

Proširenje frekvencijskog opsega za eksperimentalno određivanje disperzione relacija talasa savijanja u štapovima primenom korelacione metode

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Disperziona relacija je ključ za proučavanje prostiranja talasa. Eksperimentalno određivanje disperzione relacije merenjem talasnih svojstava u različitim tačkama u prostoru otežava prostorni alijasing, koji pogoršava numerička nestabilnost proračuna. Ovaj rad predstavlja i razmatra koncept čiji je cilj prevazilaženje problema prostornog alijasinga u merenjima disperzione relacije mehaničkih talasa koji se šire kroz šipku. Koncept se zasniva na svojstvima disperzione relacije i može se proširiti na sve ostale slučajeve talasa sa monotonom disperzionom vezom.

Ključne reči: Mehanički talas, Disperziona relacija, Frekvencijski opseg

1. UVOD

1.1. Disperziona relacija

Disperziona relacija (u nastavku teksta "DR") je odnos između talasnog broja talasa k (ili njegove talasne dužine λ) i njegove ugaone frekvencije ω (ili frekvencija f , ili čak perioda T). Pošto se fazna brzina talasa c može izraziti kao $c = \omega/k$, a grupna brzina talasa se može izraziti kao $v = d\omega/dk$, DR omogućava izračunavanje faze i grupne brzine talasa, što predstavlja glavni alat za izračunavanje širine talasa. Nedavna istraživanja prostiranja talasa kroz 3D štampane periodične strukture [1], predstavljaju primer sa-vmene primene DR.

DR može biti izvedena i izražena u eksplicitnom obliku za prostiranje talasa kroz strukture jednostavne geometrije (kao što su slobodan prostor, štapove, ploče, itd.), a izrazi uključuju viskoelastična (npr., modul elastičnosti i faktor gubitaka materijala) i inercijalna (npr., gustina) svojstva materijala. Iz tog razloga, eksperimentalno određivanje DR se ponekad koristi i za izračunavanje elastičnih svojstava materijala, naročito za merenje faktora gubitka materijala, koji je inače teško izmeriti [2][3].

Kao naročito jednostavna geometrija, koja ograničava prostiranje talasa na jednu dimenziju, tanki štapovi predstavljaju najpogodniji objekat za razvoj postupka eksperimentalnog određivanja DR i za izučavanje različitih primena DR, kao što je otkrivanje pukotina [4]. U zavisnosti od lokacije i smera pobude, kroz štapove se mogu širiti različiti tipovi mehaničkih talasa, kao što su longitudinalni talasi podužne deformacije ili torzioni talasi, koji imaju DR sa jednostavnim linearnom formom, $\omega = (E/\rho)^{1/2} \cdot k$ za longitudinalne talase, i $\omega = (G/\rho)^{1/2} \cdot k$ za torzione talase, gde je ρ gustina, E Jungov modul i G modul klizanja materijala štapa [5]. Najjednostavniji mehanički talasi za pobuđivanje i detekciju su talasi savijanja koji imaju parabolični oblik DR [5]:

$$\omega = \sqrt{\frac{EI}{\rho A}} k^2 \quad (1)$$

gde I označava moment inercije, a A površinu, poprečnog preseka štapa.

1.2. Eksperimentalno određivanje DR

Najvažniji koncept za eksperimentalno određivanje DR sastoji se u merenju odziva (obično ubrzanja) na pobudu (obično impulsnu ili harmonijsku) u više tačaka duž štapa. Odzivi u tim tačkama na određenoj frekvenciji pobude (dobijen direktnim merenjem ili spektralnom analizom) se koristi za formiranje talasnih polja sa poznatim talasnim brojevima koja imaju minimalno odstupanje od izmerenih odziva. Postoji nekoliko metoda za formiranje talasnih polja sa minimalnim odstupanjem od izmerenih odziva, koje se razlikuju se po tačnosti, matematičkoj složenosti i opsegu primenljivosti.

Jedna grupa metoda zasniva se na Pronijevoj metodi za razlaganje funkcija u niz prigušenih sinusoida, sa idejom sličnom razlaganjem na sinusoidu u Furijeovoj analizi. Kako je Pronijeva metoda osetljiva na merni šum, za povećanje stabilnosti proračuna se koristi metoda najmanjih kvadrata [6], dekompozicija pomoću singularnih vrednosti („singular value decomposition“) [7] i modifikovana Pronijeva metoda za konačne strukture [8]. Iako su ove metode efikasne, one ne koriste nikakva prethodna znanja o strukturi koja vibrira niti o talasima koji se kroz nju prostiru, i imaju dva ozbiljna nedostatka: prvo, primena je ograničena samo na ravnomerno raspoređene merne tačke, i drugo, potreban je veliki broj mernih tačaka za uspešnu primenu ovih metoda.

Druga grupe metoda je osmišljena tako da razrešava suštinske nedostatke prethodnih metoda, čak i po cenu brzine. Ideja je da se sukcesivno ispituju različita talasna polja u potrazi za skupom parametara kojim se postiže najbolja saglasnost između nekog talasnog polja i eksperimentalnih podataka. Različite metode koje pripadaju ovoj grupi razlikuju se prema izboru oblika talasnog polja, meri saglasnosti sa eksperimentalnim podacima i postupku pronalaženja optimalnih parametara talasnog polja. Neke od mogućnosti postupka su primena metode najmanjih kvadrata i različite algoritmi za nelinearnu optimizaciju [2][3].

1.3. Korelaciona metoda za eksperimentalno određivanje DR

Metoda koja je predmet ovog rada je korelaciona metoda [9], koja koristi jednostavan oblik talasnog polja, i prosto sekvencijalno pretraživanje prostora talasnih brojeva kako bi se pronašla vrednost koja pruža maksimalnu korelaciju između izmerenih podataka i talasnog polja, koja se koristi kao mera njihove saglasnosti.

U svom najjednostavnijem obliku, koji se koristi za proučavanje talasa koji se prostire duž štapova, korelaciona metoda za eksperimentalno određivanje DR (u daljem tekstu samo "korelaciona metoda") koristi talasno polje formirano na osnovu progresivnog harmonijskog talasa sa izabranim talasnim brojem k_l i ugaonom frekvencijom ω , dok uzdužna osa štapa predstavlja x -osu. Ako se progresivni harmonijski talas pobudi silom koja deluje na jednom kraju štapa, izraženom u kompleksnom obliku kao $F(t) = \underline{F}_\omega \cdot \exp(-i\omega t)$, ubrzanja tačaka na štapu usled talasnog polja mogu se iskazati kao $\underline{a}_l(x,t) = \underline{a}_\omega \cdot \exp(k_l x - i\omega t)$, a talasno polje odziva, akcelarance $\underline{w}_l(x,t) = \underline{a}_l(x,t)/F(t)$ može biti opisano njenom kompleksnom amplitudom:

$$\underline{w}_l(x, \omega) = \underline{w}_{l\omega} \cdot \exp(k_l x). \quad (2)$$

Kompleksne amplitude akcelarance talasnog polja za izabrani talasni broj k_l se upoređuju sa kompleksnim amplitudama akcelaranci $\underline{w}(x, \omega)$ izmerenim u L mernih tačaka x_l ($l = 1, 2, \dots, L$) duž štapa, a mera slaganja između dva skupa podataka je njihova korelacija, definisana izrazom:

$$\begin{aligned} \underline{W}_\omega(k_l) &= \int_{-\infty}^{+\infty} \underline{w}(x, \omega) \underline{w}_l^*(x, \omega) dx = \\ &= \underline{w}_{l\omega}^* \int_{-\infty}^{+\infty} \underline{w}(x, \omega) e^{-ik_l x} dx \end{aligned} \quad (3)$$

što se, zbog diskretne prirode ulaznih podataka, ocenjuje kao:

$$\hat{W}_\omega(k_l) = \underline{w}_{l\omega}^* \sum_{l=1}^L \underline{w}(x_l, \omega) e^{-ik_l x_l} \Delta x_l = \underline{w}_{l\omega}^* \sum_{l=1}^L \underline{w}_{l\omega} e^{-ik_l x_l} \Delta x_l \quad (4)$$

Gde Δx_l predstavlja prostorni interval oko tačke x_l , uvodeći notaciju $\underline{w}(x_l, \omega) = \underline{w}_{l\omega}$. Izraz za korelaciju dva skupa podataka dat jednačinom (3) navodi na zaključak da će, ako je talas koji se širi duž štapa progresivan harmonijski talas, moduli procenjene korelacije imati maksimalnu vrednost kada je talasni broj formiranog talasnog polja k_l jednak talasnom broju prostiranja talasa k . Da bi se odredila DR talasa koji se širi duž štapa, talasni broj se traži u opsegu $k_{min} < k_l < k_{max}$ za svaku ugaonu frekvenciju ω , a vrednost k_l koja dovodi do maksimalnog modula procenjene korelacije usvaja se kao talasni broj koji odgovara ugaonoj frekvenciji ω , označeno kao $k(\omega)$. S obzirom da je korelacija proporcionalna kompleksnom faktoru amplitude $\underline{w}_{l\omega}^*$, koji ne utiče na položaj maksimuma procenjene korelacije prema jednačini (3), DR se može odrediti pronalazanjem talasnih brojeva koji maksimiziraju izraz:

$$Y_\omega = \left| \sum_{l=1}^L \underline{w}_{l\omega} \exp(-ik_l x_l) \Delta x_l \right|, \quad (5)$$

koja će se u daljem tekstu nazivati "korelaciona funkcija". Kao što se može videti, korelaciona metoda ne uvodi nikakvu pretpostavku, niti nameće bilo kakvo ograničenje

na broj i položaj tačaka merenja x_l , što, zajedno sa jednostavnošću, predstavlja osnovnu prednost metode. U slučaju da su merne tačke jednako udaljene $x_l = l \cdot d$, korelaciona funkcija poprima oblik:

$$Y_\omega = \left| \sum_{l=1}^L \underline{w}_{l\omega} \exp(-i \cdot l \cdot d \cdot k_l) \right|, \quad (6)$$

gde je sa d označeno je rastojanje između mernih tačaka.

Metoda je uspešno korišćena za određivanje DR u čeličnim i kompozitnim pločama [9], a kasnija modifikacija metode [10], koja koristi prigušene ravanske talase za formiranje ispitivanih talasnih polja, uspešno se koristi kod materijala sa velikim gubicima u materijalu.

1.4. Problem i motivacija istraživanja

Primena korelacione metode se susreće sa dva problema koji su posledica a) konačnog broja mernih tačaka, tj. posledica diskretizacije jednačine (3) u prostoru; b) konačan broj ispitivanih talasnih brojeva, tj. diskretizacija jednačine (3) u inverznom prostoru (prostoru talasnih brojeva).

a) Diskretizacija jednačine (3) u prostoru koristi jednačine (4)-(6) koje dovode do „efekta ograde“ („picket-fence effect“), odnosno pojava da se dva signala ne mogu razlikovati ako se uzorkuju u konačnom broju mernih tačaka gde imaju jednake vrednosti. Sveprisutan u diskretnoj analizi podataka [11], efekat ograde je najpoznatiji kao uzrok alijasinga u frekvencijskoj analizi. U ovom slučaju, efekat "ograde" znači da za bilo koju vrednost talasnog broja k_l , postoji beskonačan broj talasnih brojeva $k_l' \neq k_l$ takvih da za svaku mernu tačku x_l važi:

$$\exp(-ik_l' x_l) = \exp(-ik_l x_l) \quad (l = 1, 2, \dots, L), \quad (7)$$

Usled toga, talasna polja sa kompleksnim amplitudama datim jednačinom (2), formirana korišćenjem talasnih brojeva k_l i k_l' se ne razlikuju u mernim tačkama, i zato imaju jednake ocene korelacije sa eksperimentalnim podacima. To dalje znači da u intervalu $k_l^{min} < k_l < k_l^{max}$ može biti nekoliko vrednosti talasnih brojeva koji imaju jednaku korelaciju sa mernim podacima kao i bilo koji odabrani k_l , odnosno, unutar datog intervala inverznog prostora može biti više od jednog lokalnog maksimuma sa istom vrednošću korelacione funkcije. Jedan od načina da se obezbede tačni rezultati korelacione metode je sužavanje opsega ispitivanja talasnih brojeva, što znači sužavanje odgovarajućeg frekvencijskog opsega za koji se određuje DR.

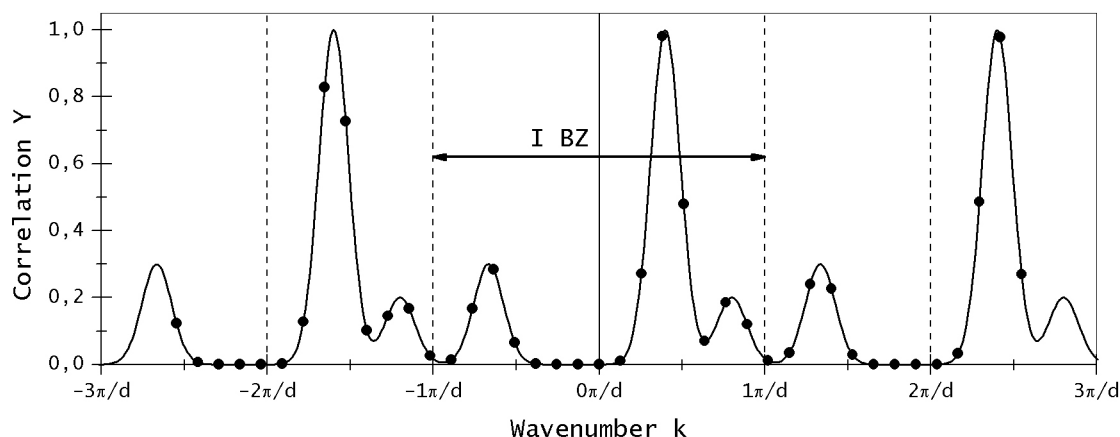
Posledice „efekta ograde“ su jasnije kada su merne tačke jednako udaljene jer je tada jednačina (7) zadovoljena za svaku mernu tačku ako je $k_l' - k_l = z(2\pi/d)$, gde je z ceo broj. Prema tome, korelacija između rezultata merenja u jednako udaljenim mernih tačkama i formiranog talasnog polja, definisana korelacionom funkcijom (6), je u tom slučaju periodična u inverznom prostoru, što prikazuje puna linija grafikona na Slici 1. Broj jednakih lokalnih maksimuma u opsegu $k_l^{min} < k_l < k_l^{max}$ je $(k_l^{max} - k_l^{min})d/2\pi$. Budući da opseg za probne vrednosti talasnih brojeva treba da opisuje talase koji se šire u oba smera štapa (pozitivne i negativne vrednosti talasnog broja), direktna primena korelacione metode je svedena na opseg inverznog prostora $-\pi/d < k_l < \pi/d$, koji je u teoriji prostiranja talasa u periodičnim strukturama, poznat kao prva Briluenova zona (skraćeno kao BZ). Primenom jednačine (1), ograničenje opsega probnih talasnih brojeva

na I BZ smanjuje frekvencijski opseg za određivanje DR na:

$$f < f_{BZ} = \frac{1}{2\pi} \sqrt{\frac{EI}{\rho A}} \left(\frac{\pi}{d} \right)^2 = \frac{\pi}{2d^2} \sqrt{\frac{EI}{\rho A}} \quad (8)$$

Ako merne tačke nisu na jednakim rastojanjima, tada je korelaciona funkcija neperiodična u inverznom prostoru, ali sistem jednačina (7) još uvek ima beskonačno mnogo rešenja za bilo koju izabranu vrednost k_r , a prosečna razlika između rešenja je bliska $(2\pi/d)$, gde je d prosečna udaljenost između mernih tačaka [13]. Problem je donekle sličan problemu aproksimacije niza N mernih tačaka polinomom, budući da postoji neograničen broj polinoma (sa redom većim od $N-1$) koji imaju vrednosti jednake izmerenim podacima u mernim tačkama.

b) Diskretizacija jednačine (3) u inverznom prostoru nastaje tokom procesa pretraživanja intervala $k_{min} < k_r < k_{max}$ pri traženju maksimalne korelacije: pretraživanje je diskretan proces, dobijen izračunavanjem vrednosti Y_ω korišćenjem jednačine (6) u nizu od J tačaka $k_r^{(j)}$ ($j = 1, 2, \dots, J$). Dobijeni niz tačaka $Y_\omega^{(j)} = Y_\omega(k_r^{(j)})$ ($j = 1, 2, \dots, J$) nazvaćemo "korelacioni niz" u nastavku teksta. Opisani pristup pronalaženja maksimuma korelacione funkcije susreće sa problemom nedostajanja ("preskakanja") one vrednosti talasnog broja koji maksimizira izraz (6). Kao što je objašnjeno, unutar pretraženog opsega inverznog prostora može biti više od jednog maksimuma sa istom vrednošću korelacione funkcije, pa „preskakanje“ nekog od maksimuma može dovesti do pogrešne identifikacije talasnog broja koji odgovara frekvenciji za koju se određuje DR.



Slika 1: Periodična korelaciona funkcija u slučaju jednako udaljenih mernih tačaka (linija) i korelacioni niz kao njegova aperiodična diskretizacija jednako udaljenih tačaka u inverznom prostoru (tačke)

2. METODA

DR talasa savijanja u štapovima, data jednačinom (1), je parna, neprekidna i monotona funkcija. Sa druge strane, kao što je ranije objašnjeno, korelaciona funkcija određuje talasne brojeve koji odgovaraju frekvencijama talasa, što dovodi do DR u inverznom obliku:

$$k = \pm \left(\frac{\rho A}{EI} \right)^{1/4} \sqrt{\omega}. \quad (9)$$

Ovaj oblik DR predstavlja rešenja jednačine (1) za talasni broj k , a ta jednačina ima dva rešenja, jedno pozitivno i jedno negativno. Ta dva rešenja imaju različita fizička značenja, predstavljajući progresivne talase u suprotnim smerovima, što se može videti po suprotnim znakovima odgovarajućih faznih brzina. Prema tome, ta

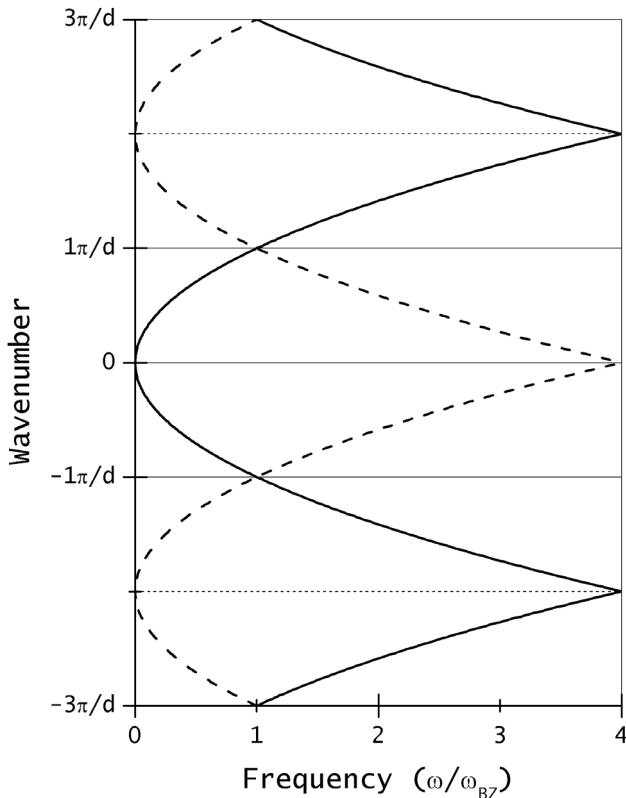
U slučaju jednako udaljenih mernih tačaka, korelacioni niz nema opisanu periodičnost sa periodom $(2\pi/d)$, kao što ilustruje grafikon niza (prikazan crnim tačkama) na Slici 1. Gubitak periodičnosti javlja se usled neizbežne greške pri digitalnom predstavljanju koja je posledica transcendentne prirode π , koji se ne može predstaviti konačnim brojem cifara u binarnom brojnem sistemu (ili bilo kojem drugom brojnem sistemu sa racionalnom osnovom). Iz tog razloga, bar jednu od vrednosti k ili $k + \pi/d$ ne možemo predstaviti u binarnom sistemu, pa niz tačaka dobijen diskretizacijom korelacione funkcije (5) ne može biti periodičan. Gubitak periodičnosti znači da se tokom pretraživanja intervala, $k_r^{min} < k_r < k_r^{max}$, može otkriti najviše jedan od ekvivalentnih lokalnih maksimuma. S obzirom da je razlika između lokalnog maksimuma i najbliže vrednosti korišćene tokom ispitivanja $k_r^{(j)}$ slučajna, to znači da vrednost $k_r^{(j)}$ koja ima maksimalnu vrednost $Y_\omega(k_r^{(j)})$ ne mora biti talasni broj koji odgovara frekvenciji ω , što je osnovna ideja korelacione metode.

Ovaj rad predstavlja algoritam za analizu vrednosti korelacionog niza $Y_\omega^{(j)} = Y_\omega(k_r^{(j)})$ koji omogućava proširenje frekvencijskog opsega za određivanje DR talasa savijanja u štapovi koristeći svojstva DR. Predložena metoda je upoređena sa sa DR određenom na osnovu maksimalnih vrednosti korelacione funkcije date jednačinom (5), i pokazano je da metoda predložena u radu može proširiti frekvencijski opseg za određivanje DR do granica frekvencijskog opsega koji se koristi u eksperimentu.

dva rešenja opisuju prenos energije u suprotnim smerovima, jer grupne brzine imaju iste znakove kao i fazne brzine. U zavisnosti od brzine talasa savijanja i faktora gubitka materijala, oba talasa se mogu primetiti tokom eksperimentalnog određivanja DR za talase savijanja u štapovima. Sa druge strane, korelaciona metoda dovodi do jedne (a ne dve) vrednosti talasnog broja koja odgovara određenoj frekvenciji, i nije moguće unapred znati predznak vrednosti talasnog broja sa najvećom vrednošću korelacije. Prema tome, interval $k_r^{min} < k_r < k_r^{max}$, koji se pretražuje kako bi se pronašao odgovarajući talasni broj mora da sadrži pozitivne i negativne vrednosti broja k_r . Zbog postojanja dva rešenja jednačine (1), svaki algoritam za primenu korelacione metode treba da se završi sa pozitivnim i negativnim vre-

dnostima talasnih brojeva odgovarajućih frekvencija, koje se dobijaju množenjem apsolutnih vrednosti određenih talasnih brojeva sa plus jedan i minus jedan.

Kao što je pokazano u uvodu, diskretna priroda skupa mernih tačaka (efekat ograde) uzrokuje nerazlikovanje vrednosti korelacione funkcije za neke veće i manje vrednosti talasnih brojeva, a u specijalnom slučaju mernih tačaka na jednakim rastojanjima, korelaciona funkcija $Y_\omega(k)$ je periodična funkcija talasnog broja. Dalja posledica efekta ograde je da postoji beskonačan broj grana zavisnosti talasnog broja od frekvencije koje imaju iste vrednosti korelacione funkcije (5) kao dve grane DR dobijene jednačinom (9), koje tražimo korelacionom metodom. Za slučaj mernih tačaka sa jednakim rastojanjima, grane ekvivalentnih maksimuma su prikazane na Slici 2. Treba naglasiti da ove grane imaju sve moguće kombinacije znakova faznih i grupnih brzina.



Slika 2: Grane talasnih brojeva sa istom vrednošću korelacione funkcije sa talasnim poljem progresivnog talasa diskretizovanom u jednako udaljenim tačkama; grane koje počinju u neparnim BZ predstavljene su punim linijama, a grane koje počinju u parnim BZ predstavljene su isprekidanim linijama

2.1. Diskretizacija korelacione funkcije i ekvivalentni maksimumi korelacione funkcije

U uvodu je objašnjeno da je osnova metode korelacije traženje vrednosti k_t koja ima maksimalnu vrednost korelacione funkcije u intervalu $k_t^{\min} < k_t < k_t^{\max}$. Dalja analiza uticaja efekta ograde na korelacionu funkciju (5), kao na Slici 2, dovela je do zaključka da za svaku frekvenciju ω korelaciona funkcija ima nekoliko jednakih maksimuma talasnog broja u intervalu $k_t^{\min} < k_t < k_t^{\max}$. U slučaju jednako udaljenih mernih tačaka, svaki od maksimuma odgovara po jednoj grani prikazanoj na

Slici 2, tako da je broj jednakih maksimuma u intervalu $k_t^{\min} < k_t < k_t^{\max}$ je ograničen na

$$K(k_t^{\max}, k_t^{\min}) \leq \left\lceil \frac{k_t^{\max} - k_t^{\min}}{\pi/d} \right\rceil + 1. \quad (10)$$

Pošto efekat ograde ima isti uticaj na korelacionu funkciju čak i ako merenja nisu na jednakima rastojanjima, jednačina (10) se takođe može koristiti i tom u slučaju za procenu maksimalnog broja maksimuma korelacione funkcije u pretraživanom opsegu talasnih brojeva.

Pošto je za implementaciju traženja maksimuma potrebna diskretizacija korelacione funkcije, koja, kako je objašnjeno u uvodu, može dovesti do gubitka ("preskakanja") nekih maksimuma, algoritam za pronalaženje odgovarajućeg maksimuma zahteva neke modifikacije. Kao što je prikazano na Slici 1, diskretizacija korelacione funkcije ne otkriva sve talasne brojeve sa maksimalnim vrednostima korelacione funkcije, ali svaki od maksimuma korelacione funkcije $Y_\omega(k)$ vodi do lokalnog maksimuma u korelacionom nizu $Y_\omega^{(j)}$. Kako lokalni maksimumi u nizu $Y_\omega^{(j)}$ koji odgovaraju maksimumima korelacione funkcije $Y_\omega(k)$ ne moraju imati jednake vrednosti, njih ćemo zvati "ekvivalentni maksimumi" u tekstu koji sledi. S obzirom da u intervalu $k_t^{\min} < k_t < k_t^{\max}$ nema više od $K(k_t^{\min}, k_t^{\max})$ maksimuma korelacione funkcije, u korelacionom nizu ne sme biti više od $K(k_t^{\min}, k_t^{\max})$ ekvivalentnih maksimuma. Međutim, korelacioni niz može imati (a u praksi ih ima mnogo) dodatnih lokalnih maksimuma koji ne odgovaraju maksimumima korelacione funkcije, pa prosta pretraga svih lokalnih maksimuma u intervalu $k_t^{\min} < k_t < k_t^{\max}$ nije dovoljna za određivanje ekvivalentnih maksimuma u intervalu. Ekvivalentni maksimumi mogu se izvući iz skupa svih lokalnih maksimuma sortiranjem skupa po veličini korelacione funkcije i korišćenjem dva njegova svojstva: 1) da ekvivalentni maksimumi imaju znatno veće vrednosti korelacione funkcije od ostalih lokalnih maksimuma, i 2) da u intervalu $k_t^{\min} < k_t < k_t^{\max}$ ne postoji više od $K(k_t^{\min}, k_t^{\max})$ maksimuma korelacione funkcije. Kada se odrede ekvivalentni maksimumi za frekvenciju ω , niz talasnih brojeva koji odgovaraju tim ekvivalentnim maksimumima, $k_\omega^{(m)}$ ($m = 1, 2, \dots, M \leq K$ – ovaj niz nazivaće se "niz kandidata" u nastavku teksta) mora da se analizira kako bi se ustanovio talasni broj koji odgovara to frekvenciji u DR. Pošto su razlike među vrednostima korelacione funkcije različitih ekvivalentnih maksimuma slučajne, izbor talasnog broja koji odgovara frekvenciji vse vrši na osnovu svojstava DR opisane jednačinom (9).

2.2. Svojstva DR

Izraz (9) predstavlja uniju dve matematičke funkcije (zване "grane kvadratnog korena") sa sledećim svojstvima: obe funkcije su definisane samo za pozitivne vrednosti frekvencije, obe su neprekidne i monotone, ali je pozitivna grana monotono rastuća, dok je negativna grana monotono opadajuća, matematička funkcija. Ova matematička svojstva se mogu koristiti za eksperimentalno određivanje DR jer predstavljaju ograničenja koja odgovarajuće vrednosti talasnih brojeva i frekvencija u DR moraju da se zadovolje.

Neprekidnost DR može se opisati ograničenjem:

$$\lim_{\omega \rightarrow \omega_0} k(\omega) \quad (11)$$

za sve pozitivne frekvence, što u suštini znači da beskonačno bliskim frekvencijama odgovaraju beskonačno bliski talasni brojevi.

Različiti karakteri monotonosti dva dela DR mogu se izraziti ograničenjima:

$$\omega_2 > \omega_1 \Rightarrow \begin{cases} k(\omega_2) > k(\omega_1), k > 0 \\ k(\omega_2) < k(\omega_1), k < 0 \end{cases} \quad (12)$$

2.3. Diskretizovana DR i primena ograničenja neprekidnosti

Razvoj bilo kakvog algoritma koji se oslanja na svojstva DR, međutim, mora da uzme u obzir da teorijski izraz (9) predstavlja dve neprekidne funkcije talasnog broja $k(\omega)$, dok DR određena korelacionom metodom predstavlja niz $k_n = k(\omega_n)$, ($n = 1, 2, \dots, N$), određen za izabrani skup frekvencija ω_n . Konačno razlike između frekvencija ω_n znače da diskretizovana DR ne može biti neprekidna u smislu jednačine (11), te da se neprekidnost mora tumačiti na drugačiji način.

Za svrhu primene metode korelacije, objašnjenje posle jednačine (11) da "neprekidnost u suštini znači da beskonačno bliskim frekvencijama odgovaraju beskonačno bliski talasni brojevi" može se protumačiti da za primenjeni algoritam treba da umanjuje rastojanje između dve susedne tačke diskretizovane DR.

Primena takvog tumačenja je sasvim jasna i direktna u okviru jedne BZ, jer to znači da su tačke na nekoj grani bliže od tačaka različitih grana sa istom vrednošću korelacije. Što su susedne frekvencije bliže, to će tumačenje biti tačnije. Međutim, problem nastaje u blizini tačaka preseka različitih grana, gde različite grane imaju iste vrednosti, i gde blizina tačaka ne podrazumeva nužno neprekidnost, s obzirom da dve bliske tačke mogu pripadati različitim granama DR sa istom vrednošću korelacije.

2.4. Greške merenja i digitalizacije i primena ograničenja monotonosti

Za razliku od ograničenja neprekidnosti, ograničenje monotonosti (12) važi i za diskretizovanu DR, što može biti korisno za identifikaciju talasnog broja koji odgovara određenoj frekvenciji, a posebno pri frekvencijama koje se nalaze u blizini preseka dve grane, s obzirom da se jedna od grana koje se seku monotono povećava, a druga se monotono smanjuje.

Problem primene ograničenja monotonosti su eksperimentalne greške nastale tokom merenja ubrzanja, kao i greške digitalizacije nastale tokom dugog procesa izračunavanja koja sadrže mnogo sabiranja (prvo izračunavanje DFT izmerenih signala, a zatim izračunavanje korelacije). Zbog ovih grešaka, talasni brojevi određeni kao odgovarajući za susedne tačke DR možda neće zadovoljavati ograničenje monotonosti, što može dovesti do pogrešne identifikacije odgovarajućeg talasnog broja, i posledičnog "preskoka" sa jedne grane DR na drugu granu sa istom vrednošću korelacije.

Rizik od gubitka monotonosti raste sa smanjenjem razlike između susednih talasnih brojeva, koji se javlja iz dva razloga: 1) usled smanjenja razlike između frekvencija, što je suprotno zahtevima ograničenja neprekidnosti, i 2) zbog smanjenja prvog izvoda DR u obliku (9), koji opada sa porastom frekvencije.

2.5. Izbor početne tačke diskretizovane DR

Primena svojstva neprekidnosti i monotonosti za određivanje DR obuhvata analizu niza kandidatskih talasnih brojeva u odnosu na prethodno utvrđenu tačku diskretizovane DR. Međutim, takav postupak nije moguć za početnu tačku diskretizovane DR, pošto ne postoje prethodna tačka koja bi mogle da se koristi kao referenca za analizu, te se mora osmisliti drugačiji postupak.

Početna tačka DR je koordinatni početak, s obzirom da nulta vrednost talasnog broja odgovara nultoj frekvenciji. Međutim u praksi, male vrednosti talasnog broja podrazumevaju duge talasne dužine odgovarajućeg talasnog polja, i u takvim slučajevima greške merenja postaju uporedive sa razlikama između amplituda ubrzanja u mernim tačkama, što znači da korelaciona metoda nije primenljiva.

Algoritam predstavljen u ovom radu koristi pretpostavku da je najniža frekvencija diskretizovane DR niža od f_{BZ} , tako da odgovarajući talasni broj pripada I BZ. U tom slučaju, odgovarajući talasni broj je član niza kandidata sa najmanjom amplitudom, tj. talasni broj koji odgovara kandidatu člana niza koji je najbliži koordinatnom početku.

2.6. Algoritam

Algoritam za određivanje DR korelacionom metodom koristi sledeće ulazne podatke:

- Rastući niz DR frekvencija ω_n ($n = 1, 2, \dots, N$); ovaj niz može biti definisan na nekoliko načina, kao što je početnom vrednošću, korakom frekvencije i brojem frekvencija N ;
- Rastući niz mernih tačaka x_l ($l = 1, 2, \dots, L$);
- Niz nizova (matrica) kompleksnih amplituda izmerenih funkcija frekventnog odziva w_{ln} ($l = 1, 2, \dots, L$ i $n = 1, 2, \dots, N$);
- Rastući niz talasnih brojeva koji će se koristiti kao probne vrednosti, $k_t^{(j)}$ ($j = 1, 2, \dots, J$); ovaj niz takođe možemo definisati na nekoliko načina, kao što je početna vrednost, korak talasnog broja i broj talasnih brojeva J ;
- Maksimalna relativna razlika između korelacija ekvivalentnih maksimuma ε_T ($0 < \varepsilon_T < 1$); ova vrednost omogućava izbor niza kandidata među otkrivenim lokalnim maksimumima;
- Granica bliskosti ε_k ($0 < \varepsilon_k < 1$), koja omogućava izbor tačke DR među članovima niza kandidata;

Algoritam za određivanje DR korelacionom metodom sastoji se od sledećih koraka:

1) Pripremni koraci

1.1) Odrediti gornju i donju granicu talasnog broja koje treba tražiti $k_t^{max} = k_t^{(J)}$ i $k_t^{min} = k_t^{(1)}$;

1.2) Izračunati prosečnu udaljenost između mernih tačaka $d = (x_L - x_1)/(L-1)$;

1.3) Izračunati maksimalan broj ekvivalentnih maksimuma u opsegu testiranih talasnih brojeva, K , primenom jednačine (10);

2) Ponavljati postupak za određivanje DR ($n = 1, 2, \dots, N$)

2.1) Izračunati korelacioni niz $Y_{\omega}^{(j)}$ ($j = 1, 2, \dots, J$) koristeći jednačinu (5) i $\omega = \omega_n$ za sve testirane tačke talasnog broja $k_t^{(j)}$ ($j = 1, 2, \dots, J$);

2.2) Pronaći K najvećih lokalnih maksimuma u korelacionom nizu $Y_{\omega}^{(j)}$ i sačuvati njima odgovarajuće talasne brojeve i vrednosti korelacione

funkcije kao pomoćne paralelne nizove $k_m^{(m)}$ i $Y_m^{(m)}$ ($m = 1, 2, \dots, K$);

2.3) Sortirati pomoćne paralelne nizove $k_m^{(m)}$ i $Y_m^{(m)}$ prema veličini $Y_m^{(m)}$;

2.4) Odrediti broj ekvivalentnih maksimuma u korelacionom nizu M kao broj lokalnih maksimuma tako da relativna razlika između njihove vrednosti korelacije i maksimalne vrednosti korelacije $1 - Y_m^{(m)}/Y_m^{(1)}$ bude manja od ε_Y ; niz $k_m^{(m)}$ ($m = 1, 2, \dots, M$) predstavlja niz kandidata;

2.5) Izabrati talasni broj koji odgovara frekvenciji ω u DR:

2.5.1) Ako se traži prva tačka DR ($n = 1$), tada je odgovarajući talasni broj najmanja apsolutna vrednost talasnog broja među tačkama kandidatima, a algoritam se nastavlja korakom 2.6;

2.5.2) Ako ($n > 1$) onda

2.5.2.1. Naći najmanji element kandidata niza veći od talasnog od broja prethodne tačke k_p – veći kandidat $k^{(Hi)}$;

2.5.2.2 Naći najveći element niza kandidata niže od talasnog broja prethodne tačke k_p – manji kandidat $k^{(Lo)}$;

2.5.2.3. Ako jedan od elemenata $k^{(Hi)}$ ili $k^{(Lo)}$ ne postoji, onda je odgovarajući talasni broj onaj kandidat koji postoji, a algoritam se nastavlja korakom 2.6;

2.5.2.4 Izračunati faktor bliskosti r , tj. količnik rastojanja prethodne tačke od kandidata i udaljenosti između kandidata,

$$r(k) = \frac{|k - k_p|}{k^{(Hi)} - k^{(Lo)}} \quad (13)$$

za oba kandidata Hi i Lo, $r^{(Hi)} = r(k^{(Hi)})$ i $r^{(Lo)} = r(k^{(Lo)})$

2.5.2.5 Izabrati talasni broj koji odgovara frekvenciji ω koristeći dobijene vrednosti $k^{(Lo)}$, $k^{(Hi)}$, $r^{(Lo)}$ i $r^{(Hi)}$ i jedan od dole opisanih postupaka;

2.6) Utvrđenu tačku postaviti (k_n , ω_n) kao novu prethodnu tačku (k_p , ω_p) = (k_n , ω_n);

Izlaz iz algoritma je DR u obliku paralelnih nizova ω_n , k_n ($n = 1, 2, \dots, N$).

U zavisnosti od primene ograničenja neprekidnosti i monotonosti, može se osmisлити nekoliko načina za izbor talasnog broja koji odgovara frekvenciji ω koristeći vrednosti $k^{(Lo)}$, $k^{(Hi)}$, $r^{(Lo)}$ i $r^{(Hi)}$ određene u koraku 2.5.2.5 algoritma.

Najjednostavniji pristup (metoda “CloseHi”) predstavlja direktnu primenu ograničenja neprekidnosti i monotonosti izborom većeg kandidata $k^{(Hi)}$ (najbliži element kandidata niza sa talasnim brojem većim od prethodne tačke) kao talasnog broja koji odgovara frekvenciji ω . Iako je pristup jednostavna i ima fizičko opravdanje, on je osetljiv na greške digitalizacije i merenja koje eksperimentalnu DR čine nemonotonim nizom.

Radi razrešenja uticaja nemonotonog ponašanja eksperimentalne DR, član niza kandidata najbliži prethodnoj tački (metoda “Close”) može biti izabran kao talasni broj koji odgovara frekvenciji ω (dakle, ako $r^{(Lo)} < r^{(Hi)}$ onda $k_n = k^{(Lo)}$, inače $k_n = k^{(Hi)}$). Iako je ovaj metod manje osetljiv na greške digitalizacije i merenja, osetljiv je

na “preskoke” između grana talasnih brojeva sa istim vrednostima korelacione funkcije, predstavljene na Slici 2, koji se mogu javiti u blizini granicama BZ, gde različite grane imaju bliske vrednosti. Kada je $k^{(Lo)}$ odabran unutar takvog područja, manji kandidat može lako pripadati monotono opadajućoj grani lokalnog maksimuma korelacione funkcije.

Sa ciljem otkrivanja dvosmislenosti izazvanih preseccima različitih grana talasnih brojeva sa istim vrednostima korelacione funkcije, uvodi se granica bliskosti ε_k kao mera bliskosti kandidata prethodnoj tački. Uvođenje granice bliskosti predstavlja dalji test za talasni broj otkriven metodom “Close”. Ako je faktor bliskosti kandidata niži od granice bliskosti, $r_n = r(k_n) < \varepsilon_k$, tada se određena vrednost talasnog broja k_n prihvata kao talasni broj koji odgovara frekvenciji ω . Međutim, ako je faktor bliskosti kandidata veći od granice bliskosti, tada se niz kandidata smatra dvosmislenim, a izbor talasnog broja koji odgovara frekvenciji ω zahteva dalju analizu.

Mogu se zamisliti bar tri načina za odlučivanje o dvosmislenim kandidatima niza :

- Automatski preskakanje određivanja talasnog broja koji odgovara frekvenciji ω („Auto-Skip“ postupak);
- Automatski izbor vrednosti dobijene nekim usrednjavanjem nižeg i višeg kandidata, koja se smatra vrednošću talasnog broja koja odgovara frekvenciji ω („Auto-Mid“ postupak); iako se postupak čini kao dobar izbor, iza njega ne stoji čvrsto fizičko obrazloženje, i nakon što probni pokušaji nisu pokazali obećavajuće rezultate, od postupka se odustalo;
- Ručni izbor, gde program prikazuje korisniku kompletan niz kandidata (paralelni nizovi talasnih brojeva i odgovarajućih korelacija $k_m^{(m)}$ i $Y_m^{(m)}$, $m = 1, 2, \dots, M$), a korisnik bira talasni broj koji odgovara frekvenciji ω na osnovu poznavanja osobina DR; ovaj pristup može dati dobre rezultate u slučaju da je poznata DR, i koristan je za eksperimentalno određivanje viskoelastičnih svojstava materijala; međutim, dobijeni rezultati su subjektivni i neće biti predstavljeni u ovom radu.

3. EKSPERIMENT

Da bi se proučavao predloženi algoritam za primenu korelacione metode, sprovedene su dve serije eksperimenata. Svaki od eksperimenata obuhvata niz uzastopnih merenja ubrzanja $a_x(t)$ u jednoj tački na štapu, koja je pobuđena modalnim čekićem.

3.1. Objekat merenja

Štap je čelična šipka dužine $D \approx 1,65$ m, i približno kvadratnog poprečnog preseka stranice $b \approx 1$ cm. Krajevi štapa, sa dužinom oko 25 cm, oslanjeni su na meke sundere, sa ciljem da oponaša štap sa slobodnim krajevima.

3.2. Pobuda

Vibracije su pobuđene modalnim čekićem B&K 8204 sa osetljivošću od 30,89 mV/N, a pobuda je merena B&K Pulse sistemom sa CCLD ulaznim opsegom od $\pm 10V$, tako da je merni opseg sile bio oko 300 N. Modalni čekić je lak i mali, opremljen je laganom udarnom glavom,

tako da je širina frekvencijskog opsega pobude udarnog čekića do 10 kHz.

Automatsko otkrivanje dvostrukog udarca nije bilo obezbeđeno, tako da je svaka pobuda praćena i uvidom u vremensku zavisnost udarne sile $F(t)$, a dvostruki udarci su odbaćeni.

Štap je udaran u blizini njegovog početka, u području $x = 0-1$ cm udaljenom od početka štapa. Na udaranu površinu postavljeno je parče lepljive trake koje je prigušilo udarce u izvlesnoj meri, što je smanjilo propusni opseg na oko 6 kHz, a takođe smanjilo i prekomerno ubrzanje koje je sprećavalo snimanje odziva softverom korišćenim za prikupljanje podataka. Maksimalna udarna sila je bila u opsegu 8-12 N.

3.3. Odziv

Odziv je meren pomoću akcelerometra B&K 4507Bx sa osetljivošću $10.055 \text{ mV/ms}^{-2}$ i B&K Pulse akvizicionim sistemom sa CCLD ulaznim opsegom $\pm 10\text{V}$, tako da je meri opseg ubrzanja bio oko $\pm 1000 \text{ m/s}^2$.

3.4. Eksperimentalni postupak

Za svaku mernu tačku merenje je izvršeno 10 puta. Tokom svakog merenja zabeležene su vremenske zavisnosti pobudne sile i ubrzanja. Posle svakog merenja, softver je izračunao snagu spektralne gustine sile i ubrzanja, funkciju frekvencijskog odziva (FRF) akcelerance, te odgovarajuću koherentnost, na osnovu izmerenih podataka.

3.5. Merne tačke

Merenja su organizovana u dve serije, nazvanih eksperimenti, sa ciljem da ispituju slučaj jednako udaljenih mernih taćaka, koji je lakše primeniti i protumaćiti, kao i slučaj nejednako udaljenih mernih taćaka, koja je od velike praktiće važnosti.

U prvom eksperimentu, merne taćke su bile ravnomerno raspoređene na 10 jednako udaljenih pozicija sa rastojanjima $d \approx 15$ cm, pri ćemu je prva merna taćka na rastojanju d od početka šipke a poslednja na rastojanju d od kraja šipke. Prema tome, raspored mernih taćaka je bio simetrićan u odnosu na središte šipke.

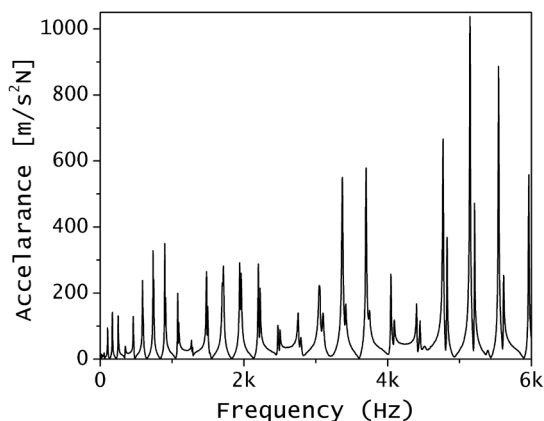
U drugom eksperimentu, 10 mernih taćaka je izabrano tako da razdaljina od početka šipke (pobuđeni kraj), koje su iznosile 20 cm, 28 cm, 44 cm, 52 cm, 68 cm, 76 cm, 92 cm, 116 cm, 124 cm i 148 cm, budu proporcionalne nizu prostih brojeva 5, 7, 11, 13, 17, 19, 23, 29, 31 i 37. Izabrani skup mernih taćaka približno pokriva isti deo štapa kao i u prvom slučaju, dok proporcionalnost rastojanja nizu prostih brojeva smanjuje, i praktiće eliminiše, uticaj alijasinga usled konaćne dućine niza mernih taćaka.

4. REZULTATI

4.1. Spektar frekvencijskog odziva

Amplitudni spektri akcelerance pokazuju očekivano rezonantno ponašanje (Slika 3). Rezonantne frekvencija prisutne u svim spektrima su iste, ali amplitude FRF merene u različitim taćkama pokazuju različito ponašanje. Frekvencija sa maksimalnom amplitudom pokazuje zavisnost od merne taćke. Rezonantni maksimumi na niskim frekvencijama (14 Hz – 590 Hz) su singleti, dok se na višim frekvencijama ćini da su rezonantni maksimumi

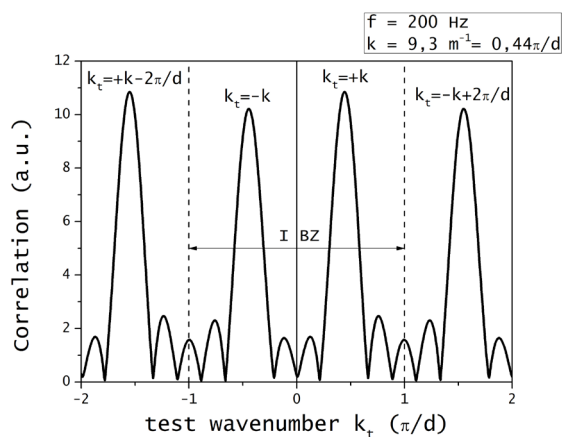
dubleti, sa razlikom frekvenci maksimuma koja se povećava sa rezonantnom frekvencijom.



Slika 3: FRF akcelerance taćke udaljene $x = 90$ cm od početka štapa

4.2. Korelaciona funkcija

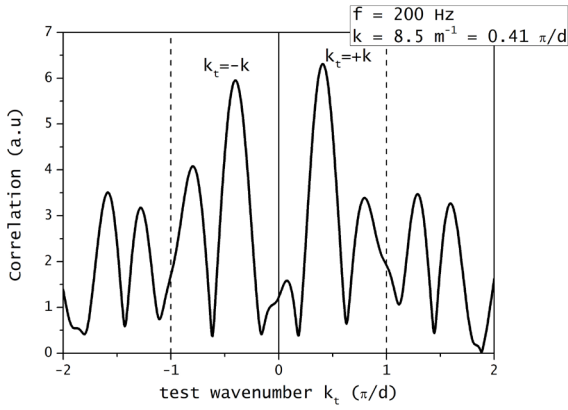
Izračunate korelacione funkcije pokazuju predvićeno ponašanje, odnosno periodićnost sa periodom $2\pi/d$ za jednako udaljene merne taćke (Slika 4). Kako pozitivno, tako i negativno, rešenje jednaćine (9) predstavljaju snaćne lokalne maksimume korelacione funkcije, koji se ponavljaju u svakoj od BZ.



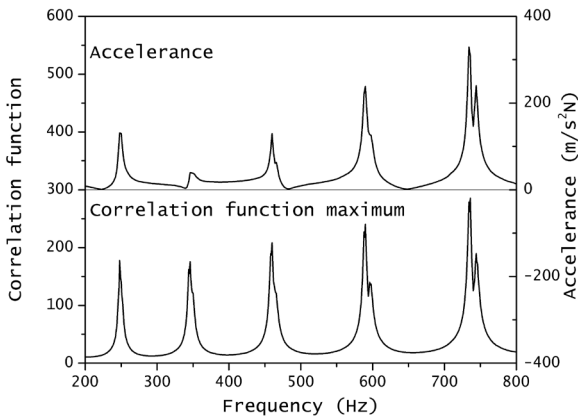
Slika 4: Korelaciona funkcija za jednako udaljena merenja

Ćak i u slučaju kada merne taćke nisu na jednakim rastojanjima, i pozitivna i negativna rešenja jednaćine (9) i dalje pokazuju lokalne maksimume, i pored toga što maksimumi nisu toliko oštri kako u slučaju jednako udaljenih mernih taćaka (Slika 5). Iako u tom slučaju korelaciona funkcija nije periodićna, treba reći da i su *pozicije lokalnih maksimuma korelacione funkcije* (koji su relativni za korelaciju metodu) i dalje približno periodićne.

S obzirom da vrednosti lokalnih maksimuma korelacione funkcije zavise od akcelerance prema jednaćini (6), maksimum korelacione funkcije takođe pokazuju isto rezonantno ponašanje kao ubrzanje, što je prikazano na Slici 6.



Slika 5: Korelaciona funkcija za nejednako udaljene merne tačke



Slika 6: Zavisnost FRF akcelarence i maksimuma korelacione funkcije od frekvencije

4.3. Direktna primena korelacione metode

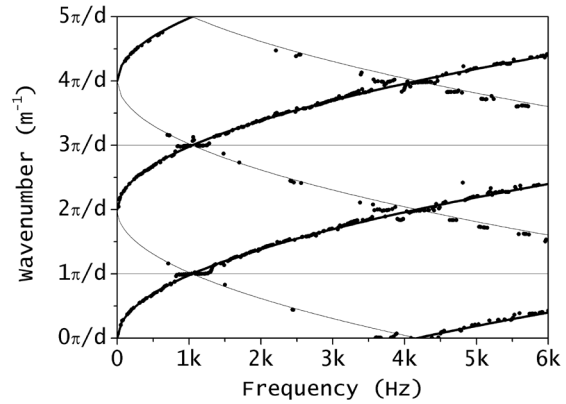
U ovom poglavlju su prikazani rezultati direktne primene korelacione metode, koja se sastoji u izboru talasnog broja koji odgovara najvećoj vrednosti korelacione funkcije kao talasnog broja koji odgovara frekvenciji ω u DR.

4.3.1. Merne tačke na jednakim rastojanjima

Rezultati dobijeni direktnom primenom korelacione metode na merenja izvršenim u mernim tačkama na jednakim rastojanjima prikazani su na Slici 7, gde je izračunata korelaciona funkcija za ispitivane vrednosti talasnog broja u opsegu $(0, +5\pi/d)$. Slika jasno pokazuje da dobijene vrednosti talasnih brojeva koje odgovaraju frekvencijama korišćenim u eksperimentu pripadaju svim granama maksimuma koje postoji u testiranom opsegu talasnih brojeva, koje ova metoda nije sposobna da razlikuje. Većina otkrivenih talasnih brojeva opisuje prenos energije u upadnom pravcu (pozitivna grupna brzina), ali neki od talasnih brojeva opisuju prostiranje energije u pravcu refleksije (negativna grupna brzina).

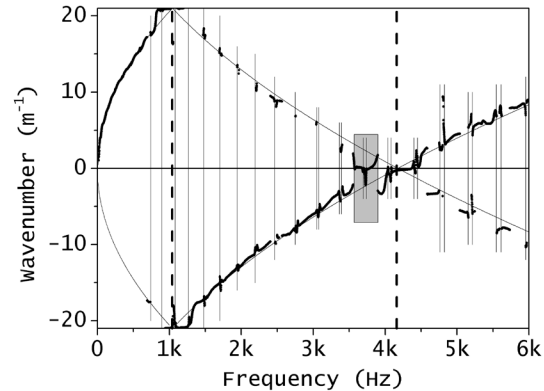
Iako Slika 7 takođe pokazuje linije koje ukazuju na grane maksimuma korelacione funkcije, koje se mogu protumačiti kao grane DR, nije moguće primeniti postupak fitovanja na dobijene rezultate, jer uzastopne tačke grafikona u opštem pripadanju različitim granama. Prema tome, linije na Slici 7 su samo vizuelni vodiči, a formirane su pomoću zavisnosti $k = \pm b_{kf} f^{1/2} + z(2\pi/d)$ predložene jednačinom (9), sa parametrom $b_{kf} \approx 0.65 \text{ 1/(mHz}^{1/2})$,

određenim probanjem, kako bi se dobilo dobro vizuelno slaganje sa eksperimentalnim tačkama. Vrednosti se inače dobro slažu sa teorijskim predviđanjem $b_{kf} \approx 0.66 \text{ 1/(mHz}^{1/2})$, dobijenim iz jednačine (9) primenjujući vrednosti $E = 200 \text{ GPa}$ i $\rho = 7800 \text{ kg/m}^3$ za Jungov modul i gustinu čelika.



Slika 7: Talasni brojevi određeni direktnom primenom korelacione metode na merenja na jednakim rastojanjima ($\pi/d \approx 21 \text{ m}^{-1}$)

Radi boljeg proučavanja dobijenih rezultata, opseg testiranih talasnih brojeva smanjen je na IBZ, a rezultati su prikazani na Slici 8.



Slika 8: Disperziona relacija sa Slike 7 svedena na prvu Brilluenovu zonu ($\pi/d \approx 21 \text{ m}^{-1}$)

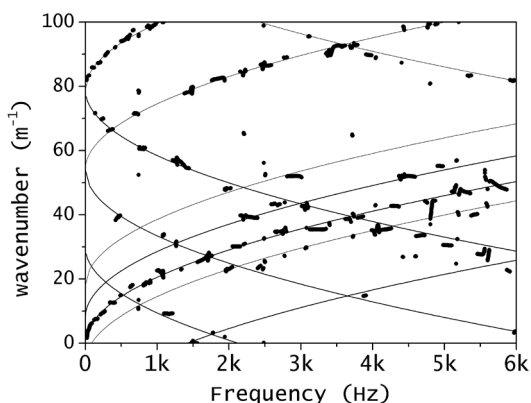
Svođenje talasnih brojeva na IBZ gotovo uspostavlja neprekidnost eksperimentalno određene DR. Diskontinuiteti izazvani “preskakanjem” na negativnu granu grupne brzine odgovaraju rezonantnim frekvencijama (obeležene tankim punim vertikalnim linijama na Slici 8). Samo postojanje dubleta, koje teorija ne predviđa, ukazuje na to da, pod rezonantnim uslovima, eksperimentalna postavka ne zadovoljava pretpostavke teorijskog modela.

Osim “preskoka” na granu sa negativnom grupnom brzinom, redukcija talasnih brojeva na IBZ vraća monotoni karakter eksperimentalno otkrivenoj DR, sa značajnim izuzetkom na granicama Brilluenove zone (obeležene debelim vertikalnim isprekidanim linijama na Slici 8), gde su eksperimentalno otkriveni talasni vektori nezavisni od frekvencije (tj. grupna brzina je jednaka nuli, što znači da se prenos energije ne događa, slično kao kod stojećih talasa). Pošto su Brilluenove zone samo posledica jednakih rastojanja među mernim tačkama, uočena odstupanja od monotonosti takođe moraju biti artefakt

nastao usled eksperimentalne postavke. Zapravo, kada je talasni broj blizu granice Brillueneve zone, $k = z(\pi/d)$, rastojanje između mernih tačaka je celobrojni umnožak polovine talasne dužine ($d = z\lambda/2$), pa progresivni talasi koji se kreću u suprotnim smerovima ne mogu da se razlikuju u mernim tačkama, što dovodi do nemogućnosti merenja da, u tom slučaju, otkrije smer prenosa energije.

4.3.2. Merne tačke na nejednakim rastojanjima

Rezultati direktne primene korelacione metode na merenja izvršena u tačkama na nejednakim rastojanjima prikazani su na Slici 9, pri čemu je korelaciona funkcija proračunavana za ispitivane vrednosti talasnog broja u opsegu (0, 100 m^{-1}), koji je približan opsegu korišćenom u slučaju jednako udaljenih mernih tačaka. Određene vrednosti talasnih brojeva koji odgovaraju frekvencijama korišćenim u eksperimentu pripadaju mnogo većem broju grana maksimuma nego u slučaju jednako udaljenih mernih tačaka. Te grane ne pokazuju periodičnost, ali nejednako udaljene merne tačke sprečavaju pojavu pogrešnog otkrivanja stojećih talasa na granicama Brillueneve zone, kao u slučaju jednako udaljenih mernih tačaka. Sa druge strane, dobijeni rezultati su generalno manje jasni nego u slučaju mernih tačaka na jednako udaljenom rastojanjima.



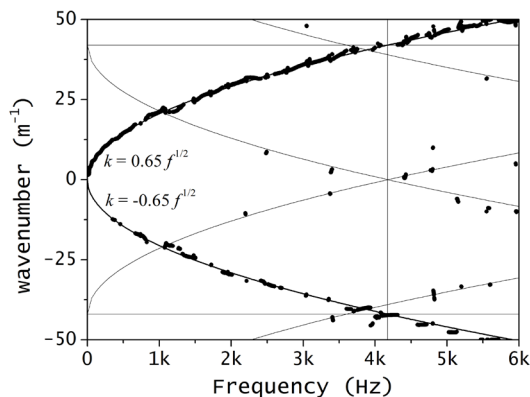
Slika 9: Talasni brojevi određeni direktnom primenom korelacione metode na nejednakom rastojanju mernih tačaka

4.4. Povećan broj mernih tačaka

Slika 10 predstavlja rezultate istovremene neposredne primene korelacione metode na rezultate kako jednako udaljenih, tako i nejednako udaljenih, mernih tačaka (dakle 20 mernih tačaka), gde se korelaciona funkcija proračunava za test vrednosti talasnog broja u opsegu (-50 m^{-1} , +50 m^{-1}), koji obuhvata očekivane vrednosti opsega za talasni broj na osnovu jednačine (9).

Slika prikazuje da određeni talasni brojevi pretežno pripadaju dvema granama koje su opisane jednačinom (9), sa retkim "preskakanjem" na ostale grane lokalnih maksimuma. Dok većina otkrivenih talasnih brojeva opisuje prostiranje talasa u upadnom smeru, gotovo trećina otkrivenih talasnih brojeva opisuje prostiranje talasa u smeru odbijanja. Iako skup mernih tačaka koji se sastoji od jednako udaljenih i od nejednako udaljenih tačaka po sebi predstavlja skup nejednako udaljenih tačaka, može se uočiti da su talasni brojevi otkriveni u blizini talasnog

broja koji odgovara vrednosti $k = \pi/d_{av} \approx 42 m^{-1}$ (frekvencija blizu 4100 Hz), gde d_{av} predstavlja prosečno rastojanje između mernih tačaka, nezavisni od frekvencije, što ukazuje na nemogućnost otkrivanja smera prenosa energije, slično slučaju jednako udaljenih mernih tačaka. Uopšte uzev, određivanje talasnih brojeva u blizini preseka između različitih grana maksimuma korelacione funkcije predstavlja kritični problem korelacione metode.



Slika 10: Talasni broj određen direktnom primenom korelacione metode na rezultate merenja u svim mernim tačkama

4.5. Primena algoritma za proširenje frekvencijskog opsega

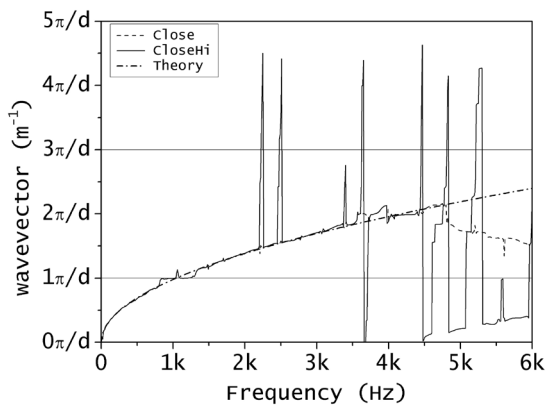
Ovaj odeljak predstavlja rezultate primene varijacija algoritma objašnjenih u odeljku 2.6 ovog rada. Algoritam je namenjen poboljšanju primenljivosti korelacione metode izvan IBZ, pa ključni aspekt koji treba uzeti u obzir predstavljaju rezultati dobijeni u opsegu talasnih brojeva $k > \pi/d$, dakle iznad frekvencije $f_{BZ} \approx 1$ kHz. Poređenja radi, na sledećim slikama linija crta-tačka prikazuje teorijska predviđanja dobijena korišćenjem jednačine (9) sa vrednostima $E = 200$ GPa i $\rho = 7800$ kg/m^3 za Jungov modul i gustinu čelika.

4.5.1. Merenja na jednakim rastojanjima

Rezultati primene dve metode koje koriste jedno od svojstva DR u obliku (9), "CloseHi" metode (koja favorizuje monotonost, puna linija) i "Close" metode (koja favorizuje neprekidnost, isprekidana linija) su predstavljene na Slici 11.

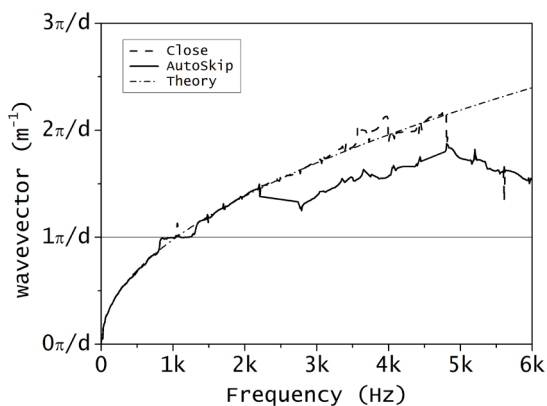
Dobijeni rezultati pokazuju da se varijacije algoritma mogu koristiti u frekvencijskom opsegu iznad f_{BZ} , ali takođe jasno pokazuje i slabosti oba postupka objašnjena u odeljku 2.6.

Zbog osetljivosti na grešku digitalizacije, "CloseHi" metoda "preskače" na više grane ekvivalentnih maksimuma u slučajevima kada je kandidata $k^{(Lo)}$ manji od prethodne vrednosti k_p samo zbog greške digitalizacije. Kada se dogodi "preskok", algoritam se vraća na tačne vrednosti samo kada vrednost $k^{(Hi)}$ nije više u ispitivanom opsegu. Takvo ponašanje dovodi do "oštrica" koje se opažaju u opsegu 2-4 kHz, kao i na oscilacije između grana ekvivalentnih maksimuma primećenih u opsegu 4.5-6.0 kHz.



Slika 11: Poređenje DR dobijenih metodama "Close" i "CloseHi" sa teorijskim predviđanjima u slučaju jednako udaljenih mernih tačaka

"Close" metoda prati teorijska predviđanja do 5 kHz, ali, kako je predviđeno u odeljku 2.6, ne uspeva da pravilno otkrije odgovarajuće vrednosti talasnog broja blizu granica BZ kada se smanji prvi izvod DR: razlika između talasnih brojeva uzastopnih tačaka na granici IBZ i dalje je dovoljna da omogući tačan izbor između tačaka kandidata, ali smanjenjem prvog izvoda sa daljim porastom frekvencije otežava izbor između tačaka različitih grana u blizini preseka vrednosti $2\pi/d$ (blizu 4.1 kHz, vidi Sliku 7). Kada se algoritam prebaci na negativnu granu grupne brzine, njegovo favorizovanje neprekidnosti rezultira izborom talasnih brojeva koji odgovaraju toj grani u opsegu iznad 5 kHz.



Slika 12: Poređenja DR dobijenih metodama "Close" i "AutoSkip" sa teorijskim predviđanjima u slučaju jednako udaljenih tačaka

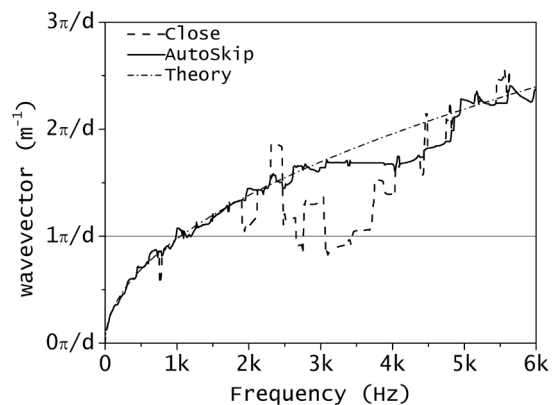
Slika 12 predstavlja poređenje rezultata primenom metode "AutoSkip" (puna linija), u poređenju sa rezultatima metodom "Close" (isprekidana linija). Zbog preskakanja jedne tačke, ili čak niza tačaka, "AutoSkip" metoda zadržava talasni broj prethodne tačke na niskoj vrednosti u poređenju sa sledećim tačkama; prema tome, ekvivalentni lokalni maksimum koji pripada nižoj grani može postati sledeća bliska tačka, što dovodi do "preskakanja" na odgovarajuću granu; posle takvog "preskakanja", donja grana se prati na osnovu zahteva monotonosti i neprekidnosti.

Opseg konstantnih vrednosti talasnih brojeva oko frekvencija 1 kHz i 4.1 kHz pokazuje da su sve predstavljene metode osetljive na pogrešno otkrivanje stojećih talasa u blizini preseka između grana

ekvivalentnih maksimuma sa pozitivnim i negativnim grupnim brzinama. Taj nedostatak je svojstven eksperimentalnim podacima dobijenim korišćenjem jednako udaljenih tačaka merenja: s obzirom da takva eksperimentalna postavka dovodi do sličnih eksperimentalnih rezultata za talase koji se prostiru u oba smera duž štapa, nijedan algoritam ne može naći razliku između podataka.

4.5.2. Merenja na nejednakim rastojanjima

S obzirom da su nedostaci metoda koji strogo koriste jedno od svojstava DR doveli do neuspeha čak i u slučaju jednako udaljenih tačaka, ovaj odeljak razmatra samo primenu metoda "Close" (pune linije) i "AutoSkip" (isprekidane linije) na merenja kod nejednako raspoređenih tačaka, sa rezultatima predstavljenim na Slici 13.



Slika 13: Poređenja DR dobijenih metodama "Close" i "AutoSkip" sa teorijskim predviđanjima u slučaju nejednako udaljenih tačaka

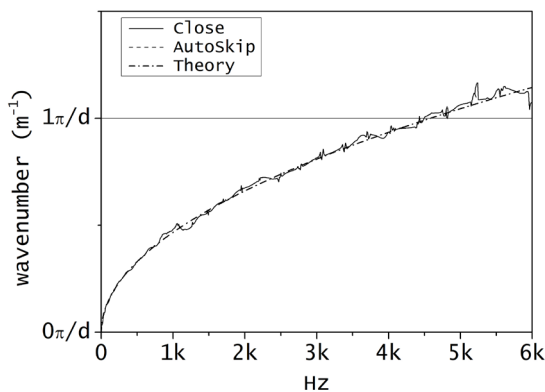
Predstavljeni rezultati pokazuju predviđenu i očekivanu otpornost prema pogrešnom otkrivanju stojećih talasa, što predstavlja važnu prednost upotrebe postavke za nejednako udaljene tačke. Nažalost, dobijene DR pokazuju da se obe metode, i to u širokim frekventnim opsezima, ne slažu sa teorijskim predviđanjima. Zanimljivo je primetiti da u slučaju nejednako udaljenih mernih tačaka metoda "Auto-Skip" daje bolje rezultate (dakle, suprotno slučaju merenja u jednako udaljenim tačkama), s obzirom da je u stanju da prati teorijska predviđanja do 3.2 kHz, dok je metoda "Close" ima prihvatljiva predviđanja samo do 2 kHz.

Sa druge strane, takođe se može primetiti da primena metoda na merenja u nejednako udaljenim tačkama dovodi do stabilnijih rezultata nego što je to slučaj sa merenjima u jednako udaljenim tačkama. Štaviše, primena merenja u nejednako udaljenim tačkama dovodi do konvergencije rezultata prema teorijskim predviđanjima u visokofrekventnom opsegu, suprotno slučaju primene merenja u jednako udaljenim tačkama.

4.5.3. Povećan broj mernih tačaka

Da bi se predložena metoda uporedila sa rezultatima direktne primene korelacione metode, Slika 14 predstavlja rezultate primene metoda "Close" i "AutoSkip" na podatke uzete u jednako udaljenim i nejednako udaljenim mernim tačkama, analogno rezultatima predstavljenim u odeljku 4.4 i na Slici 10.

S obzirom ne postoji “preskakanje” na negativne grane grupne brzine, koja se mogu uočiti na Slici 10, može se zaključiti da je predložena metoda u ovom slučaju superiorna u odnosu na direktnu primene korelacione metode. Sa druge strane, upotreba povećanog broja mernih tačaka dovodi do proširenja IBZ do frekvencije od 4.1 kHz, koja pokriva veliki deo frekventijskog opsega koji se koristi u merenjima, i ne daje uvid u primenljivost predloženih metoda za proširenje frekventijskog opsega izvan granica IBZ, što je tema rada.



Slika 14: Poređenje DRs dobijenih primenom “Close” i “AutoSkip” koristeći sve metode mernih tačaka sa teorijskim predviđanjima

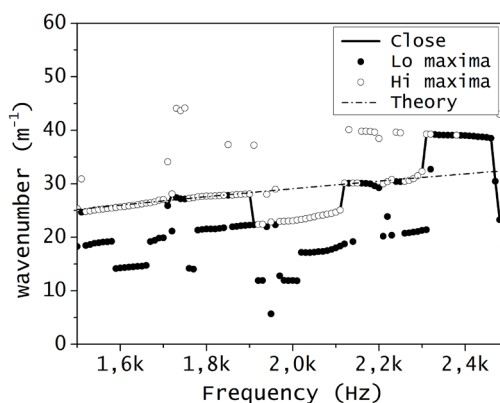
5. DISKUSIJA

Odsustvo artefakata uzrokovanih prostornom periodičnošću snažno favorizuje izbor podataka dobijenih nejednako raspoređenim mernim tačkama kao izvor podataka za primenu korelacione metode. Međutim, u slučaju nejednako udaljenih mernih tačaka, nijedna od razmatranih varijacija metoda za izbor odgovarajućih talasnih brojeva iz niza kandidata nije dala dobre rezultate u celom opsegu frekvencija korišćenih u eksperimentu. Kako bi se razvila bolja metoda za izbor odgovarajućeg talasnog broja iz niza kandidata, treba razumeti mehanizam neuspeha razmatranih metoda.

Glavni razlog posmatranog ponašanja je taj što je broj grana ekvivalentnih maksimuma u slučaju mernih tačaka na nejednakim rastojanjima mnogo veći nego u slučaju jednakih rastojanja, o čemu svedoče Slika 7 i Slika 9. Veći broj grana znači manje razdvajanje između njih, a bilo kojoj metodi je lakše da “preskoči” između bliskih grana. Štaviše, grane dobijene korišćenjem nejednako udaljenih mernih tačaka ne pokazuje periodično ponašanje kao grane dobijene korišćenjem jednako udaljenih mernih tačaka, i sigurno je teže razviti algoritam koji sprečava “preskakanje” u prvom slučaju. U cilju proučavanja mehanizma pogrešnog određivanja, na slikama 15-17 predstavljeni su najbliži članovi niza kandidata $k^{(Lo)}$ (pun krug) i $k^{(Hi)}$ (prazan krug), označavajući izbor punom linijom, a teorijska predviđanja isprekidanom linijom.

Slika 15 otkriva razlog neuspeha metode “Close” u frekventijskom opsegu 1.5-2.5 kHz. Greška se javlja na frekvenciji 1910 Hz, kada prethodna tačka ima frekvenciju $f_p = 1900$ Hz i talasni broj $k_p = 28.12$ m⁻¹, a kandidati imaju talasne brojeve $k^{(Lo)} = 22.34$ m⁻¹ i $k^{(Hi)} = 37.21$ m⁻¹. Sa $r^{(Lo)} = 39\%$ i $r^{(Hi)} = 61\%$, nijedan od kandidata nije blizu prethodne tačke, ali se metoda “Close” pomera na

bližu, nižu vrednost. Međutim, niža vrednost pripada drugoj grani ekvivalentnih maksimuma, pa i naredni određeni talasni broj pripada toj grani, sve dok slične okolnosti ne dovedu do novog “preskoka” pri frekvenciji 2120 Hz. Jasno je, pa čak i trivijalno, ta metoda “Close” ne uspeva kada niz kandidata ne sadrži tačku koja je blizu prethodne tačke, tj. tačku koja pripada istoj grani ekvivalentnih maksimuma kao i prethodna tačka. Treba primetiti da Slika 15 prikazuje tri grane ekvivalentnih maksimuma do frekvencije $f_p = 1900$ Hz (niža, sa vrednostima oko 20 m⁻¹, grana DR sa vrednostima oko 30 m⁻¹, i viša, vrednosti oko 40 m⁻¹). Odsustvo tačke iz grane DR na frekvenciji $f = 2310$ Hz je izuzetak, s obzirom da postoje tri ekvivalentna maksimuma na frekvenciji $f = 2320$ Hz sa talasnom brojevima 21.7 m⁻¹, 32.7 m⁻¹ i 39.4 m⁻¹, ali metoda “Close” nije u stanju da ponovo “preskoči” nazad na granu DR, jer je vrednosti 39.4 m⁻¹ najbliža vrednosti 39.2 m⁻¹, koja je u prethodnom koraku pogrešno identifikovana kao odgovarajući talasni broj. Verovatni razlozi za odsustvo tačke iz grane DR na frekvenciji $f = 2310$ Hz su greške merenja i digitalizacije, zbog čega su lokalni maksimumi korelacionog niza koji odgovaraju DR mali u poređenju sa lokalnim maksimumom koji odgovaraju višim i nižim granama ekvivalentnih maksimuma. Svako poboljšanje metode “Close” mora da reši pojavu povremenih gubitaka lokalnih maksimuma korelacionog niza koji odgovara DR.

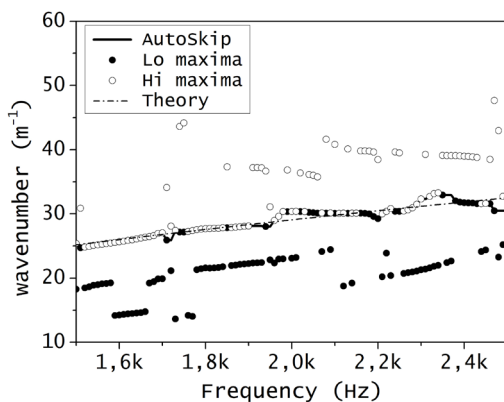


Slika 15: Primena metode “Close” za izbor odgovarajućih talasnih brojeva korišćenjem eksperimentalnih podataka u frekventijskom opsegu 1.5-2.5 kHz

Slika 16 pokazuje sposobnost metode “AutoSkip” da se izbori sa izolovanim gubicima tačaka iz DR, jer jednostavno prelazi na sledeću tačku, dosledno prateći granu DR lokalnih maksimuma u celom frekventijskom opsegu 1.5-2.5 kHz.

Metoda “AutoSkip”, međutim, ne uspeva u frekventijskom opsegu 3.5-4.0 kHz, gde ona “gubi” postojeću granu DR ekvivalentnih maksimuma. Greška nastaje nakon frekvencije $f_p = 3420$ Hz i talasnog broja $k_p = 35.39$ m⁻¹, kada su talasni brojevi kandidata za sledeću tačku, $f = 3420$ Hz, su $k^{(Lo)} = 30.47$ m⁻¹ i $k^{(Hi)} = 38.28$ m⁻¹. Sa $r^{(Lo)} = 63\%$ i $r^{(Hi)} = 37\%$, nijedan od kandidata nije zaista blizu prethodne tačke, tako da metoda “AutoSkip” preskače tu tačku i prelazi na sledeću frekvenciju. Međutim, isti uslovi važe do frekvencije 3890 Hz, gde metoda “preskače” na donju granu ekvivalentnih maksimuma. Razlog za neuspeh mehanizma “AutoSkip” je drugačiji, a za razliku od neuspeha metode

“Close”, ne leži u frekventijskog opsegu gde se pojavljuje, već mnogo ranije. Slika 17 jasno pokazuje da su u razmatranom frekventijskom području stalno prisutne tri grane ekvivalentnih maksimuma (niža, sa vrednostima oko 35 m^{-1} , grana DR sa vrednostima oko 40 m^{-1} , i višim, vrednostima oko 45 m^{-1}). Međutim, otkriveni maksimumi sve tri grane su prilično konstantne vrednosti u frekventijskom opsegu 3.0-3.4 kHz, a zatim imaju nagli “skok” na frekvenciji 3430 Hz, vraćajući se monotonom porastu na višim frekvencijama. Nagli “skok” vrednosti uzrokuje preskakanje posmatrane tačke, ali i “gubitak” veze sa granom DR koji se ne može oporaviti: zbog monotonog porasta dve grane ekvivalentnih maksimuma, donja grana će se neizbežno približiti referentnoj tački na $f_p = 3420 \text{ Hz}$, i algoritam će “preskočiti” na donju granu, umesto grane DR ekvivalentnih maksimuma. Metoda “AutoSkip” je, prema tome, osetljiva na nagle promene nagiba grana ekvivalentnih maksimuma.



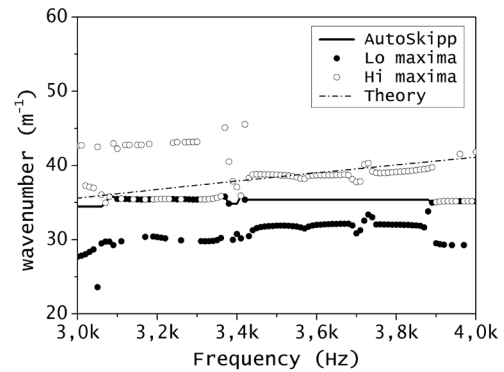
Slika 16: Primena metode “Auto-Skip” za određivanje odgovarajućeg talasnog broja koristeći eksperimentalne podatke u frekventijskom opsegu 1.5-2.5 kHz

Predstavljena diskusija objašnjava uočeno ponašanje predloženih algoritama za proširenje opsega merenja pri eksperimentalnom određivanju DR korelacionom metodom: osetljivost metode “AutoSkip” na nagle promene nagiba čini je slabijim rešenjem za eksperimente sa jednako udaljenim mernim tačkama, kada određena DR ima frekventijski opseg sa konstantnom vrednostima zbog pogrešnog otkrivanja stojećih talasa; sa druge strane, metoda “Close” je lošije rešenje za nejednako udaljene merne tačke, kada se zbog složene prirode grana ekvivalentnih maksimuma javljaju povremeni gubici tačaka grane ekvivalentnih maksimuma DR.

6. ZAKLJUČAK

U radu je predstavljen koncept za određivanje disperzione relacije talasa savijanja u štapovima primenom korelacione metode. Koncept je izveden iz neprekidnosti i monotonosti disperzione relacije, a implementira se kao algoritam koji može koristiti različite metode za određivanje talasnog broja koji odgovaraju frekvenciji talasa, koji se zasniva na te dve osobine disperzione relacije. Ideja koncepta je prevazilaženje ograničenja usled prostornog alijasinga prouzrokovanih diskretnom prirodom mernih podataka, a time i doprinos proširenju frekventijskog opsega primene korelacione metode za određivanje disperzione relacije. Prostorni alijasing

ograničava primenljivost direktne primene korelacione metode u eksperimentima sa jednako udaljenim merenjima na prvu Brilluenuvu zonu diskretnog prostora definisanog tačkama merenja položaja, što je predstavljeno na Slici 8. U eksperimentu analiziranom u ovom radu, prostorni alijasing ograničava direktnu primenu korelacione metode na frekventijski opseg do 1 kHz.



Slika 17: Primena metode “Auto-Skip” za izbor odgovarajućeg talasnog broja korišćenjem eksperimentalnih podataka u opsegu 3-4 kHz

U radu su razmatrane tri metode za određivanje talasnog broja koji odgovara frekvenciji, kako u slučaju jednako udaljenih, tako i nejednako udaljenih, mernih tačaka. Metode su “CloseHi” – koja favorizuje monotonost DR, “Close” – koja favorizuje neprekidnost DR, i “AutoSkip” metoda – koja uzima u obzir okolnosti koje nisu uračunate u prve dve metode. Analiza rezultata pokazala je da je, zbog manje osetljivosti na sistematske i slučajne greške pri merenju, kao i na grešku digitalizacije, metoda “Close” bolja od metode “CloseHi”, kako za jednako udaljena tako i za nejednako udaljene merne tačke. Metoda “Close” je takođe pokazala bolje rezultate od metode “AutoSkip” u slučaju jednako udaljenih mernih tačaka, gde je metoda “Close” omogućila izračunavanje disperzione relacije u širokom frekventijskom opsegu (do 5 kHz), osim u frekventijskom opsegu oko granica Brilluenuve zone. Sa druge strane, metoda “AutoSkip” pokazuje bolje rezultate od metode “Close” za nejednako udaljene merne tačke, pri čemu je metoda “AutoSkip” omogućila izračunavanje disperzione relacije u frekventijskom opsegu do 3.2 kHz, dok je metoda “Close” omogućila proširenje opsega merenja do 2 kHz.

Rezultati su takođe pokazali da su, u slučaju jednako udaljenih mernih tačaka, sve metode podložne pogrešnom otkrivanju stojećih talasa u širokom frekventijskom opsegu oko granica Brilluenuve zone. Pošto korelaciona metoda ne zahteva jednako udaljene merne tačke, postavke sa nejednako udaljenim mernim tačkama čine se poželjnim izborom za eksperimentalno određivanje disperzione relacije.

Problem postavke sa nejednako udaljenim mernim tačkama je složenija priroda dobijenih korelacionih funkcija. Eksperimentalne rezultate je teže interpretirati i osetljivi su na greške merenja i digitalizacije, što – pre ili kasnije – dovodi do neuspeha svih analiziranih metoda, jer eksperimentalni podaci nisu u skladu sa pretpostavkama koje su korišćene pri razvoju metoda.

Jednostavan i efikasan pristup za proširivanje primenljivosti razmatranih metoda je proširenje frekvencijskog opsega primenljivosti korelacione metode povećanjem broja mernih tačaka. Međutim, ovaj pristup nije uvek moguć, pa je razvoj novih metoda za analizu eksperimentalnih podataka vredan istraživanja. Jedan od mogućih pravaca za dalje proučavanje je razvijanje metoda koje uključuju informacije o svim prethodno utvrđenim tačkama disperzione relacije (njenom ukupnom ponašanju) u proces određivanja novih tačaka, bilo korišćenjem matematičke analize ili mašinskog učenja u tu svrhu.

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Extension of the Frequency Range for Experimental Determination of Dispersion Relationship of Flexural Waves in Beams by Correlation Method

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The dispersion relation is the key for studies of wave propagation. The experimental determination of a dispersion relation by measurements of wave properties in different points in space meets the challenge of spatial aliasing, which is further augmented by numerical instability of calculations. This paper presents and discusses a concept aimed to overcome the spatial aliasing problem in measurements of dispersion relation of mechanical waves propagating through beams. The concept is based on the properties of the dispersion relationship and it may be extended to the case of all other waves with monotonous dispersion relationship.

Keywords: Mechanical waves, Dispersion relationship, Frequency range

1. INTRODUCTION

1.1. Dispersion relationship

Dispersion relationship (abbreviated as “DR” in the following text) is the relationship between the wavenumber of a wave k (or its wavelength λ) and its angular frequency ω (or frequency f , or even period T). Since the phase velocity c of a wave may be expressed as $c = \omega/k$, and the group velocity of a wave may be expressed as $v = d\omega/dk$, the DR enables calculation of both phase and group velocity of the wave, thus representing an important tool for wave propagation studies. The recent studies of wave propagation through 3D printed periodic and aperiodic structures [1], represent an example of modern application of DRs.

The DRs may be derived and expressed in explicit form for wave propagation through media with simple geometries (such as free space, beams, plates), and the expressions include viscoelastic (e.g., elastic moduli and material loss factors) and inertial (e.g., density) properties of the material. For that reason, the experimental determination of DRs is sometimes also used for calculation of elastic properties of matter, particularly for measurement of material loss factor, which is otherwise difficult to measure [2][3].

As a particularly simple geometry, which confines the wave propagation to one dimension, thin beams represent the most convenient object for development of methodologies for experimental determination of DRs, as well as for studies of various applications of DRs, such as crack detection [4]. Depending on location and direction of the excitation, different types of mechanical waves may propagate through beams, such as longitudinal compression waves or torsional waves, which have DRs with simple linear forms, $\omega = (E/\rho)^{1/2} \cdot k$ for longitudinal waves, and $\omega = (G/\rho)^{1/2} \cdot k$ for torsional waves, with ρ standing for density, E for Young’s modulus and G for shear modulus of the material of the beam [5]. The simplest mechanical waves for excitation and detection are flexural waves, which have parabolic form of DR [5]:

$$\omega = \sqrt{\frac{EI}{\rho A}} k^2 \quad (1)$$

with I standing for the area moment of inertia, and A for the area, of the cross-section of the beam.

1.2. Experimental determination of DR

The main concept for experimental determination of the DR consists in measurement of the response (usually acceleration) to an excitation (usually impulse or harmonic) in multiple points along the beam. The responses at these points at a particular excitation frequency (obtained by direct measurement or spectral decomposition) are then used to construct wave fields with known wavenumbers that have maximal deviation from the measured responses. There are several methods for construction of the wavefields with maximal deviation from the measured responses, and they differ in accuracy, mathematical complexity and range of applicability.

One group of methods are based on Prony method for decomposition of a function into a series of damped sinusoids, an idea analogous to use of sinusoids in Fourier analysis. As the Prony method is very sensitive to measurement noise, the methods that use least-squares-fitting method [6], singular-value-decomposition method [7] and modifications of Prony method for finite structures [8] are used to increase stability of the calculation. While these methods are computationally efficient, they do not use any previous knowledge about the vibrating structure or propagating waves and they have two serious disadvantages: they are limited only to evenly spaced measurement points, and they require a large number of measurement points.

The other group of methods intends to resolve the substantial disadvantages of the previous methods, even if the cost is the speed of the methods. The idea is to repetitively test different wavefields looking for a set of parameters that provide the best fit between a wavefield and the experimental data. Different methods that belong to this group differ by the selection of the form of wavefield, the criteria selected as measure of the quality of

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the fit, and the procedure for finding the optimal wavefield parameters. One choice for the procedure is use of least-squares-method and different algorithms for non-linear optimization [2][3].

1.3. The correlation method for experimental determination of DR

The method that is the subject of this paper is the correlation method [9] that uses a simple form of the wavefield, and the simple sequential search of wavenumber space to find the value that provides maximum of correlation between the measured data and wavefield, which is used as the measure of their similarity.

In its simplest form, used to study the waves propagating along a beam, the correlation method for experimental determination of DR (in further text just “*correlation method*”) uses wavefield constructed on the basis of a harmonic progressive wave with selected wavenumber k_l and angular frequency ω , and the longitudinal axis of the beam as the x -axis. If a harmonic progressive wave is generated by force acting at one end of the beam, expressed in a complex form as $F(t) = F_\omega \cdot \exp(-i\omega t)$, the accelerations of beam points due to the wavefield may be expressed as $\underline{a}_l(x, t) = \underline{a}_\omega \exp(k_l x - i\omega t)$, and the wavefield of the response, the accelerances $\underline{w}_l(x, t) = \underline{w}_\omega(x, t)/F(t)$ will be described by its complex amplitudes:

$$\underline{w}_l(x, \omega) = \underline{w}_{l\omega} \cdot \exp(k_l x). \quad (2)$$

The complex amplitudes of the accelerances of the wavefield for selected wavenumber k_l are compared to complex amplitudes of accelerances $\underline{w}(x, \omega)$ measured in L measurement points x_l ($l = 1, 2, \dots, L$) along the beam, and the measure of the fit between two datasets is their correlation defined in the form:

$$\begin{aligned} \underline{W}_\omega(k_l) &= \int_{-\infty}^{+\infty} \underline{w}(x, \omega) \underline{w}_l^*(x, \omega) dx = \\ &= \underline{w}_{l\omega}^* \int_{-\infty}^{+\infty} \underline{w}(x, \omega) e^{-ik_l x} dx \end{aligned} \quad (3)$$

which is, due to the discrete nature of input data, estimated as:

$$\hat{W}_\omega(k_l) = \underline{w}_{l\omega}^* \sum_{l=1}^L \underline{w}(x_l, \omega) e^{-ik_l x_l} \Delta x_l = \underline{w}_{l\omega}^* \sum_{l=1}^L \underline{w}_{l\omega} e^{-ik_l x_l} \Delta x_l \quad (4)$$

where Δx_l represents the space interval around the point x_l , and introducing the notation $\underline{w}(x_l, \omega) = \underline{w}_{l\omega}$. The expression for correlation between the two datasets given by (3) leads to the conclusion that, if the wave propagating along the beam is a progressive harmonic wave, then the modulus of the estimated correlation will have maximal value when the wavenumber of the constructed wavefield k_l is equal to the wavenumber of the propagating wave k . In order to determine the DR of waves propagating along the beam, the wavenumber space is scanned in the range $k_{min} < k_l < k_{max}$ for each angular frequency ω , and the value of k_l that leads to maximal modulus of the estimated correlation is assumed to be the wavenumber that corresponds to the angular frequency ω , denoted as $k(\omega)$. Since the correlation is proportional to the complex amplitude factor $\underline{w}_{l\omega}^*$, it does not affect the position of the maxima of the estimated correlation (3), and the DR may be determined by finding the wavenumbers that maximize the expression:

$$Y_\omega = \left| \sum_{l=1}^L \underline{w}_{l\omega} \exp(-ik_l x_l) \Delta x_l \right|, \quad (5)$$

which will be called “*the correlation function*” in further text. As it may be seen, the correlation method does not make any assumption, nor imposes any limitation on the number and position of data measurement points x_l , which, together with its simplicity, represents the fundamental advantage of the method. In the case of equidistant measurement points $x_l = l \cdot d$, the correlation function takes the form

$$Y_\omega = \left| \sum_{l=1}^L \underline{w}_{l\omega} \exp(-i \cdot l \cdot d \cdot k_l) \right|, \quad (6)$$

with d being the distance between the measurement points.

The method was successfully used for determination of DR in steel and composite plates [9], and a later modification of the method [10], which uses damped plane waves for construction of the test wavefields, was successfully used for materials with high material losses.

1.4. The problem and motivation for the research

The implementation of the correlation method meets two problems that are consequences of a) finite number of measurement points, i.e. a consequence of discretization of equation (3) in space; b) finite number of wavenumber trial points, i.e. of discretization of equation (3) in inverse (wavenumber) space.

a) The discretization of the equation (3) in space using the equations (4) - (6) leads to “picket-fence” effect, the phenomenon that two signals become indistinguishable if they are sampled in a finite number of measurement points where they have equal values. Omnipresent in discrete data analysis [11], the “picket-fence” effect is best known as the cause of the aliasing in frequency analysis. In this case, the “picket-fence” effect means that for any given value of wavenumber k_l , there is an infinite number of wavenumbers $k_l' \neq k_l$ such that for each measurement point x_l holds:

$$\exp(-ik_l' x_l) = \exp(-ik_l x_l) \quad (l = 1, 2, \dots, L), \quad (7)$$

As a consequence, the wavefields with complex amplitudes given by equation (2), constructed using wavenumbers k_l and k_l' are indistinguishable in the measurement points, and hence they have equal estimations for correlation with experimental data. It further means that in an interval $k_l^{min} < k_l < k_l^{max}$ there may be several wavenumber values with equal correlation with the measurement data as any selected k_l , and, in particular, that there may be more than one local maximum with the same correlation function value within the interval. One way to ensure that the correlation method leads to correct results is to reduce the scanning wavenumber range, and consequentially the respective frequency range where the DR is determined.

The consequences of the “picket-fence” effect are particularly clear when the measurement points are equidistant because the equation (7) is then satisfied for each measurement point if $k_l' - k_l = z(2\pi/d)$, where z is an integer. Therefore, the correlation between the equidistant measurement data and the constructed wavefield, defined by correlation function (6), is in that case periodic in inverse space, as illustrated by the line graph in Figure 1.

The number of the equal local maxima in a range $k_i^{min} < k_i < k_i^{max}$ is $(k_i^{max} - k_i^{min})d/2\pi$. Since the range for trial values of wavenumbers should describe the waves propagating in both directions of the beam (positive and negative values of wavenumbers), the direct application of the correlation method is thus reduced to the inverse space range $-\pi/d < k_i < +\pi/d$, which is, in the theory of propagation of waves in periodic structures, known as the first Brillouin zone (abbreviated as BZ). Using the equation (1), the restriction of the range of trial wavenumbers to the 1 BZ reduces the frequency range for determination of the DR to

$$f < f_{BZ} = \frac{1}{2\pi} \sqrt{\frac{EI}{\rho A}} \left(\frac{\pi}{d} \right)^2 = \frac{\pi}{2d^2} \sqrt{\frac{EI}{\rho A}} \quad (8)$$

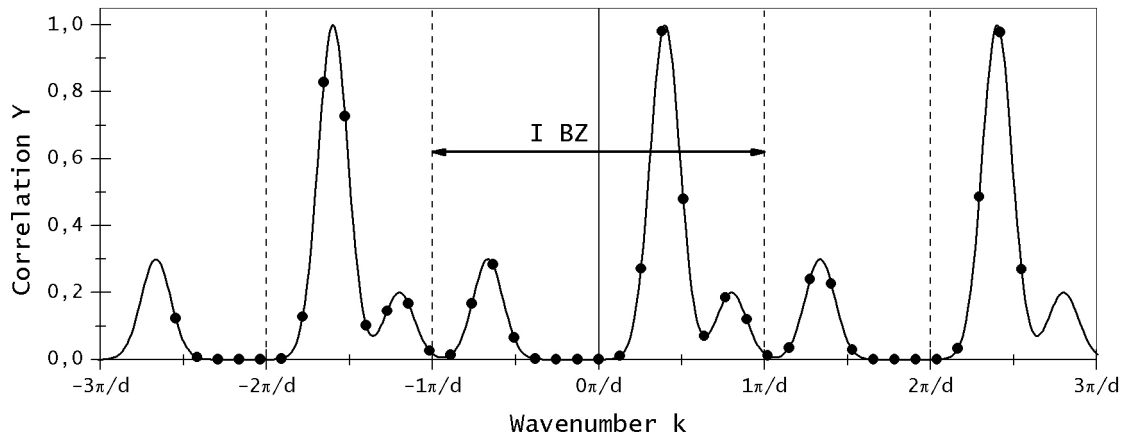


Figure 1: Periodic correlation function in the case of equidistant measurement points (line) and the correlation array as its aperiodic discretization by equidistant points in inverse space (dots)

b) The discretization of the equation (3) in inverse space is made during the process of searching the interval $k_{min} < k_i < k_{max}$ for the maximal correlation: the search is a discrete process, made by calculation of values of Y_ω using equation (6) in an array of J points $k_i^{(j)}$ ($j = 1, 2, \dots, J$). The obtained array of points $Y_\omega^{(j)} = Y_\omega(k_i^{(j)})$ ($j = 1, 2, \dots, J$) will be called “the correlation array” in the following text. The described approach to finding the maxima of correlation function meets the problem of missing (“skipping over”) the actual wavenumber values that maximize the expression (6). As it is explained that there may be more than one maximum with the same value of correlation function within the searched range of inverse space, missing of a maximum may lead to misidentification of the wavenumber that corresponds to a frequency.

In the case of equidistant measurement points, the correlation array does not have the previously explained periodicity with period $(2\pi/d)$, as illustrated by the scatter graph (black dots) in Figure 1. The loss of the periodicity arises due to the inevitable digital representation round-off error that is a consequence of transcendent nature of π , which cannot be represented in binary (or any other rational) number system using finite number of digits. For that reason, at least one of the values k and $k + \pi/d$ cannot be represented in binary system, and the array of points obtained by discretization of correlation function (5) cannot be periodic. The loss of the periodicity means that at most one of the equivalent local maxima may be detected during the search of the interval, $k_i^{min} < k_i < k_i^{max}$. Since the difference between a local maximum and the

If the measurement points are not equidistant, then the correlation is not periodic in wavenumber space, but the system of equations (7) still has an infinite number of solutions for any selected value of k_i , and the average difference between the solutions is close to $(2\pi/d)$, where d is the average distance between the measurement points [13]. The problem is somewhat analogous to the problem of approximation of N measurement data by a polynomial function, since there is an infinite number of polynomial functions (with the order higher than $N-1$) that have the values equal to the measured data in measurement points.

nearest test value $k_i^{(j)}$ is random, it means that the $k_i^{(j)}$ value that has maximal value of $Y_\omega(k_i^{(j)})$ does not have to be the wavenumber which corresponds to the frequency ω , which was the basic idea of the correlation method.

This paper presents an algorithm for analysis of the correlation array values $Y_\omega^{(j)} = Y_\omega(k_i^{(j)})$ that enables extension of the frequency range for determination of DR of flexural waves in beams using the properties of DR. The proposed method is compared to determination of DR using maximal values of the correlation function given by equation (5), and it is shown that the method proposed in the paper may extend the frequency range for determination of the DR to extents of the limits of the frequency range used in the experiment.

2. METHOD

The DR of flexural waves in beams, given by equation (1), is an even, continuous and monotonous function of wavenumber. On the other hand, as explained above, the correlation method determines the wavenumbers that correspond to frequencies of the waves, meaning that it leads to the DR in the inverse form,

$$k = \pm \left(\frac{\rho A}{EI} \right)^{1/4} \sqrt{\omega}. \quad (9)$$

This form of the DR represents a solution of the equation (1) for wavenumber k , and the equation has two solutions, one positive and one negative. The two solutions have different physical meanings, representing progressive waves propagating in opposite directions, which may be

seen by the opposite signs of corresponding phase velocities.

Furthermore, the two solutions describe energy transfer in opposite directions, because the group velocities have the same signs as phase velocities. Depending on the speed of flexural waves and material loss factor, both of the waves may be observed during the experimental determination of DR for flexural waves in beams. On the other hand, the correlation method leads to a single value of the wavenumber corresponding to a certain frequency, and it is not possible to know in advance the sign of the wavenumber value with the highest correlation function value. Therefore, the interval $k_t^{\min} < k_t < k_t^{\max}$, which is searched to find the corresponding wavenumber, should contain both positive and negative values of k_t . Due to the existence of two branches of the solutions of (1), any algorithm for implementation of the correlation method should close by outputting both the positive and negative values of the wavenumbers corresponding to a frequency, obtained by multiplying the absolute values of the determined wavenumbers by plus one and minus one, respectively.

As is it is shown in the introduction, the discrete nature of the set of measurement points (the “picket-fence-effect”) causes indistinguishability of the correlation function values for some higher and lower values of wavenumbers, and in the special case of equidistant measurement points, the correlation function $Y_\omega(k)$ is a periodic function of the wavenumber. The further consequence of the picket-fence-effect is that there is an infinite number of branches of wavenumber-frequency functions that have the same values of correlation functions (5) as the two branches of the DR given by the equation (9), which are sought for by the correlation method. The branches for the case of equidistant measurement points are shown in the Figure 2. It should be noted that the branches have all possible combinations of signs of phase and group velocities.

2.1. The discretization of the correlation function and equivalent maxima of the correlation function

In the introduction was explained that the basis of the correlation method is search for a k_t value that has maximal value of correlation function within an interval $k_t^{\min} < k_t < k_t^{\max}$. The further analysis of the influence of the picket-fence-effect to the correlation function (5), as well as the Figure 2, led to conclusion that for each frequency ω the correlation function should have several equal maxima in a wavenumber interval $k_t^{\min} < k_t < k_t^{\max}$. In the case of equidistant measurement points, each of the maxima correspond to a branch presented in Figure 2, so that the number of equal maxima in the interval $k_t^{\min} < k_t < k_t^{\max}$ is limited to

$$K(k_t^{\max}, k_t^{\min}) \leq \left\lceil \frac{k_t^{\max} - k_t^{\min}}{\pi/d} \right\rceil + 1. \quad (10)$$

Since the picket-fence-effect has the same influence on correlation function even if the measurements are not equidistant, the equation (10) may be also used in that case for estimation of the maximal number of correlation function maxima within the scanned wavenumber range.

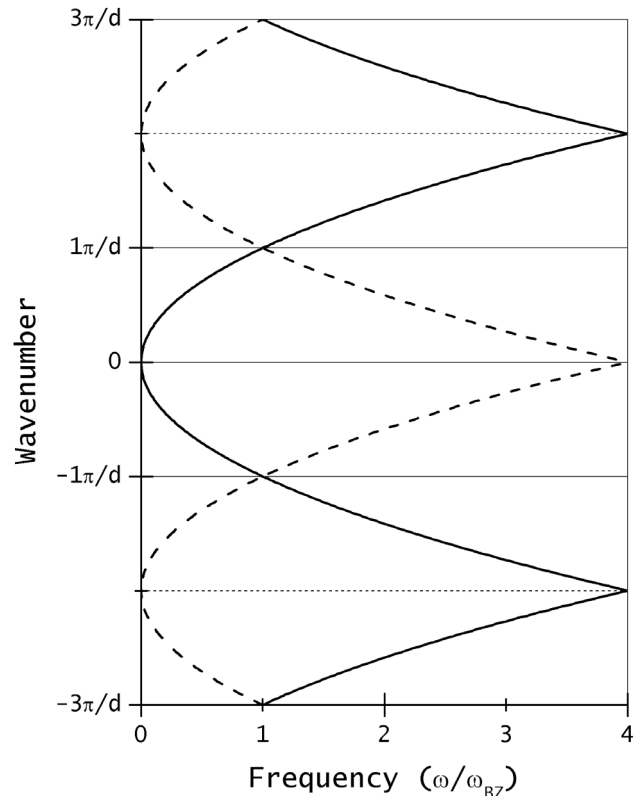


Figure 2: The branches of wavenumbers with the same value of correlation function with a wavefield of a progressive wave discretized in equidistant points; the branches that start in odd BZs are presented with solid lines, and the branches that start in even BZs are presented by dotted lines

Since the implementation of the search for maxima requires the discretization of correlation function, which, as explained in the introduction, may lead to loss (“skipping”) of some maxima, the algorithm for identification of the maxima requires some modifications. As it is illustrated in the Figure 1, the discretization of the correlation function does not reveal all the wavenumbers with maximal values of correlation function, but each of the maxima of the correlation function $Y_\omega(k)$ leads to a local maximum in the correlation array $Y_\omega^{(j)}$. As the local maxima in the array $Y_\omega^{(j)}$ that correspond to the maxima of correlation function $Y_\omega(k)$ do not have to have equal values, they will be called “the equivalent maxima” in the text that follows. Since there are no more than $K(k_t^{\min}, k_t^{\max})$ maxima of correlation function in an interval $k_t^{\min} < k_t < k_t^{\max}$, there may be no more than $K(k_t^{\min}, k_t^{\max})$ equivalent maxima in correlation array. However, the correlation array may have (and in practice has a lot of) additional local maxima that do not correspond to maxima of correlation function, and a simple search for all local maxima in an interval $k_t^{\min} < k_t < k_t^{\max}$ is not sufficient to determine the equivalent maxima in the interval. The equivalent maxima may be extracted from the set of all local maxima by sorting the set by magnitude of correlation function and by using two of their properties: 1) that the equivalent maxima have substantially higher values of correlation function than the other local maxima, and 2) that there is no more than $K(k_t^{\min}, k_t^{\max})$ maxima of correlation function in the interval $k_t^{\min} < k_t < k_t^{\max}$. Once the equivalent maxima for a frequency ω are determined, the

array of wavenumbers corresponding to them, $k_{\omega}^{(m)}$ ($m = 1, 2, \dots, M \leq K$ – this array will be called “*the candidates array*” in the following text), has to be analysed to determine the wavenumber that corresponds to the frequency in the DR. Since the differences between the values of correlation function of the equivalent maxima are random, the selection of the wavenumber that corresponds to a frequency is made on basis of properties of DR described by the equation (9).

2.2. The properties of the DR

The expression (9) represents a union of two mathematical functions (called “*branches of square root*”) with the following properties: both functions are defined only for positive values of the frequency, both are continuous and monotonous, but the positive branch is monotonously increasing, while the negative branch is monotonously decreasing function. These mathematical properties may be used to improve the algorithms for experimental determination of DR by providing constraints that the determined corresponding values of wavenumbers and frequency have to satisfy.

The continuous character of the DR may be described by the constraint:

$$\lim_{\omega \rightarrow \omega_0} k = k(\omega_0) \quad (11)$$

for all positive frequencies, which essentially means that the infinitesimally close frequencies correspond to infinitesimally close wavenumbers.

The different monotonous characters of the two branches of the DR may be expressed by the constraints:

$$\omega_2 > \omega_1 \Rightarrow \begin{cases} k(\omega_2) > k(\omega_1), k > 0 \\ k(\omega_2) < k(\omega_1), k < 0 \end{cases} \quad (12)$$

2.3. The discretized DR and the implementation of the continuity constraint

The design of any algorithm that relies on the properties of the DR, however, has to consider that the theoretical expression (9) represents two continuous functions of wavenumber $k(\omega)$, while the DR determined by the correlation method represents an array $k_n = k(\omega_n)$, ($n = 1, 2, \dots, N$), calculated for the selected set of frequencies ω_n . The finite differences between the frequencies ω_n mean the discretized DR cannot be continuous in the meaning of the equation (11), and that the continuity has to be interpreted in a different manner.

For the purposes of the implementation of the correlation method, the explanation after the equation (11) that “the continuity essentially means that the infinitesimally close frequencies correspond to infinitesimally close wavenumbers” may be interpreted that the respective algorithm should maximize the distance between the two subsequent points of the discretized DR.

The implementation of such an interpretation is quite clear and straightforward within the BZs, as it means that the points on a branch are closer than the points of different branches with the same correlation value. The closer the subsequent frequencies are the more will the interpretation be correct. However, the problem arises close to the points of intersection of different branches, where the different branches have same values, and where the proximity of points does not necessarily imply

continuity, since two close points may belong to different branches of DR with the same correlation value.

2.4. The experimental and the round-off errors and the implementation of monotonicity constraint

Unlike the constraint of the continuity, the constraint of the monotonicity (12) holds also for the discretized DR, which may be useful for identification of the wavenumber corresponding to a certain frequency, and particularly for the frequencies close to the intersection of two branches, since one of the intersecting branches is monotonously increasing, while the other branch is monotonously decreasing.

The problem with the implementation of the monotonicity constraint, however, are the experimental errors made during the measurement of accelerations, and the round-off errors made during the long process of calculations that comprises many additions (first calculation of the DFTs of measured signals, and then calculation of correlation). Due to the errors, the wavenumbers determined as corresponding to subsequent points of DR may not satisfy the monotonicity constraint, which may lead to misidentification of the corresponding wavenumber and consequent “switch” from one branch of the DR to another branch with the same correlation value.

The risk of monotonicity loss increases with decrease of the difference between the subsequent wavenumbers, which occurs due to two reasons: 1) due to decrease of the difference between the frequencies, which is opposite to the requests of the continuity constraint, and 2) due to decrease of the first derivative of the DR in the form (9), which decreases with increase of the frequency.

2.5. The selection of the initial point of the discretized DR

The application of properties of continuity and monotonicity for determination of DR comprises analysis of candidate wavenumbers with respect to previously determined points of the discretized DR. However, such procedure is not possible for the initial point of the discretized DR, as there are no previous points to be used as the reference for the analysis, and another procedure has to be devised.

The initial point of a DR is the origin, since the zero wavenumber corresponds to zero frequency. In practice, however, the low wavenumber values mean long wavelengths of the corresponding wavefield, and in such cases measurement error becomes comparable to the differences between the amplitudes of accelerations in measurement points, which means that the correlation method is not applicable.

The algorithm presented in this paper uses the assumption that the lowest frequency of the discretized DR is lower than f_{BZ} , so that the corresponding wavenumber belongs to IBZ. In that case, the corresponding wavenumber is the member of the candidate array with the lowest magnitude, i.e. the wavenumber which corresponds to the member of the candidate array which is closest to origin.

2.6. The algorithm

The algorithm for the determination of the DR by the correlation method uses the following input data:

- the increasing array of DR frequencies ω_n ($n = 1, 2, \dots, N$); this array may be defined in several ways, such as by the initial value, the frequency step and number of frequencies N ;
- the increasing array of measurement points x_l ($l = 1, 2, \dots, L$);
- the array of arrays (matrix) of complex amplitudes of measured frequency response functions w_{ln} ($l = 1, 2, \dots, L$ and $n = 1, 2, \dots, N$);
- the increasing array of wavenumbers to be used as test values, $k_t^{(j)}$ ($j = 1, 2, \dots, J$); this array also may be defined in several ways, such as by the initial value, the wavenumber step and number of wavenumbers J ;
- the maximal relative difference between the correlations of equivalent maxima ε_Y ($0 < \varepsilon_Y < 1$); this value enables selection of candidate array among the detected local maxima;
- the proximity limit ε_k ($0 < \varepsilon_k < 1$), which enables selection of a DR point among the members of the candidate array;

The algorithm for the determination of the DR by the correlation method consists of the following steps:

1) Preparation steps

1.1) Determine the lower and upper limits of wavenumber range to be searched as $k_t^{max} = k_t^{(J)}$ and $k_t^{min} = k_t^{(1)}$;

1.2) Calculate the average distance between the measurement points $d = (x_L - x_1)/(L-1)$;

1.3) Calculate the maximal number of equivalent maxima in the testing wavenumber range, K , using the equation (10);

2) Repeat the procedure for determination of the points of the DR ($n = 1, 2, \dots, N$)

2.1) Calculate the correlation array $Y_{\omega}^{(j)}$ ($j = 1, 2, \dots, J$) using equation (5) and $\omega = \omega_n$ for all test wavenumber points $k_t^{(j)}$ ($j = 1, 2, \dots, J$);

2.2) Find the K largest local maxima in the correlation array $Y_{\omega}^{(j)}$ and store their corresponding wavenumber and correlation function values as the auxiliary parallel arrays $k_m^{(m)}$ and $Y_m^{(m)}$ ($m = 1, 2, \dots, K$);

2.3) Sort the auxiliary parallel arrays $k_m^{(m)}$ and $Y_m^{(m)}$ according to the magnitude of $Y_m^{(m)}$;

2.4) Determine the number of equivalent maxima in the correlation array M as number of the local maxima such that relative difference between their correlation value and maximal correlation value $1 - Y_m^{(m)}/Y_m^{(1)}$ is smaller than ε_Y ; the array $k_m^{(m)}$ ($m = 1, 2, \dots, M$) represents the candidate array;

2.5) Select the wavenumber that corresponds to the frequency ω in the DR:

2.5.1) If it is the first point of the DR ($n = 1$), then the corresponding wavenumber is the smallest absolute value of the wavenumbers of the candidate points, and algorithm proceeds with the step 2.6;

2.5.2) If ($n > 1$) then

2.5.2.1. Find the smallest element of candidate array higher than the wavenumber of the previous point k_p – the Hi candidate $k^{(Hi)}$;

2.5.2.2 Find the largest element of the candidate array lower than the wavenumber of the previous point k_p – the Lo candidate $k^{(Lo)}$;

2.5.2.3. If one of the elements $k^{(Hi)}$ or $k^{(Lo)}$ does not exist, then the corresponding wavenumber is the existing one, and the algorithm proceeds with the step 2.6;

2.5.2.4 Calculate the *proximity ratio* r , i.e. the ratio of the distance between the previous point and the candidate point to the distance between the candidate points,

$$r(k) = \frac{|k - k_p|}{k^{(Hi)} - k^{(Lo)}} \quad (13)$$

for both Hi and Lo candidate, $r^{(Hi)} = r(k^{(Hi)})$ and $r^{(Lo)} = r(k^{(Lo)})$

2.5.2.5 Select the wavenumber corresponding to the frequency ω using the determined values $k^{(Lo)}$, $k^{(Hi)}$, $r^{(Lo)}$ and $r^{(Hi)}$ and one of the approaches described below;

2.6) Set the determined point (k_n, ω_n) as the new previous point $(k_p, \omega_p) = (k_n, \omega_n)$;

The output of the algorithm is the DR in the form of parallel arrays ω_n, k_n ($n = 1, 2, \dots, N$).

Depending on implementation of the continuity and monotonicity constraints, several approaches may be devised to select the wavenumber corresponding to the frequency ω using the determined values $k^{(Lo)}$, $k^{(Hi)}$, $r^{(Lo)}$ and $r^{(Hi)}$ within the step 2.5.2.5 of the algorithm.

The simplest approach (the “CloseHi” method) represents a straightforward implementation of the continuity and monotonicity constraints by selecting the “Hi” candidate $k^{(Hi)}$ (the closest element of the candidate array with wavenumber higher than the previous point) as the wavenumber corresponding to the frequency ω . While the approach is simple and has physical justification, it is sensitive to round-off and measurement errors that make experimental DR a non-monotonous array.

In order to deal with the influence with the non-monotonous behaviour of experimental DR, the member of the candidate array closest to the previous point (the “Close” method) may be selected as the wavenumber corresponding to the frequency ω (therefore, if $r^{(Lo)} < r^{(Hi)}$ then $k_n = k^{(Lo)}$, otherwise $k_n = k^{(Hi)}$). While this method is less sensitive to round-off and measurement errors, it is sensitive to “switches” between the branches of wavenumbers with the same values of correlation function, presented in Figure 2, which may occur at the BZ boundaries, where different branches have close values. Once $k^{(Lo)}$ is selected within such an area, the following closes points may easily belong to a monotonously decreasing branch of local maxima of correlation function.

With the aim to detect the ambiguities caused by intersections of different branches of branches of wavenumbers with the same values of correlation function, the proximity limit ε_k is introduced as a measure of proximity of a candidate point to the previous point. The introduction of the proximity limit represents a further test for the wavenumber detected by the “Close” method. If the proximity ratio of the candidate point is lower than the proximity limit, $r_n = r(k_n) < \varepsilon_k$, then the determined value of wavenumber k_n is accepted as the wavenumber

corresponding to the frequency ω . However, if the proximity ratio of the candidate point is higher than the proximity limit, then the candidate array is considered ambiguous, and the selection of the wavenumber corresponding to the frequency ω needs further elaboration.

At least three approaches to handling of the ambiguous candidate arrays may be considered:

- automated skipping of selection of the wavenumber corresponding to the frequency ω (Auto-Skip method);
- automated selection of the weighted average of Hi- and Lo- candidate points, which is further considered as the value of the wavenumber corresponding to the frequency ω (Auto-Mid approach); while the method seems like a reasonable choice, there is no firm physical reasoning behind it, and after the initial tests did not show promising results, it is abandoned;
- manual selection, where the program lists the complete candidate array (parallel arrays of wavenumbers and the corresponding correlations $k_m^{(m)}$ and $Y_m^{(m)}$, $m = 1, 2, \dots, M$), and the user makes an educated guess of the wavenumber corresponding to the frequency ω ; this approach may give good results in the case since DR is known, and is useful for experimental determination of viscoelastic properties of materials; however, the obtained results are somewhat artificial and it will not be presented in this paper.

3. EXPERIMENT

In order to study the proposed algorithm for implementation of correlation method, two series of experiments were carried out. Each of the experiments comprised series of subsequent measurements of time history of acceleration $a_x(t)$ at a single point of a beam excited by impact hammer.

3.1. Measurement object

The beam was a steel rod with length $D \approx 1,65$ m, and roughly square cross-section with side $b \approx 1$ cm. The end parts of the rod, with the length of around 25 cm, were resting on soft sponges, with the aim to emulate a beam with free ends.

3.2. Excitation

The vibrations were excited by the impact hammer B&K 8204 with sensitivity 30.89 mV/N, and the excitation was measured by the B&K Pulse system with CCLD input with range $\pm 10V$, so that force measurement range was around 300 N. The impact hammer is light and small, and it is equipped with a light and hard impact head, so that the bandwidth of the impact hammer is broad, up to 10 kHz.

Automatic double-hit detection was not provided, so that each excitation hit was monitored by inspecting the time history of impacting force $F(t)$, and the double-hits were discarded.

The rod was hit close to the beginning, at the area $x = 0-1$ cm from the beginning of the rod. The hitting area was covered by a piece of scotch-tape that provided a bit of dampening to the hit, which reduced the bandwidth to around 6 kHz, but also reduced excessive accelerations that prevented recording of the response by the software

used for data acquisition. The maximal impact force was in the range 8-12 N.

3.3. Response

The response was measured using the accelerometer B&K 4507Bx with sensitivity 10.055 mV/ms⁻² and B&K Pulse system with CCLD input with range $\pm 10V$, so that so that acceleration measurement range was around ± 1000 m/s².

3.4. Experimental procedure

For each of the measurement points the measurements of acceleration were taken 10 times. During each of the measurements were recorded time histories of excitation force and acceleration. After each of the measurements, the data acquisition software calculated power spectral density of force and acceleration, frequency response function (FRF) of acceleration, and the respective coherence.

3.5. Measurement points

The measurements were organized in two series – experiments, with the aim to test both the case of equidistant measurement points, which is easier to implement and interpret, and the case of non-equidistant measurement points, which is great practical importance.

In the first case, the measurement points were uniformly distributed at 10 equidistant positions with distances $d \approx 15$ cm, with the first measurement point being at the distance d from the beginning of the rod and the last being at the distance d from the end of the rod. Therefore, the arrangement of the measurement points was symmetric with respect to the centre of the rod.

In the second case, the 10 measurement points were selected to have the distances to the beginning of the rod (the excited end) being 20 cm, 28 cm, 44 cm, 52 cm, 68 cm, 76 cm, 92 cm, 116 cm, 124 cm and 148 cm, which is proportional to prime numbers sequence 5, 7, 11, 13, 17, 19, 23, 29, 31 and 37. The selected set of measurement points roughly covers the same part of the rod as in the first case, while the proportionality of the distances to the prime numbers reduces – and practically eliminates – the influence of the aliasing due to the finite length of the measurement points array.

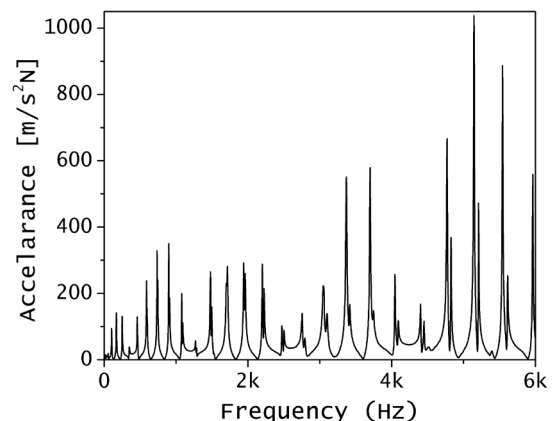


Figure 3: FRF of acceleration at point $x = 90$ cm from beginning of the rod

4. RESULTS

4.1. Frequency response spectra

The accelerance amplitude spectra revealed the expected resonant behaviour (Figure 3). The resonant frequencies present in all spectra are the same, but the amplitudes of the FRFs at different resonant points show different behaviours at different points. The frequency with maximal amplitude shows dependence on the measurement point. The resonant peaks at low frequencies (14 Hz – 590 Hz) are singlets, while at higher frequencies the resonant peaks appear to be doublets, with the frequency split of the doublets increasing with resonant frequency.

4.2. Correlation function

The calculated correlation functions show the predicted behavior, showing periodicity with period $2\pi/d$ for equidistant measurement points (Figure 4). Both positive and negative solution of the equation (9) represent strong local maxima of correlation function, which are reproduced in each of the BZ.

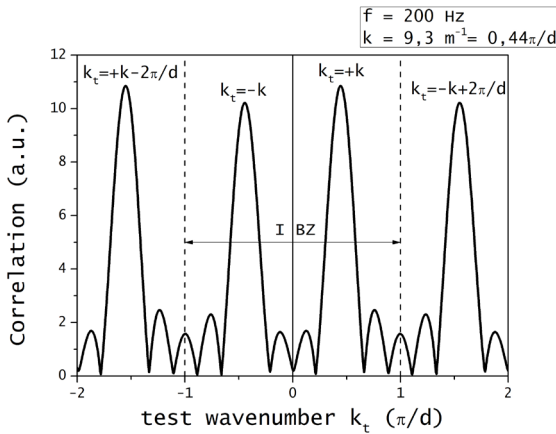


Figure 4: Correlation function for equidistant measurement points

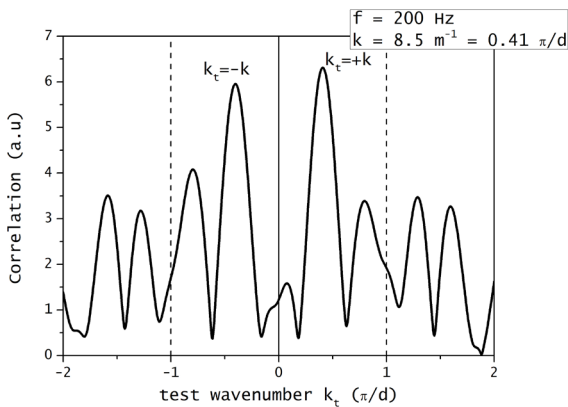


Figure 5: Correlation function for non-equidistant measurement points

Even in the case when the measurement points are not equidistant, both the positive and negative solution of the equation (9) still represent local maxima, although the maxima are not that sharp as in the case of equidistant measurement points (Figure 5). While the correlation function is not periodic in that case, it should be noted that the positions of the local maxima of correlation function

(which are relevant for correlation method) still show approximate periodicity.

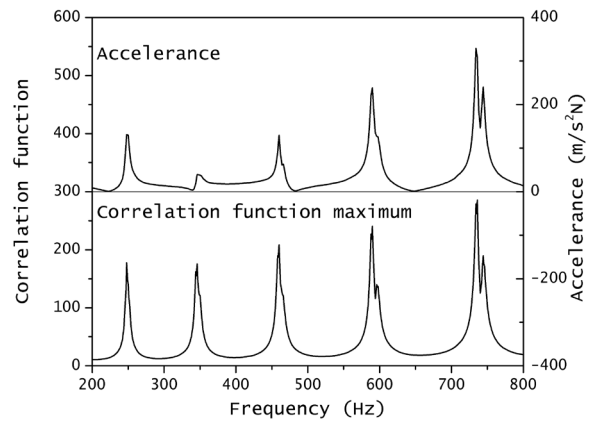


Figure 6: Dependencies of FRF of accelerance and the maximum of the correlation function on frequency

Since the values of the local maxima of correlation function depend on the accelerance according to the equation (6), the maxima of the correlation function also show the same resonant behaviour as accelerance, as illustrated in the Figure 6.

4.3. Straightforward application of the correlation method

In this section are presented results of the straightforward application of the correlation method, which consists in selection of the wavenumber corresponding to highest correlation as the wavenumber corresponding to frequency ω in the DR.

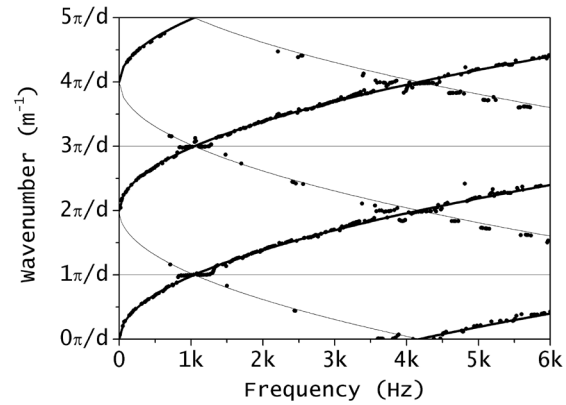


Figure 7: Wavenumbers detected by the straightforward implementation of correlation method to equidistant measurements ($\pi/d \approx 21 \text{ m}^{-1}$)

4.3.1. Equidistant measurement points

The results of the straightforward application of the correlation method to the measurements taken in equidistant measurement points are shown in Figure 7, where the correlation function was calculated for test values of wavenumbers in the range $(0, +5\pi/d)$. The figure clearly shows that the detected values of wavenumbers corresponding to frequencies used in the experiment belong to all branches of maxima existing in the test wavenumbers range, and that the method is not capable of distinguishing them. Majority of the detected wavenumbers describe energy transfer in the direction of incidence (positive group velocity), but some of the

wavenumbers describe energy propagation in the direction of reflection (negative group velocity).

Although the Figure 7 also shows lines indicating the branches of correlation function maxima, which may be interpreted as branches of DR, it is not possible to apply fitting procedure to the obtained results, because consequent points of the graph in general belong to different branches. Therefore, the lines in the Figure 7 are just guide for eyes, and they are constructed using the $k = \pm b_{kf} f^{1/2} + z(2\pi/d)$ dependences suggested by the equation (9), with the parameter $b_{kf} \approx 0.65 \text{ 1}/(\text{mHz}^{1/2})$, selected using trial-and-error method to have a good visual agreement with the experimental points. The value is in a fairly good agreement with the theoretical prediction $b_{kf} \approx 0.66 \text{ 1}/(\text{mHz}^{1/2})$, obtained from the equation (9) using the values $E = 200 \text{ GPa}$ and $\rho = 7800 \text{ kg/m}^3$ for Young modulus and density of steel.

In order to better study the obtained results, the range of test wavenumbers is reduced to the IBZ, and the results are shown in the Figure 8.

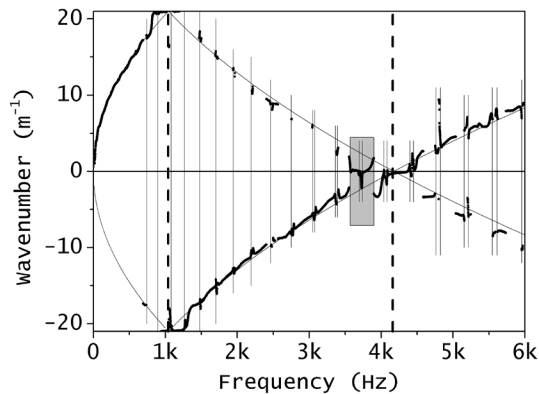


Figure 8: Dispersion relationship from Fig. 7 reduced to the first Brillouin zone ($\pi/d \approx 21 \text{ m}^{-1}$)

The reduction of the wavenumbers to the IBZ almost restores the continuity of the experimentally determined DR. The discontinuities caused by “switching” to negative group velocity branch correspond to resonant frequencies (marked by the thin solid vertical lines in the Figure 8). The mere existence of the doublets not predicted by the theory indicates that, under resonant conditions, the experimental setup does not satisfy the assumptions requested by the theoretical model.

Apart from the “switching” to the negative group velocity, the reduction of the wavenumbers to the IBZ restores the monotonous character of the experimentally detected DR, with the notable exception of borders of the Brillouin zones (marked by the thick vertical dotted lines in the Figure 8), where the experimentally detected wavevectors are almost independent of the frequency (i.e. the group velocity is zero, meaning that the energy transfer does not occur, similar to the standing waves). Since the Brillouin zones are only the consequence of equidistant distribution of measurement points, the observed departure from monotonicity has also to be an artefact introduced by the experimental setup. In fact, when the wavenumber is close to the border of the Brillouin zones $k = z(\pi/d)$, the distance between the measurement points is multiple of the half-wavelength ($d = z\lambda/2$), and the progressive waves moving in the opposite directions cannot be distinguished in the measurement points, which leads to inability of the

measurement to detect the direction of the energy transfer in that case.

4.3.2. Non-equidistant measurement points

The results of the straightforward application of the correlation method to the measurements taken in non-equidistant measurements are shown in Figure 9, where the correlation function was calculated for test values of wavenumbers in the range $(0, 100 \text{ m}^{-1})$, which is close to the range used on the case of equidistant measurement points. The detected values of wavenumbers corresponding to frequencies used in the experiment belong to many more branches of maxima than in the case of equidistant measurements. The branches do not show periodicity, and the non-equidistant measurement points prevent fault detection of standing waves at the borders of Brillouin zones, which are observed in the case of equidistant measurement points. On the other hand, the obtained results are in general less clear than in the case of equidistant measurement points.

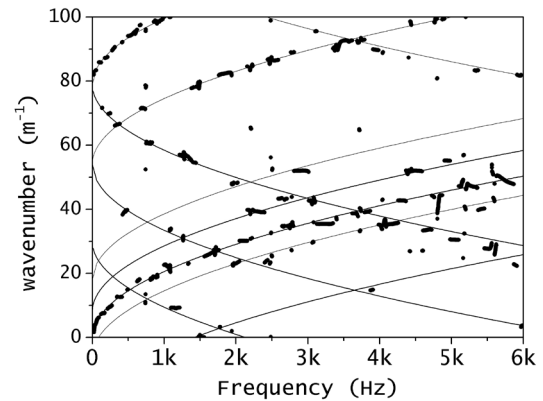


Figure 9: Wavenumbers detected by the straightforward implementation of correlation method to non-equidistant measurement points

4.4. Increased number of measurement points

Figure 10 presents the results of the straightforward application of the correlation method to the measurements taken in both equidistant and non-equidistant measurement points (20 measurement points), where the correlation function was calculated for test values of wavenumbers in the range $(-50 \text{ m}^{-1}, +50 \text{ m}^{-1})$, which is close to the range of values expected for the wavenumbers according to the equation (9).

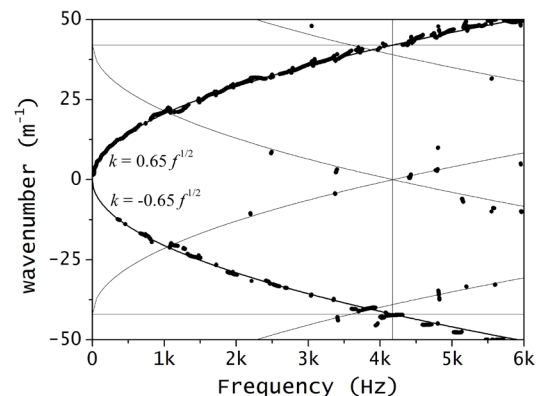


Fig. 10: Wavenumbers detected by the straightforward implementation of correlation method to measurements data taken in all measurement points

The figure shows that the detected wavenumbers predominantly belong to the two branches described by the equation (9), with “switches” to the other branches of local maxima being rare. While majority of detected wavenumbers describe the wave propagation in direction of incidence, almost one third of detected wavenumbers are describing the wave propagation in direction of reflection. Although the measurement points set consisting of both equidistant and non-equidistant points represents a non-equidistant set of points, it may be noticed that in the vicinity of the wavenumber value $k = \pi/d_{av} \approx 42 \text{ m}^{-1}$ (frequencies close to 4100 Hz) where d_{av} represents the average distance between the measurement points, the detected wavenumbers are independent on frequency, indicating the inability to detect direction of energy transfer, similar to the case of equidistant measurement points. In general, detection of the wavenumbers in vicinity of intersections between different branches of correlation function maxima represents the critical problem of the correlation method.

4.5. Application of the algorithm for extension of the frequency range

This section presents the results of the application of the variations of the algorithm proposed in section 2.6 of this paper. The algorithm is intended to improve the applicability of the correlation method beyond the IBZ, so the key aspect to be considered are the results obtained for wavenumbers range $k > \pi/d$, hence frequencies above $f_{BZ} \approx 1 \text{ kHz}$. For the sake of comparison, in the following figures the dash-dot line presents the theoretical predictions obtained using the equation (9) with the values $E = 200 \text{ GPa}$ and $\rho = 7800 \text{ kg/m}^3$ for Young modulus and density of steel.

4.5.1. Equidistant measurements

The results of the application of two methods that strictly use one of the properties of the DR in form (9), the “CloseHi” method (that favours monotonicity, solid line) and the “Close” method (that favours continuity, dashed line) are presented in the Figure 11.

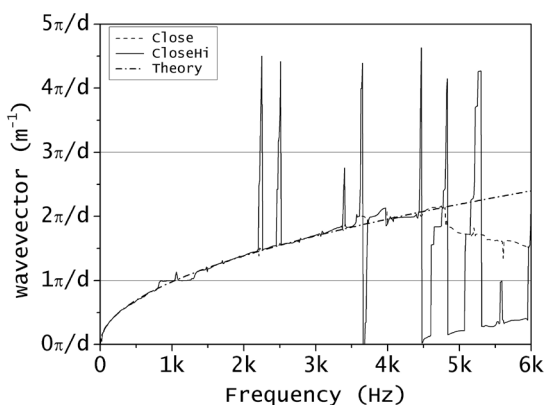


Fig. 11: Comparison of the DRs obtained by “Close” and “CloseHi” methods to theoretical predictions in the case of equidistant measurement points

The obtained results show that the variations of the algorithm may be used in the frequency range beyond the f_{BZ} but also clearly demonstrate the weaknesses of both approaches explained in the section 2.6.

Due to its sensitivity to round-off errors, the “CloseHi” method “switches” to higher branches of equivalent maxima in cases where the candidate point $k^{(Lo)}$ is smaller than the previous value k_p only due to the round-off error. Once such “switch” occurs, the algorithm returns to correct values only when value $k^{(Hi)}$ is above the range of tested wavenumbers. Such behaviour leads to the “glitches” observed in the range 2-4 kHz, as well as to oscillations between various branches of equivalent maxima observable in the range 4.5-6.0 kHz.

The “Close” method follows the theoretical predictions almost to 5 kHz, but, as predicted in the section 2.6, fails to correctly detect the corresponding value of wavenumber close to the BZ borders when the first derivative of the DR decreases: the difference between the wavenumbers of successive points on the border of IBZ is still sufficient to enable correct selection between the candidate points, but the decrease of the first derivative with increase of frequency makes selection between the points of different branches difficult in vicinity of intersection at value $2\pi/d$ (close to 4.1 kHz, see Figure 7). Once the algorithm switches to negative group velocity branch, its preference for continuity results in selection of the wavenumbers corresponding to that branch in the range above 5 kHz.

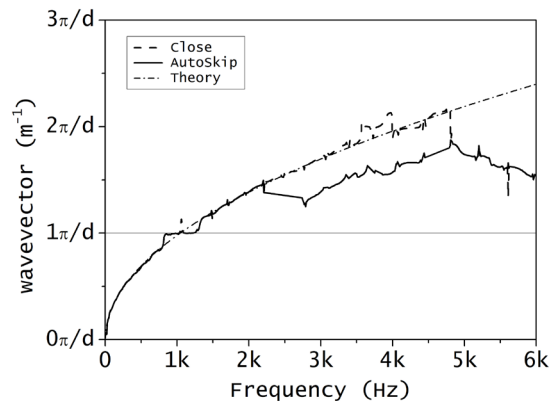


Fig. 12: Comparison of the DRs obtained by “Close” and “AutoSkip” methods to theoretical predictions in the case of equidistant measurement points

Figure 12 presents comparison of the results of the application of the “AutoSkip” method (solid line), in comparison to the results of the “Close” method (dashed line). Due to the skipping of a data point, or even a sequence of data points, the “AutoSkip” method keeps the wavenumber of the previous point at low value in comparison to the following points; therefore, an equivalent local maximum that belong to a lower branch may become the next close point, which in turn leads to “switch” to the respective branch; after the “switch” the lower branch is followed using the requests for monotonicity and continuity.

The ranges of constant wavenumber values around frequencies 1 kHz and 4.1 kHz show that all presented methods are sensitive to the fault detection of standing waves in vicinity of intersections between branches of equivalent maxima with positive and negative group speeds. That deficiency is inherent to the experimental data obtained using equidistant measurement points: since such experimental setup leads to similar experimental

results for waves propagating in both directions of a beam, no algorithm may find a difference between the data.

4.5.2. Non-equidistant measurements

Since the deficiencies of the methods that strictly use one of the properties of the DR led them to failures even in the case of equidistant measurement points, this section considers only application of “Close” (solid line) and “AutoSkip” (dashed line) methods to results of measurements in non-equidistant points, presented in the Figure 13.

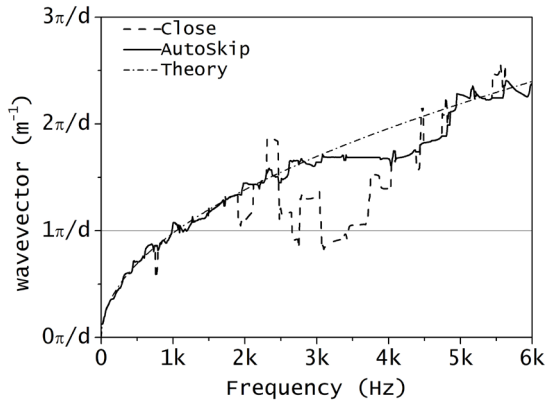


Fig. 13: Comparison of the DRs obtained by “Close” and “AutoSkip” methods to theoretical predictions in the case of non-equidistant measurement points

The presented results show the intended and expected resistance to fault detection of standing waves, which represents an important advantage of use of non-equidistant points setups. Unfortunately, the obtained DR show that both methods, and in wide frequency ranges, fail to follow the theoretical predictions. It is interesting to notice that in the case of non-equidistant measurement points the “Auto-Skip” method gave better results (therefore, opposite to the case of measurements in equidistant points), as it was able to follow the theoretical predictions until 3.2 kHz, while the “Close” method failed around 2 kHz.

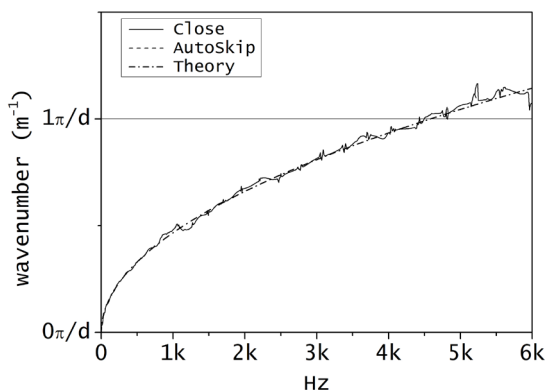


Fig. 14: Comparison of the DRs obtained by application of “Close” and “AutoSkip” using all measurement points methods to theoretical predictions

On the other hand, it may also be observed that the application of the methods to the measurements in non-equidistant points leads to more stable results than it is the case with measurements in equidistant points. Furthermore, the use of measurements of non-equidistant points

leads to convergence of results towards the theoretical predictions in high-frequency range, opposite to the case of use of measurements in equidistant points.

4.5.3. Increased number of measurement points

In order to compare the proposed method to the results of straightforward application of correlation method, Figure 14 presents the results of application of the “Close” and “AutoSkip” method to data taken in both equidistant and non-equidistant measurement points, analogously to the results presented in section 4.4 and Figure 10.

Since there are no “switches” to negative group velocity branches that may be observed in the Figure 10, it may be concluded that the proposed method is superior to the straightforward application of correlation method in this case. On the other hand, the use of the increased number of measurement points leads to extension of the IBZ up to frequencies of 4.1 kHz, which covers a large part of the frequency range used in measurements and does not provide insight of applicability of the proposed methods for extension of the frequency range beyond the limits of IBZ, which is the topic of the paper.

5. DISCUSSION

The absence of the artefacts caused by spatial periodicity strongly favours the selection of non-equidistant measurement data points as a source of data for the application of correlation method. However, in the case of non-equidistant measurement points, none of the considered variations of methods for selection of the corresponding wavenumbers from the candidate array gave good results within a whole frequency range used in the experiment. In order to develop a better method for selection of corresponding wavenumber from candidate array, one should understand the mechanism of failure of the considered methods.

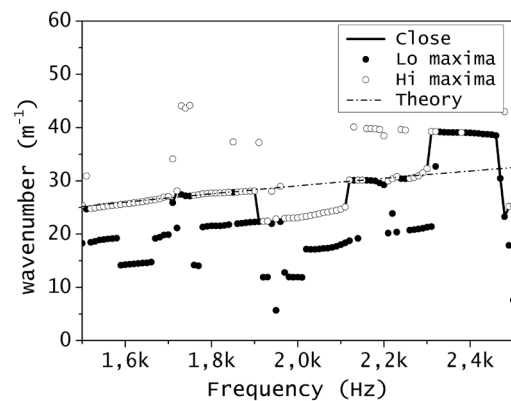


Fig. 15: Application of the “Close” method for selection of the corresponding wavenumber using the experimental data in the frequency range 1.5-2.5 kHz

The main reason for the observed behaviour is that the number of equivalent maxima branches in the case of non-equidistant measurement points is much higher than in the case of equidistant measurement points, as evidenced by comparison of Figure 7 and Figure 9. The larger number of branches means smaller separation between them, and it is easier for any method to undergo “switch” between the branches. Furthermore, the branches obtained using non-equidistant measurement

points do not show predictable periodic behaviour as the branches obtained using the equidistant measurement points, and it is certainly more difficult to develop an algorithm that prevents “switching” in the former case. In order to study the failure mechanisms, in the figures 15-17 are presented the closest members of the candidate array $k^{(Lo)}$ (full circles) and $k^{(Hi)}$ (empty circles), indicating the selection by solid line and theoretical predictions by dash-dot line.

The Figure 15 reveals the reason for failure of the “Close” method in the frequency range 1.5-2.5 kHz. The failure occurs at frequency 1910 Hz, when the previous point has frequency $f_p = 1900$ Hz and wavenumber $k_p = 28.12 \text{ m}^{-1}$, and the candidate points have wavenumbers $k^{(Lo)} = 22.34 \text{ m}^{-1}$ and $k^{(Hi)} = 37.21 \text{ m}^{-1}$. With $r^{(Lo)} = 39\%$ and $r^{(Hi)} = 61\%$, none of the candidate points is indeed close to the previous point, but the “Close” method moves to the closer, lower value. However, the lower value belongs to another branch of equivalent maxima, and the following detected wavenumbers belong to that branch, until similar circumstances lead to “switch-back” at frequency 2120 Hz. It is clear, and even trivial, that “Close” method fails when the candidate array does not contain a point that is close to the previous point, i.e. a point that belongs to the same equivalent maxima branch as the previous point. It should be noticed, however, that the Figure 15 shows three equivalent maxima branches up to frequency $f_p = 1900$ Hz (lower, with values around 20 m^{-1} , DR branch with values around 30 m^{-1} , and higher, values around 40 m^{-1}). The absence of a point from DR branch at the frequency $f = 2310$ Hz is a singular event, since there are again three equivalent maxima at frequency $f = 2320$ Hz with wavenumbers 21.7 m^{-1} , 32.7 m^{-1} and 39.4 m^{-1} , but the “Close” method is not able to “switch-back” to DR branch, as the value 39.4 m^{-1} is close to the value 39.2 m^{-1} , which was falsely identified as the corresponding wavenumber in the previous step. Probable reasons for the absence of a point from DR branch at the frequency $f = 2310$ Hz are experimental errors and round-off errors that made the local maxima of correlation array corresponding to DR small compared to the local maxima corresponding to the higher and lower equivalent maxima branches. Any improvement of the “Close” method has to handle the occasional occurrences of loss of the local maxima of correlation array corresponding to DR.

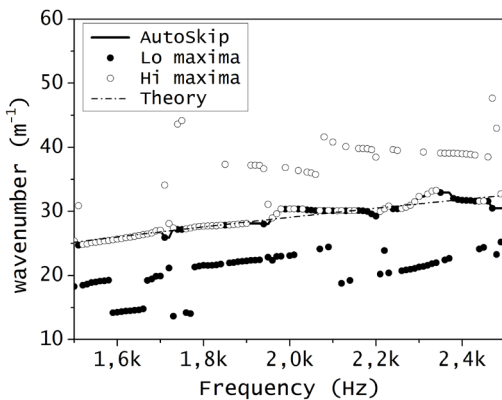


Fig. 16: Application of the “Auto-Skip” method for selection of the corresponding wavenumber using the experimental data in the frequency range 1.5-2.5 kHz

Figure 16 demonstrates the ability of the “AutoSkip” method to cope with such isolated losses of points from the DR, as it simply passes to the next frequency point, consequently following the DR branch of local maxima within the whole 1.5-2.5 kHz frequency range.

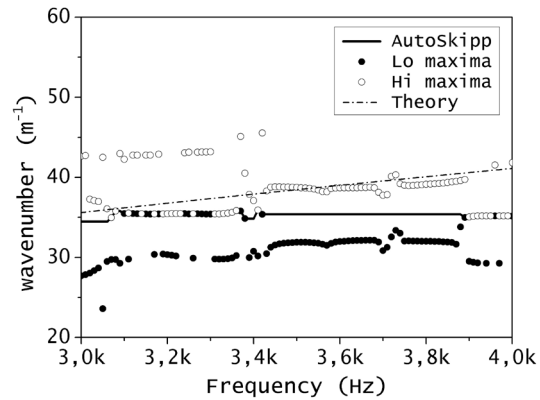


Fig. 17: Application of the “Auto-Skip” method for selection of the corresponding wavenumber using the experimental data in the frequency range 3-4 kHz

The “AutoSkip” method fails within frequency range 3.5-4.0 kHz, where it “loses” an existing DR branch of the equivalent maxima. The failure arises after frequency $f_p = 3420$ Hz and wavenumber $k_p = 35.39 \text{ m}^{-1}$, when the candidate wavenumbers for the following point, $f = 3420$ Hz, are $k^{(Lo)} = 30.47 \text{ m}^{-1}$ and $k^{(Hi)} = 38.28 \text{ m}^{-1}$. With $r^{(Lo)} = 63\%$ and $r^{(Hi)} = 37\%$, none of the candidate points is indeed close to the previous point, so that “AutoSkip” method skips the point and moves to the next frequency. However, the same conditions hold until frequency 3890 Hz, where the method “switches” to a lower branch of the equivalent maxima. The reason for failure of the “AutoSkip” mechanism is different, and unlike the failure of the “Close” method, does not lie in a problem at the frequency where it arises, but much earlier. Figure 17 clearly shows that in the considered frequency region are permanently present the three branches of the equivalent maxima (lower, with values around 35 m^{-1} , DR branch with values around 40 m^{-1} , and higher, values around 45 m^{-1}). However, the detected maxima of all three branches are fairly constant in the frequency range 3.0-3.4 kHz, and then have a sudden “jump” at frequency 3430 Hz, returning to monotonous rise at higher frequencies. The sudden “jump” causes the skipping of the considered point, but also “loss” of connection to the DR branch, which cannot be recovered: due to the monotonous rise of two branches of equivalent maxima, the lower branch will inevitably become closer to the reference point at $f_p = 3420$ Hz, and the algorithm will “switch” to a lower branch, instead of DR branch of equivalent maxima. The “AutoSkip” method is therefore sensitive to sudden changes of the slope of branches of equivalent maxima.

The presented discussion explains the observed behaviour of the proposed algorithms for extension of the measurement range for experimental determination of DR by correlation method: the sensitivity of the “AutoSkip” method to sudden changes of slope makes it the worse solution for the experiments with equidistant measurement points, when the determined DR has frequency ranges

with constant value due to the fault detection of standing waves; on the other hand, the “Close” method is worse solution for non-equidistant measurements, when, due to the complex nature of equivalent maxima branches, occur occasional losses of points of DR equivalent maxima branch.

6. CONCLUSION

The paper presented a concept for determination of dispersion relationship of flexural waves in beams using correlation method. The concept is derived from continuity and monotonicity of the dispersion relationship, and it is implemented as an algorithm that may use various methods for determination of wavenumber corresponding to frequency of the waves, which are based on those two properties of the dispersion relationship. The aim of the concept is to overcome limitations due to spatial aliasing caused by discrete nature of the measurement data, and thus to contribute to extension of the frequency range of application of correlation method for determination of dispersion relationship. The spatial aliasing restricts applicability of straightforward implementation of correlation method in experiments with equidistant measurements to the first Brillouin zone of discrete space defined by positions measurement points, as presented in the Figure 8. In the experiment analysed in this paper, the spatial aliasing limited the straightforward implementation of correlation method to the frequency range up to 1 kHz.

In the paper were considered three methods for determination of wavenumber corresponding to frequency, both in the cases of equidistant and non-equidistant measurements. The methods are called “CloseHi” – favouring monotonicity of DR, “Close” – favouring continuity of DR, and “AutoSkip” method – which accounts for circumstances not accounted by the first two methods. The analysis of the results proved that, due to the smaller sensitivity to systematic and random measurement errors as well as to round-off errors, the “Close” method is superior to “CloseHi” method both for the equidistant and non-equidistant measurements. “Close” method also showed better results than the “AutoSkip” method in the case of equidistant measurement points, where the “Close” method enabled calculation of the dispersion relationship in a wide frequency range (up to 5 kHz), except in frequency ranges around borders of the above-mentioned Brillouin zones. On the other hand, the “AutoSkip” method showed better results than the “Close” method for non-equidistant measurement points, where the “AutoSkip” method enabled calculation of the dispersion relationship in the frequency range up to 3.2 kHz, while the “Close” method enabled extension of the measurements range up to 2 kHz.

The results also revealed that, in the case of equidistant measurement points, all the methods were susceptible to false detection of standing waves in wide frequency ranges around borders of Brillouin zones. Since correlation method does not request equidistant measurement points, the setups with non-equidistant measurement points seem like a preferable choice for experimental determination of dispersion relationship.

The problem of setups with non-equidistant measurement points is more complex nature of the obtained correlation functions. The experimental results

are harder to interpret and sensitive to measurement and round-off errors, which – sooner or later – lead to failure of all analysed methods because the experimental data do not comply with assumptions used for development of the methods.

The simple and efficient approach to extend the applicability of the considered methods is to extend the frequency range of applicability of correlation method by increase of measurement points. However, this approach is not always possible, and development of new methods for analysis of experimental data are worthy of investigation. One possible direction for further study is to develop methods that include information about all previously determined points of dispersion relationship (its overall behaviour) into the process of prediction of new points, either by using calculus (derivatives) or machine learning for the purpose.

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