

## Robusna identifikacija linearnih modela u prostoru stanja u prisustvu otkaza komponenti i senzora

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*Procena stanja i vremenski promenljivih parametara linearnih modela u prostoru stanja je od praktičnog značaja za dijagnostiku otkaza i upravljanje koje je tolerantno na otkaze. Ovaj rad razmatra robusnu identifikaciju linearnih modela u prostoru stanja u prisustvu otkaza komponenti i senzora. Sa druge strane, prethodni radovi na ovu temu nisu razmatrali procenu stanja i parametara linearnih sistema u prisustvu autlajera. Oni mogu značajno pokvariti karakteristike linearnih rekurzivnih algoritama, koji su projektovani da rade u prisustvu Gausovih šumova. Zbog svojih dobrih osobina u robusnoj filtraciji, modifikovani Masreliez-Martinov filter predstavlja kamen temeljac za realizaciju robusnog algoritma za procenu stanja i parametara linearnih vremenski promenljivih stohastičkih sistema u prisustvu negausovih šumova. Dobre karakteristike predloženog robusnog algoritma za procenu stanja i parametara linearnih vremenski promenljivih stohastičkih sistema ilustrovane su simulacijama.*

**Ključne reči:** linearni modeli u prostoru stanja, procena parametara i stanja, robusna identifikacija, vremenski promenljivi parametri, negausovi šumovi

### 1. UVOD

Dobro je poznato da je veoma teško odrediti veliki broj fizičkih parametara koji su sastavni deo kompleksnih sistema. Uprkos činjenici da su mnogi parametri sistema dostupni sa razumnom preciznošću, veliki broj parametara je poznat u određenom opsegu, dok su neki parametri potpuno nepoznati jer proizvođači te podatke smatraju kao vlasničke informacije [1]. Na primer, precizno određivanje parametara sistema, kao što su dimenzije pojedinih komponenti, koeficijenti curenja, koeficijenti trenja, kao i statičke i dinamičke sile trenja usled nemogućnosti direktnog merenja ili proračuna izazivaju velike poteškoće u upravljanju pneumatskih aktuatora. Precizno poznavanje parametara i stanja sistema je presudno za uspešnu realizaciju mnogih upravljačkih tehnika. Takođe, filtriranje stanja kao i procena parametara mogu biti ključni faktori za performanse, stabilnost i tačnost sistema.

Pošto je Rudolf Kalman objavio svoj čuveni rad [2], Kalmanov filter (KF) postao je osnova mnogih procena procesa u različitim oblastima primene. Poslednjih godina, obnovljen je interes za Kalmanovim filtrom, zbog sve većeg spektra primena [3-6]. Precizno poznavanje parametara i stanja sistema je presudno za uspešnu realizaciju mnogih tehnika upravljanja. Mnoge moderne inženjerske primene kao što su autonomna vozila [7], predviđanje deformacija za zamor [8] ili robotski manipulativni zadaci [9] zahtevaju Kalmanovo okruženje za filtriranje linearnih modela u realnom vremenu.

Obično je suviše skupo da se direktno mere stanja sistema. Metode procene stanja za samostalnu primenu pretpostavljaju da su parametri sistema konstantni. U realnom svetu, ovi parametri se uvek menjaju (npr. koeficijenti trenja, temperatura, pritisak ili protok).

Procedura procene stanja sa konstantnim parametrima će rezultirati velikim greškama prilikom promene parametara. Takođe je poznato da se dinamičko ponašanje složenih sistema obično opisuje linearnim stohastičkim modelom u prostoru stanja sa vremenski promenljivim parametrima [10]. Stoga su potrebne metode pomoću kojih se u isto vreme mogu dobiti procene parametara i stanja.

Procena parametara i stanja je od velike praktične važnosti za dijagnostiku kvarova i upravljanje koje je tolerantno na kvarove. Jedan od najvećih izazova u projektovanju sistema za upravljanje letom je zahtev da se let aviona bezbedno oporavi od strukturnih oštećenja i/ili kvarova sistema. Bez obzira na to da li je vazduhoplov opremljen posebnom sposobnošću rekonfiguracije upravljanja, pouzdane dijagnostičke informacije o kvarovima su izuzetno važne za pilota. Glavni izazov je detekcija i izolacija početnih kvarova u prisustvu neodređenosti modela i šuma [11]. Uključivanje nepoznatih parametara u vektor stanja omogućava jednostavnu implementaciju algoritma procene, jer se problem procene parametara u ovom slučaju rešava pomoću standardne teorije filtriranja. Takođe, izvedeni rekurzivni algoritam procene omogućava i offline i online realizaciju.

S druge strane, ne postoji nijedno takvo rešenje za linearne sisteme u prisustvu negausovih merenja. Prisustvo autlajera može uništiti dobre osobine linearno rekurzivnih algoritama koji su projektovani za procenu u prisustvu Gausovih šumova. Naime, poznata je činjenica da merenja imaju nekonzistentne opservacije sa glavninom populacije opservacija (outlieri). Njihovo prisustvo može uništiti dobre osobine rekurzivnih algoritama pretpostavljajući prisustvo Gausovih šumova. Zbog toga je

veoma važno projektovati robusan algoritam koji bi bio malo osetljiv na ekstremne vrednosti – autlajere. Huberova teorija robusnih statistika je ključna za projektovanje algoritma čija se robusnost postiže uvođenjem nelinearne transformacije greške predikcije (Huberova funkcija), [12]. Dakle, Huberova teorija robusnih statistika je ključna za projektovanje robusnog algoritma [13].

Ovaj rad predlaže strategiju za procenu stanja i parametara. Procena stanja i identifikacija parametara su objedinjuje kroz procenu parametara i stanja. Potpuno je prirodno staviti nepoznate, u opštem slučaju vremenski promjenjive, parametre u vektor stanja, nakon čega se problem svodi na klasični problem filtriranja. Razmatrana je robusna procena stanja i parametara linearnih stohastičkih sistema sa matricama čiji su elementi nezavisni od parametara. U ovom slučaju prošireni sistem ostaje linearan, tako da Masreliez-Martinov filter može biti kamen temeljac za robusnu procenu parametara i stanja.

Projektovani estimator uzima u obzir i otpornost na šumove i osetljivost na greške parametara. Dobre karakteristike predloženog robusnog algoritma ilustrovane su simulacijama.

## II. ALGORITAM PROCENE STANJA I PARAMETARA U PRISUSTVU OTKAZA KOMPONENTI I SENZORA

Klasa sistema za procenu stanja i vremenski promjenjivih parametara, koja se razmatrana u ovom radu, data je u obliku:

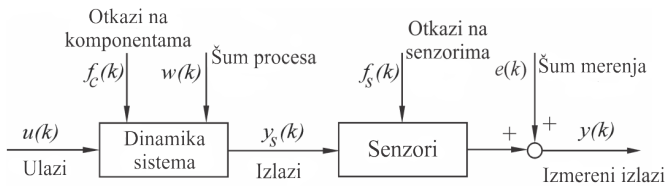
$$x(k+1) = A(k)x(k) + B(k)u(k) + C(k)\theta(k) + w(k) \quad (1)$$

$$y(k) = D(k)x(k) + \Gamma(k)\theta(k) + e(k) \quad (2)$$

gde su  $x(k) \in R^n$  i  $\theta(k) \in R^p$  nepoznati vektori stanja i parametara, respektivno. Kada su u sistemu prisutni otkazi senzora i komponenti (što je najčešća situacija), model sistema je opisan pomoću (1)-(2). Kvarovi komponenti i senzora su matematički opisani na sledeći način, vidi Sliku 1:

$$f_c(k) = C(k)\theta(k) \quad (3)$$

$$f_s(k) = \Gamma(k)\theta(k) \quad (4)$$



Slika 1: Otvoreno kolo sistema sa kvarovima komponenti i senzora

Kvar komponente predstavlja slučaj kada se neki uslov promeni u sistemu, čineći dinamički odnos nevažecim, na primer curenje u pneumatskom cilindru [14]. Generalno govoreći, stvarni izlazi sistema  $y_s(k) = D(k)x(k)$  nisu direktno dostupni, pa se onda koriste senzori za merenje izlaza sistema.

Senzori su najvažnije komponente za upravljanje letenjem i sigurnost aviona zbog svoje uloge u upravljanju letenjem i navigaciji. Svaki kvar senzora mora se otkriti što je ranije moguće kako bi se sprečila ozbiljna nesreća.

Za dijagnostiku početnih grešaka, sistemi za dijagnostiku kvarova moraju biti robusni u odnosu na modeliranje nesigurnosti i šuma [15]. Opšta forma promene parametara stohastičkih linearnih sistema je predstavljena kao  $\theta(k+1) = G\theta(k) + \eta(k)$  pri čemu je  $G$  a priori poznata nesingularna matrica koja je pogodna za uključenje apriorne informacije fenomena koji se identifikuje. Stohastički proces  $\eta(k)$  je beli šum nulte sredine i kovarijacione matrice  $\Phi(k)$ . Vektori ulaza i merenog izlaza sistema su  $u(k) \in R^m$  i  $y(k) \in R^r$ , respektivno, dok su  $A(k), B(k), C(k), D(k)$  i  $\Gamma(k)$  poznate, u opštem slučaju, vremenski promenljive matrice odgovarajućih dimenzija. Pretpostavljeno je da šum procesa beli šum nulte srednje vrednosti  $w(k) : \mathcal{N}(0, Q(k))$ , pri čemu je  $Q(k)$  kovarijaciona matrica. Šum merenja  $e(k)$  ima negausovu raspodelu sa približno normalnim klasama raspodele:

$$\mathcal{P}_\varepsilon = \{p(e) : p(e) = (1-\varepsilon)p_1(e) + \varepsilon p_2(e)\} \quad (5)$$

pri čemu funkcija gustine raspodele  $p(e)$  predstavlja mešavinu primarne gustine verovatnoće  $p_1(e) : \mathcal{N}(0, R_1(k))$  i kontaminirajuće gustine verovatnoće  $p_2(e) : \mathcal{N}(0, R_2(k))$  gde je stepen kontaminacije  $\varepsilon$  u opsegu  $0 < \varepsilon < 1$ , dok su  $R_1(k)$  i  $R_2(k)$  kovarijacione matrice primarnog i kontaminirajućeg člana negausove raspodele (5), respektivno.

Očigledno lak pristup proceni vektora stanja  $x(k)$  i vektora parametara  $\theta(k)$  sistema (1)-(2) je razmatranje proširenog sistema:

$$z(k+1) = \begin{bmatrix} A(k) & C(k) \\ 0 & I \end{bmatrix} z(k) + \begin{bmatrix} B(k) \\ 0 \end{bmatrix} u(k) + \xi(k) \quad (6)$$

$$y(k) = \begin{bmatrix} D(k) & \Gamma(k) \end{bmatrix} z(k) + e(k) \quad (7)$$

Ili u kompaktnijoj formi:

$$z(k+1) = F(k)z(k) + \bar{B}(k)u(k) + \xi(k) \quad (8)$$

$$y(k) = H(k)z(k) + e(k) \quad (9)$$

pri čemu su blok matrice  $H(k) = \begin{bmatrix} D(k) & \Gamma(k) \end{bmatrix}$ ,

$$\bar{B}(k) = \begin{bmatrix} B(k)^T & 0^T \end{bmatrix}^T,$$

$$F(k) = \begin{bmatrix} A(k) & C(k) \\ 0 & I \end{bmatrix}, \quad \text{prošireni vektor stanja}$$

$$z(k) = \begin{bmatrix} x(k)^T & \theta(k)^T \end{bmatrix}^T \quad \text{dok}$$

$$\xi(k) = \begin{bmatrix} w^T(k) & \eta^T(k) \end{bmatrix}^T \text{ označava prošireni vektor}$$

$$\text{poremećaja, gde je } \xi(k) : \mathcal{N}(0, \Xi(k)) \text{ sa}$$

$$\Xi(k) = \text{diag}(Q(k), \Phi(k)).$$

Koristeći združenu formulaciju stanja i parametara, postignuta je njihova jedinstvena procena. Pošto je prošireni sistem (8)-(9) još uvek linearni sistem, Masreliez-Martinov filter [16] se primenjuje za procenu vektora stanja  $x(k)$  i vektora parametara  $\theta(k)$ . Potpuno je prirodno uvrstiti parametre u vektor stanja, nakon čega se

problem svodi na klasični problem filtriranja  $k$ -th reda jedinstvenog sistema (8)-(9) (gde je  $k = n + p$ , pri čemu je  $n$  broj stanja koja se procenjuju a  $p$  je broj parametara koji se procenjuju).

Masreliez i Martin su predložili robusan Kalmanov filter za pomenutu situaciju [17]. Ovaj filter ima malu osetljivost na prisustvo autlajera u poređenju sa standardnim Kalmanovim filtrom izvedenim za slučaj kada veličine  $w(\cdot)$  i  $v(\cdot)$  imaju Gaussovu raspodelu.

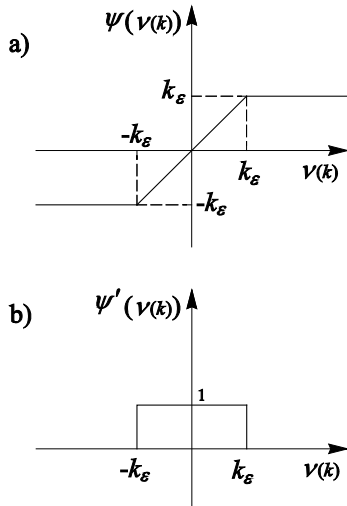
Naš cilj je da izvedemo robusni algoritam za procenu stanja i parametara stohastičkih linearnih sistema u prisustvu autlajera koji obezbeđuje nisku osetljivost pri pojavi autlajera. Za klasu  $\varepsilon$ -kontaminiranih raspodela verovatnoća, nelinearna transformacija greške predikcije  $\psi(\cdot)$  (Huberova funkcija) dobija se kao:

$$\psi(v(k)) = \min\{|v(k)|, k_\varepsilon\} \operatorname{sgn}(v(k)), \quad (10)$$

i njen izvod:

$$\psi'(v(k)) = \begin{cases} 1 & |v(k)| < k_\varepsilon, \\ 0 & \text{otherwise.} \end{cases} \quad (11)$$

pri čemu je  $k_\varepsilon$  podesno definisan parametar Huberove funkcije, videti Sliku 2.



Slika 2: Huberova funkcija i njen izvod

Izvorno predloženi Masreliez-Martinov filter uključuje član  $E_{P_\varepsilon} \{\psi'[v(k)]\}$  u a posteriori kovarijacionoj matrici, koji nije lako odrediti u praktičnim uslovima [16]. Da bi se poboljšala njegova primenljivost, umesto člana  $E_{P_\varepsilon} \{\psi'[v(k)]\}$  uvedena je realizacija  $\psi'[v(k)]$ . Intenzivne simulacije opravdale su takve intervencije. Može se pokazati da su matematičko očekivanje i kovarijaciona matrica transformisanih reziduala  $\varepsilon(k) = y(k) - H(k)\hat{x}(k-1|k-1)$ :

$$E\{\psi(v(k))\} = T(k)E\{\varepsilon(k)\} = 0 \quad (12)$$

$$E\{v^2(k)\} = T^2(k) \left[ H(k)P(k|k-1)H^T(k) + R_1(k) \right]^{-\frac{1}{2}} \quad (13)$$

Dakle, u slučaju da je transformacija  $T(k)$ :

$$T(k) = \left[ H(k)P(k|k-1)H^T(k) + R_1(k) \right]^{-1/2}, \quad (14)$$

kovarijaciona matrica transformisanih reziduala će biti  $I$ .

Važno je naglasiti da ova modifikacija povećava brzinu konvergencije tako modifikovanog robusnog filtra u početnim iteracijama. Naime, pojačanje robusnog Kalmanovog filtra je dat:

$$K(k) = F(k-1) \left[ P(k|k-1) - P(k|k-1)H(k)^T \right] \cdot T(k)^2 H(k)^T P(k|k-1) \psi'[\varepsilon(k)] \cdot F(k-1)^T H(k)^T T(k) + Q(k-1)H(k)^T T(k) \quad (15)$$

Ako je  $|v(k)| > k_\varepsilon$ , relacija (15) postaje:

$$K(k) = F(k-1)P(k|k-1)F(k-1)^T \cdot H(k)^T T(k) + Q(k-1)H(k)^T T(k) \quad (16)$$

jer je  $\psi'[v(k)] = 0$  kada  $|v(k)| > k_\varepsilon$  (videti Sliku 2).

Lako je zaključiti da je pojačanje  $K(k)$  iz (16) veće od pojačanja  $K(k)$  iz relacije (15). To znači da veće greške u proceni dovode do većeg pojačanja filtra što posledično dovodi do veće brzine konvergencije.

Zbog svojih dobrih osobina u robusnoj filtraciji, takav modifikovani Masreliez-Martinov filter se koristi kao osnova za formulisanje estimatora stanja i parametara linearnih stohastičkih sistema, kao što sledi:

$$\begin{aligned} \hat{z}(k|k-1) &= F(k-1)\hat{z}(k-1|k-1) + \bar{B}(k-1)u(k-1) \\ P(k|k-1) &= F(k-1)P(k-1|k-1)F^T(k-1) + \Xi(k-1) \\ K(k) &= [N(k) \quad M(k)]^T = P^T(k|k-1)H^T(k)T^T(k) \\ v(k) &= T(k)[y(k) - H(k)\hat{x}(k-1|k-1)] \\ \hat{x}(k|k) &= A(k-1)\hat{x}(k-1|k-1) + B(k-1)u(k-1) + \\ &+ C(k-1)\hat{\theta}(k-1|k-1) + N(k)\Psi(v(k)) \end{aligned} \quad (17)$$

$$\begin{aligned} \hat{\theta}(k|k) &= \hat{\theta}(k-1|k-1) + M(k)\Psi(v(k)) \\ P(k|k) &= P(k|k-1) - K(k)\Psi'(v(k))K^T(k) \\ \Psi'(v(k)) &= \operatorname{diag}(\psi'(v_1(k)), \dots, \psi'(v_r(k))) \end{aligned}$$

$$T(k) = \left[ H(k)P(k|k-1)H^T(k) + R_1(k) \right]^{-\frac{1}{2}}$$

$$\text{pri početnim uslovima: } \hat{z}_0 = 0 \text{ i } P_0 = \begin{bmatrix} P(x_0) & 0 \\ 0 & P(\theta_0) \end{bmatrix}.$$

Na ovaj način je izveden robusni algoritam za procenu stanja i parametara linearnih stohastičkih sistema.

## IV. REZULTATI SIMULACIJA

Određivanje tačnih vrednosti vektora stanja i parametara predstavlja osnovni uslov za projektovanje modela visokog kvaliteta, što uslovljava bolje performanse upravljanja. Prednosti predloženih robusnih algoritama za procenu stanja i parametara stohastičkih linearnih vremenski promenljivih sistema ilustrovane su kroz intenzivne simulacije. Ovi rezultati pokazuju superiornost predloženog robusnog algoritma (RKF) u odnosu na algoritme za procenu stanja i parametara zasnovanih na široko korišćenom Kalmanovom filtru (KF) i Masreliez-Martinovom filtru (MMF). Ponašanje algoritama razmatra se na:

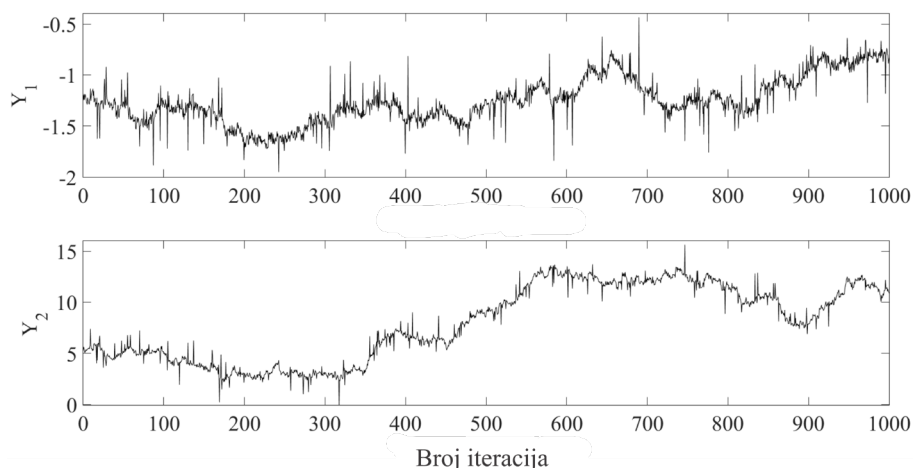
$$\begin{aligned}
 \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} &= \begin{bmatrix} 0.9 & 0 \\ 0 & 0.8 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \\
 &+ \begin{bmatrix} 0.1 & 1 & 0 \\ 0.5 & 0.3 & 1 \end{bmatrix} \begin{bmatrix} p_1(k) \\ p_2(k) \\ p_3(k) \end{bmatrix} + \begin{bmatrix} w_1(k) \\ w_2(k) \end{bmatrix}
 \end{aligned} \quad (18a)$$

$$\begin{aligned}
 \begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \\
 &+ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_1(k) \\ p_2(k) \\ p_3(k) \end{bmatrix} + \begin{bmatrix} e_1(k) \\ e_2(k) \end{bmatrix}
 \end{aligned} \quad (18b)$$

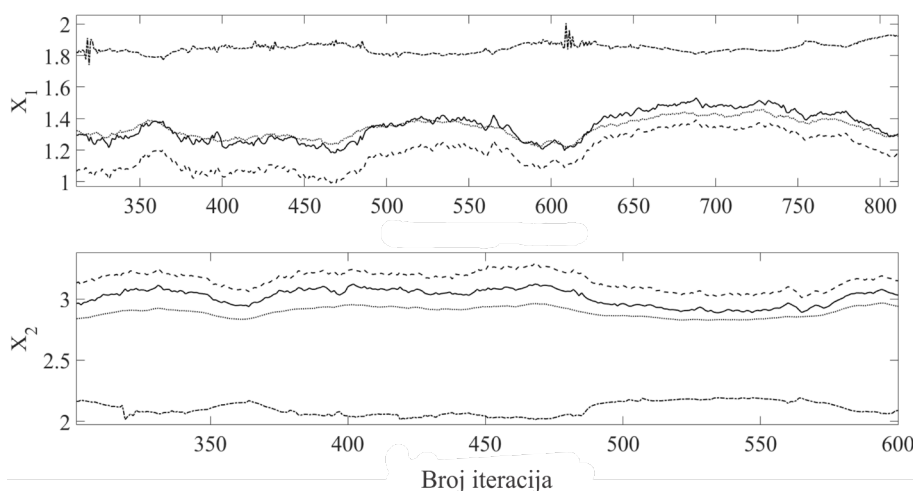
Karakteristike predloženog robusnog algoritma razmatrane su na modelu čiji vremenski promenljiv vektor parametara ima očekivanu vrednost  $\bar{\theta} = [0.96 \quad -1.88 \quad 2.23]^T$ . Šum procesa  $w(k)$  je beli šum sa nultom srednjom vrednošću i sa kovarijacionom matricom  $Q(k) = \text{diag}(0.0015, 0.001)$ . Kovarijaciona matrica parametara je data kao  $\Phi(k) = \text{diag}(0.003, 0.003, 0.02)$ . Negausova raspodela šuma merenja je data kao [13]:

$$\mathcal{P}_\varepsilon = \left\{ \begin{aligned} p(v_1) &= (1 - \varepsilon_1) \cdot \mathcal{N}(0; 0.005) + \varepsilon_1 \cdot \mathcal{N}(0; 0.5), \\ p(v_2) &= (1 - \varepsilon_2) \cdot \mathcal{N}(0; 0.01) + \varepsilon_2 \cdot \mathcal{N}(0; 1) \end{aligned} \right\}. \quad (19)$$

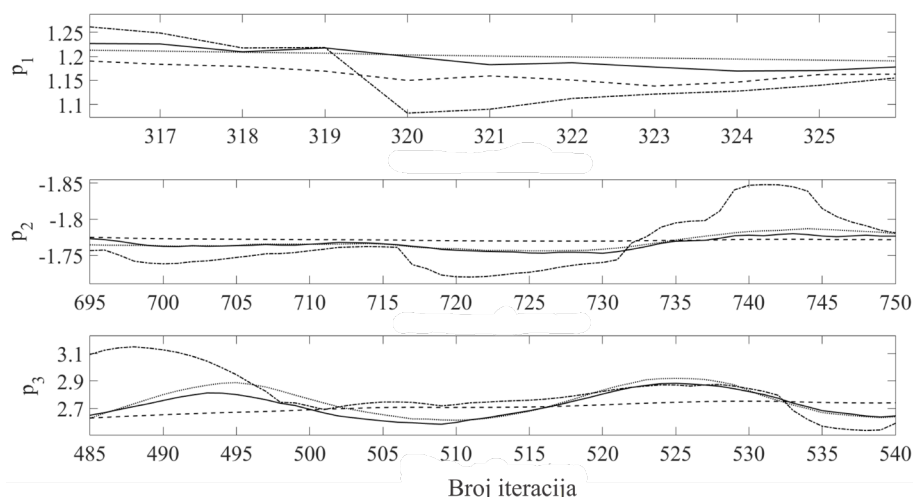
Izlazi sistema kao i procene stanja i parametara, u slučaju kada kontaminacije imaju vrednosti  $\varepsilon_1 = \varepsilon_2 = 0.1$  prikazane su na Slikama 3 do 5.



Slika 3: Izlazni signali sistema



Slika 4: Procene stanja (puna linija: Predloženi robusni algoritam procene stanja i parametara, isprekidana linija: Algoritam za procenu stanja i parametara zasnovan na Masreliez-Martinovom filtru, Crta-tačka: Algoritam za procenu stanja i parametara zasnovan na Kalmanovom filtru, tačkasta linija: Tačne vrednosti stanja).

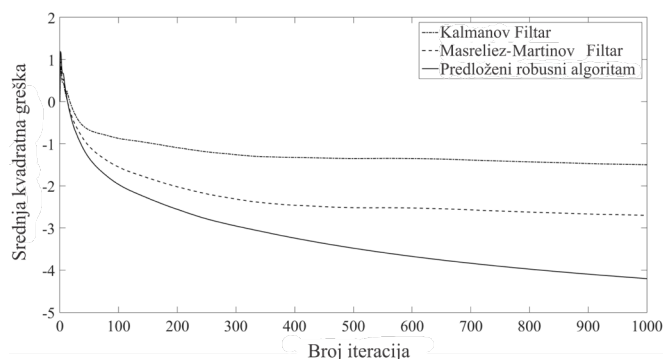


Slika 5: Procene parametara (puna linija: Predloženi robusni algoritam procene stanja i parametara, isprekidana linija: Algoritam za procenu stanja i parametara zasnovan na Masreliez-Martinovom filtru, Crta-tačka: Algoritam za procenu stanja i parametara zasnovan na Kalmanovom filtru, tačkasta linija: Tačne vrednosti parametara).

U cilju ilustrovanja kvaliteta procene stanja i parametara, koristi se srednja kvadratna greška (MSE) na sledeći način:

$$MSE = \ln \left( E \left\| \hat{z}(k) - z(k) \right\|^2 \right). \quad (20)$$

Srednje kvadratne greške za razmatrane algoritme prikazane su na Slici 6.



Slika 6: Srednje kvadratne greške

Prikazani rezultati su pokazali da je algoritam procene stanja i parametara zasnovan na široko korišćenom Kalman filteru veoma osetljiv na prisustvo negausovih šumova, za razliku od predloženog robusnog algoritma procene stanja i parametara.

Da bi se pokazala robusnost predloženog robusnog algoritma za procenu stanja i parametara u odnosu na druge konvencionalne i široko korišćene algoritme za procenu stanja i parametara, algoritmi su pokrenuti nezavisno po 30 puta, za različite stepene kontaminacije. Na osnovu 1000 iteracija, srednje, najbolje, najgore vrednosti i varijanse su prikazane u Tabeli 1.

Iz Tabele 1 može se videti da su čak i najgori rezultati, koji su dobijeni predloženim robusnim algoritmom procene stanja i parametara (RKF) bolji od najboljih rezultata dobijenih algoritmom procene stanja i parametara zasnovane na Masreliez-Martinovom filtru (MMF), kao i

na algoritmu procene stanja i parametara zasnovanom na Kalmanovom filtru (KF), pri određenom stepenu kontaminacije. Štaviše, iz Tabele 1 može se jasno videti da je superiornost predloženog robusnog algoritma (RKF) veća pri višim stepenima kontaminacije. Takođe, može se pokazati da je RKF, iz Tabele 1, robusniji od Masreliez-Martinovog algoritma.

Algoritmi procene	Srednja vrednost	Najbolja	Najgora	Varijansa
<b>Slučaj I</b> <i>Stepen kontaminacije <math>\varepsilon=0.05</math></i>				
KF	-1.614	-1.808	-1.460	0.010
MMF	-2.854	-3.050	-2.641	0.008
RKF	-4.215	-4.351	-4.001	0.007
<b>Slučaj II</b> <i>Stepen kontaminacije <math>\varepsilon=0.1</math></i>				
KF	-1.121	-1.327	-0.912	0.011
MMF	-2.230	-2.382	-2.049	0.009
RKF	-3.854	-4.023	-3.734	0.007
<b>Slučaj III</b> <i>Stepen kontaminacije <math>\varepsilon=0.2</math></i>				
KF	-0.715	-0.912	-0.371	0.011
MMF	-2.989	-3.228	-2.835	0.009
RKF	-3.641	-3.798	-3.396	0.008

Tabela 1: Srednje kvadratne greške pri različitim stepenima kontaminacije.

## IV. ZAKLJUČAK

Predloženi su robusni algoritmi procene stanja i parametara stohastičkih, linearnih, vremenski promenljivih sistema, u prisustvu negausovih šumova. Predloženi algoritam je korišćen za rešavanje problema procene stanja i parametara linearnih, stohastičkih modela gde konvencionalni pristupi ne uspeavaju. Projektovani estimator uzima u obzir i robusnost na šumove i osetljivost na otkaze senzora i komponenti. Zbog svojih dobrih osobina u robusnom filtriranju, modifikovani MMF je upotrebljen kao osnova za formulisanje robusnog estimatora procene stanja i parametara linearnih, stohastičkih sistema.

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# Robust Identification of Linear State-space Models in Presence of Component and Sensor Faults

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*Joint estimation of states and time-varying parameters of linear state space models is of practical importance for fault diagnosis and fault tolerant control. This paper considers robust identification of linear state-space models with component and sensor faults. On the other side, previous works on this topic have not considered joint estimation of linear systems in presence of outliers. They can significantly make worse the properties of linearly recursive algorithms, which are designed to work in the presence of Gaussian noises. Because of their good features in robust filtering, the modified Masreliez-Martin filter represents a cornerstone for realization of the robust algorithm for joint state-parameter estimation of linear time-varying stochastic systems in presence of non-Gaussian noises. The good features of the proposed robust algorithm for joint estimation of linear time-varying stochastic systems is illustrated by simulations.*

**Keywords:** linear state-space models, joint estimation, robust identification, time-varying parameters, non-Gaussian noises

## 1. INTRODUCTION

It is well known that it is very difficult to determine a large number of physical parameters which are integral part of complex systems. Despite the fact that many system parameters are available with some reasonable accuracy, a large number of parameters are known within a certain range, while some parameters are entirely unknown because manufacturers consider these data as proprietary information [1]. For example, precise determination of system parameters such as dimensions of certain components, leakage coefficients, friction coefficients, as well as static and dynamic friction forces due to impossibility of direct measurement or calculation causes great difficulty in control of pneumatic actuators. Precise knowledge of system parameters and states is crucial for successful realization of many control techniques. Also, states filtering as well as parameters estimation can be key factors for performances, stability and accuracy of the systems.

Since Rudolf Kalman published his famous paper [2], the Kalman filter (KF) has become the basis of many estimation processes in different application areas. In recent years, KF has encountered renewed interest, due to an increasing range of applications [3-6]. Precise knowledge of system parameters and states is crucial for successful realization of many control techniques. Many modern engineering applications such as autonomous vehicles [7], strain prediction for fatigue [8] or robotic manipulation tasks [9] require real-time Kalman filtering framework with linear models.

It is usually too expensive to measure directly the system states. Self-applied state estimation methods assume that the system parameters are constant. In the real world, these parameters always change (e.g. friction coefficients, temperature, pressure, or flow). The states

estimation procedure with constant parameters will result in large errors when changing parameters. It's also known that the dynamic behaviour of complex systems is usually described by a linear stochastic state space model with time-varying parameters [10]. Therefore, methods by which parameter and state estimation can be obtained at the same time are required.

Joint estimation is of great practical importance for fault diagnosis and fault tolerant control. One of the biggest challenges in the design of flight control systems is a requirement for the flight of the aircraft to recover safely from structural damage and/or system faults. Regardless of whether the aircraft is equipped with a special control reconfiguration capability, reliable fault diagnostic information is extremely important to the pilot. The main challenge is the detection and isolation of incipient faults in the presence of modeling uncertainty and noise [11]. Inclusion of unknown parameters in the state vector allows easy implementation of the estimation algorithm, because the problem of parameters estimation in this case is solved using the standard filtering theory. Also, thus derived recursive estimation algorithm enables both offline and online realization.

On the other side, there is no such solution available for linear systems in presence of non-Gaussian measurements. The presence of outliers can destroy the good features of linearly recursive algorithms which are designed for estimation in the presence of Gaussian noises. Namely, the known fact is that the measurements have inconsistent observations with the largest part of the observation population (outliers). Their presence can significantly destroy good characteristics of recursive algorithms assuming the presence of Gaussian noises. Therefore, it is very important to design a robust algorithm which would be a little sensitive to outliers.

Huber's theory of robust statistics is crucial for the algorithm design whose robustness is achieved by introducing a nonlinear transformation of prediction error (Huber's function), [12]. Hence, Huber's theory of robust statistics is crucial for the robust algorithm design [13].

This paper proposes the strategy to estimate the state and parameter jointly. The state estimation and parameter identification are united by the joint parameter and state estimation. It is completely natural to put unknown, generally time-varying, parameters in the vector of states, after which the problem is reduced to the classic filtering problem. It is considered joint robust estimation of a linear stochastic systems with parameter-independent matrices. In this case the extended system remains linear, so Masreliez-Martin filter can be a cornerstone for joint state-parameter robust estimation.

The designed estimator considers both robustness against noises and sensitivity to component and sensor faults. The good features of the proposed robust algorithm is illustrated through simulations.

## 2. JOINT ESTIMATION ALGORITHM IN PRESENCE OF COMPONENT AND SENSOR FAULTS

The class of systems considered in this paper for joint estimation of states and time-varying parameters is in the form of:

$$x(k+1) = A(k)x(k) + B(k)u(k) + C(k)\theta(k) + w(k) \quad (1)$$

$$y(k) = D(k)x(k) + \Gamma(k)\theta(k) + e(k) \quad (2)$$

where  $x(k) \in R^n$  and  $\theta(k) \in R^p$  are unknown state and parameter vectors, respectively. When the system has all possible sensor and component faults (this is the most common situation), the system model is described as (1)-(2). Component faults and sensor faults are described mathematically as follows, see Figure 1:

$$f_C(k) = C(k)\theta(k) \quad (3)$$

$$f_S(k) = \Gamma(k)\theta(k) \quad (4)$$

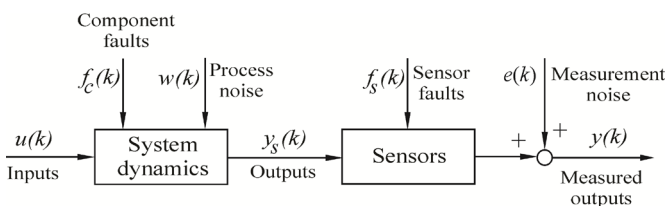


Figure 1: Open-loop system with component and sensor faults

The component fault represents the case when some condition changes in the system, rendering the dynamic relation invalid, for example a leak in a pneumatic cylinder [14]. Generally speaking, the actual outputs of the system  $y_S(k) = D(k)x(k)$  are not directly accessible, and sensors are then used to measure the system output.

Sensors are the most important components for flight control and aircraft safety due to its roles in flight control and navigation. Any sensor fault must be detected as early as possible to prevent serious accident. To diagnose incipient faults, a fault diagnosis systems have to be made robust against modeling uncertainty and noise [15]. The general form of parameters changing of the stochastic linear system is  $\theta(k+1) = G\theta(k) + \eta(k)$  in which  $G$  is a

priori known nonsingular matrix which is convenient for inclusion of a priori information on the phenomenon which is identified. The stochastic process  $\eta(k)$  is zero-mean white noise with covariance matrix  $\Phi(k)$ . Input and measured output vector of the system are  $u(k) \in R^m$  and  $y(k) \in R^r$ , respectively, while  $A(k)$ ,  $B(k)$ ,  $C(k)$ ,  $D(k)$  and  $\Gamma(k)$  are known, in general case, time-varying matrices with appropriate dimensions. It is assumed that the process noise is zero-mean Gaussian white noise  $w(k): \mathcal{N}(0, Q(k))$ , in which  $Q(k)$  is the covariance matrix. The measurement noise  $e(k)$  has non-Gaussian distribution with approximately normal distribution classes:

$$\mathcal{P}_\varepsilon = \{p(e) : p(e) = (1-\varepsilon)p_1(e) + \varepsilon p_2(e)\} \quad (5)$$

in which the probability density  $p(e)$  represents a mixture of primary probability density  $p_1(e) : \mathcal{N}(0, R_1(k))$  and contaminating probability density  $p_2(e) : \mathcal{N}(0, R_2(k))$  where contamination degree  $\varepsilon$  is in range  $0 < \varepsilon < 1$ , while  $R_1(k)$  and  $R_2(k)$  are covariance matrices of primary and contaminating term in non-Gaussian distribution (5), respectively.

An obviously easy approach to the joint estimation of  $x(k)$  and  $\theta(k)$  for system (1)-(2) is to consider the extended system:

$$z(k+1) = \begin{bmatrix} A(k) & C(k) \\ 0 & I \end{bmatrix} z(k) + \begin{bmatrix} B(k) \\ 0 \end{bmatrix} u(k) + \xi(k) \quad (6)$$

$$y(k) = [D(k) \quad \Gamma(k)] z(k) + e(k) \quad (7)$$

or in a more compact form:

$$z(k+1) = F(k)z(k) + \bar{B}(k)u(k) + \xi(k) \quad (8)$$

$$y(k) = H(k)z(k) + e(k) \quad (9)$$

in which block matrices are  $H(k) = [D(k) \quad \Gamma(k)]$ ,

$$\bar{B}(k) = \begin{bmatrix} B(k)^T & 0^T \end{bmatrix}^T,$$

$$F(k) = \begin{bmatrix} A(k) & C(k) \\ 0 & I \end{bmatrix}, \quad \text{extended state vector is}$$

$$z(k) = \begin{bmatrix} x(k)^T & \theta(k)^T \end{bmatrix}^T \quad \text{and}$$

$\xi(k) = [w^T(k) \quad \eta^T(k)]^T$  denotes extended disturbance vector, where  $\xi(k) : \mathcal{N}(0, \Xi(k))$  with  $\Xi(k) = \text{diag}(Q(k), \Phi(k))$ .

Using the joint state-parameter formulation, a unified estimation of states and parameters has been achieved. Since extended system (8)-(9) is still a linear system, the Masreliez-Martin filter [16] is applicable to the joint estimation of  $x(k)$  and  $\theta(k)$ . It is completely natural to put the parameters in the vector of states, after which the problem is reduced to the classic filtering problem of  $k$ -th order unified system (8)-(9) (where  $k = n + p$ , in which  $n$  is a number of estimated states and  $p$  is a number of estimated parameters).



Masrelicz and Martin have proposed the robust Kalman filter for the mentioned situation [17]. This filter has small sensitivity to the presence of outliers in comparison with the standard Kalman filter deduced for the case when the values  $w(\cdot)$  and  $v(\cdot)$  have Gaussian distribution.

Our goal is to derive the robust algorithm for joint state and parameter estimation of stochastic linear systems in the presence of outliers which maintains a low sensitivity in appearance of outliers. For the class of  $\varepsilon$ -contaminated distributions of probabilities, the nonlinear transformation of prediction error  $\psi(\cdot)$  (Huber's function), is obtained as:

$$\psi(v(k)) = \min\{|v(k), k_\varepsilon|\} \operatorname{sgn}(v(k)), \quad (10)$$

and its derivative:

$$\psi'(v(k)) = \begin{cases} 1 & |v(k)| < k_\varepsilon, \\ 0 & \text{otherwise.} \end{cases} \quad (11)$$

in which  $k_\varepsilon$  is appropriately defined parameter of Huber's function, see Figure 2.

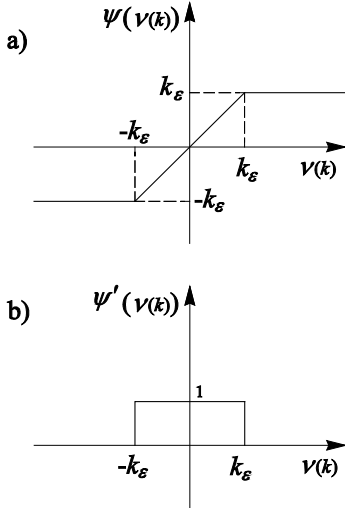


Figure 2: Huber's function and its derivative

The originally proposed Masrelicz-Martin filter include the member in the a posteriori covariance matrix  $E_{P_\varepsilon} \{\psi'[v(k)]\}$  which is not easy to determine in practical conditions [16]. In order to improve its applicability, the realization of  $\psi'[v(k)]$  is introduced instead the member  $E_{P_\varepsilon} \{\psi'[v(k)]\}$ . Intense simulations justified such interventions. It can be shown that the mathematical expectation and the covariance matrix of transformed residuals  $\varepsilon(k) = y(k) - H(k)\hat{x}(k-1|k-1)$  are:

$$E\{v(k)\} = T(k)E\{\varepsilon(k)\} = 0 \quad (12)$$

$$E\{v^2(k)\} = T^2(k) \left[ H(k)P(k|k-1)H^T(k) + R_1(k) \right]^{-\frac{1}{2}} \quad (13)$$

Therefore, in the case that transformation  $T(k)$  is:

$$T(k) = \left[ H(k)P(k|k-1)H^T(k) + R_1(k) \right]^{-1/2}, \quad (14)$$

the covariance matrix of the transformed residuals will be  $I$ .

It is important to emphasize that this modification rises the convergence rate of so modified robust filter in initial iterations. Namely, the robust Kalman filter gain is given by:

$$K(k) = F(k-1) \left[ P(k|k-1) - P(k|k-1)H(k)^T \right. \\ \left. T(k)^2 H(k)^T P(k|k-1) \psi'[\varepsilon(k)] \right] \cdot \\ \cdot F(k-1)^T H(k)^T T(k) + Q(k-1)H(k)^T T(k) \quad (15)$$

If  $|v(k)| > k_\varepsilon$ , the relation (15) becomes:

$$K(k) = F(k-1)P(k|k-1)F(k-1)^T \cdot \\ \cdot H(k)^T T(k) + Q(k-1)H(k)^T T(k) \quad (16)$$

because  $\psi'[v(k)] = 0$  when  $|v(k)| > k_\varepsilon$  (see Figure 2).

It is easy to conclude that the gain  $K(k)$  from (16) is higher than  $K(k)$  from relation (15). It means that the bigger estimation errors cause the higher filter gain which consequently leads to the higher convergence rate.

Because of its good features in robust filtering, such modified Masrelicz-Martin filter is used as a basis in formulating the joint state and parameter estimator of linear stochastic systems, as follows:

$$\begin{aligned} \hat{z}(k|k-1) &= F(k-1)\hat{z}(k-1|k-1) + \bar{B}(k-1)u(k-1) \\ P(k|k-1) &= F(k-1)P(k-1|k-1)F^T(k-1) + \Xi(k-1) \\ K(k) &= [N(k) \ ; \ M(k)]^T = P^T(k|k-1)H^T(k)T^T(k) \\ v(k) &= T(k)[y(k) - H(k)\hat{x}(k-1|k-1)] \\ \hat{x}(k|k) &= A(k-1)\hat{x}(k-1|k-1) + B(k-1)u(k-1) + \\ &+ C(k-1)\hat{\theta}(k-1|k-1) + N(k)\Psi(v(k)) \\ \hat{\theta}(k|k) &= \hat{\theta}(k-1|k-1) + M(k)\Psi(v(k)) \\ P(k|k) &= P(k|k-1) - K(k)\Psi'(v(k))K^T(k) \\ \Psi'(v(k)) &= \operatorname{diag}(\psi'(v_1(k)), \dots, \psi'(v_r(k))) \\ T(k) &= \left[ H(k)P(k|k-1)H^T(k) + R_1(k) \right]^{-\frac{1}{2}} \end{aligned} \quad (17)$$

with initial conditions:  $\hat{z}_0 = 0$  and  $P_0 = \begin{bmatrix} P(x_0) & 0 \\ 0 & P(\theta_0) \end{bmatrix}$ .

In this way, the robust algorithm for the joint states-parameters estimation of linear stochastic systems has been derived.

### 3. SIMULATION RESULTS

Determination of exact values of state and parameter vectors presents a basic condition for a high-quality model design, which causes better control performances. The benefits of the proposed robust algorithms for joint estimation of stochastic linear time-varying systems are illustrated through intensive simulations. These results demonstrate superiority of the proposed robust algorithm (RKF) in relation to the joint estimation algorithms based on widely used Kalman filter (KF) and Masreliez-Martin filter (MMF). Behavior of the algorithms will be considered on:

$$\begin{aligned} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} &= \begin{bmatrix} 0.9 & 0 \\ 0 & 0.8 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \\ &+ \begin{bmatrix} 0.1 & 1 & 0 \\ 0.5 & 0.3 & 1 \end{bmatrix} \begin{bmatrix} p_1(k) \\ p_2(k) \\ p_3(k) \end{bmatrix} + \begin{bmatrix} w_1(k) \\ w_2(k) \end{bmatrix} \end{aligned} \quad (18a)$$

$$\begin{aligned} \begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \\ &+ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_1(k) \\ p_2(k) \\ p_3(k) \end{bmatrix} + \begin{bmatrix} e_1(k) \\ e_2(k) \end{bmatrix} \end{aligned} \quad (18b)$$

The features of the proposed robust algorithm are considered on the model whose time-varying parameter vector has expected value  $\bar{\theta} = [0.96 \quad -1.88 \quad 2.23]^T$ . The process noise  $w(k)$  is zero-mean white noise with covariance matrix  $Q(k) = \text{diag}(0.0015, 0.001)$ . The covariance matrix of parameters is given by  $\Phi(k) = \text{diag}(0.003, 0.003, 0.02)$ . The non-Gaussian distribution of the measured noise is given by [13]:

$$\mathcal{P}_\varepsilon = \left\{ \begin{aligned} p(v_1) &= (1 - \varepsilon_1) \cdot \mathcal{N}(0; 0.005) + \varepsilon_1 \cdot \mathcal{N}(0; 0.5), \\ p(v_2) &= (1 - \varepsilon_2) \cdot \mathcal{N}(0; 0.01) + \varepsilon_2 \cdot \mathcal{N}(0; 1) \end{aligned} \right\}. \quad (19)$$

The system outputs as well as estimates of states and parameters, in the case when contaminations have values  $\varepsilon_1 = \varepsilon_2 = 0.1$  are shown on Figures 3 to 5.

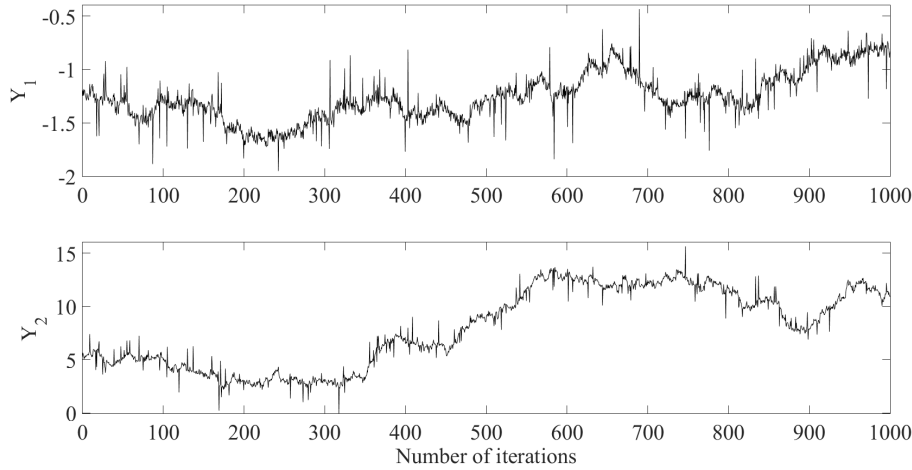


Figure 3: System output signals

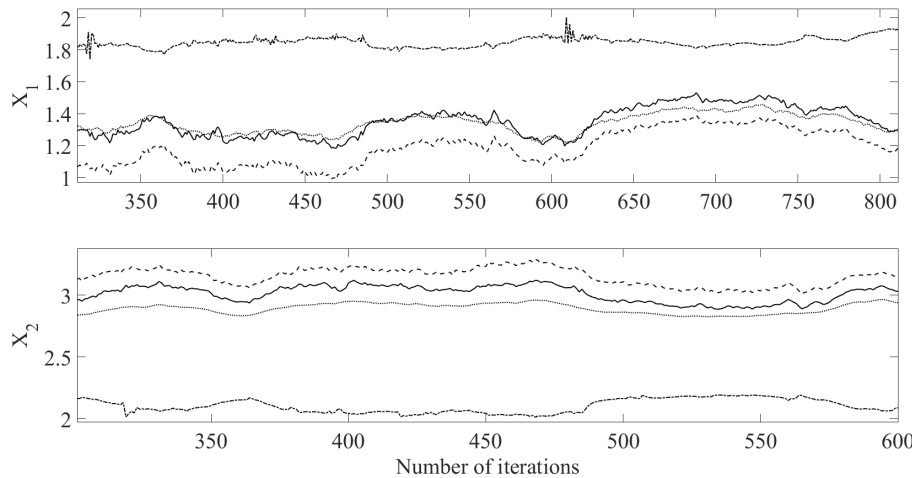


Figure 4: States estimates (solid line: Proposed joint robust algorithm, dashed line: Joint algorithm based on Masreliez-Martin filter, dash-dot: Joint algorithm based on Kalman filter, dotted line: True values of states).

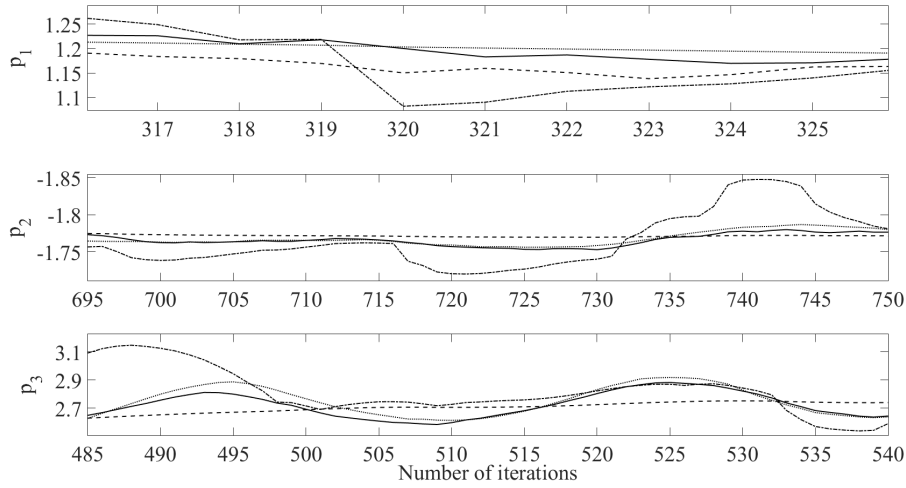


Figure 5: Parameters estimates (solid line: Proposed joint robust algorithm, dashed line: Joint algorithm based on Masreliez-Martin filter, dash-dot: Joint algorithm based on Kalman filter, dotted line: True values of parameters).

For the purpose of illustrating estimation quality, mean square error (MSE) is used as follows:

$$MSE = \ln \left( E \left\| \hat{z}(k) - z(k) \right\|^2 \right). \quad (20)$$

Mean square errors for considered algorithms are shown in Figure 6.

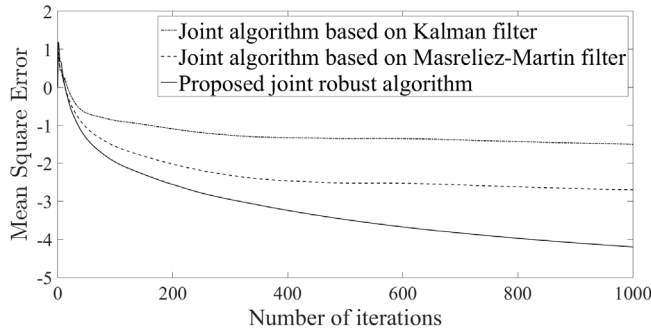


Figure 6: Mean square errors

The presented results have shown that the joint estimation algorithm based on widely-used Kalman filter is very sensitive to the presence of non-Gaussian noises, as opposed to the proposed robust joint algorithm.

In order to show robustness of the proposed robust joint algorithm in relation to the other conventional and widely-used joint estimation algorithms, the algorithms are run 30 times independently, for different contamination degrees. Based on 1000 iterations, the mean, best, worst, and variance values are shown in Table 1.

From Table 1, it can be seen that the worst results obtained by proposed robust joint estimation algorithm (RKF) is even better than the best result obtained by joint algorithm based on Masreliez-Martin filter (MMF) as well as joint algorithm based on Kalman filter (KF), at a certain contamination degree. Furthermore, from Table 1, it can be clearly seen that the superiority of the proposed robust algorithm (RKF) is greater in higher degrees of contamination. Also, it can be shown that RKF, from

Table 1, is more robust than Masreliez-Martin based algorithm.

Table 1: Mean square errors for different degrees of contaminations.

Joint estimation algorithms	Mean	Best	Worst	Variance
<b>Case I</b> Contamination degree $\varepsilon=0.05$				
KF	-1.614	-1.808	-1.460	0.010
MMF	-2.854	-3.050	-2.641	0.008
RKF	-4.215	-4.351	-4.001	0.007
<b>Case II</b> Contamination degree $\varepsilon=0.1$				
KF	-1.121	-1.327	-0.912	0.011
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RKF	-3.854	-4.023	-3.734	0.007
<b>Case III</b> Contamination degree $\varepsilon=0.2$				
KF	-0.715	-0.912	-0.371	0.011
MMF	-2.989	-3.228	-2.835	0.009
RKF	-3.641	-3.798	-3.396	0.008

## 4. CONCLUSION

The joint state and parameter robust estimation algorithms for stochastic linear time-varying systems, in presence of non-Gaussian noises have been proposed. The proposed algorithm has been used to solve the joint estimation problem of linear stochastic models where the conventional approaches fails. The designed estimator consider both robustness against noises and sensitivity to component and sensor faults. Because of their good features in robust filtering, the modified MMF was used as a basis in formulating the joint robust estimator of linear stochastic systems.

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