

Mathematica software graphical simulation of Iodine isotopes nuclear decay for teaching purposes

Milentije Luković¹, Vesna Veličković^{*1} and Vanja Luković¹

¹ University of Kragujevac, Faculty of technical sciences, Čačak, Serbia

* vesna.velickovic@ftn.kg.ac.rs

Abstract: *The aim of this work is to show the simulation of the nuclear decay of Iodine isotopes using the Mathematica software package. The nuclear decay of Iodine occurs most often during nuclear accidents in various types of nuclear facilities, during which radioactive isotopes are produced. Radioactive Iodine isotopes lead to the manifestation of harmful side effects, but with the development of science, it has been established that some of them can be used for diagnostic purposes and to detect certain types of carcinoma. The software package itself makes it possible to actively change the parameters in real time that are characteristic for the nuclear decay of Iodine. Graphically by displaying the exponential curves for decayed and non-decayed nuclei, the ratio of the number of undecayed nuclei as a function of the elapsed time can be directly calculated. The model can be implemented in classes and enables students to better understand nuclear decay in the undergraduate courses of Physics, Physics 2 and Computer simulation of physical phenomena.*

Keywords: *Mathematica software; graphical simulation; nuclear decay; Iodine; isotopes*

1. INTRODUCTION

Radioactive isotopes of Iodine, which are produced as a result of nuclear decay, represent one of the main pollutants during nuclear processes and nuclear incidents. Nuclear incidents are occurrences of sudden and uncontrolled release and effects of ionizing radiation isotopes along with other accompanying phenomena. They are very often unintentional and arise as a result of the human factor (ignorance, carelessness, technical failures, outdated technology, non-observance of safety measures at work and others) [1]. Damages and breakdowns on nuclear reactors in nuclear power plant as well as other objects such as artificial satellites, containers with nuclear fuel and radioactive material lead to many ecological and biological unfathomable consequences [2-4].

Radioactive isotopes of Iodine are interesting from the aspect of our research, because they represent a potential source of pollutants during the nuclear process. Given properties such as high volatility, and therefore potentially high speed of spreading into the surrounding environment, radioactive isotopes of Iodine represent the primary danger of exposure to a radioactive cloud in the first stages of a nuclear accident. For example, in the first days immediately after the nuclear accident in Chernobyl, the dominant influence on the total measured radiation in the air was given by Iodine isotopes I-127 and I-131 [5].

There are 37 known isotopes of Iodine (⁵³I) from I-108 to I-144. All undergo radioactive decay

except I-127, which makes Iodine a monoisotopic element [6].

The longest-lived radioactive isotope of Iodine is I-129, which has a Half-life ($T_{1/2}$) of 15.7 million years, which is too short to exist as a primordial nuclide. Most of the I-129 present on earth, made by man are an unwanted long-lived byproduct of early nuclear tests and accidents caused by uncontrolled nuclear fission [6,7].

It is important to note that in addition to side effects, some of the Iodine isotopes show great applicability in therapeutic practice and treatment. Decades of experience indicate that due to its beneficial properties due to its biophysical characteristics, I-131 compared to other isotopes has been used in the treatment of, above all, some kind of cancer, although it has been established that there may be certain harmful effects in addition to beneficial ones, which occur as a result of radiation damage to other tissues and organs [8]. Iodine isotope I-131 was used in the diagnosis and for the first time for the treatment of thyroid gland diseases in 1942 by Hertz and Roberts [9]. This radionuclide was introduced into clinical practice as a drug for the treatment of hyperthyroidism and some forms of cancer and it has an important role in detection of diseases [10-11].

Based on all of the above, the numerous negative but also positive properties of Iodine isotopes, and their application, various programs of simulative nuclear decay of Iodine are used. This program predicts the ratio of the number of decayed and

undecayed nuclei when different parameters are changed, such as the elapsed time, that is, the number of Half-lives. It is also useful for students to better understand nuclear decay and can be applied in the undergraduate courses of Physics, Physics 2 and Computer simulation of physical phenomena.

2. SELECTION THE TYPE OF GRAPHICS PRESENTATION

For the simulation of the nuclear decay of the Iodine isotope, the *Mathematica* software package was selected, which is actively used in various undergraduate courses. *Mathematica* can also be used as a standalone programming language that supports procedural, functional and object-oriented constructions, because it has great connections to other program packages (C, JAVA, SQL, OpenOffice, Acrobat, etc). Also, a large number of simulations related to different physical phenomena were written in it [12].

Mathematica software package has various functions that we used to actively change the parameters of decay. It is particularly convenient to use a *Manipulate* function that allows students to actively change different parameters, such as:

- *Graphics* - there are *Plot* and *BarChart* graphical options;
- *Isotopes* - where the radioactive elements of iodine isotope are selected;
- *Starting number of nuclei N_0* - the initial number of nuclei;
- *Time (days)* - the elapsed time since the beginning of the measurement;
- *Half-life ($T_{1/2}$)* - which represents the time of semi-decay, i.e. the time it takes to decay half of the radioactive nuclei of the selected radioactive element;
- *Number of $T_{1/2}$* - number of Half-life at *BarChart* presentation graphics.

2.1. Choosing the type of Iodine isotope

At the beginning of the simulation, students have options to choose from *Plot* and *BarChart* graphical presentation (Fig. 1). *Plot* graphical presentation by function *Plot*, offers graphical display in the form of curves view of the number of remaining and decayed nuclei, while *BarChart* graphical presentation offers a so-called bar chart for graphical display of the remaining nuclei.

The next option offered to students is a selection of radioactive Iodine isotopes. The selection of isotopes that is performed on the drop-down list isotopes and brings with it a selection of different types of isotopes of radioactive Iodine ^{53}I : I-124, I-125, I-126, I-130, I-131 and I-133 (Fig. 2). Each of these radioactive elements has different characteristics reflected in different $T_{1/2}$. They range from 0,515 days for I-130 to 59.4 days for I-125.

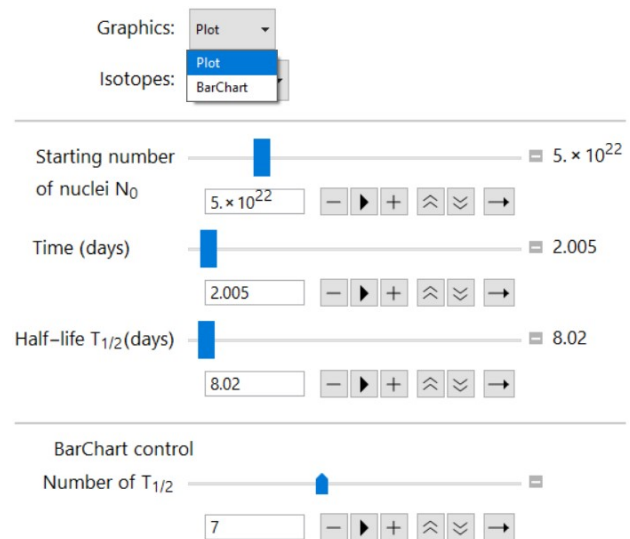


Figure 1. Option for selection *Plot* or *BarChart* graphical display type, along with options for changing different parameters: Starting number of nuclei, Time, Half-life $T_{1/2}$, and Number of $T_{1/2}$

In fact, option *Isotopes* only provides the option for choosing isotopes which Half-life are expressed in days (Table 1).

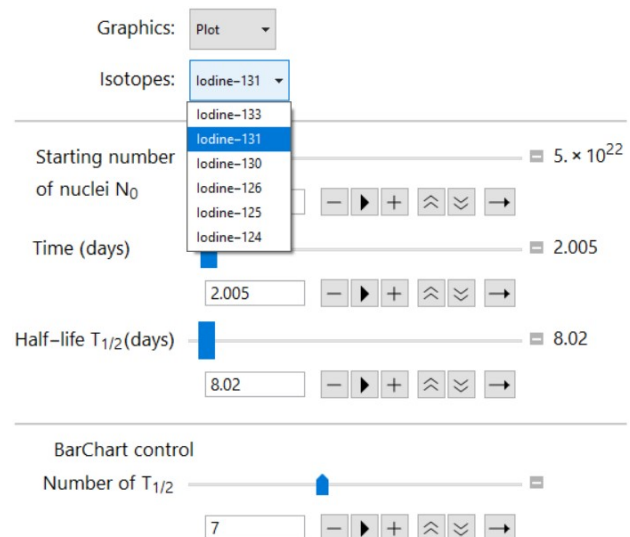


Figure 2. Option to select type of radioactive Iodine isotope

Other radioactive Iodine isotopes have small times of Half-life and are not in the long-term health-environmentally significant.

Table 1. Radioactive isotopes of Iodine ^{53}I which are used in simulation

symbol	Mass number	Half-life $T_{1/2}$ (days)
$^{53}\text{I}^{124}$	124	4.18
$^{53}\text{I}^{125}$	125	59.4
$^{53}\text{I}^{126}$	126	12.93
$^{53}\text{I}^{130}$	130	0.515
$^{53}\text{I}^{131}$	131	8.02
$^{53}\text{I}^{133}$	133	0.867

3. CURVE GRAPHICS PRESENTATION AND NUCLEI CALCULATION

After choosing type of graphical display (*Curve* or *BarChart*) and the type of radioactive Iodine isotope, students are offered to set the number of nuclei with *Starting number of nuclei* N_0 slider. This number ranges from some starting value of 10^{22} nuclei to the end value of $20 \cdot 10^{22}$ nuclei, but the initial number of nuclei is a set to be $5 \cdot 10^{22}$, unless a student selects the otherwise.

This choice is made possible by using the *Manipulate* function, where the interval of the number of nuclei is set to be comparable to the number of radioactive isotope samples corresponding to the real situations of removing samples from soil, food and water. That is, since one moll contains $6,022 \cdot 10^{23}$ nuclei, the selection of the number of nuclei is indirectly and the initial number of molls ranges from approximately 0.0166 to 0.332 molls.

By using the *Manipulate* function in software package *Mathematica*, students are also given the ability to animate or linearly change the value of parameters over time. Below each slider is a minus sign, a right arrow, a plus sign, an acceleration of changing the value of a given parameter, option to reduce it value, and ultimately choosing it direction of change - right to increase or left to decrease (Fig. 2). Pressing a minus or plus character changes the value of the selected parameter with a step defined by the user in the code itself. Pressing the arrow between the minus sign and plus starts the animation, which can be further accelerated or slowed down. At the end of a row is an arrow that indicates the direction of the change of a given parameter: from the minimum value to the maximum - direction to the right, or vice versa from maximum to minimum value - direction to the left. The next option is to select the time given in days - *Time (days)* that represents elapsed time since the beginning of measuring the number of decayed and undecayed (remaining) nuclei. And here there is the possibility of moving the slider from the initial moment to the maximum of the time parameter, which is 6 times the Half-life of the selected isotope. Also, by moving the slider that changes the value of time, a shaded blue area appears on the graphic, indicating the elapsed time (Fig. 3).

In Fig. 3, two curves can be observed: one that indicates an exponential decline in the number of nuclei as a function of time; and another that indicates an exponential increase in the number of decayed nuclei as a function of time. The equation for calculating the number of remaining nuclei is

$$N_n(t) = N_0 \cdot e^{-\lambda \cdot t} \quad (1)$$

and it represents the so-called Radioactive Decay Law, while the equation for calculating the number of decayed nuclei is

$$N_r(t) = N_0 \cdot (1 - e^{-\lambda \cdot t}) \quad (2)$$

Parameter λ is a constant of radioactive decay and is closely linked to the isotope Half-life over the equation

$$\lambda = \frac{\ln 2}{T_{1/2}} \quad (3)$$

The top part of the graphics in Fig. 3 also shows the number of remaining and decayed nuclei over time that are displayed in percentage.

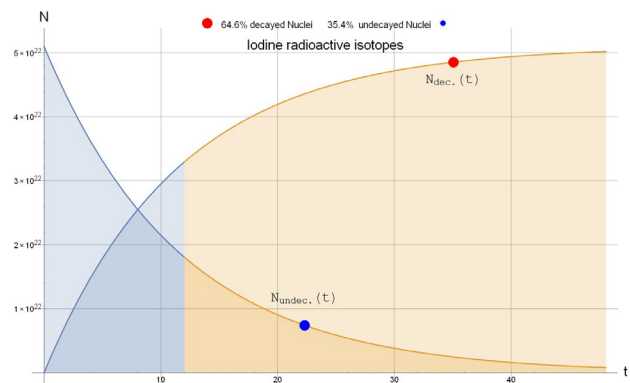


Figure 3. Curve graphics with curves that represents undecayed (remaining) and decayed nuclei as a function of the elapsed time

By introducing *DynamicModule* and *LocatorPane* functions, it is possible to use the mouse to move the locator and select individual dots on the graphics whose values we want to read. In Fig. 3, they're marked with blue dot for remaining nuclei, and with red dot for decayed nuclei. Graphical presentation of these values and volume ratio of remaining and decayed nuclei is shown on Fig. 3 which displays curve graphic type. In Fig. 4, which represent the right side of the Fig. 3, in addition to the elapsed time, the number of remaining and decayed nuclei is also being display.

This graphics presentation is closely linked to *DynamicModule* and *LocatorPane* functions in the simulation and is not associated with a *Time* slider. The slider, which is related to the time of nuclear decay, displays volume ratio of remaining and decayed nuclei, as shown in Fig. 4.

In addition to the two previous parameters given by *Manipulate* function, there is a third parameter in the *Curve* graphics option that can also be selected. It's a time that is represented in Half-life's (*Half-life* $T_{1/2}$) with unites represented in days. It is a characteristic of the radioactive isotope itself and represent time required to decay half of the total number of radioactive nuclei. The program automatically selects a certain Half-time that corresponds to the selected radioactive isotope, but students have option to move slider in order to change this parameter and observe changes to the graphics that occur as a consequence of that.

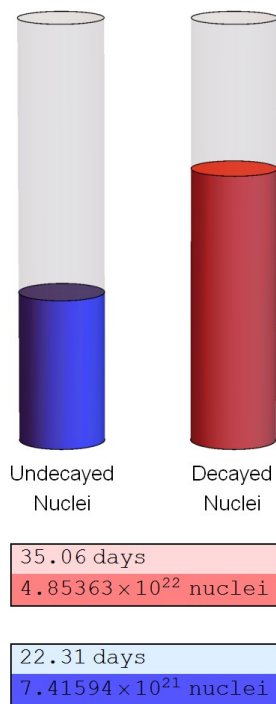


Figure 4. Representation of the volume ratio of remaining (blue color) and decayed (red color) nuclei; elapsed time and number of nuclei

4. BARCHART GRAPHICS PRESENTATION

At the beginning of the simulation in the *Graphics* option, beside the possibility of selecting the curve representation of the graphic, there is also a possibility to select graphic view using bars - *BarChart* option. With this option, using the *Manipulate* function, the slider is set to change the number of bar columns - *Barchart control* which is connected to parameter *Number of $T_{1/2}$* (Fig. 2). The set value is number 7, but if necessary, it can be changed from 1 to 16 (Fig. 5).

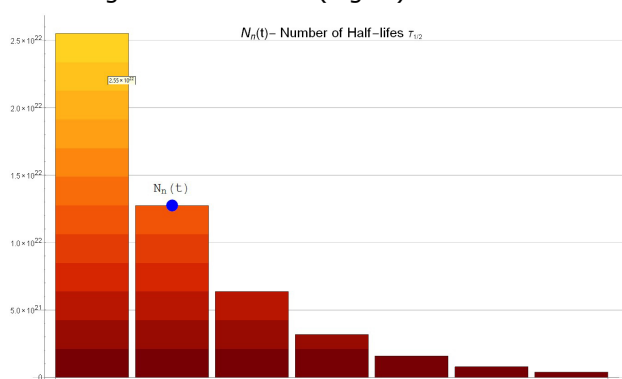


Figure 5. BarChart graphic for time period of 7 $T_{1/2}$ for radioactive isotope I-131 and starting number of nuclei $N_0 = 5,2 \cdot 10^{22}$

Each column of bars represents one Half-life, that is, 16 columns of bars will represent a time equal to 16 $T_{1/2}$, while the number of remaining undecayed nuclei is related to the height of the bars (Fig. 6).

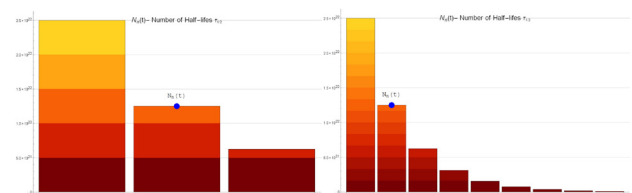


Figure 6. Example BarChart graphics presentation for $T_{1/2}$ (left) and 9 $T_{1/2}$ (right)

As with drawing a curve graph in the *Curve* option, with the *BarChart* graph there is the possibility of moving the locator on which the elapsed time and the number of nuclei are read. As for the previous graphics, this was achieved by introducing the *DynamicModule* and *LocatorPane* functions into the simulation. The elapsed time (in $T_{1/2}$) and the number of undecayed nuclei are printed on the right side next to the graph (Fig. 7).

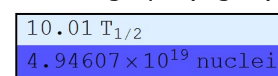


Figure 7. Displaying of the elapsed time in $T_{1/2}$ units and the number of undecayed nuclei on the BarChart graphic

Moving the locator and reading the data for each individual *BarChart* allows students to gain a better insight into the change in the number of undecayed nuclei values that can be represented by the equation

$$N_n(t) = N_0 \cdot 2^{-\frac{t}{T_{1/2}}} \quad (4)$$

, which is obtained from expression (1) as follows

$$\left. \begin{aligned} N_n(t) &= N_0 \cdot e^{-\lambda t} \\ \lambda &= \frac{\ln 2}{T_{1/2}} \end{aligned} \right\} \Rightarrow N_n(t) = N_0 \cdot 2^{-\frac{t}{T_{1/2}}} \quad (5)$$

5. MATHEMATICA SOFTWARE SIMULATION CODE

The code for this graphical simulation, which was done in the *Mathematica* software package, is as follows:

```
ClearAll["Global`*"]
Manipulate[If[uslov==1,
DynamicModule[
{pt={2.2, 1.4*10^22},pt1={2.0, 3.8*10^22}},
Row[{LocatorPane[Dynamic[{pt, pt1}],
Plot[Evaluate[{
If[c<t,{N0-N0*e^(-Log[2]/tpol*c),N0*e^(-Log[2]/tpol*c)},
If[c>t,{N0-N0*e^(-Log[2]/tpol*c),N0*e^(-Log[2]/tpol*c)}]}],
{c, 0, 6tpol}, Filling->Axis,
AxesLabel->{"t (days)","N"},
PlotLabel->PaddedForm[Row
```

```

[Graphics[Red,Disk[{0,0},1-e- $\frac{\text{Log}[2]}{\text{tpol}}$ c}]],
PlotRange->1.4, ImageSize->30,
BaselinePosition->(Center->Center)],
100N[1-e- $\frac{\text{Log}[2]}{\text{tpol}}$ c}], "% decayed Nuclei ",
100N[e- $\frac{\text{Log}[2]}{\text{tpol}}$ c}], "% undecayed Nuclei",
Graphics[Blue,Disk[{0,0},e- $\frac{\text{Log}[2]}{\text{tpol}}$ c}]],
PlotRange->1.4, ImageSize->30,
BaselinePosition->(Center->Center)],
Iodine radioactive isotopes
}],{3,1}],
LabelStyle->Darker[Black],
GridLines->Automatic, ImageSize->{850,600},
Epilog->{PointSize[0.018], Point[{
Dynamic[
{First[pt],(N0-N0*Exp[-(Log[2]/tpol)*First[pt]])}],
Dynamic[
{First[pt1],(N0*Exp[-(Log[2]/tpol)*First[pt1]])}],
}],
VertexColors->{Red,Blue}],
Appearance->{"Ndec.(t)","Nundec.(t)"},
Column[{Graphics3D[{Opacity[.10],Gray,
Cylinder[{{-8,0,0},{-8,0,10}},1],
Opacity[1],Lighter[Blue],
Cylinder[{{-8,0,10*Exp[-(Log[2]/tpol)*t}],
{-8,0,0}},1],
{Black,
Text[Style["Undecayed
Nuclei",12],{-8,0,-1.2}],
Text[Style["Decayed
Nuclei",12],{-4,0,-1.2}]
}],
Opacity[.10],Gray,
Cylinder[{{-4,0,0},{-4,0,10}},1],
Opacity[1],Lighter[Red],
Cylinder[{{-4,0,0},{-4,0,Piecewise[
{{(10-(10*Exp[
(-(Log[2]/tpol)*t))},t<=100000)}}}],1],
Opacity[1]},ViewPoint->{0,-10,2},
Boxed->False,ImageSize->{160,320}],
Dynamic[Column[{First[pt]"days ",
(N0-N0*Exp[-(Log[2]/tpol)*First[pt]])"nucleus"},
Background->{{LightRed,Pink}},
Frame->True]]," ",
Dynamic[Column[
{First[pt1]"days ",

```

```

N0*Exp[-(Log[2]/tpol)*First[pt1]]"nucleus"},
Background->{{LightBlue,Lighter[Blue]}},
Frame->True]]
}],
}],
DynamicModule[{pt2={2,1.4*1022}},
Row[{LocatorPane[Dynamic[pt2],
k=Table[2i,{i,1,n}];
BarChart[N0/k,
ChartElementFunction-
>ChartElementDataFunction["SegmentScaleRectangle",
"Segments"->1.75n,
"ColorScheme"->"SolarColors"],
ImageSize->{850,600},
PlotLabel->"
Nn(t)- Number of Half-lives T1/2
",
LabelStyle->Darker[Black],GridLines->Automatic,
Epilog->{PointSize[0.02],
Point[
Dynamic[
{First[pt2],(N0*Exp[-(Log[2]/1)*First[pt2]])}],
VertexColors->Blue}],
Background->None,
Appearance->"Nn(t)",
Dynamic[
Column[{First[pt2]"T1/2",
N0*Exp[-(Log[2]/1)*First[pt2]]"nucleus"},
Background->{{LightBlue,Lighter[Blue]}},
Frame->True]]
}],
}],
{uslov,1,"Graphics:"},
{1->"Plot",
2->"BarChart"},ControlType->PopupMenu},
{
{tpol,8.02,"Isotopes:"},
{0.8667->"Iodine-133",
8.02->"Iodine-131",
0.515->"Iodine-130",
12.93->"Iodine-126",
59.4->"Iodine-125",
4.18->"Iodine-124"},
ControlType->PopupMenu},
Delimiter,
{{N0,5*1022,"Starting number
of nucleus N0"},1*1022,20*1022,0.1*1022,
Appearance->"Labeled"}},

```

```
{
{t, tpol/4, "Time (days)      "}, 0, 6tpol,
Appearance->"Labeled"},
{
{tpol, tpol, "Half-life T1/2(days)"}, .1, 250,
Appearance->"Labeled"},
Delimiter, Item["      BarChart control",
Alignment->Left],
{
{n, 7, "Number of T1/2"}, 1, 16, 1,
Appearance->"UpArrow"}
]
```

6. CONCLUSION

The presented simulation in the *Mathematica* software package enables students to better understand nuclear decays, which are studied in the undergraduate courses Physics, Physics 2 and Computer simulation of Physical phenomena. In this simulation, students are given the opportunity to change certain parameters that are characteristic of nuclear decay in real time. The graphic representation is given in two forms - *Curve* and *BarChart* graphics presentation, each of which in its own way contributes to the understanding of nuclear physics and the dynamics of nuclear decay. *Curve* graphic presentation with exponential curves for undecayed and decayed nuclei, and a *BarChart* graphic presentation that directly gives the relationship of the number of undecayed nuclei with the elapsed time represented by the number of Half-lives. In this way, students can understand much better the mechanism and pace of the nuclear decay process. Also, they can draw conclusions about the number of undecayed and decayed nuclei as a function of the elapsed time and Half-life time, for which they could previously draw conclusions only by performing calculations using the appropriate equations.

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