Chapter

Computation of Numerical Solution via Non-Standard Finite Difference Scheme

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Abstract

The recent COVID-19 pandemic has brought attention to the strategies of quarantine and other governmental measures, such as lockdown, media coverage on social isolation, strengthening of public safety, etc. All these strategies are because to manage the disease as there is no vaccine and appropriate medicine for treatment. The mathematical model can assist to determine whether these intervention options are the most effective ones for illness control and how they might impact the dynamics of the disease. Motivated by this, in this manuscript, a classical order nonlinear mathematical model has been proposed to analyze the pandemic COVID-19. The model has been analyzed numerically. The suggested mathematical model is classified into susceptible, exposed, recovered, and infected classes. The non-standard finite difference scheme (NSFDS) is used to achieve the approximate results for each compartment. The graphical presentations for various compartments of the systems that correspond to some real facts are given via MATLAB.

Keywords: nonlinear dynamical system, COVID-19, approximate solution, NSFDS

1. Introduction

Many diseases have affected the human population throughout history, the most dangerous of which are viral diseases. Measles, TB, Malaria, HBV, HCV, Dengue fever, Malignant Malignancies, Spanish flu, and other diseases have resulted in millions of deaths. People have learned a memorable lesson from history. So, for controlling and reducing the rate of infections in their communities, they have established different strategies. Among the aforesaid diseases, one of the infectious diseases is COVID-19.

COVID-19 is a threatful outbreak that arose in China [1, 2] and spread throughout the globe very rapidly. It is an infectious disease caused by the virus, severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2). The disease started at a seafood market in Wuhan, a big city in China, in December 2019. The disease spread in the entire city during February and March 2020. At that time, infected people were nearly 0.84 million and more than 5000 have died. Also a considerable number of infected people recovered from the said disease. The disease COVID-19 has become a pandemic due to several reasons. Some of them are (i) high transmission rate of the disease, (ii) lack of suitable vaccine and exact medicine, and (iii) the exact nature of the SARS-CoV-2 virus is still unknown. The incubation period can range from 2 to 14 days [3–12]. The majority of COVID-19 symptoms are mild, although this may increase when variants arise.

Although there were 5.94 million COVID-19 deaths that were officially reported between January 1,2020, and December 31, 2021, the excess mortality caused by the COVID-19 pandemic resulted in 18.2 million deaths globally during that time. The COVID-19 pandemic caused an excess mortality rate of 120.3 fatalities per 100,000 people worldwide. The regions of south Asia, the Middle East, north Africa, and eastern Europe had the highest number of additional deaths brought on by COVID-19. At the national level, Mexico (798, 000), Brazil (792, 000), Indonesia (736, 000), and Pakistan (664, 000) were expected to have the largest total excess mortality from COVID-19, followed by the United States (1.13 million), Russia (1.07 million), and India (4.07 million). The excess mortality rate among these nations was highest in Mexico (325.1 per 100,000) and Russia (374.6 per 100,000), and it was comparable in Brazil (186.9 per 100,000) and the USA (179.3 per 100,000).

COVID-19 symptoms differ from one person to the next. In fact, some infected people show no signs or symptoms (asymptomatic). Cough, shortness of breath or difficulty breathing, fever or chills, headaches, weariness, muscular or body aches, sore throat, loss of taste or smell, congestion or runny nose, diarrhea, and nausea or vomiting are some of the symptoms people with COVID-19 infection [9]. It's also possible that some will have additional symptoms. Many researchers, doctors, and policymakers are trying to prevent the disease from spreading. One important factor in the spreading of said disease is the migration of affected persons from one locality to another. This affects more people and hence plays a major role in the spreading. Therefore, the primary step taken by most countries is to announce city-wide lockdowns. So that some protective measures should be taken to minimize the greatest possible loss of human lives [13]. On an international level, banned air traffic for an unknown period of time. Keeping in mind that in the past such outbreak not only led to the greatest loss of human lives but also damaged the economy very badly throughout the world. Therefore, scientists and researchers are trying their best to put their part in the investigation of a cure for the COVID-19 outbreak. It is clear from a medical engineering point of view that infectious diseases can be better understood by using the mathematical model. In the last many decades, mathematical modeling is one of the important areas of research [14–55]. To understand the dynamics of COVID-19, it is essential to formulate mathematical models that can assist in the estimation of the transmissibility and dynamic of the virus transmission. Also, the majority of real-world problems, such as infectious diseases, are nonlinear in nature. As a result, nonlinear mathematical models that describe a variety of real-world issues have piqued interest for decades. In this regard, various models were formulated or updated. Also, several types of research focusing on mathematical modeling of COVID-19 have been considered recently. Some models that have recently been considered in this regard are [56–59]. Motivated by the above work, we are going to investigate the COVID-19 mathematical model (see 4) numerically under NSFDS.

2. Preliminaries

In numerical analysis, NSFDS is a general set of methods that gives numerical solutions to differential equations by discretizing the data. Many real-life problems are modeled by differential equations, for which analytical solutions are difficult to find out efficiently. Several researchers have tried different ways (e.g., via Finite Element Methods, Standard Finite Difference Methods, Spline Approximation Methods, etc). Nowadays, NSFDS is playing an important role in solving the real-life problems governed by ODEs and/or by PDEs. In science and engineering, many differential models for which the existing methodologies do not give reliable results, NSFDS are solving them competitively.

Here we derive the suggested scheme for simple problems as let

$$\frac{dy}{dt} = f(t, y) \tag{1}$$

then NSFD equation is

$$\frac{y_{k+1} - y_k}{h} = f(t, y(k)),$$
$$y_{k+1} = y_k + hf(t, y(k)).$$

Definition 1. *A successful example of a NSFD equation is one setup for a combustion model*

$$\frac{dw}{dt} = w^2(1 - w). \tag{2}$$

The NSFD equation would be

$$\frac{w_{k+1} - w_k}{h} = w_k^2 - w_k^3. \tag{3}$$

3. Formulation of proposed model

A model is formulated that further divides the entire population into different classes given as:

individuals who have high chance of getting an infection are placed in susceptible class S, individuals who are in close contact with COVID-19 environment are placed in exposed class E, individuals having the symptoms of COVID-19 are placed in infected class I and R recovered class includes recovered individuals. A mathematical model of COVID-19 is described by the following system of differential eqs. [30].

$$\begin{cases} \dfrac{d}{dt}S(t) = \gamma - k(1 + \alpha I(t))S(t)I(t) - \varepsilon S(t), \\ \dfrac{d}{dt}E(t) = k(1 + \alpha I(t))S(t)I(t) - (\varepsilon + \delta)E(t), \\ \dfrac{d}{dt}I(t) = \eta + \delta E(t) - (\upsilon + \varepsilon + \beta)I(t), \\ \dfrac{d}{dt}R(t) = \beta I(t) - \varepsilon R(t). \end{cases} \tag{4}$$

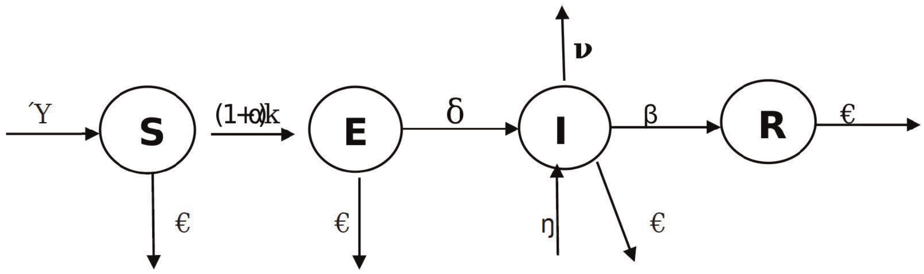

Figure 1.
A flow chart of the proposed model.

With initial conditions given by

$$S(0) = S_0,\ E(0) = E_0,\ I(0) = I_0,\ R(0) = R_0.$$

The description of above model is given in **Figure 1**.

4. Algorithm for approximate solution of the considered model

To compute the required approximate solution, using general form of NSFD on (4), we have

$$\begin{cases} \dfrac{S_{n+1}(t) - S_n(t)}{h} = \gamma - k(1 + \alpha I_n(t))S_n(t)I_n(t) - \varepsilon S_n(t), \\ \dfrac{E_{n+1}(t) - E_n(t)}{h} = k(1 + \alpha I_n(t))S_n(t)I_n(t) - (\varepsilon + \delta)E_n(t), \\ \dfrac{I_{n+1}(t) - I_n(t)}{h} = \eta + \delta E_n(t) - (\upsilon + \varepsilon + \beta)I_n(t), \\ \dfrac{R_{n+1}(t) - R_n(t)}{h} = \beta I_n(t) - \varepsilon R_n(t). \end{cases} \tag{5}$$

$$\begin{cases} S_{n+1}(t) = S_n(t) + h(\gamma - k(1 + \alpha I_n(t))S_n(t)I_n(t) - \varepsilon S_n(t)), \\ E_{n+1}(t) = E_n(t) + h(k(1 + \alpha I_n(t))S_n(t)I_n(t) - (\varepsilon + \delta)E_n(t)), \\ I_{n+1}(t) = I_n(t) + h(\eta + \delta E_n(t) - (\upsilon + \varepsilon + \beta)I_n(t)), \\ R_{n+1}(t) = R_n(t) + h(\beta I_n(t) - \varepsilon R_n(t)). \end{cases} \tag{6}$$

Now putting n = 0, 1, 2 …. in (6), we get few terms of the approximate solution as

$$\begin{cases} S_1(t) = S_0(t) + h(\gamma - k(1 + \alpha I_0(t))S_0(t)I_0(t) - \varepsilon S_0(t)), \\ E_1(t) = E_0(t) + h(k(1 + \alpha I_0(t))S_0(t)I_0(t) - (\varepsilon + \delta)E_0(t)), \\ I_1(t) = I_0(t) + h(\eta + \delta E_0(t) - (\upsilon + \varepsilon + \beta)I_0(t)), \\ R_1(t) = R_0(t) + h(\beta I_0(t) - \varepsilon R_0(t)). \end{cases} \tag{7}$$

$$\begin{cases} S_2(t) = S_1(t) + h(\gamma - k(1 + \alpha I_1(t))S_1(t)I_1(t) - \varepsilon S_1(t)), \\ E_2(t) = E_1(t) + h(k(1 + \alpha I_1(t))S_1(t)I_1(t) - (\varepsilon + \delta)E_1(t)), \\ I_2(t) = I_1(t) + h(\eta + \delta E_1(t) - (\upsilon + \varepsilon + \beta)I_1(t)), \\ R_2(t) = R_1(t) + h(\beta I_1(t) - \varepsilon R_1(t)). \end{cases} \tag{8}$$

$$\begin{cases} S_3(t) = S_2(t) + h(\gamma - k(1 + \alpha I_2(t))S_2(t)I_2(t) - \varepsilon S_2(t)), \\ E_3(t) = E_2(t) + h(k(1 + \alpha I_2(t))S_2(t)I_2(t) - (\varepsilon + \delta)E_2(t)), \\ I_3(t) = I_2(t) + h(\eta + \delta E_2(t) - (\upsilon + \varepsilon + \beta)I_2(t)), \\ R_3(t) = R_2(t) + h(\beta I_2(t) - \varepsilon R_2(t)). \end{cases} \tag{9}$$

and so on. Similarly, the other terms may be computed.

5. Numerical interpretation

To present the concerned approximate solutions computed above of the model under consideration, we use numerical values for the parameters in given in **Table 1**. Based on reported data, the initial condition is set as [45]

$$(\mathbb{S}(0), \mathbb{E}(0), \mathbb{I}(0), \mathbb{R}(0)) = (32.37 million, 12 million, 0.001523 million, 0.005025 million).$$

After putting the numerical values in Eq. (6), we obtained the following results.
Case (1) n = 0

$$\begin{cases} \mathbb{S}_1(t) = 3.2018 \times 10^7, \\ \mathbb{E}_1(t) = 1.3738 \times 10^6, \\ \mathbb{I}_1(t) = 4.5718 \times 10^3, \\ \mathbb{R}_1(t) = 5.0163 \times 10^3. \end{cases} \tag{10}$$

Parameters	Description of parameters	Numerical value
γ	Tested negative population	0.250281×10^{-6}
η	Tested positive population	0.006656×10^{-6}
k	The infection rate	0.000024
α	Rate of individual lose immunity	0.01182
ε	Natural death rate	0.0000004×10^{-6}
ν	Death rate due to C0VID-19	0.016
δ	Infected rate	0.025
β	Recovered rate	0.75

Table 1.
Numerical values of parameters.

And similarly from Eqs. (8) and (9), we get
Case (2) n = 1

$$\begin{cases} S_2(t) = 2.9958 \times 10^7, \\ E_2(t) = 3.3015 \times 10^6, \\ I_2(t) = 4.9285 \times 10^3, \\ R_2(t) = 5.0305 \times 10^3. \end{cases} \tag{11}$$

Case (3) n = 2

$$\begin{cases} S_3(t) = 2.7741 \times 10^7, \\ E_3(t) = 5.3871 \times 10^6, \\ I_3(t) = 5.7629 \times 10^3, \\ R_3(t) = 5.0473 \times 10^3. \end{cases} \tag{12}$$

Case (4) n = 3

$$\begin{cases} S_4(t) = 2.4980 \times 10^7, \\ E_4(t) = 8.0161 \times 10^6, \\ I_4(t) = 7.1091 \times 10^3, \\ R_4(t) = 5.0703 \times 10^3. \end{cases} \tag{13}$$

Case (5) n = 4

$$\begin{cases} S_5(t) = 2.1258 \times 10^7, \\ E_5(t) = 1.1606 \times 10^7, \\ I_5(t) = 9.0967 \times 10^3, \\ R_5(t) = 5.1033 \times 10^3. \end{cases} \tag{14}$$

In **Figures 2–5**, we have provided a graphical representation of different classes for the proposed model. We concluded that by taking a few terms of the series solutions we can efficiently describe the proposed model. We see in the figures that the

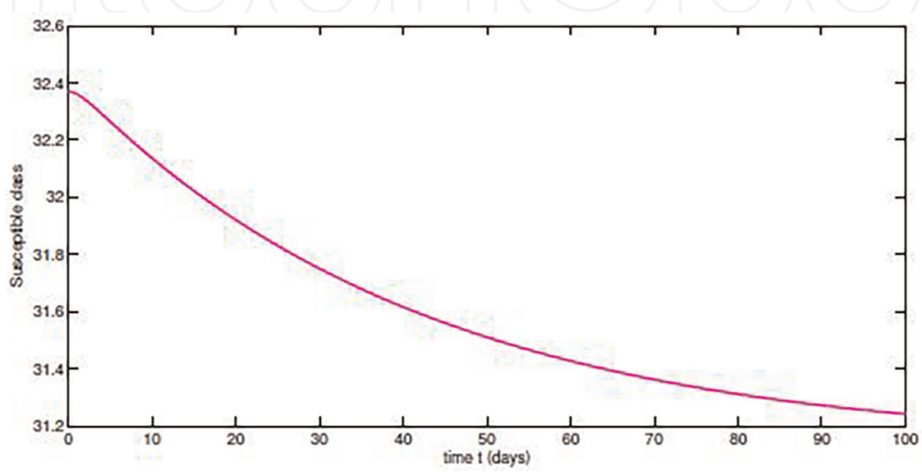

Figure 2.
Dynamics of susceptible class.

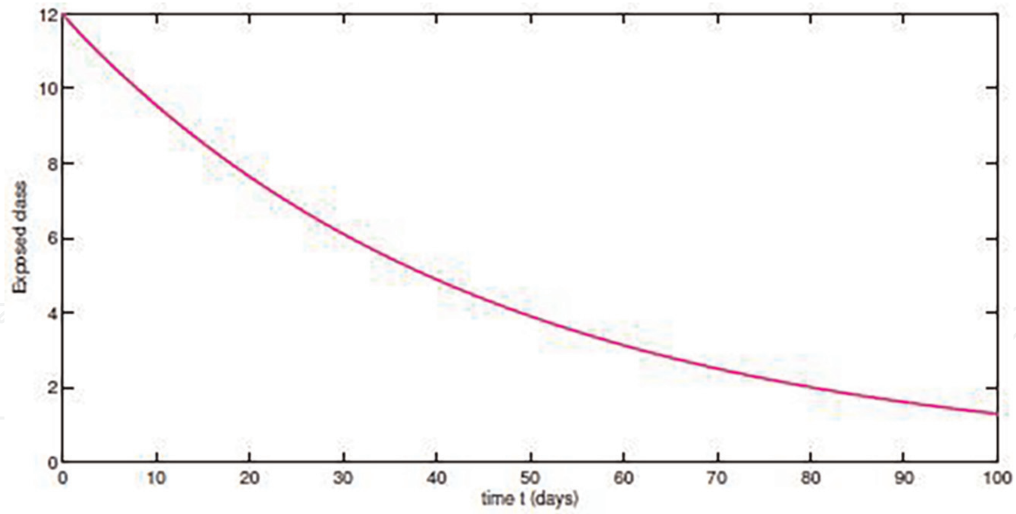

Figure 3.
Dynamics of exposed class.

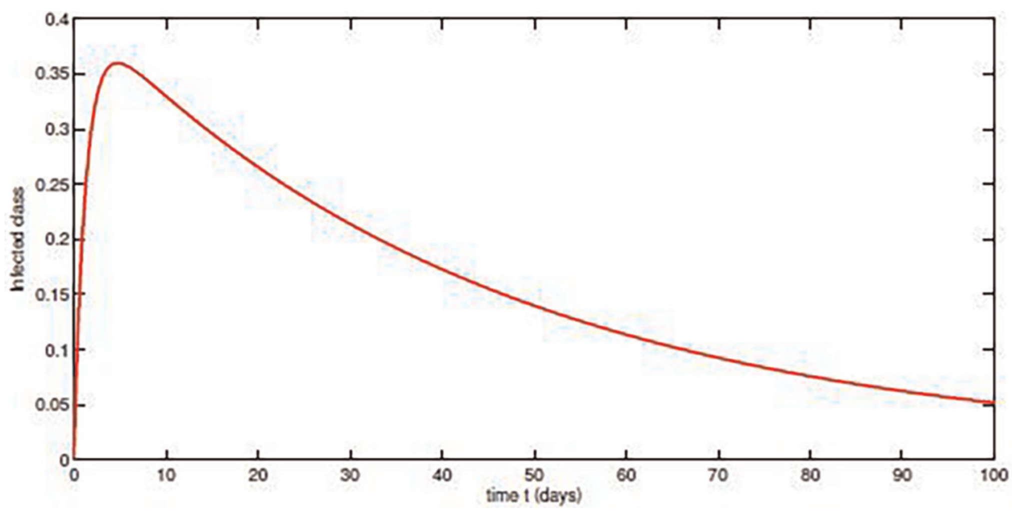

Figure 4.
Dynamics of infected class.

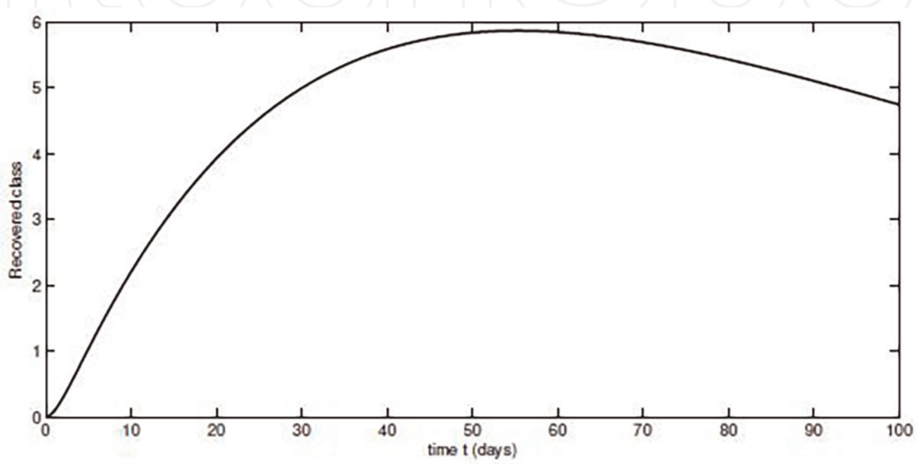

Figure 5.
Dynamics of recovered class.

susceptible class is decreasing as a result increase in infection occurred but due to vaccination and other precautions there occurred an increase in the recovered class. Further, we compare our results with the usual RK4 method numerical results for the given data in **Table 1** in **Figures 6–9** respectively. We see that the solution through the NSFDS and RK4 method agrees very well.

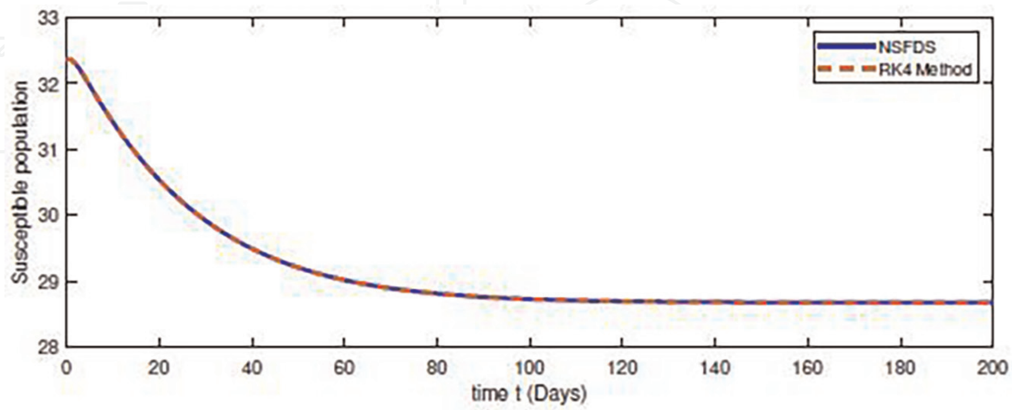

Figure 6.
Comparison of the approximate solution for the susceptible class at NSFS and RK4.

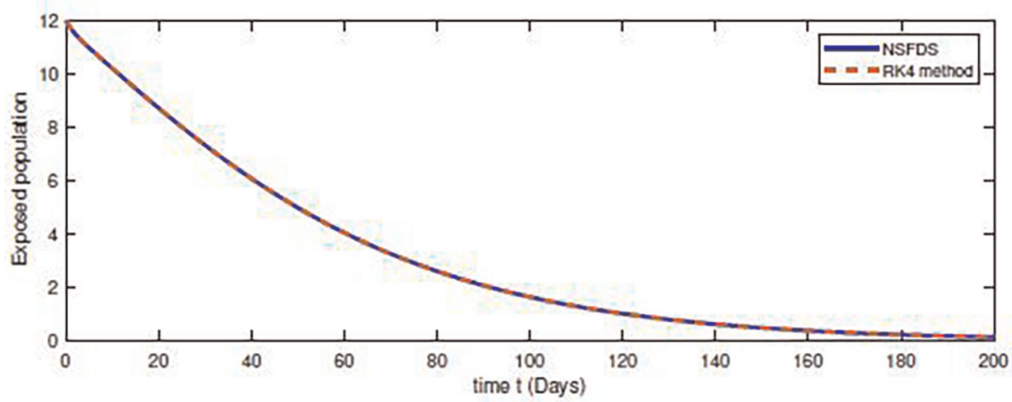

Figure 7.
Comparison of the approximate solution for the exposed class at NSFS and RK4.

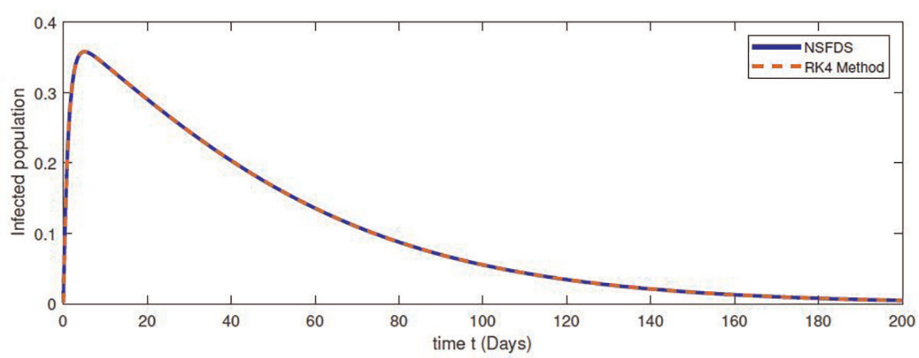

Figure 8.
Comparison of the approximate solution for the infected class at NSFS and RK4.

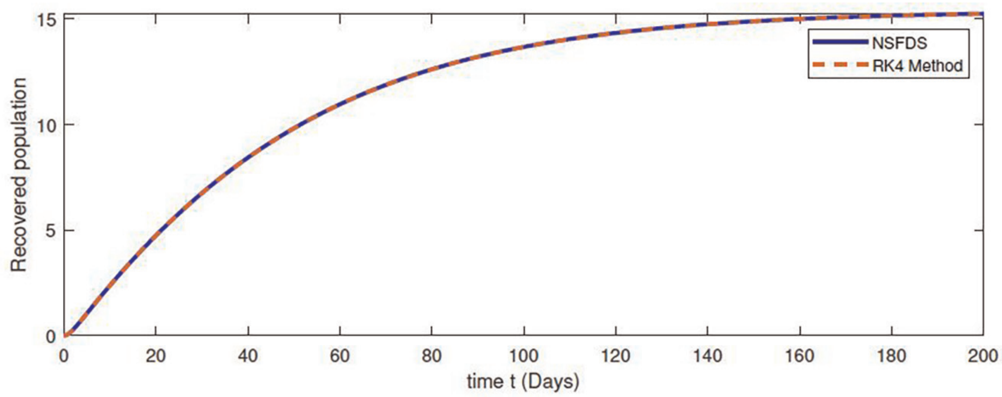

Figure 9.
Comparison of the approximate solution for the recovered class at NSFS and RK4.

6. Some explanation and concluding remarks

In this work, we have studied a four-compartmental mathematical model based on a system of ordinary differential equations to study the dynamics of COVID-19 through the NSFDS method. With the help of the said technique, we develop an algorithm to discretize the data to find an approximate solution to the proposed problem. Using some real values for the parameters and initial data, we compute a few terms and approximate solutions corresponding to a different compartment. We plot our approximate solutions for different compartments graphically using MATLAB. We concluded that by taking a few terms of the solutions, we can efficiently describe the proposed model. As compared to RK4 and Euler methods, NSFD method is easy to implement. The computational cost is low and also good for time-saving in the future, one can extend the current study for mathematical models under nonsingular type derivatives. Finally, we have given a comparison between the approximate solution at NSFD method and RK4 method. We see that both solutions agreed very well.

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