Semi-markov Resource Flow as a Bit-Level Model of Traffic

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Abstract. In this paper, we consider semi-Markov flow as a bit-level model of traffic. Each request of the flow brings some arbitrary distributed amount of information to the system. The current paper aims to investigate the amount of information received in semi-Markov flow during time unit. We use the asymptotic analysis method under the limit condition of growing time of observation to derive the limiting probability distribution of the amount of information received in the flow and build the approximation of its prelimit distribution function.

Keywords: semi-Markov flow · asymptotic analysis · Gaussian approximation · traffic modeling

1 Introduction

In telecommunication systems, the models of arrivals usually capture the structure of traffic from a packet-level point of view. Despite the interest in traffic models, few studies take into account packet length. Traffic modeling is focused on capturing such properties of telecommunication flows as burstiness, self-similarity and long-range dependence [5,13–15].

The idea of modeling arrivals together with the size of packets described in paper [4]. Authors use batch Markovian arrival process (BMAP) to model packet size as a size of the batch. In paper [12], authors build the model of traffic based on discrete-time BMAP model using two counting processes: the number of arriving packets and the number of bytes in those packets. Both processes in the model are affected by the state of the underlying Markov chain. More ideas of using packet size in traffic modeling are described in [3]. In some cases, for example, in papers [10,11], the model cannot be investigated when the input process describe only the number of received packets.

Resource flows are applicable in such area of research as queueing systems with random resource requirements. In such systems, each request of the flow

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has some random requirement on the resources [1,2,6]. Similar resource systems are described in papers [7,8].

We propose semi-Markov flow as a model of bit-level traffic, which allows us to take into account the length of packets in telecommunication systems. In our model, packets arrivals are driven by the semi-Markov process and the lengths of packets follow the arbitrary distribution. To research the model, we use the asymptotic analysis method under the limit condition of the growing time of the flow observation. We build a Gaussian approximation of the cumulative distribution function of the amount of information received in the flow.

We have organized the paper as follows. In Sect. 2, we present a mathematical model of semi-Markov flow. Section 3 is devoted to the derivation of the balance equation for the probability distribution of the process describing the amount of information received in the flow. In Sect. 4, we investigate the model using the asymptotic analysis method under the limit condition of growing time and build a Gaussian approximation. In Sect. 5, we show the numerical experiments and the area of applicability of the approximation. Section 6 is dedicated to the concluding remarks.

2 Mathematical Model of Semi-markov Flow

Semi-Markov flow is determined by semi-Markov matrix $\mathbf{A}(x)$. Elements $A_{k\nu}(x)$ of the matrix has the following from:

$$A_{k\nu}(x) = P\{\xi(n+1) = \nu, \tau(n+1) < x | \xi(n) = k\}. \tag{1}$$

We also take into account that

$$\mathbf{P} = \mathbf{A}(\infty), \tag{2}$$

where $\mathbf{P}$ is the transition matrix of embedded Markov chain $\xi(n)$ at the moments of state changes of the semi-Markov process. Moments t_n of arrivals in semi-Markov flow we determine as follows:

$$t_{n+1} = t_n + \tau(n+1).$$

Further, we use semi-Markov process $k(t)$, which is defined by equality

$$k(t) = \xi(n+1), \text{if } t_n < t \le t_{n+1} = t_n + \tau(n+1). \tag{3}$$

Each request of the flow brings some random amount of information with arbitrary distribution given by cumulative distribution function $B(x)$.

We denote $S(t)$ as the amount of information received in semi-Markov flow during time t. The problem is to derive the probability distribution of process $S(t)$.

We also denote $z(t)$ as the residual time of next arrival in the flow and consider three-dimensional process $\{k(t), S(t), z(t)\}$.

3 Balance Equation for the Probability Distribution of the Flow State

Three-dimensional process $\{k(t), S(t), z(t)\}$ is Markovian. Thus, we consider the function

$$P_k(s,z,t) = P\{k(t) = k, S(t) < s, z(t) < z\}$$

and derive balance equation

$$\frac{\partial P_k(s,z,t)}{\partial t} = \frac{\partial P_k(s,z,t)}{\partial z} - \frac{\partial P_k(s,0,t)}{\partial z} + \sum_{\nu=1}^{K} \int_0^s \frac{\partial P_\nu(s-x,0,t)}{\partial z} dB(x) A_{\nu k}(z), \tag{4}$$

where $\frac{\partial P_k(s,0,t)}{\partial z} = \frac{\partial P_k(s,z,t)}{\partial z}\Big|_{z=0}$.

We introduce partial characteristic functions

$$H_k(u,z,t) = \int_0^\infty e^{jus} d_s P_k(s,z,t)$$

and denote vector characteristic function

$$\mathbf{H}(u,z,t) = \{H_1(u,z,t), H_2(u,z,t), ..., H_K(u,z,t)\},$$

identity matrix $\mathbf{I}$ and vector of ones $\mathbf{e}$. After that, we rewrite Eq. (4) together with additional equation obtained taking the limit by $z \to \infty$

$$\frac{\partial \mathbf{H}(u,z,t)}{\partial t} = \frac{\partial \mathbf{H}(u,z,t)}{\partial z} - \frac{\partial \mathbf{H}(u,0,t)}{\partial z}\{\mathbf{I} - \mathbf{A}(z)B^*(u)\},$$

$$\frac{\partial \mathbf{H}(u,t)}{\partial t}\mathbf{e} = \frac{\partial \mathbf{H}(u,0,t)}{\partial z}\{B^*(u) - 1\}\mathbf{e}, \tag{5}$$

where $B^*(u) = \int_0^\infty e^{jux} dB(x)$ is the characteristic function of the amount of information in one request of the semi-Markov flow and $\mathbf{H}(u,t) = \mathbf{H}(u,\infty,t)$.

We cannot solve system (5) directly. Thus, we use asymptotic analysis method to investigate the amount of information received in the flow per time unit.

4 Asymptotic Probability Distribution

We introduce the equality $t = \tau T$, where $\tau \geq 0$ and T is an infinite parameter, as the limit condition of growing time. Solving system (5) in the limit by $T \to \infty$, we formulate the following theorem.

Theorem 1. *For characteristic function* $H(u,t) = \mathbb{E}e^{juS(t)} = \mathbf{H}(u,t)\mathbf{e}$ *in the limit condition of growing time, the following equality holds:*

$$\lim_{t\to\infty} \left\{ H(u,t) - \exp\left(ju\kappa_1 t + \frac{(ju)^2}{2}\kappa_2 t \right) \right\} = 0, \tag{6}$$

where

$$\kappa_1 = \frac{b_1}{\mathbf{r}\mathbf{A}_1\mathbf{e}}, \tag{7}$$

$$\kappa_2 = \frac{b_2}{\mathbf{r}\mathbf{A}_1\mathbf{e}} + 2b_1\mathbf{g}'(0)\mathbf{e}. \tag{8}$$

Here b_1 *and* b_2 *are the first and second raw moments of distribution function* $B(x)$*, matrices* $\mathbf{A}_1$ *and* $\mathbf{A}_2$ *are determined by formulas*

$$\mathbf{A}_1 = \int_0^\infty (\mathbf{P} - \mathbf{A}(x))dx,$$

$$\mathbf{A}_2 = \int_0^\infty x^2 d\mathbf{A}(x).$$

Vector $\mathbf{g}'(0)$ *is the solution of the inhomogeneous system of equations*

$$\mathbf{g}'(0)(\mathbf{I} - \mathbf{P}) = \kappa_1(\mathbf{r} - \mathbf{R}),$$

$$\mathbf{g}'(0)\mathbf{A}_1\mathbf{e} = \frac{b_1}{2}\frac{\mathbf{r}\mathbf{A}_1\mathbf{e}}{(\mathbf{r}\mathbf{A}_2\mathbf{e})^2} - b_1.$$

Vector $\mathbf{r}$ *is the steady state probability distribution of embedded Markov chain* $\xi(n)$*, which is the solution of the system*

$$\mathbf{r} = \mathbf{r}\mathbf{P},$$

$$\mathbf{r}\mathbf{e} = 1.$$

Vector $\mathbf{R}$ *is the steady-state probability distribution of semi-Markov process* $k(t)$*, which is given by formula*

$$\mathbf{R} = \frac{\mathbf{r}\mathbf{A}_1}{\mathbf{r}\mathbf{A}_1\mathbf{e}}.$$

Proof. In system (5), we denote $\frac{1}{T} = \varepsilon$ and make the following substitutions:

$$\tau = \varepsilon t, \;\; u = \varepsilon w, \;\; \mathbf{H}(u,z,t) = \mathbf{F}(w,z,\tau,\varepsilon). \tag{9}$$

We obtain

$$\varepsilon\frac{\partial\mathbf{F}(w,z,\tau,\varepsilon)}{\partial\tau} - \frac{\partial\mathbf{F}(w,z,\tau,\varepsilon)}{\partial z} = \frac{\partial\mathbf{F}(w,0,\tau,\varepsilon)}{\partial z}\left\{\mathbf{A}(z)B^*(\varepsilon w) - \mathbf{I}\right\},$$

$$\varepsilon\frac{\partial \mathbf{F}(w,z,\tau,\varepsilon)}{\partial \tau}\mathbf{e}=\frac{\partial \mathbf{F}(w,z,\tau,\varepsilon)}{\partial z}\left\{B^{*}(\varepsilon w)-1\right\}\mathbf{e}. \quad (10)$$

After that, we take the limit by $\varepsilon \to 0$ in the first equation of system (10) taking into account that $B^{*}(0)=1$, which yields

$$\frac{\partial \mathbf{F}(w,z,\tau)}{\partial z}=\frac{\partial \mathbf{F}(w,0,\tau)}{\partial z}\left\{\mathbf{I}-\mathbf{A}(z)\right\}.$$

The idea of the asymptotic analysis method, which is outlined in paper [9], is to present the solution of the last equation in the following form:

$$\mathbf{F}(w,z,\tau)=\Phi(w,\tau)\mathbf{R}(z), \quad (11)$$

where $\mathbf{R}(z)$ is the steady-state distribution of two-dimensional process $\{k(t),\ z(t)\}$, which satisfies the equality

$$\mathbf{R}(z)=\mathbf{R}'(0)\int_{0}^{z}(\mathbf{P}-\mathbf{A}(x))dx.$$

Here

$$\mathbf{R}'(0)=\frac{\mathbf{r}}{\mathbf{r}\mathbf{A}_1\mathbf{e}},$$

matrix $\mathbf{A}_1$ is given by

$$\mathbf{A}_1=\int_{0}^{\infty}(\mathbf{P}-\mathbf{A}(x))dx,$$

vector $\mathbf{r}$ is the steady-state distribution of the embedded Markov chain, which is the solution of the system

$$\mathbf{r}=\mathbf{r}\mathbf{P},$$
$$\mathbf{r}\mathbf{e}=1. \quad (12)$$

Consider the second equation of system (10), making the decomposition of $B^{*}(\varepsilon w)$ into the Taylor series up to $O(\varepsilon^2)$:

$$\varepsilon\frac{\partial \mathbf{F}(w,\tau,\varepsilon)}{\partial \tau}\mathbf{e}=jw\varepsilon b_1\frac{\partial \mathbf{F}(w,0,\tau,\varepsilon)}{\partial z}\mathbf{e}+O(\varepsilon^2),$$

where b_1 is the mean packet length. Substituting the solution (11) into the last equation, we take the limit by $\varepsilon \to 0$ and obtain

$$\frac{\partial \Phi(w,\tau)}{\partial \tau}=jwb_1\Phi(w,\tau)\mathbf{R}'(0)\mathbf{e}.$$

It is easy to see that the solution of the last equation is given by

$$\Phi(w,\tau)=e^{jw\kappa_1\tau}.$$

Here κ_1 has the following form:

$$\kappa_1 = \frac{b_1}{\mathbf{r}\mathbf{A}_1\mathbf{e}},$$

which coincides with (7).

Making substitutions $w = \dfrac{u}{\varepsilon}$ and $\tau = \varepsilon t$ reverse to (9), we obtain the equality

$$e^{jw\kappa_1\tau} = e^{ju\kappa_1 t}.$$

For the more detailed analysis, we make the following substitution in system (5):

$$\mathbf{H}(u,z,t) = e^{ju\kappa_1 t}\mathbf{H}_1(u,z,t). \tag{13}$$

Substituting (13) into system (5), we obtain the system of equations for characteristic function $\mathbf{H}_1(u,z,t)$:

$$\frac{\partial \mathbf{H}_1(u,z,t)}{\partial t} + ju\kappa_1\mathbf{H}_1(u,z,t) = \frac{\partial \mathbf{H}_1(u,z,t)}{\partial z} + \frac{\partial \mathbf{H}_1(u,z,t)}{\partial z}\left\{\mathbf{A}(z)B^*(u) - \mathbf{I}\right\},$$

$$\frac{\partial \mathbf{H}_1(u,z,t)}{\partial t}\mathbf{e} + ju\kappa_1\mathbf{H}_1(u,z,t)\mathbf{e} = \frac{\partial \mathbf{H}_1(u,z,t)}{\partial z}\left\{B^*(u) - 1\right\}\mathbf{e}. \tag{14}$$

We denote $\dfrac{1}{T} = \varepsilon^2$ and make the following substitutions in system (14):

$$\tau = \varepsilon^2 t, u = \varepsilon w, \mathbf{H}_1(u,z,t) = \mathbf{F}_1(w,z,\tau,\varepsilon). \tag{15}$$

We obtain the system of equations

$$\varepsilon^2\frac{\partial \mathbf{F}_1(w,z,\tau,\varepsilon)}{\partial \tau} + j\varepsilon w\kappa_1\mathbf{F}_1(w,z,\tau,\varepsilon)$$

$$= \frac{\partial \mathbf{F}_1(w,z,\tau,\varepsilon)}{\partial z} - \frac{\partial \mathbf{F}_1(w,0,\tau,\varepsilon)}{\partial z}\left\{\mathbf{I} - \mathbf{A}(z)B^*(\varepsilon w)\right\},$$

$$\varepsilon^2\frac{\partial \mathbf{F}_1(w,\tau,\varepsilon)}{\partial \tau}\mathbf{e} + j\varepsilon w\kappa_1\mathbf{F}_1(w,\tau,\varepsilon)\mathbf{e} = \frac{\partial \mathbf{F}_1(w,0,\tau,\varepsilon)}{\partial z}\left\{B^*(\varepsilon w) - 1\right\}\mathbf{e}. \tag{16}$$

We will seek the solution of system (16) in the following form:

$$\mathbf{F}_1(w,z,\tau,\varepsilon) = \Phi(w,\tau)\left\{\mathbf{R}(z) + j\varepsilon w\mathbf{f}(z)\right\} + O(\varepsilon^2), \tag{17}$$

which we substitute into (16):

$$j\varepsilon w\kappa_1\mathbf{R}(z) = \mathbf{R}'(z) + j\varepsilon w\mathbf{f}'(z) - \mathbf{R}'(0)\left\{\mathbf{I} - \mathbf{A}(z)(1 + j\varepsilon w b_1)\right\}$$

$$-j\varepsilon w\mathbf{f}'(0)\left\{\mathbf{I} - \mathbf{A}(z)\right\} + O(\varepsilon^2).$$

After that, we present the last equation as follows:

$$\mathbf{f}'(z) - \mathbf{f}'(0)\left\{\mathbf{I} - \mathbf{A}(z)\right\} = \kappa_1\left[\mathbf{R}(z) - \mathbf{r}\mathbf{A}(z)\right]. \tag{18}$$

According to the superposition principle, we present the solution of Eq. (18) as the sum:

$$\mathbf{f}(z) = C\mathbf{R}(z) + \mathbf{g}(z), \tag{19}$$

which we substitute into (18):

$$\mathbf{g}'(z) - \mathbf{g}'(0)\left\{\mathbf{I} - \mathbf{A}(z)\right\} = \kappa_1\left[\mathbf{R}(z) - \mathbf{r}\mathbf{A}(z)\right]. \tag{20}$$

Since $\mathbf{g}(z)$ by virtue of (19) is a particular solution of (18), then we assume that it satisfies the additional condition $\mathbf{g}(\infty)\mathbf{e} = 0$. We take the limit by $z \to \infty$ in Eq. (20) and obtain

$$\mathbf{g}(\infty) = \int\limits_0^\infty \mathbf{g}'(0)\left\{\mathbf{I} - \mathbf{A}(z)\right\}dz - \kappa_1\int\limits_0^\infty (\mathbf{r}\mathbf{A}(z) - \mathbf{R}(z))dz.$$

For the improper integral, we set the integrand as $z \to \infty$ equal to zero:

$$\mathbf{g}'(0)(\mathbf{I} - \mathbf{P}) - \kappa_1(\mathbf{r} - \mathbf{R}) = 0, \tag{21}$$

where $\mathbf{R} = \mathbf{R}(\infty)$ is the vector of steady-state distribution of semi-Markov process $k(t)$, which satisfies the system of equations

$$\mathbf{R} = \frac{\mathbf{r}\mathbf{A}_1}{\mathbf{r}\mathbf{A}_1\mathbf{e}},$$
$$\mathbf{R}\mathbf{e} = 1. \tag{22}$$

Taking back to Eq. (21), we represent it as follows:

$$\mathbf{g}'(0)(\mathbf{I} - \mathbf{P}) = \kappa_1(\mathbf{r} - \mathbf{R}).$$

The obtained system of linear algebraic equations has unlimited number of solutions. Thus, we apply the additional condition, which we derive from the equality

$$0 = \mathbf{g}(\infty)\mathbf{e} = \int\limits_0^\infty \left\{\mathbf{g}'(0)(\mathbf{I} - \mathbf{A}(z)) - \kappa_1(\mathbf{r}\mathbf{A}(z) - \mathbf{R}(z))\right\}dz\ \mathbf{e}.$$

Taking (21) into account, we can transform the last equality:

$$0 = \mathbf{g}(\infty)\mathbf{e} = \int\limits_0^\infty \left\{\mathbf{g}'(0)(\mathbf{I} - \mathbf{A}(z)) + \kappa_1\mathbf{r}(\mathbf{P} - \mathbf{A}(z)) + \kappa_1(\mathbf{R}(z) - \mathbf{R})\right\}dz\mathbf{e}$$

$$= \mathbf{g}'(0)\int\limits_0^\infty (\mathbf{P} - \mathbf{A}(x))dx\mathbf{e} + \kappa_1\int\limits_0^\infty (\mathbf{R}(x) - \mathbf{R})dx\mathbf{e} + \kappa_1\mathbf{r}\int\limits_0^\infty (\mathbf{P} - \mathbf{A}(x))dx\mathbf{e}$$

$$= \mathbf{g}'(0)\mathbf{A}_1\mathbf{e} - \kappa_1\int\limits_0^\infty (\mathbf{R} - \mathbf{R}(x))dx\mathbf{e} + b_1.$$

Here the integral can be transformed as follows:

$$\int_0^\infty (\mathbf{R} - \mathbf{R}(x))dx = (\mathbf{R} - \mathbf{R}(x))x\Big|_0^\infty + \int_0^\infty x d\mathbf{R}(x)$$

$$= \mathbf{R}'(0)\int_0^\infty x(\mathbf{I} - \mathbf{A}(x))dx = \mathbf{R}'(0)\int_0^\infty (\mathbf{I} - \mathbf{A}(x))d\frac{x^2}{2}$$

$$= \mathbf{R}'(0)\left\{(\mathbf{I} - \mathbf{A}(x))\frac{x^2}{2}\Big|_0^\infty + \int_0^\infty \frac{x^2}{2}d\mathbf{A}(x)\right\} = \frac{1}{2}\mathbf{R}'(0)\mathbf{A}_2 = \frac{\mathbf{r}\mathbf{A}_2}{\mathbf{r}\mathbf{A}_2\mathbf{e}}.$$

Here matrix $\mathbf{A}_2$ is given by

$$\mathbf{A}_2 = \int_0^\infty \frac{x^2}{2}d\mathbf{A}(x).$$

Finally, we have the system of linear algebraic equations with a solution

$$\mathbf{g}'(0)(\mathbf{I} - \mathbf{P}) = \kappa_1(\mathbf{r} - \mathbf{R}),$$

$$\mathbf{g}'(0)\mathbf{A}_1\mathbf{e} = \frac{b_1}{2}\frac{\mathbf{r}\mathbf{A}_2\mathbf{e}}{(\mathbf{r}\mathbf{A}_1\mathbf{e})^2} - b_1. \quad (23)$$

After that, we consider the second equation of system (16), in which we substitute decomposition (17):

$$\varepsilon^2\frac{\partial\Phi(w,\tau)}{\partial\tau} + jw\varepsilon\kappa_1\Phi(w,\tau)(1 + jw\varepsilon C)$$

$$= \Phi(w,\tau)\left\{\mathbf{R}'(0)\left[jw\varepsilon b_1 + \frac{(jw\varepsilon)^2}{2}b_2\right] - jw\varepsilon\mathbf{f}'(0)(-jw\varepsilon b_1)\right\}\mathbf{e} + O(\varepsilon^3).$$

By simple transformations, we obtain

$$\frac{\partial\Phi(w,\tau)}{\partial\tau} + (jw)^2\kappa_1\Phi(w,\tau)C = \Phi(w,\tau)\left\{\frac{(jw)^2}{2}\mathbf{R}'(0)b_2 + (jw)^2 b_1\mathbf{f}'(0)\right\}\mathbf{e}.$$

By the virtue of (19), we can write

$$\frac{\partial\Phi(w,\tau)}{\partial\tau} + (jw)^2\kappa_1\Phi(w,\tau)C$$

$$= \Phi(w,\tau)\left\{\frac{(jw)^2}{2}\mathbf{R}'(0)b_2 + (jw)^2 b_1(C\mathbf{R}'(0) + \mathbf{g}'(0))\right\}\mathbf{e},$$

from which we obtain

$$\frac{\partial\Phi(w,\tau)}{\partial\tau} = \Phi(w,\tau)\frac{(jw)^2}{2}\left\{\frac{b_2}{\mathbf{r}\mathbf{A}_1\mathbf{e}} + 2b_1\mathbf{g}'(0)\mathbf{e}\right\}.$$

Denoting

$$\kappa_2 = \frac{b_2}{\mathbf{r}\mathbf{A}_1\mathbf{e}} + 2b_1\mathbf{g}'(0)\mathbf{e}, \tag{24}$$

which coincides with (8), we derive the solution of differential equation above

$$\Phi(w,\tau) = \exp\left\{\frac{(jw)^2}{2}\kappa_2\tau\right\}.$$

From substitutions (15), we make the reverse substitutions

$$w = \frac{u}{\varepsilon},\ \tau = \varepsilon^2 t,$$

which yields

$$\Phi(w,\tau) = \exp\left\{\frac{(jw)^2}{2}\kappa_2\tau\right\} = \exp\left\{\frac{(ju)^2}{2\varepsilon^2}\kappa_2\varepsilon^2 t\right\} = \exp\left\{\frac{(ju)^2}{2}\kappa_2 t\right\}.$$

Finally, in (13), we set $z \to \infty$ and obtain the asymptotic characteristic function

$$h_1(u,t) = \exp\{ju\kappa_1 t\}\exp\left\{\frac{(ju)^2}{2}\kappa_2 t\right\} = \exp\left\{ju\kappa_1 t + \frac{(ju)^2}{2}\kappa_2 t\right\}.$$

As we can see, the distribution of the amount of information received in semi-Markov flow is asymptotically Gaussian with mean $\kappa_1 t$ and variance $\kappa_2 t$.

We note that by setting $b_1 = 1$ and $b_2 = 1$, we obtain the case when the amount of information in a packet is deterministic and equal to one. Thus, the obtained result is valid for the number of packet arrivals in the flow.

Since Gaussian distribution allows negative values, we propose the following approximation for distribution function of the amount of information received in the flow during time t:

$$F_{Approx}(x,t) = \frac{G(x,t) - G(0,t)}{1 - G(0,t)}, \tag{25}$$

where $G(x,t)$ is the Gaussian distribution function with mean $\kappa_1 t$ and variance $\kappa_2 t$.

5 Numerical Example

We set semi-Markov matrix as follows:

$$\mathbf{A}(x) = \mathbf{P} \circ \mathbf{G}(x),$$

where $\mathbf{P}$ is the transition matrix of the embedded Markov chain $\xi(n)$ and $\mathbf{G}(x)$ is the matrix of conditional distributions of the process $\tau(n)$, operation $\circ$ is Hadamard product of matrices.

Matrix $\mathbf{P}$ is given by

$$\mathbf{P} = \begin{bmatrix} 0.95 & 0.05 \\ 0.8 & 0.2 \end{bmatrix}.$$

The elements of matrix $\mathbf{G}(x)$ are gamma distribution functions with shape parameters $\alpha_{11} = 0.005$, $\alpha_{12} = 0.01$, $\alpha_{21} = 0.1$, $\alpha_{22} = 1$ and scale parameter $\beta = 1$. We assume that the amount of information in one packet is deterministic and equals to $b_1 = 1.5$.

Figures 1, 2, 3 show the distribution function of the amount of information received in semi-Markov flow via simulation (solid line) compared with asymptotic results (dash line) for $t = 20$, $t = 50$ and $t = 75$.

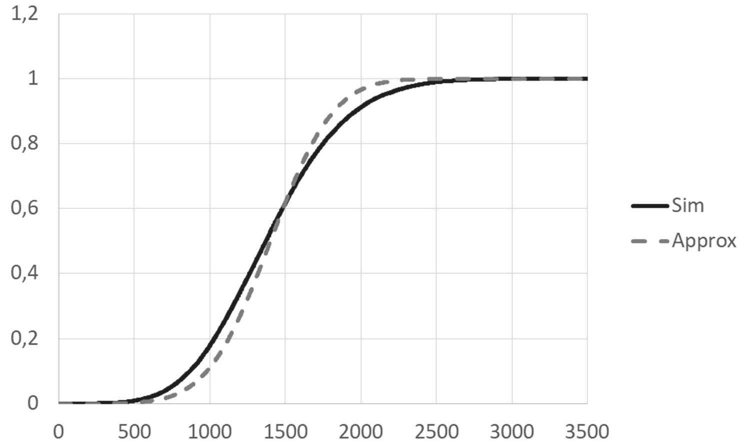

Fig. 1. The distribution function of the amount of information received in semi-Markov flow and its asymptotic approximation for $t = 20$

Table contains the values of Kolmogorov distance

$$\Delta = \max_{0 \leq x < \infty} \left| F_{Sim}(x,t) - F_{Approx}(x,t) \right|$$

between empirical distribution function obtained via simulation $F_{Sim}(x,t)$ and asymptotic cumulative distribution function $F_{Approx}(x,t)$ of the amount of information received in the flow during time t given by (25) (Table 1).

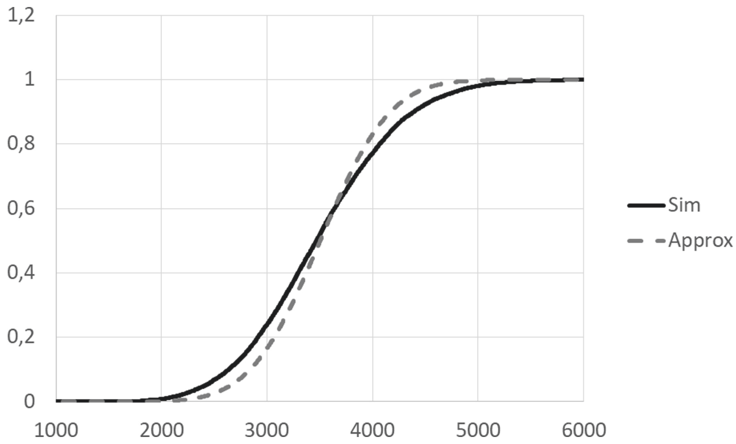

Fig. 2. The distribution function of the amount of information received in semi-Markov flow and its asymptotic approximation for $t = 50$

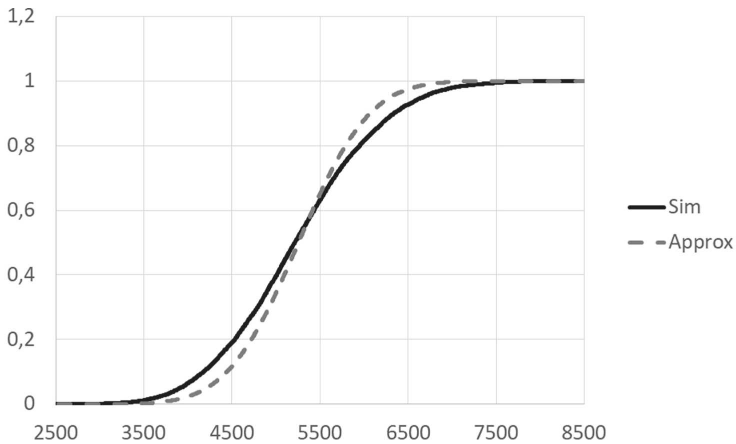

Fig. 3. The distribution function of the amount of information received in semi-Markov flow and its asymptotic approximation for $t = 75$

Table 1. Kolmogorov distance between empirical distribution function of the amount of information in the buffer and its asymptotic approximation

	$t = 10$	$t = 20$	$t = 50$	$t = 75$	$t = 100$
Δ	0.0817	0.0768	0.0766	0.0758	0.0752

6 Conclusion

We have considered the bit-level traffic model in form of semi-Markov flow. For the amount of information received in the flow, we have obtained the limiting probability distribution under the limit condition of growing time of observation. We have derived the explicit formula for the mean and variance of Gaussian distribution. Since the distribution of the packet length in the model is arbitrary, the results are applicable for the number of packets arrivals when we set the size of each packet is equal to one.

References

1. Galileyskaya, A., Lisovskaya, E., Fedorova, E.: Resource queueing system with the requirements copying at the second phase. In: Vishnevskiy, V.M., Samouylov, K.E., Kozyrev, D.V. (eds.) DCCN 2019. CCIS, vol. 1141, pp. 352–363. Springer, Cham (2019). https://doi.org/10.1007/978-3-030-36625-4_28
2. Galileyskaya, A., Lisovskaya, E., Pagano, M.: On the total amount of the occupied resources in the multi-resource QS with renewal arrival process. In: Dudin, A., Nazarov, A., Moiseev, A. (eds.) ITMM 2019. CCIS, vol. 1109, pp. 257–269. Springer, Cham (2019). https://doi.org/10.1007/978-3-030-33388-1_21
3. Gao, J., Rubin, I.: Multifractal analysis and modeling of long-range-dependent traffic. In: 1999 IEEE International Conference on Communications (Cat. No. 99CH36311), vol. 1, pp. 382–386. IEEE (1999)
4. Klemm, A., Lindemann, C., Lohmann, M.: Modeling IP traffic using the batch Markovian arrival process. Perform. Eval. **54**(2), 149–173 (2003)
5. Li, M.: Long-range dependence and self-similarity of teletraffic with different protocols at the large time scale of day in the duration of 12 years: Autocorrelation modeling. Physica Scripta **95**(6), 065222 (2020)
6. Lisovskaya, E., Moiseeva, S., Pagano, M.: Multiclass GI/GI/∞ queueing systems with random resource requirements. In: Dudin, A., Nazarov, A., Moiseev, A. (eds.) ITMM/WRQ -2018. CCIS, vol. 912, pp. 129–142. Springer, Cham (2018). https://doi.org/10.1007/978-3-319-97595-5_11
7. Naumov, V., Samouylov, K., Yarkina, N., Sopin, E., Andreev, S., Samuylov, A.: Lte performance analysis using queuing systems with finite resources and random requirements. In: 2015 7th International Congress on Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT), pp. 100–103. IEEE (2015)
8. Naumov, V.A., Samuilov, K.E., Samuilov, A.K.: On the total amount of resources occupied by serviced customers. Autom. Remote Control **77**(8), 1419–1427 (2016). https://doi.org/10.1134/S0005117916080087
9. Nazarov, A., et al.: Multi-level MMPP as a model of fractal traffic. In: Dudin, A., Nazarov, A., Moiseev, A. (eds.) ITMM 2020. CCIS, vol. 1391, pp. 61–77. Springer, Cham (2021). https://doi.org/10.1007/978-3-030-72247-0_5
10. Nazarov, A., et al.: Mathematical model of scheduler with semi-markov input and bandwidth sharing discipline. In: 2021 International Conference on Information Technology (ICIT), pp. 494–498. IEEE (2021)
11. Peng, X., Bai, B., Zhang, G., Lan, Y., Qi, H., Towsley, D.: Bit-level power-law queueing theory with applications in lte networks. In: 2018 IEEE Global Communications Conference (GLOBECOM), pp. 1–6. IEEE (2018)

12. Salvador, P., Pacheco, A., Valadas, R.: Modeling IP traffic: joint characterization of packet arrivals and packet sizes using BMAPs. Comput. Netw. **44**(3), 335–352 (2004)
13. Willinger, W., Taqqu, M.S., Leland, W.E., Wilson, D.V., et al.: Self-similarity in high-speed packet traffic: analysis and modeling of ethernet traffic measurements. Stat. Sci. **10**(1), 67–85 (1995)
14. Yang, T., Zhao, R., Zhang, W., Yang, Q.: On the modeling and analysis of communication traffic in intelligent electric power substations. IEEE Trans. Power Delivery **32**(3), 1329–1338 (2016)
15. Yang, X., Petropulu, A.P.: The extended alternating fractal renewal process for modeling traffic in high-speed communication networks. IEEE Trans. Signal Process. **49**(7), 1349–1363 (2001)