateral vibration occurs due to the yawing, which has an inconvenient effect on the movement of the towing vehicle, as it impairs handling and increases fuel consumption and wear of the tires and joint links.

When modelling vehicles, and therefore trailers, models of different structures [10-19] can be used, but in practice, the principle that their complexity is such that it covers only the variables being analysed [12] is very commonly used. This is justified by the fact that more complex models require the use of more geometric and inertial parameters, which are often unknown. Their approximate selection leads to bigger errors in the process of dynamic simulation, so it is justified to use as simple models as possible.

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With this in mind, a single-axle passenger car trailer was considered in this paper with the following constraints [5,6]:

- the connection between the trailer and the towing vehicle (coupling device) is articulated,
- the vertical vibration of the trailer is neglected and
- the movement of the joint is straight.

It is known from mechanics that the bodies that move in a plane have three degrees of freedom, i.e. three differential equations need to be written to describe their motion. However, as a towing vehicle moving in a straight line at a constant speed is observed in this case, it is justified to observe only the yawing of the trailer.

Figure 2 shows the movement of the trailer, where Ψ represents its yawing relative to the rectilinear (straight-line) motion of the joint *C* (coupling device).

The following labels were used in Figure 2:

- F_{cx} and F_{cy} - components of the force at the joint,
- R_{vx}, R_{vy} - the air resistance of the trailer in the direction of movement and in the lateral direction of the towing vehicle respectively.

The values of the observed air resistance components were used in this paper:

$$R_{vx} = K_{vx} A_x v^2 \sin\psi \qquad R_{vy} = K_{vy} A_y v^2 \cos\psi \tag{1}$$

where

- K_{vx}, K_{vy} - coefficients of air resistance of the trailer in the direction of its motion and perpendicular to it, respectively, and
- v - the towing vehicle speed.
- R – Resulting trailer motion resistance force. In this case, the relation used was:

$$R = G\sin\alpha + Gf\cos\alpha + R_{vx}\cos\psi + R_{vy}\sin\psi + R_j \tag{2}$$

- G - weight of the trailer,
- α - the angle of inclination of the road,
- R_j-resistance of inertial forces (in case of steady motion equal to zero).
- Y - lateral ground reaction (acts for value "c" behind the wheel centre [5,6,10-19]),
- P - point on the floor of the trailer above the axle,
- $\dot{\psi}$ - angular velocity and
- v_p^C - relative velocity of point *P* with respect to point *C*.

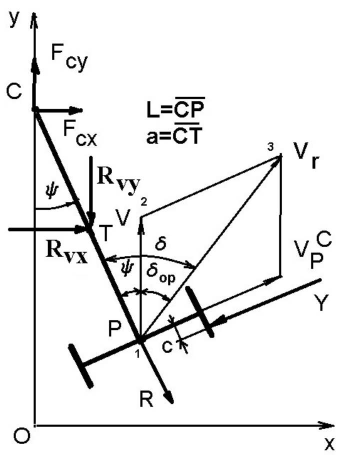

Figure 2: Schematic of the action of forces on the trailer

The speed of v_p^C can be expressed as:

$$v_P{}^C = L\dot{\psi} \tag{3}$$

The absolute velocity of point *P* is given by the expression:

$$\overrightarrow{v_P} = \vec{v} + \overrightarrow{v_p{}^C} \tag{4}$$

The angle this velocity has with the axle axis represents the actual tire steer angle, i.e.:

$$\delta = \psi + \delta_{op} \tag{5}$$

To calculate the magnitude of the angle, a triangle with vertices 123, and corresponding angles $\delta_{op}, \gamma, \varepsilon$ is observed. From the Figure 2 the obvious relation is:

$$\psi + \delta_{op} + \varepsilon = 90^o \tag{6}$$

Applying the sine theorem to the observed triangle:

$$\frac{v}{\sin\varepsilon} = \frac{v_P{}^C}{sin\delta_{op}} \tag{7}$$

Bearing in mind the expression (7):

$$\frac{v}{\cos(\psi + \delta_{op})} = \frac{L\dot{\psi}}{\sin\delta_{op}} \tag{8}$$

Applying the addition theorem:

$$\cos(\psi + \delta_{op}) = \cos\psi\cos\delta_{op} - \sin\psi\sin\delta_{op} \tag{9}$$

For small angles can be written:

$$\sin\psi\sin\delta_{op} \approx 0$$

$$\cos\psi\cos\delta_{op} \approx 1$$

Bearing in mind that δ_{op} has small value, it can be assumed that the angle is approximately equal to the sine of the angle, so one can finally write an expression for steer angle of the tire:

$$\delta = \psi + \frac{L\dot{\psi}}{v} \tag{10}$$

As already noted, the trailer, as a rigid body moving in a plane, has three degrees of freedom of movement. In this paper, only yawing is analysed, so the differential equation describing this motion is:

$$\left(J_z + m_p a^2\right)\ddot{\psi} = -Y(L+c) + R_{vx} a\cos\psi - R_{vy} a\sin\psi$$

(11)

where the sum in brackets represents the moment of inertia of the semitrailer with respect to point *C*, and

- M_s - tire stabilization moment [5,6,10-19].

For small angles, differential equation (11) can be written in the form:

$$\left(J_z + m_p a^2\right)\ddot{\psi} = -YL - M_s + R_{vx} a - R_{vy} a\psi \tag{12}$$

There are several tire models in the literature [1-4]. It was considered appropriate to use lateral tire performance models from [4] in this paper:

$$Y = (x_1 + x_2\delta + x_3\delta^2 + x_4\delta^3)(x_5 + x_6 Z)$$
$$M_s = (x_7 + x_8\delta + x_9\delta^2 + x_{10}\delta^3)(x_{11} + x_{12} Z) \tag{13}$$

where

- x_i i = 1-12 tire model parameters,
- δ - calculated tire steer angle (term 8), a

- Z-dynamic ground reaction (in this case approximately equal to the static ground reaction).

In doing so, a constraint in the form of a boundary lateral force was introduced, which was defined by the adhesion force [5,6,10-19]:

$$Y_{\max} = \varphi Z \qquad (14)$$

where

- Z – dynamic ground reaction, and
- φ - coefficient of adhesion.

Given the character of the lateral force and the stabilization moment (expression 13), the differential equation (12) is nonlinear, with constant parameters, so it can only be solved numerically.

In order to verify (calibrate) the model, a dynamic simulation of the trailer with 145SR13 tires was performed, the lateral characteristics of which are known [4] and their model's parameters shown in expression (13) are given in Table 2.

Table 2: Parameters of Test Tire 145SR13

x_1, -	-0.699
x_2, rad^{-1}	17.80
x_3, rad^{-2}	-1.17
x_4, rad^{-3}	202
x_5, N	23.00
x_6, -	0.0045
x_7, -	-0.23
x_8, rad^{-1}	1.40
x_9, rad^{-2}	-0.23
x_{10}, rad^{-3}	0.0101
x_{11}, Nm	-20
x_{12}, m	0.0141

Integration of differential equation (12) - previously transferred to a system of two first-order differential equations, was performed by the Runge–Kutta method, in steps of 0.01 s, at 10000 points, which ensures the reliability of results within the range 0.01 to 50 Hz, which is acceptable from the point of view of oscillatory comfort and vehicle behaviour on the road [12,20].

For illustration, Figure 3 shows the time-lapse record of a loaded trailer at a speed of 20 m/s. Analysing the data in Figure 3, it can be seen that the test tire parameters give acceptable trailer model results. It should be noted that the dynamic excitation of the trailer in lateral direction during the simulation, came from the longitudinal and lateral air resistance components. Testing was performed with a loaded and unloaded trailer, and it was considered appropriate to calculate the effective yawing values given in Table 3.

By analysing the data in Table 3, it can be established that the effective value of the trailer's yawing angle is affected by its load and speed. In this case, the yawing angles are higher in the case of a loaded condition and the higher speed, which indicates the need to make a choice of parameters of the lateral characteristics of the tires for these operating conditions because 175/70R13 tires [9] are provided for this trailer based on the payload whose lateral characteristics are unknown.

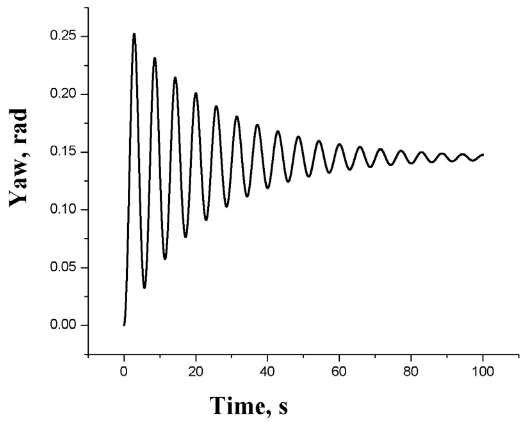

Figure 3: Yawing of a loaded trailer with 145 SR13 tires, at a speed of 20 m/s

Table 3: Dependence of trailer yawing on load and speed of movement

Load state	Speed of movement, m/s	Effective value of yawing angle, rad
Unloaded	10	0.06572
Unloaded	20	0.12957
Loaded	10	0.07544
Loaded	20	0.14679

3. METHOD

In order to select the optimal set of parameters of the tires lateral characteristics, the Hooke-Jeeves optimization method was used, which is explained in detail in [21]. To illustrate, a block diagram of the optimization method is shown in Figure 4. As described in detail in [12], it will not be discussed further here. The limit values for the interval of acceptable tire parameters are defined by the introduction of external penalty functions, by defining the value of the goal function, outside the parameter definition area, 1020. The program was implemented in Pascal, on a personal computer.

In this case, the goal function is defined as the effective value of the trailer's yawing angle, i.e.:

$$z = \frac{1}{T}\sqrt{\int_0^T \psi^2 dt} \qquad (15)$$

where

- z - goal function,
- ψ- trailer yawing angle,
- t - time, a
- T-yawing observation period.

The program was implemented with the limit values of tire parameters introduced:

$$-10 < x_i < 20,\ i = 1 \div 12$$

As all nonlinear programming methods converge to the closest minimum of the goal function from the starting point, the problem of finding the global minimum is very difficult [21]. Therefore, in this paper, it was considered appropriate to perform optimization with three groups of initial values of the optimizing parameters:

- the middle of the interval of the parameter definition area,

- 0.8 of the upper limit values and
- 1.3 of the lower limit values.

The goal function search was performed with 0.1 steps, and the iterative process was completed when the difference between the two adjacent values of the goal function was less than 10-20. The optimal parameter values were calculated for the loaded and unloaded condition of the trailer, with the results shown in Tables 4 and 5.

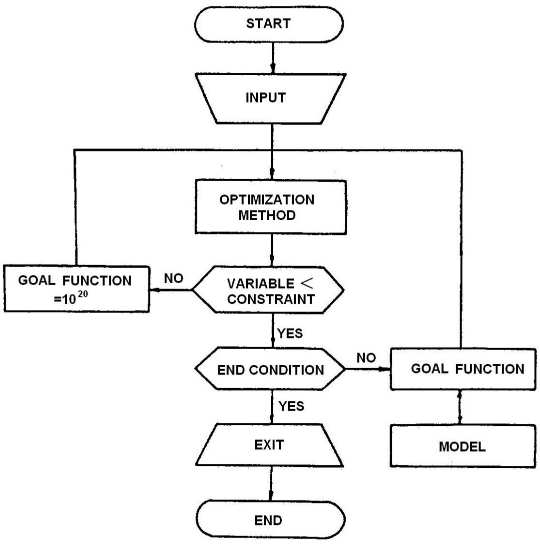

Figure 4: Block diagram of the method used to select tire parameters

Table 4: Optimal tire parameters for unloaded trailer

Value	$0.5(x_d+x_g)$
x_1, -	1.051936315832336E-014
x_2, rad^{-1}	1.879999999999998E+001
x_3, rad^{-2}	9.500000000000218E+000
x_4, rad^{-3}	1.839999999999998E+001
x_5, N	1.789999999999998E+001
x_6, -	1.559999999999995E+001
x_7, -	3.999999999999895E-001
x_8, rad^{-1}	1.200000000000003E+000
x_9, rad^{-2}	1.859999999999996E+001
x_1, rad^{-3}	4.200000000000003E+000
x_{11}, Nm	5.799999999999989E+000
x_{12}, m	3.999679999993426E-001
Goal function	2.163335731040705E-021
Number of iterations	7135

Table 5: Optimal tire parameters for loaded trailer

Value	$0.5(x_d+x_g)$
x_1, -	1.051936315832336E-014
x_2, rad^{-1}	1.329999999999997E+001
x_3, rad^{-2}	1.999999999999999E+001
x_4, rad^{-3}	-9.999999999999984E+000
x_5, N	-9.900000000000075E+000
x_6, -	1.299999999999997E+001
x_7, -	-3.999999999999897E-001
x_8, rad^{-1}	1.999999999999999E+001
x_9, rad^{-2}	-9.999999999999973E+000
x_1, rad^{-3}	2.000000000000000E+001
x_{11}, Nm	1.990000000000003E+001
x_{12}, m	-1.339848440140420E-001
Goal function	5.062664962514430E-008
Number of iterations	10285

4. DATA ANALYSIS AND DISCUSSION

By analysing the data in Tables 4 and 5, it can be concluded that the optimization method used converges only in the case of initial values, in the middle of the interval, of the area of definition of the optimizing parameters. When starting the optimization process with initial values that were 30% less than the upper limit values, i.e. higher than the lower limit values, no real results were obtained, so these values are not even shown in Tables 4 and 5. Therefore, it is concluded that it is justified to start the iterative optimization process with the middle of the interval of the parameter definition area.

In this specific case, the goal function values for the loaded trailer are higher than for the unloaded trailer, but it is considered justified to adopt the parameters of lateral characteristics of the provided tires in the case of the loaded trailer. Therefore, the lateral force was calculated for this case, based on which the final choice of tire characteristics (primarily the coefficients of lateral stiffness and air pressure, as their dimensions were defined in project) should be made. For illustration, the lateral force is shown in Figure 5, which shows the lateral characteristics that the 175/70SR13 trailer tire must satisfy. It is noted that the choice of tires should be guided by the fact that the curves of the actual tires to be as closely as possible to the calculated. Therefore, it is not possible to discuss about the optimum, but acceptable characteristics of a tire.

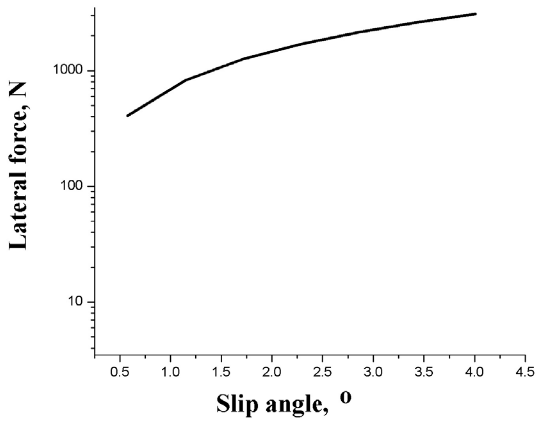

Figure 5: The desired lateral characteristics of the tire, for the predicted static tire load

5. CONSCLUSION

The selection of type and size of trailer tires is made based on the prescribed payload. The choice of lateral characteristics parameters is made in the later stages of design, using models or experimental methods. Based on the performed research, it can be concluded that the developed method can be used in practice to select the most acceptable lateral characteristics of tires in the previous stage of trailer design.

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