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A Methodological Framework for Parametric Combat Analysis

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A METHODOLOGICAL FRAMEWORK FOR PARAMETRIC COMBAT ANALYSIS

DISSERTATION

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DEPARTMENT OF THE AIR FORCE
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DISSERTATION

Presented to the Faculty

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A METHODOLOGICAL FRAMEWORK FOR PARAMETRIC COMBAT ANALYSIS

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AFIT-ENV-DS-22-S-085

Abstract

This work presents a taxonomic structure for understanding the tension between certain factors of stability for game-theoretic outcomes such as Nash optimality, Pareto optimality, and balance optimality and then applies such game-theoretic concepts to the advancement of strategic thought on spacepower. This work successfully adapts and applies combat modeling theory to the evaluation of cislunar space conflict. This work provides evidence that the reliability characteristics of small spacecraft share similarities to the reliability characteristics of large spacecraft. Using these novel foundational concepts, this dissertation develops and presents a parametric methodological framework capable of analyzing the efficacy of heterogeneous force compositions in the context of space warfare. This framework is shown to be capable of predicting a stochastic distribution of numerical outcomes associated with various modes of conflict and parameter values. Furthermore, this work demonstrates a general alignment in results between the game-theoretic concepts of the framework and Media Interaction Warfare Theory in terms of evaluating force efficacy, providing strong evidence for the validity of the methodological framework presented in this dissertation.

Acknowledgments

Thank you to Gina, Phil, Gonzo, Pete, Eugene, and Foster for being great teammates.

Thank you to the committee for teaching me and helping me graduate.

Thank you to my family for their love and support.

Maranatha.

Table of Contents

Page

List of Figures

List of Tables

A METHODOLOGICAL FRAMEWORK FOR PARAMETRIC COMBAT ANALYSIS

Chapter 1: Introduction

1.1 Background and Motivation

The United States now contends with peer competitors within a multipolar geopolitical order in the context of an emerging space warfare domain and as such the United States requires an effective force composition in order to deter or win a potential future space conflict. The development of a military which presents an effective force composition is an immense, multidimensional undertaking with many efforts including combat modeling. Combat modeling in and of itself is a vast field including such spheres as agent-based modeling, high-fidelity physics-based modeling, and parametric modeling. A specific and important niche within parametric combat modeling is that field of parametric combat modeling which utilizes differential equations to characterize the numerical outcomes of conflicts – here referred to as differential parametric combat modeling. The importance of differential parametric combat modeling lies in its elegance which allows it to produce results quickly without the use of computationally intensive, sophisticated, expensive, or licensed software or hardware. Such elegance allows differential parametric combat modeling to be executed diffusely in an operationally and strategically-relevant timeframe.

The original developer of differential parametric combat modeling, Lanchester, published his seminal work, *Aircraft in Warfare: The Dawn of the Fourth Arm*, in 1916. (Lanchester, 1916) Within the next decade, Lotka and Volterra were researching biomathematics and theoretical ecology, independently developing models which used differential equations to characterize predator-prey interactions and interspecific competition. (Kingsland, 2015) By 1959, Brackney developed models in the same mold to integrate the fog of war concept and directly map physical phenomena to parameter values. (Brackney, 1959) Despite this progress, all of these models relied on homogenous force structures within each belligerent to be effective. In 1970, Bonder and Farrell attempted to solve the heterogeneous force structure problem but, by their own admission, were not able to close the gap despite a noble effort which made valuable contributions to the field. (Bonder & Farrell, 1970)

Differential parametric combat modeling is a form of aggregated combat modeling since, as Washburn and Kress assert, "Combat models are sometimes described as 'aggregated' or 'high resolution,' but aggregation should really be measured on a continuous scale… To the extent that dissimilar things are treated as if they were identical, the model is said to be more or less aggregated." (Washburn & Kress, 2009) The aforementioned works of Lanchester, Lotka, Volterra, Brackney, Bonder, and Farrell certainly utilize a significant degree of aggregation. As a developer or user of combat modeling pushes a particular model down the continuum towards greater aggregation, it is of vital importance that the aggregation is conducted in such a manner so as to not warp the accurate representation of the underlying physical phenomena. While the past luminaries of differential parametric combat modeling were unable to effectively aggregate heterogeneous force structures, this dissertation effectively addresses heterogeneity using game-theoretic mixed strategies as population parameters. Such an approach preserves the value of aggregation within combat modeling without sacrificing the accurate representation of the underlying phenomena. Naturally, this methodology requires the establishment of those population parameters. Probability theory, specifically reliability modeling, is utilized within this work to determine the necessary population parameters for the game-theoretic approach. The methodological framework then, is a mathematical construct which ties the aforesaid elements together in a coherent manner so as to enable effective parametric combat analysis. Throughout this dissertation, the elements of the framework and the framework itself are contextualized to the space warfare domain.

1.2 Problem Statement

Current approaches within differential parametric combat modeling cannot effectively distill heterogeneous force structures into the necessary parameter values. Given the heterogeneous nature inherent to modern militaries, such a gap limits the effectiveness of differential parametric combat modeling in analysis, wargaming, and game-theoretic system design.

1.3 Research Objective

The research objective of this work is to create a methodological framework for the analysis of space conflict between two heterogeneous belligerents using differential parametric combat modeling.

1.4 Research Questions

The research questions which arose from this objective throughout the pertinent course of work include:

1. How can game theory inform spacepower?
2. How can aggregated combat models be applied to space conflict?
3. What similarities exist between the reliability models of large and small spacecraft?
4. How can game theory, aggregated combat models, and reliability models be integrated to provide analysis for space combat?

1.5 Assumptions and Limitations

1.5.1 Understanding the Difference between the Framework, Analysis, Wargaming, and Game-Theoretic System Design.

The framework presented in this work, when used skillfully, is useful for analysis, wargaming, and game-theoretic system design while being definitively distinct from those entities. The framework is a tool whereas the latter three are processes for which the tool may be used. The thrust of this dissertation is the development of the framework rather than the execution of those three processes. Despite the use of notional analyses to demonstrate the framework, the distinction between the framework and the analysis should always be kept in mind. Furthermore, although the framework could be used in wargaming or game-theoretic system design, those processes are not used to demonstrate the framework and are distinct from both each other as well as analysis. Combat analysis provides descriptive results based on some set of conditions and may serve to implicitly inform the user. Wargaming is an abstraction of warfare which requires dynamic in-the-loop human decision-making. (Perla & Curry, 2011) As the name suggests, game-theoretic system design is the use of game theory to design a system and is an explicitly prescriptive process.

1.5.2 The Abstraction of Warfare.

The framework presents a mathematical construct which asserts the ability to conduct combat analysis and predict a distribution of quantitative outcomes. This work stands on the intellectual shoulders of such luminaries as Lanchester, Brackney, Lotka, Volterra, and Bonder while seeking to advance their seminal concepts. To that end, this work assumes that, to an extent, the chaos and complexities of combat may be distilled to a mathematical representation. Of course, mathematical constructs are not capable of perfectly representing combat; as the adage goes, all models are wrong but some are useful.

1.5.3 Different Spheres of Modeling are Complimentary.

The parametric nature of the framework compliments rather than competes with agent-based modeling and high-fidelity physics-based modeling. Agent-based modeling is the programming of specific behavior into various agents and collecting the results of the emergent behavior within the system. High-fidelity physics-based modeling emphasizes accurately representing the physical behavior of a system in a relatively comprehensive fashion. These various modeling approaches do not supplant each other but could potentially enhance each other. For example, a high-fidelity physics-based model could inform the values of a parametric model; a parametric model could provide an initial direction for the development of a more complex agent-based model.

1.5.4 (The Lack of) Just War Theory.

This work does not attempt to say why a belligerent should fight or even prescribe victory conditions – although someone might use this framework in a discussion on just war theory, such a conversation is beyond the scope of this work.

1.5.5 Media Interaction Warfare Theory.

This work asserts that the framework and Media Interaction Warfare Theory (MIWT), a contemporary parametric modeling methodology, serve to validate each other. Given the historical research required for the parameter values used in MIWT, MIWT should not be considered as a replacement for any part of the framework.

Reference Figure 1.1 for a visual representation of the relationships discussed in the Assumptions and Limitations section.

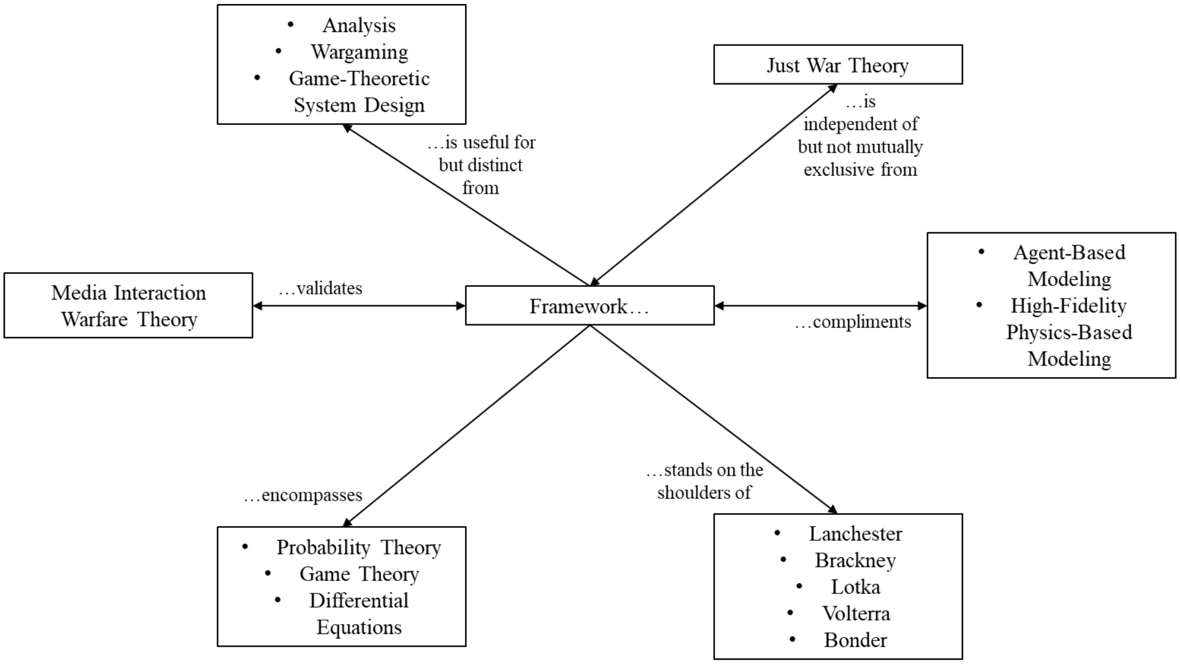


Figure 1.1. Framework Relationships Mind Map

1.6 Dissertation Outline

This work is organized in a scholarly article format. Chapter 1 introduces the work, Chapters 2 through 6 each present a scholarly article published or submitted for publication, and Chapter 7 concludes the work. Chapters 2 through 4 focus on the foundational material of the framework, Chapter 5 focuses on the validation of the framework, and Chapter 6 integrates the framework.

Chapter 2, “Game-Theoretic System Design in the Development of Spacepower,” was published in the *Air & Space Power Journal* and answers research question one: how can game theory inform spacepower? (Hayhurst & Colombi, 2021) This work is a foundational piece of the framework because game theory is used within the framework to distill heterogeneous force structures with corresponding pairwise attrition coefficients into aggregate attrition coefficients for use within the differential parametric combat models.

Chapter 3, “Aggregated Space Combat Modeling,” was published in the *Journal of Defense Modeling and Simulation* and answers research question two: how can aggregated combat models be applied to space conflict? (Hayhurst, Colombi, & Meyer, 2021) This work is a foundational piece of the framework because it demonstrates that differential parametric combat models are effective in characterizing space combat.

Chapter 4, “Survival Analysis for Nanosatellites and Picosatellites,” was published in the proceedings of the 35th Annual Small Satellite Conference and answers research question three: what similarities exist between the reliability models of large and small spacecraft? (Hayhurst, Bettinger, & Schubert Kabban, 2021) This work is a foundational piece of the framework because the respective reliability models are used within the framework to establish a mixed-strategy as a population parameter for each belligerent.

Chapter 5, “A Game-Theoretic Evaluation of Media Interaction Warfare Theory,” is under review for publication by the *Space Force Journal*. This work is meaningful in that it validates the framework by showing a general alignment of results between the MIWT approach and the game-theoretic approach.

Chapter 6, “A Methodological Framework for Parametric Combat Analysis,” was accepted for publication in the *Space Education and Strategic Applications Journal* and answers research question four: how can game theory, aggregated combat models, and reliability models be integrated to provide analysis for space combat? As the eponymous chapter of this dissertation, Chapter 6 brings all of the preceding elements together to present them as one integrated framework.

Chapter 2: Game-Theoretic System Design in the Development of Spacepower

2.1 Introduction

The US space enterprise plays an integral role in maintaining the peace and prosperity of the nation. In times of conflict, the country depends on American space power. Leaders within the US space community advance space power through the evaluation and execution of strategically interdependent decisions. These decisions pertain to the technology development, acquisition, and operation of space systems and are analogous to moves, strategies, and payoffs in multiplayer games. Using game-theoretic models, decision-makers possess the valuable opportunity to partially manipulate game structure before stepping into the role of a player. To bolster this hypothesis, this article presents several game-theoretic system design concepts. First, this article contextualizes the spectrum of agent strategic interactions, from collaboration through competitive to more antagonistic outcomes. Second, a new taxonomy for the classification of game-theoretic models is proposed. Third, we expound on the proposed taxonomy using eight atomic game structures and exemplify their use with pertinent space applications.

2.2 Game Theory

Game theory dates back to work by John Von Neumann in 1928. With wide applications in political science, economics, biology and genetics, sociology, linguistics, and even system design, game theory is a tool to solve decision-making problems. A game involves a set number of players, strategies (decisions, possible moves, or actions), and a payoff or value that captures the outcome of each play per player. (Spaniel, 2015)

The strategy or strategies for each player can be simple and small, or complicated. Consider chess, where the number of possible moves and strategies are massive. But even for atomic games with two players and two possible moves each, one can observe interesting and counterintuitive scenarios and equilibria. Three important aspects of game theory include agent utility balance, Nash equilibrium, and the Pareto front.

Agent utility balance states that an outcome holds approximately the same utility for all agents.

Nash equilibrium relies on the conventional use of the term in the field of game theory – a set of strategies, one for each player, such that no player has an incentive to unilaterally change their current decision or move. (Spaniel, 2015) A player achieves a pure-strategy Nash equilibrium (where such equilibrium exists) by playing a single strategy. A player can achieve indifference in the other player(s) through a mixed-strategy Nash equilibrium wherein a set of pure strategies are played with some probability. (Spaniel, 2015)

Generally, Pareto optimality exists when no single criterion can be improved without diminishing at least one other criterion. In the case of a two-player game, the two-dimensional Pareto front considers each agent utility as a positive asset for maximization. The Pareto front is formed using nondominated outcomes within the game-theoretic model. (Leyton-Brown & Shoham, 2008)

2.3 The Atomic Competitive Element Taxonomy

The Atomic Competitive Element (ACE) taxonomy presents an abstract and descriptive decision space that illustrates contextually desirable attributes. Therefore, an understanding of the ACE taxonomy encompasses comprehension of that context, specifically, agent goals and the resultant behavior. While the user may frame any game-theoretic model with the ACE taxonomy, situations containing self-interested players (who nonetheless display a willingness to cooperate to achieve a mutually beneficial outcome) provide the most natural fit. Close allies with a shared goal, working toward a collaborative outcome, often diverge from the ACE taxonomy construct. Similarly, hostiles committed to self-deleterious min-max strategies frequently eschew such a framework. The span between these extremes – including self-interested cooperators, competitors, and belligerents – fit naturally into the ACE taxonomy construct.

Collaborative outcomes maximize the collective utility of the agents within the game. Close allies with a shared vision, generally common values, and a shared goal, often work toward such outcomes; each agent sees the team success as personal success. Under certain circumstances, such an approach can maximize both coalition and individual utility over the long term. By maximizing team utility, collaborative outcomes always exist on the Pareto front. Collaborative outcomes do not fit as naturally within the ACE taxonomy framework.

Cooperative, competitive, and antagonistic outcomes always use Nash equilibria as the baseline solution. Agents working toward a cooperative outcome are willing to move from a Nash equilibrium to a mutually beneficial outcome with a higher utility for both players. In a cooperative context, agents treat each other benevolently and work for the betterment of other agents as long as the respective individual agent garners a positive or neutral result. Cooperative outcomes generally fall on a Nash equilibrium or a Pareto front outcome with adequate utility balance and mutual utility improvement. They also generally maximize individual utility within a specific game. Allies with shared interests work together toward the same outcome. Importantly, agents within such a context need not demonstrate altruism (i.e., agents act in self-interest), but the agents must trust each other and act in good faith.

Naturally, competitors pursue competitive outcomes and seek to maximize individual utility through individual effort. Competitive outcomes land on Nash equilibria. Agents within such a context display indifference toward other agents – seeking neither good nor harm for fellow players.

Antagonistic outcomes display the same characteristics as competitive outcomes except that, in such a context, agents choose to harm each other when there is no cost to do so. For example, an agent given two options with the same personal utility would follow a min-max strategy to minimize the other agent's utility. Cooperative, competitive, and antagonistic outcomes, as well as the associated agent behavior, naturally fit into the ACE taxonomy framework.

In a hostile context, adversarial players engage in a pure min-max strategy wherein every choice minimizes the other agent's maximum possible utility. (Leyton-Brown & Shoham, 2008) When seeking a hostile outcome, agents pursue this min-max approach even when such a strategy presents self-detrimental consequences. Interestingly, these hostile agents are not self-interested and can be trusted to always commit the most harmful action. Hostile outcomes and belligerents do not fit into the ACE taxonomy construct. Reference Figure 2.1 for the spectrum of interaction among agents in a game.

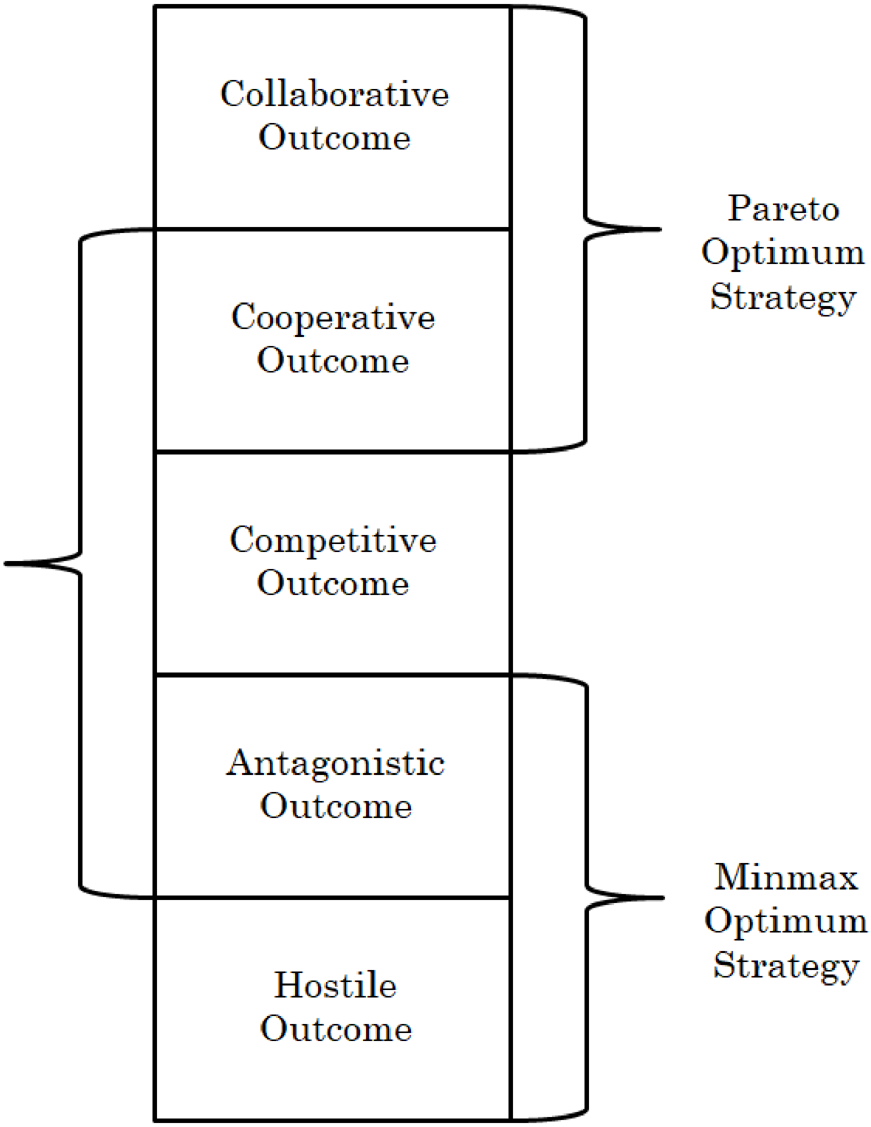


Figure 2.1. Spectrum of Interaction

The ACE taxonomy illustrates and classifies game-theoretic models according to three contextually desirable attributes (for the stability of an outcome), which may exist in a particular outcome: agent utility balance, Nash equilibrium, and the Pareto front.

The ACE taxonomy represents these three attributes with primary colors, their combinations with secondary colors, the presence of all three attributes with white, and the absence of all three attributes with gray. Reference Figure 2.2 for the Venn diagram illustrating the ACE taxonomy.

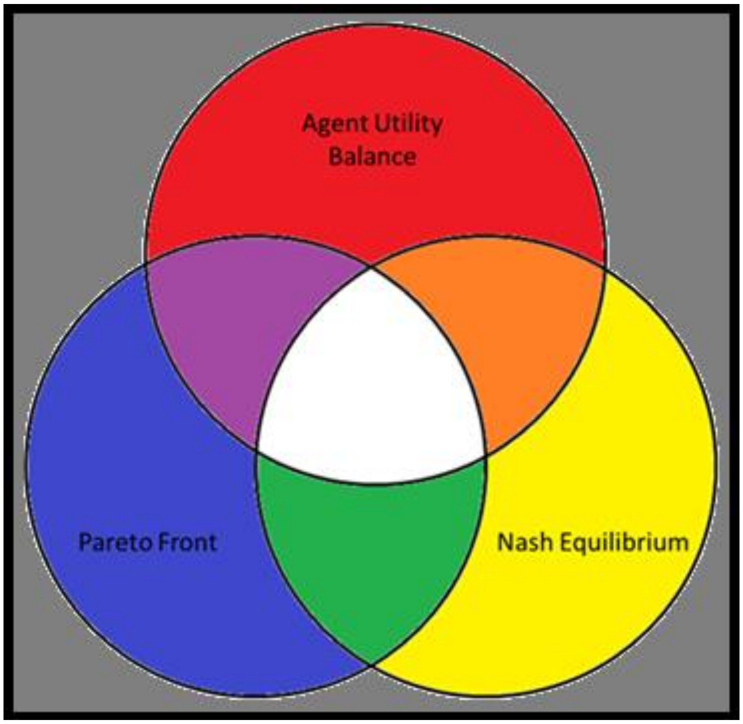


Figure 2.2. Factors of Stability in Multiagent Games

2.4 Characterization of Atomic Competitive Elements

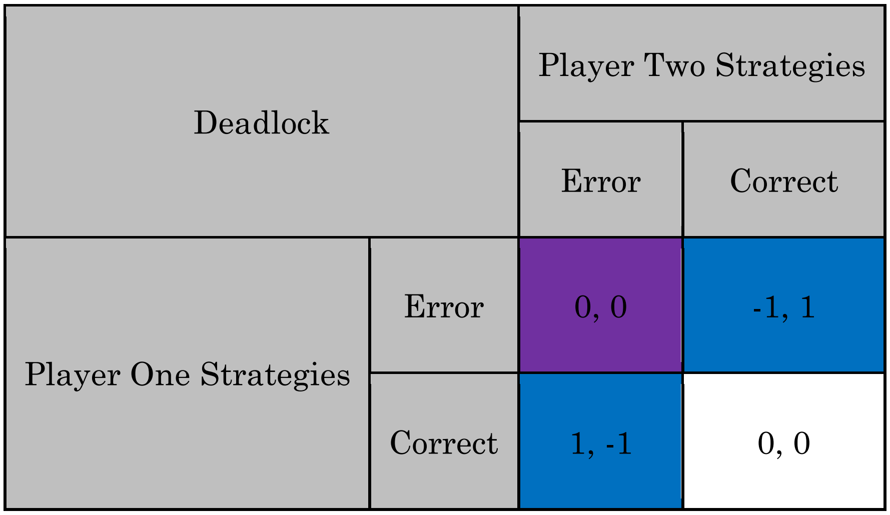
This section introduces and characterizes eight fundamental building blocks of ACE that are significant in the formation of many higher-complexity game-theoretic models. The user of this taxonomy may recognize each kind of ACE by its unique color scheme based on the three properties (agent utility balance, Nash equilibrium, and the Pareto front) present or not within each of the four outcome cells of the respective two-by-two matrix. This taxonomy does not consider game-theoretic models as unique ACE wherein the game designer may trivially rearrange the choices of the respective game to achieve a repeated color scheme. Systematically categorizing game-theoretic models at a fundamental level empowers the user to identify the scenario at hand, understand the scenario's dynamics, and draw upon heuristic solutions to maximize the utility for one or more agents within the game. Specifically, this article uses this taxonomy to address challenges and opportunities in the development of space power.

2.4.1 Deadlock.

In Deadlock, each player knows both the correct and incorrect answer and must simply choose the correct answer. If both players choose the same answer, they earn a neutral utility value. If one player makes an unforced error, the winning player achieves positive utility at the expense of the losing player. Importantly, this game, as well as the other games, are presented in a strategic form where both players must act simultaneously; players do not know what the other player will do, and prior communication or coordination is not guaranteed.

Perhaps the most stable and simple game-theoretic model, Deadlock contains a single balanced pure-strategy Nash equilibrium on the Pareto front. Deadlock presents a straightforward, intuitive scenario wherein agents converge to the Nash equilibrium with no opportunity to improve utility through cooperation. (Spaniel, 2015) Other outcomes within Deadlock represent unforced errors by one or more agents. Reference Table 2.1 for the game of Deadlock using the ACE taxonomy.

Table 2.1. Deadlock



Deadlock		Player Two Strategies	
		Error	Correct
Player One Strategies	Error	0, 0	-1, 1
	Correct	1, -1	0, 0

2.4.2 Pure Coordination.

In Pure Coordination, players must decide to stay or go. If both players choose the same answer, both players achieve a positive utility. If players differ in their choices, neither benefits. The self-explanatory Pure Coordination game-theoretic model presents an extremely stable game in the presence of effective communication with two balanced pure-strategy Nash equilibria on the Pareto front and one mixed-strategy Nash equilibrium. (Spaniel, 2015) Since the payoffs for both pure strategies hold the same utility for each agent, players of the game display indifference in the pursuit of a particular pure strategy and act amiably in the respective coordination. Reference Table 2.2 for the game of Pure Coordination, using the ACE taxonomy.

Table 2.2. Pure Coordination

Pure Coordination		Player Two Strategies	
		Stay	Go
Player One Strategies	Stay	1, 1	0, 0
	Go	0, 0	1, 1

2.4.3 Stag Hunt.

In Stag Hunt, each player must decide to hunt the stag or hunt the two hares. Hares can be caught by one player, but the stag requires both players working together to catch it. If each player hunts for hares, each will catch one hare and achieve a utility of one. If both players hunt for the stag, each will achieve a utility of three, since the stag is worth six total utility. However, if one player hunts for hares, that player will catch both hares and achieve a utility of two, while the other player will earn nothing since they will be unable to singlehandedly catch the stag.

Stag Hunt generally represents the synergistic effect of cooperative resource harvesting with one pure-strategy Nash equilibrium on the three-cell Pareto front, one pure-strategy Nash equilibrium off the Pareto front, and one mixed-strategy Nash equilibrium. (Spaniel, 2015) The Pareto front pure strategy presents high stability in the presence of effective communication and the absence of adversarial intentions. In a similar fashion to other ACE, such as Stoplight and Chicken, this game presents the opportunity for game-theoretic system design to expand the scope of the scenario to achieve a higher utility for both players. The game designer may translate the strategic form of the game to an extensive form and introduce a new branch on the first node with outcome utility less than the utility of synergistic harvesting but greater than individualistic harvesting. Given logical, sophisticated agents capable of forward induction, the players will not use the new branch and will instead converge to synergistic resource harvesting. (Spaniel, 2015) Reference Table 2.3 for the game of Stag Hunt using the ACE taxonomy.

Table 2.3. Stag Hunt

Stag Hunt		Player Two Strategies	
		Stag	Hare
Player One Strategies	Stag	3, 3	0, 2
	Hare	2, 0	1, 1

2.4.4 Matching Pennies.

In Matching Pennies, each player decides whether to play their coin heads-up or tails-up. One player wins if both coins match while the other player wins if the coins do not match.

Matching Pennies represents arguably the most unstable simple game-theoretic model with no balance, one mixed-strategy Nash equilibrium and a four-cell Pareto front that spans the entire decision space. In Matching Pennies, one agent attempts to match the metaphorical penny while the other agent works to prevent the match. (Spaniel, 2015) Reference Table 2.4 for the game of Matching Pennies using the ACE taxonomy.

Table 2.4. Matching Pennies

Matching Pennies		Player Two Strategies	
		Heads	Tails
Player One Strategies	Heads	1, -1	-1, 1
	Tails	-1, 1	1, -1

2.4.5 Stoplight.

In Stoplight, two drivers arrive at an intersection simultaneously and must decide whether to continue or stop. If one continues, that driver will gain a utility of one while the other driver will be indifferent. If both players stop, both players will be mildly annoyed and lose one utility value. If both players continue, they will cause an accident greatly detrimental to their utility values.

Stoplight represents the quintessential game-theoretic model for the application of correlated equilibrium with two unbalanced pure-strategy Nash equilibria on the Pareto front, one mixed-strategy Nash equilibrium, and two balanced, mutually deleterious outcomes off the Pareto front. (Spaniel, 2015) In the Stoplight model, logical agents use a correlated equilibrium mechanism (perceived as fair by all agents) whenever possible to maximize overall and individual utility. Reference Table 2.5 for the game of Stoplight using the ACE taxonomy.

Table 2.5. Stoplight

Stoplight		Player Two Strategies	
		Continue	Stop
Player One Strategies	Continue	-5, -5	1, 0
	Stop	0, 1	-1, -1

Fundamentally, Stoplight represents the same ACE as both the Battle of the Sexes and Volunteer’s Dilemma game-theoretic models. Stoplight addresses safe traffic flow, Battle of the Sexes addresses coordination (or lack thereof) for an entertainment venue, and the Volunteer’s Dilemma addresses costly intervention to help a crime victim. (Spaniel, 2015) Effectively, since each of these game-theoretic models represents the same kind of ACE, game agents, or the game designer may use a fair correlated equilibrium mechanism to achieve a higher utility.

2.4.6 Prisoner's Dilemma.

In the Prisoner's Dilemma, an interrogator can convict two players of minor crimes without a confession such that each player will spend one month in jail. The interrogator offers a plea bargain to both suspects where they can sell out the other player for personal leniency – if only one player takes the deal, that player will receive no time in jail while the other player will spend 12 months in jail having been successfully convicted of the more serious crime with the help of the defector's confession. However, if both players confess, their confessions are worthless, and each will receive eight months in jail on the charges of the more serious crime.

The Prisoner's Dilemma represents arguably the most famous game-theoretic model with a single pure-strategy Nash equilibrium off the Pareto front. The game demonstrates the difficulty among self-interested, untrustworthy agents in moving from the Nash equilibrium to a balanced, mutually beneficial outcome. The difficulty in establishing the mutually beneficial outcome lies in the opportunity for profitable deviation by an untrustworthy agent. (Spaniel, 2015) Reference Table 2.6 for the game of Prisoner's Dilemma using the ACE taxonomy.

Table 2.6. Prisoner’s Dilemma

Prisoner’s Dilemma		Player Two Strategies	
		Silence	Defect
Player One Strategies	Silence	-1, -1	-12, 0
	Defect	0, -12	-8, -8

The Prisoner’s Dilemma forms an important conduit to understanding other game-theoretic models such as the Optional Prisoner’s Dilemma, repeated Prisoner’s Dilemma games, the Tragedy of the Commons, the Hawk-Dove game, and duopolistic competition.

The Optional Prisoner’s Dilemma represents an exogenous manipulation of the traditional game and enables an agent to abstain when playing with a perceived defector to achieve a higher utility. Repeated Prisoner Dilemma games allow for higher levels of cooperation and more sophisticated strategies such as tit for tat; an unknown or infinite number of Prisoner Dilemma games aids the strategic enhancement for improved utility. Scenarios that permit proactive self-determined agent mixing (players may choose which agent to play with from the available pool) especially increase the utility value for cooperative agents. Robert Axelrod explored the concept of the Prisoner’s Dilemma in his developing notion of cooperation as an evolutionarily stable strategy. (Axelrod, 1981) In his work with the Prisoner’s Dilemma, Ahmed Ibrahim contended that “evolutionary mechanisms have nothing to do with conflict between the causes of the tragedy and their solutions for it, whether the solution is that of outcompeting the tragedy or its contrary.” In considering the existence of cooperation among organisms, Ibrahim asserted the presence of a conscious intervener. (Ibrahim, 2015)

The Tragedy of the Commons represents a more unwieldy N-player version of the Prisoner’s Dilemma where at least one agent exploits a common resource for personal gain to the detriment of the common resource and the community. Garrett Hardin suggested privatization and top-down regulation (mutual coercion) as remedies, implicitly assuming the existence of a strong, efficient central authority. (Hardin, 1968) Elinor Ostrom focused on bottom-up institutions and articulated conditions that fostered such cooperation: easy-to-monitor resources, moderate rates of change, robust social networks, the ability to exclude outsiders, and a strong push for self-enforcement among community members. (EconClips, 2018) The pseudonymous Satoshi Nakamoto utilized cryptography to protect a common in the form of a public ledger. (Nakamoto, n.d.)

The Hawk-Dove game exists as a superset of three simpler games wherein the Prisoner’s Dilemma fundamentally represents the manifestation of relatively low-cost conflict. The game designer, by exogenous manipulation, may significantly increase the relative cost of conflict with respect to the value of the prize to transform the Prisoner’s Dilemma into a game of Chicken. Such a transformation creates a new set of strategies as well as new pathways for game-theoretic system design.

The dynamics of the Prisoner’s Dilemma, to some degree, check the spread of collusion in duopolistic competition and preserve the health of a limited marketplace.

2.4.7 Take or Share.

In Take or Share, each player must decide whether to take the pot of money or share the pot of money worth eight dollars. If both players share, they will split the pot. If both players take, each will receive no money. If one player takes, that player will receive all the money while the other player receives nothing.

In the Hawk-Dove superset, Take or Share represents the knife-edge transition from Prisoner's Dilemma to Chicken as the relative cost of conflict increases. Outside of artificial or discretized environments, such knife-edge equilibria do not exist. Take or Share encompasses three pure-strategy Nash equilibria and infinitely many partially mixed strategy Nash equilibria. (Spaniel, 2015) Reference Table 2.7 for the game of Take or Share using the ACE taxonomy.

Table 2.7. Take or Share

Take or Share		Player Two Strategies	
		Share	Take
Player One Strategies	Share	4, 4	0, 8
	Take	8, 0	0, 0

2.4.8 Chicken.

In Chicken, two drivers drive toward each other at high speeds in a show of bravado. If both drivers swerve, nothing will happen. If both continue, each will be engulfed in a devastating accident. If one swerves, that player will be embarrassed for having lost the intimidation game, while the player who continued will gain positive utility in the form of a fearless reputation. Incidentally, the authors recommend against playing the game of Chicken.

Chicken represents arguably the most fascinating simple game-theoretic model with two unbalanced pure-strategy Nash equilibria along a three-cell Pareto front as well as one mixed-strategy Nash equilibrium. Generally, Chicken exists as an intimidation game with high-value assets at stake and represents relatively high-cost conflict in the Hawk-Dove superset. The mixed-strategy Nash equilibrium enables the use of comparative statics that demonstrate a dramatic decrease in the probability of conflict for any incremental, mutual increase in the cost of conflict. Political scientists use such results to explain the role nuclear weapons play in peacekeeping under the construct of mutually assured destruction. (Spaniel, 2015) Reference Table 2.8 for the game of Chicken using the ACE taxonomy.

Table 2.8. Chicken

Chicken		Player Two Strategies	
		Continue	Swerve
Player One Strategies	Continue	-10, -10	2, -2
	Swerve	-2, 2	0, 0

Counterintuitively, increasing the cost of conflict improves the overall payoff for an agent within the Chicken game when playing the mixed strategy. However, throwing the cost of conflict disproportionately out of balance significantly increases the chance the agents play the pure-strategy Nash equilibrium deleterious to the respective agent.

Exogenous control accounts for the cost of conflict in the game of Chicken (high-cost Hawk-Dove) where each agent makes a binary choice between conflict and peace. In a game where agents may choose a private commitment of resources to some conflict (i.e., a cost known only to the respective agent), Maynard Smith discovered the evolutionarily stable strategy of generating an exponential distribution using the value of the prize of the conflict as the beta parameter and randomly drawing from that distribution to determine the acceptable value of the cost of the commitment to conflict. Given that the expected value of the cost of the conflict equals the value of the prize of the conflict, the expected overall utility for such a stable approach equals zero. Therefore, Smith suggested the use of some credible mechanism for correlated equilibrium to improve the utility for both agents; he later learned certain animals use the ownership principle as that mechanism. (Web of Stories – Life Stories of Remarkable People, 2017)

2.5 Spacepower Applications

2.5.1 Space Debris and the Prisoner's Dilemma.

The development of space power offers each nation the opportunity to bolster its technical acumen, national prestige, and instruments of war. Among the many facets of space power, direct ascent antisatellite (DA-ASAT) weapons offer an instructive case study on the generation of space debris. Perhaps the four most pertinent events related to DA-ASAT weapons and space debris include the 1985 destruction of the US P78-1 Solwind satellite, using an air-launched ASM-135 (during the era of the Strategic Defense Initiative), the 2007 destruction of the Chinese FY-1C (Fengyun, "Wind and Cloud") satellite using a ground-launched SC-19, the 2008 destruction of the US USA-193 satellite using a sea-launched Standard Missile-3 (Operation Burnt Frost), (Grego, 2012) and the 2019 destruction of the Indian Microsat-R satellite using a ground-launched Prithvi Defense Vehicle Mark-II (Mission Shakti, "Power"). (Tellis, 2019) All four of these satellites experienced destruction at the hands of their owners, and each event caused significant orbital debris. Notably, however, the US and India conducted their tests in such a manner as to deorbit all the debris within several years and much of the debris within the first several weeks and months. In contrast, China's demonstration contributed to the formation of a perpetual low-earth orbit Kessler field.

Beyond DA-ASAT weapons, many other space activities and events contributed to the debris cloud in space. Spacefaring nations often leave spent rocket bodies and nonfunctional spacecraft in orbit, finding such an approach more economical than returning the artificial satellites to Earth. Many of these objects undergo physical explosions (e.g., explosions caused by the pressure buildup in the fuel lines) or chemical explosions (e.g., a hypergolic ignition of residual propellants, an explosion caused by severely decayed batteries, or the purposeful self-destruction of Soviet Union satellites) that further contribute to space debris pollution. Satellites often face the threat of conjunction (i.e., accidental, hypervelocity, destructive collision); the 2009 Cosmos 2251 and Iridium 33 collision provides the most destructive, polluting example. (Linville & Bettinger, 2020) The Soviet Union contributed to the space debris field with spacecraft that leaked sodium-potassium droplets (meant to cool the nuclear reactor onboard the respective satellite) into orbit. (The European Space Agency, n.d.)

In each of the aforementioned scenarios, the agents involved chose an action to maximize individual utility to the detriment (directly or indirectly) of the space community as a whole. During the era of the US and Soviet Union bipolar dichotomization of power, such events functioned within the context of a Prisoner's Dilemma. With a larger and growing community of modern spacefaring entities (to include the US, Russia, China, the European Space Agency, Japan, India, South Korea, North Korea, Iran, and Israel), the current space debris events occur in the framework of a Tragedy of the Commons. (NASIC Public Affairs Office, 2018) While nations utilize the more egregious events as political weapons within the international community, no mechanism exists to definitively prevent the creation of space debris. The 1967 Outer Space Treaty prohibits the privatization of space, and no top-down organization currently wields the power necessary to impose and enforce space debris regulations on the collective group of spacefaring nations. (US Department of State, 1967) The factors that would contribute to the effective formation of bottom-up institutions capable of addressing the space debris issue simply do not exist. The innovation of technologies capable of addressing the space debris problem (e.g., reusable rocket bodies, mechanical space debris collection devices, or lasers used to deorbit space debris) afford a worthwhile goal. The political efforts to prevent the proliferation of harmful space debris also provide an avenue for potential progress. However, the core characteristics of the Prisoner's Dilemma ACE and the associated game-theoretic models suggest the inevitability of an increasingly polluted space. Therefore, the main thrust of the US efforts in this field should be in the development of spacecraft capable of surviving and operating in such an environment—not in the attempt to prevent the formation of such an environment. Increasing the resiliency of spacecraft to hypervelocity impacts, using simpler, cost-effective replaceable spacecraft, disaggregating satellite constellation architectures, or transitioning to less-polluted orbital regimes all provide potential avenues for such an undertaking. In a polluted yet still usable space environment, spacecraft maneuver also provides a mechanism for survivability. However, the finite fuel onboard a satellite mandates the prudent use of any such maneuver. To ensure spacecraft maneuvers are conducted judiciously and effectively, the US requires a robust array of space domain awareness capabilities, including both ground-based and space-based sensors and processors.

2.5.2 Department of Defense Policy and Deadlock.

Deadlock illustrates the self-imposed damage of unforced errors by one or more agents. A plethora of policies, some worthy of several research papers, guide the personnel and technological development of the Department of Defense, including the US Space Force. Any of these policies that inadvertently cause a substantive number of talented people to exit the US military might be considered an unforced error. Furthermore, policies that neglect the development of critical technologies (e.g., cyber) might be considered unforced errors. When agents do not understand the implications of their actions or hold some other goal as a higher priority, they may fail to reach the stable equilibrium within the Deadlock ACE.

2.5.3 Conjunction, Collision, or Rendezvous and Proximity Operations.

The Pure Coordination ACE covers mutually desirable rendezvous and proximity operations in space, such as the docking of a supply vessel to the International Space Station. While the orbital dynamics and control theory of such an endeavor present a technological hurdle, the game-theoretic considerations are quite simple and require only sound communication. The Matching Pennies ACE addresses situations in which one agent desires the proximate interaction and the other agent desires the opposite. In a pertinent situation concerning the optimal pursuit of a spacecraft by a piece of space debris, David Spendel relied on the field of Differential Game Theory – specifically, the Homicidal Chauffer game-theoretic model. (Spendel, 2018)

2.5.4 Space Resource Harvesting and the Stag Hunt.

The nascent field of space resource harvesting holds tremendous potential. Lunar extraction may yield nuclear fusion fuel and rare earth metals with important technological and industrial uses on Earth. Near-earth object chondrites and achondrites may yield valuable resources for in situ utilization by manned missions or high-value precious metals. (Duke, n.d.) Given the Stag Hunt ACE framework, synergistic cooperation in the harvesting of these resources may occur naturally. In cases where there are barriers to such cooperation, an agent (acting as a game designer) may use game-theoretic system design to exogenously change the structure of the game. The agent translates the strategic form game to an extensive form information set and adds a new branch on the previous node. This new course of action strikes a balance in individual utility between synergistic cooperation and the preexisting choice to not cooperate. The respective agent will never use this new branch so long as the other agent demonstrates forward induction through the *a priori* commitment to synergistic cooperation. Perhaps counterintuitively, the more developed an entity's capacity for previous space resource harvesting, the greater trust other agents will place in that entity's commitment to cooperation. Therefore, early US investment in space resource harvesting may incur a beneficial positive feedback cycle.

2.5.5 Stoplight and Correlated Equilibrium.

The Stoplight ACE encompasses the Stoplight, Volunteer's Dilemma, and Battle of the Sexes game-theoretic models. The respective space analogs of these models are cooperative maneuvering to avoid a collision, international policing in space, and harvesting space resources in one of two locations where the utility payoff for each agent is different based on the location. Correlated equilibrium provides a natural and beneficial heuristic solution for the challenges posed in this ACE. The type of mechanism used for correlated equilibrium (e.g., memorandum of understanding alternating decision power or an international third party) is immaterial as long as all players view the mechanism as fair and effective.

2.5.6 Chicken as High-Cost Conflict or Intimidation.

The Chicken ACE manifests itself as a high-cost Hawk-Dove game-theoretic model. The space analog presents itself in one of two ways: two spacefaring entities with spacecraft on a collision course where neither will maneuver or the impending large-scale conflict between two nations encompassing the space domain. There are several game-theoretic system design approaches capable of addressing the Chicken ACE. Similar to the Stag Hunt, a game designer may exogenously translate the game into extensive form and add a branch to the previous node. This new branch acts as a commitment mechanism that turns an incredible threat into a credible threat (much like the concept of burning bridges). The commitment mechanism may exist in a technological form (a doomsday device serves as a sensational example) or in a diplomatic-political form (such as the use of a “red line”). The strength of this approach rests in the strength of the commitment mechanism; for example, if other agents do not believe in the credibility of a player’s red line, the approach will falter. To preserve credibility, red lines must be enforced even when doing so seems impractical since a failed red-line strategy will impact an agent’s credibility in any future game against a player with knowledge of the unenforced red line. If a player is unwilling to follow through with the red-line threat, the player should consider not making the red-line threat in the first place.

Another game-theoretic system design approach drives the hypothetical mutual cost of conflict so high that the comparative statics indicate that the two agents would never enter into such a conflict. Quintessentially, the space-contextual application for such an approach would be the commitment by two or more nations to disregard the Outer Space Treaty and commit to the use of nuclear weapons in space should a conflict ever occur.

A final game-theoretic system design approach encompasses an agent that reduces the individual cost of conflict or collision. If the two agents play the mixed-strategy Nash equilibrium, this approach will work to the detriment of the agent using this method. However, this approach improves the probability that the two agents will transition to the pure-strategy Nash equilibrium favorable to the player that used this taxonomy. In the space domain, a nation might enact this approach by developing lower-cost, less reliable, and less exquisite spacecraft, which the nation can affordably replenish in the event of a collision or malfunction.

2.6 Conclusion

This article asserted that decision-makers could use game-theoretic system design to understand space power challenges and opportunities better, as well as achieve better outcomes for the US space enterprise. In support of this thesis, we contextualized the spectrum of agent strategic interactions, proposed a new taxonomy for the classification of game-theoretic models, and expounded the proposed taxonomy, using eight atomic game structures with pertinent space applications. In this effort, we strive for the advancement of strategic thinking in the space domain for the enhancement of the US space security posture.

Chapter 3: Aggregated Space Combat Modeling

3.1 Abstract

The use of aggregated combat modeling in the cislunar environment has been demonstrated to inform acquisition decisions for the United States Space Force (USSF). First, the cislunar space is hypothesized as a future strategic conflict environment. As such, Lanchester, Lotka-Volterra, and Brackney models could be appropriate to describe such conflict. All models encompass a system of differential equations which parametrically capture the dynamics between friendly and hostile forces. While the Brackney model was constructed to explain two-dimensional land battle, this article adapts it for the respective three-dimensional space domain and applies it to strategic procurement. The analysis demonstrates the preeminence of Space Domain Awareness (SDA) in certain contexts while recognizing conditions in which spacecraft survivability holds greater importance.

3.2 Introduction

Combat modeling holds meaningful potential for the nascent United States Space Force (USSF). While disparate schools of thought concerning the best approaches are developing, the judicious use of multiple models will enable decision makers to thrive in the multifaceted, competitive space environment. Washburn asserts, “Combat models are sometimes described as ‘aggregated’ or ‘high resolution,’ but aggregation should really be measured on a continuous scale… To the extent that dissimilar things are treated as if they were identical, the model is said to be more or less aggregated.” (Washburn & Kress, 2009) In addition to existing along a continuum, aggregation is also multidimensional in that the level of aggregation must be considered for time, space, and the attributes of the objects within the model. Whereas high-fidelity, physics-based models such as Advanced Framework for Simulation, Integration, and Modeling (AFSIM) and Systems Tool Kit (STK) may be used to predict tactical outcomes, aggregated models provide a level of abstraction more appropriate for strategic insight. Using aggregated combat models, an opportunity exists to understand complex interactions and anticipate the strategic outcomes of space conflicts and make effective procurement decisions accordingly. To bolster this hypothesis, this article:

- Characterizes a potential future conflict environment
- Presents and explains certain landmark aggregated combat models
- Demonstrates the use of an aggregated combat model in strategic procurement

The United States (US) and China acknowledge the strategic importance of the space domain, including the Moon, by investing significant financial and political resources into the Artemis (National Aeronautics and Space Administration, 2020) and Chang'e (Myers & Chang, 2020) programs, respectively. While a multitude of factors contribute to the strategic significance of the Moon, space resources provide one potential impetus for lunar security. Section 10 of the Artemis Accords formally demonstrates the international commitment to the "extraction and utilization of space resources… from the surface or subsurface of the Moon, Mars, comets, or asteroids." (National Aeronautics and Space Administration, 2020) The National Aeronautics and Space Administration (NASA) recognizes that China controls 90% of the production of Rare Earth Metals

(REM) and that the US might one day harvest these resources from the Moon. The set of resources includes: Scandium, Yttrium, Lanthanum, Cerium, Praseodymium, Neodymium, Promethium, Samarium, Europium, Gadolinium, Terbium, Dysprosium, Holmium, Erbium, Thulium, Ytterbium, and Lutetium. The Moon may also offer a supply of Helium-3 which, when processed with Deuterium, can yield Helium-4. Helium-4 could potentially be safely contained within an electromagnetic field and provide a consistent supply of energy. Spacefaring entities could potentially harvest water from the Moon for life support, agriculture, radiation shielding, and the production of rocket fuel. (911Metallurgist, 2015) Beyond lunar extraction, the Moon offers a potential staging area for asteroid mining – either as a waypoint or as a parking orbit for mining asteroids. Chondrite asteroids may contain water and achondrite asteroids may contain both precious and industrial metals. (Glester, 2018)

The lunar environment provides a natural fit for aggregated combat modeling because of the relative homogeneity of its natural environment and its future potential force composition. Whereas the Earth presents an atmosphere with a fluctuating density (Earth's atmospheric density fluctuates with respect to time according to the solar cycle (Walterscheid, 1989) and decreases with increasing altitude for a specific time) well past 1,000 kilometers (km), the Moon presents no meaningful atmosphere with respect to spacecraft maneuvers. (National Aeronautics and Space Administration, 2020) The Earth also holds a relatively strong electromagnetic field with non-monotonic levels of radiation. The outer Van Allen radiation belt extends beyond geostationary (GEO) orbit. By contrast, the weak magnetic field of the Moon enables spacecraft in lunar orbit to experience cosmic and solar radiation relatively independent of lunar altitude. The Moon's weaker gravitational field causes lower orbital velocities and affords spacecraft greater maneuverability. (Sellers, 2005) Lastly, Low Earth Orbit (LEO) and GEO pose concentrations of spacecraft and space debris ripe for the formation of Kessler fields; cislunar space from Lagrange 1 (L1) to L2 does not yet pose such a threat. (The European Space Agency, n.d.)

The destructions of several spacecraft (P78-1, FY-1C, USA-193, (Grego, 2012) and Microsat-R) (Tellis, 2019) using Direct Ascent Antisatellite (DA-ASAT) weapons demonstrate the practical utility in using Earth-based sensors and weapons to achieve kinetic kills on LEO targets. This tight integration between ground and space forces creates a heterogeneous force composition not ideally suited for aggregated models. The distance from Earth to the cislunar environment (L1 to L2) precludes the efficient use of one-to-one DA-ASAT weapons and encourages the future bulk deployment of spacecraft to the cislunar environment. Such deployments potentially constitute relatively homogenous forces appropriate for aggregated modeling. The characterization of this potential conflict environment serves to justify the use of the aggregated combat models and establish a basis for parameter values for the strategic procurement analysis.

3.3 Literature Review

While fractionation of belligerent parties characterized certain ancient warfare and modern internecine strife, dichotomization largely characterizes large-scale modern conflict including the formation of unnatural alliances. To that end, this article considers systems of two differential equations.

The effects of nominally negative events, when delivered in proper amount, may produce a net positive result such as shown in several biological processes - eustress, osteogenic loading, or hormesis. For example, a little stress on humans results in cognitive and mental performance gains. Examples extend beyond biology into the sphere of geopolitical and military power. The United States underwent decay in strategic and conventional warfighting capabilities when it embodied the sole global superpower and focused on counterinsurgency operations (c. 1991 – 2016). Presented with near-peer adversaries, the United States again sharpened its technical acumen and focused its efforts on strategic and conventional warfighting capabilities. (O'Rourke, 2020) Many acknowledge the role that levels of adversity and the shape of the growth-adversity curve play in warfighting. In a time of intense conflict, near-peer adversaries will move each other into the fragility-zone, wherein greater adversity produces greater loss. Reference Figure 3.1 for non-monotonic notional growth-adversity curve.

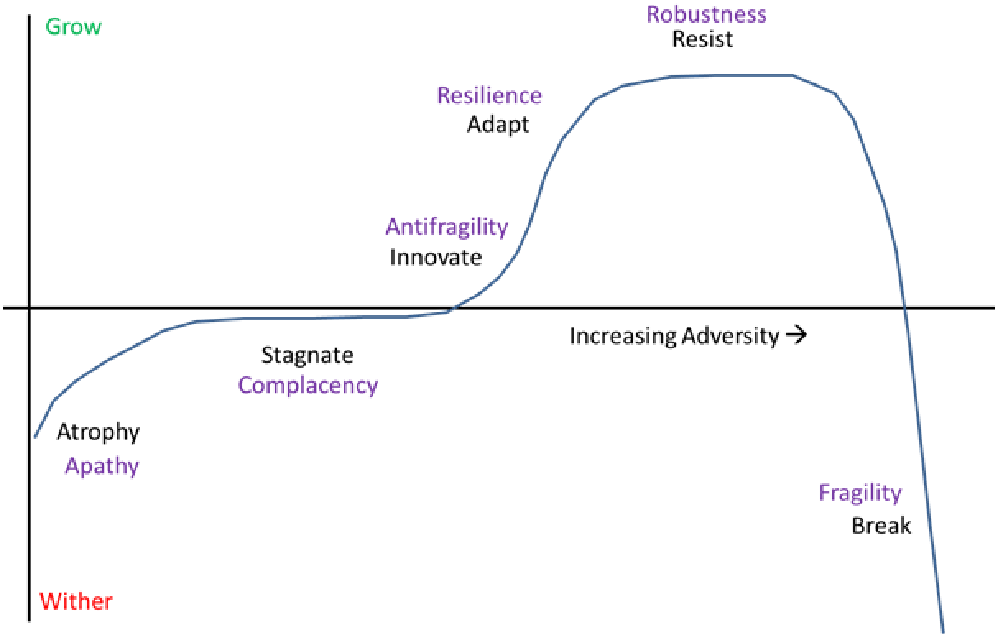


Figure 3.1. Notional Growth-Adversity Curve

The relationship of two entities may be characterized based on their effect on each other. Reference Figure 3.2 for characterization of relationships based on interaction effects.

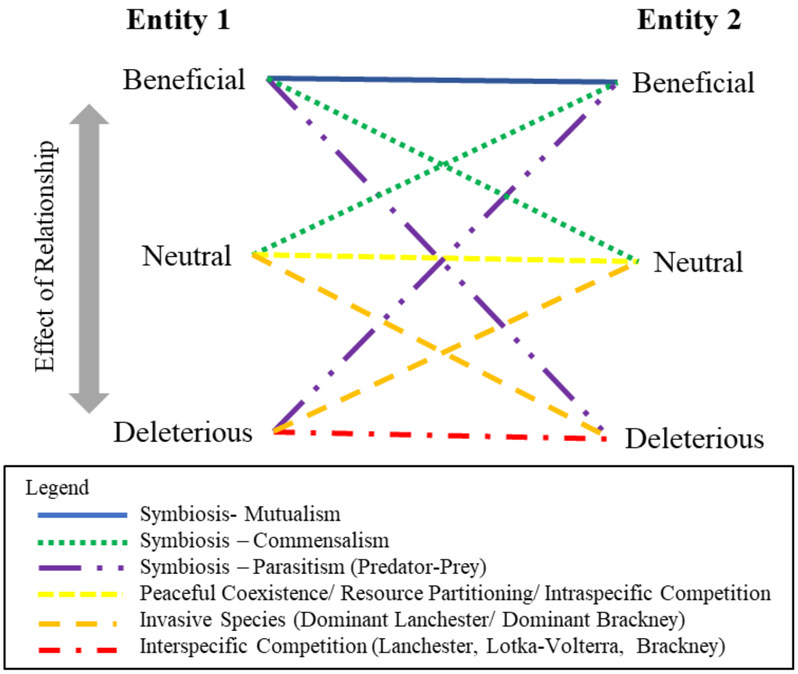


Figure 3.2. Characterizing Relationships Based on Interaction Effects

The characteristics of near-peer space conflict suggest a deleterious-deleterious relationship. The Lanchester, Lotka-Volterra, and Brackney models constitute the most significant, historical aggregated combat models developed from the beginning of the twentieth century.

Lanchester published his models in the book *Aircraft in Warfare: The Dawn of the Fourth Arm* against the backdrop of World War I to describe force attrition during aerial warfare. The Lanchester Linear Law for unaimed fire describes the attrition of red and blue forces under simplified engagement assumptions; it may be presented as

$$\frac{dB}{dt} = -rRB$$

$$\frac{dR}{dt} = -bBR$$

such that

$r \sim red\ force\ attrition\ coefficient$

$b \sim blue\ force\ attrition\ coefficient$

$B \sim size\ of\ the\ blue\ force$

$R \sim size\ of\ the\ red\ force$

This states that the attrition rate for both red and blue is a function of the size of both forces. In this case, the fighting strength of each force may be presented as

$$F_B = bB$$

$$F_R = rR$$

The Lanchester Square Law for aimed fire may be presented as

$$\frac{dB}{dt} = -rR$$

$$\frac{dR}{dt} = -bB$$

Here, the red (blue) attrition rate is a function of the blue's (red's) force size. The fighting strength of each force for the Square Law is presented as

$$F_B = bB^2$$

$$F_R = rR^2$$

For both Lanchester laws, the force with the greater fighting strength will win the conflict. Both laws depend on the homogeneity of units on either side of the conflict and assume a generally uniform distribution of fire from one side against the other. The equations do not account for superior tactics unless such tactics are captured in the attrition coefficients. (Lanchester, 1916) Conventionally, attrition coefficients for the

Lanchester Laws are determined empirically. Predictive values for attrition coefficients concerning future conflicts which are determined by a group of experts will encompass a degree of subjectivity.

Bonder attempted to overcome the need for homogeneity in the Lanchester laws by establishing a more generalized form for the coupled sets of differential equations which may be presented as

$$\frac{dB_m}{dt} = -\sum_{n=1}^{N} r_{m,n} R_n \quad for\ m = 1, 2, \ldots, M$$

$$\frac{dR_n}{dt} = -\sum_{m=1}^{M} b_{m,n} B_m \quad for\ n = 1, 2, \ldots, N$$

These equations are identical to the Lanchester Square Law except that the cumulative effects of the constituent parts of the heterogeneous forces are accounted for using the attrition coefficients within the pairwise relationships between the opposing forces. Bonder also developed quantitative processes for predicting specific attrition coefficients based on weapons systems efficacy against live and dead targets as well as the allocation procedure in assigning weapons to targets; to a lesser extent, Bonder also examined the effects of varied terrain. Bonder admitted that many of the parametric inputs for determining specific attrition coefficients could not be effectively predicted. The difficulty in establishing accurate predictions for the Bonder parameters affirms a potential strength in the simplicity of the Lanchester equations which might provide satisfactory approximations while describing mean results. (Bonder & Farrell, 1970)

Out of a keen interest in Biomathematics, Lotka and Volterra independently developed their ideas on predator-prey interactions and interspecific competition during the 1920s. While their works have stood as a cornerstone of theoretical ecology, the ideas therein were also contextualized to the domain of warfare. (Kingsland, 2015) The Competitive Lotka-Volterra Equations, modeling interspecific competition, has been modeled as

$$\frac{dN_1}{dt} = r_1 N_1 \left(\frac{K_1 - N_1 - \alpha N_2}{K_1}\right)$$

$$\frac{dN_2}{dt} = r_2 N_2 \left(\frac{K_2 - N_2 - \beta N_1}{K_2}\right)$$

where

$r \sim respective\ per\ capita\ growth\ rate$

$N \sim respective\ population\ size$

$K \sim respective\ carrying\ capacity$

$\alpha \sim the\ per\ capita\ effect\ of\ entity\ 2\ on\ carrying\ capacity\ of\ entity\ 1$

$\beta \sim the\ per\ capita\ effect\ of\ entity\ 1\ on\ the\ carrying\ capacity\ of\ entity\ 2$

These equations assume a mutually deleterious relationship between two entities and are generally used for modeling competition between two biological species.

If warfighting entities encompass certain characteristics of interspecific competition such as a carrying capacity, logistic growth, and competition over common resources, the Lanchester-Lotka-Volterra Hybrid Model may be presented as

$$\frac{dB}{dt} = B \left(\frac{K_B - B - rR}{K_B}\right)$$

$$\frac{dR}{dt} = R \left(\frac{K_R - R - bB}{K_R}\right)$$

where

K is still carrying (fighting) capacity of both entities

α and β have been replaced with attrition rates of both entities, r and b, and

R and B are force size.

In the transition from the Lanchester Equations to the Hybrid Equations, the goal of conflict shifts from direct attrition to the establishment of a dominant isocline. Fundamentally, there exist four isocline scenarios which determine the fate of the conflict. In the case of the dominant blue isocline, the following inequalities hold:

$$K_B > \frac{K_R}{b}$$

$$\frac{K_B}{r} > K_R$$

The blue force will eliminate the red force and grow to its natural carrying capacity. Likewise, dominant red isocline occurs when:

$$K_R > \frac{K_B}{r}$$

$$\frac{K_R}{b} > K_B$$

The red force will eliminate the blue force and grow to its natural carrying capacity. Interestingly, the isoclines present divergent bifurcation when

$$K_B > \frac{K_R}{b}$$

$$K_R > \frac{K_B}{r}$$

In this case, the outcome of the conflict hinges on the parameter values of the respective carrying capacities and attrition coefficients as well as the initial size of each belligerent force. Lastly, the isoclines present convergent behavior when

$$\frac{K_B}{r} > K_R$$

$$\frac{K_R}{b} > K_B$$

and the size of each force will converge to a steady equilibrium. (Olson, 2014)

Brackney published his work “The Dynamics of Military Combat” in 1959 against the backdrop of the Cold War. His article sought mathematical grounding for established combat principles and proposed a model which decomposed attrition coefficients into discoverable physical phenomena. (Brackney, 1959) The Brackney Equations may be presented as

$$\frac{dB}{dt} = -\frac{RBS_R}{T_B BS_R + V_B}$$

$$\frac{dR}{dt} = -\frac{BRS_B}{T_R RS_B + V_R}$$

such that

$S_B \sim blue\ search\ rate, search\ volume\ per\ unit\ of\ time$

$V_B \sim amount\ of\ volume\ the\ blue\ force\ may\ occupy$

$T_B \sim amount\ of\ time\ required\ for\ red\ unit\ to\ destroy\ blue\ unit$

The Brackney Equations do not present tractable expressions for fighting effectiveness. According to Washburn and Kress, (Washburn & Kress, 2009) these equations act like the Lanchester Linear Law when the *B* and *R* force sizes are large. In this case, attrition is limited by the time for destruction, *T*. Alternatively, these equations act like the Lanchester Square Law when *B* and *R* are small; attrition is instead limited by the time for search. The Brackney Equations still assume a random uniform distribution of each force throughout its respective volume. A distinct advantage of the Brackney approach is that the parameter values may be determined objectively and quantitatively from concrete, discoverable phenomena. In observing the operational envelope, time-to-kill, and search rate of a single blue and red unit, the parameters related to both forces may be discovered. Therefore, policymakers could use the Brackney Equations in a forward-looking manner to make military procurement decisions.

3.4 Methodology

This section discusses a method for strategic procurement analysis using the Brackney Equations and establishes pertinent parameter values. Python 3.7 implemented in a Spyder integrated development environment (IDE) provides the computational backbone for this analysis.

The distance from the center of the Moon to the Moon's first Lagrange point L1 is approximately 61,350 km (Maccone, 2002) and the distance to the gravitational equilibrium point (EP) between the Earth and the Moon is approximately 47,934 km. As such, a conservative estimate for modeling the envelope of conflict might be approximated as extending from the lunar surface to a semimajor axis considerably lower than the EP distance. The volume of conflict is calculated as

$$V = \left(\frac{4}{3}\right)\pi(a^3 - r_{Moon}^3)$$

such that (Pisacane, 2016)

$a \sim chosen\ semimajor\ axis$

$r_{Moon} \sim volumetric\ mean\ radius\ of\ Moon = 1737.1\ km$

This analysis assumes a finite number of opposing spacecraft occupy the same volumetric space distributed uniformly. This analysis assumes the mutual use of directed energy weapons (DEW) which do not generate additional debris. The sensors on each spacecraft are idealized as omnidirectional enabling an awareness sphere. The search rate is approximated as the amount of volume within the sphere per second. The search volume per second is presented as

$$S = \left(\frac{4}{3}\right)\pi r_{Sensor}^{3}$$

such that

$$r_{Sensor} \sim maximum\ range\ of\ the\ sensor$$

This research is meant to afford decision makers a forward-looking mechanism to improve strategic posture. Many of the technologies associated with this scenario exist at a low technology readiness level (TRL), and/or are not yet deployed en masse. As such, any parameter values concerning number of spacecraft, time-to-kill, or sensor range are purely notional and only for demonstration purposes. The value of this method exists in integrating the strategic models for use in space.

3.5 Analysis

This analysis provides an example of a simple strategic procurement decision and models a conflict which lasts one week (604,800 seconds (s)). In a theoretical contest between two belligerent forces with technical and force parity such that

$a = 5000\ km$, semimajor axis

$r_{Blue\ Sensor} = r_{Red\ Sensor} = 20\ km$, max effective sensor range

$$T_B = T_R = 100\ s\text{, destruction time}$$

$$B = R = 100\text{, force size}$$

each side will fight to a draw. Figure 3.3 shows the results of this baseline conflict.

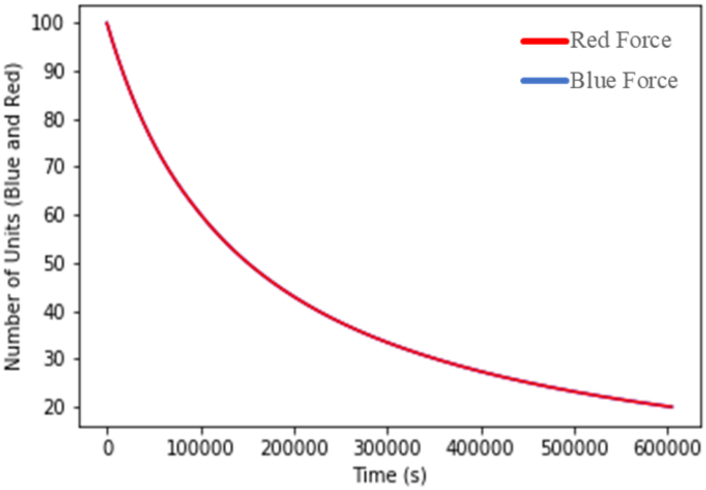


Figure 3.3. Baseline Conflict Results

However, prior to the conflict, the blue force is afforded the opportunity to upgrade one aspect of its space force by 50%. The options are:

a) increase $r_{Blue\ Sensor} = 30\ km$,

b) increase destruction time $T_B = 150\ s$, a proxy for improving survivability, or

c) increase constellation size $B = 150$.

Using the Brackney Model, decision makers determine that the $T_B = 150\ s$ upgrade makes a negligible difference. The $B = 150$ upgrade significantly improves the outcome of the conflict, shown in Figure 3.4.

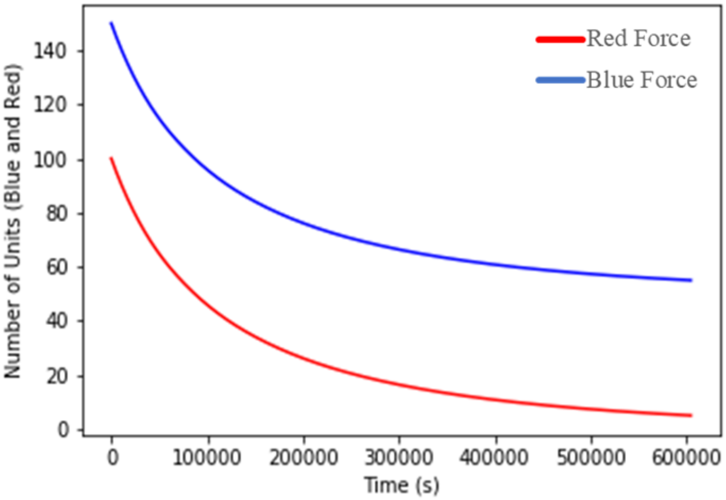


Figure 3.4. Conflict Results After B = 150 Upgrade

The $r_{Blue\ Sensor} = 30\ km$ upgrade provides an overwhelming advantage to the blue force. Figure 3.5 shows the results of this upgrade.

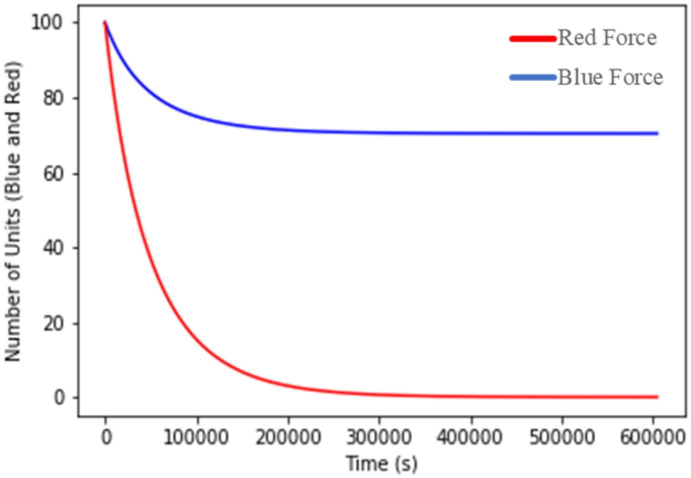
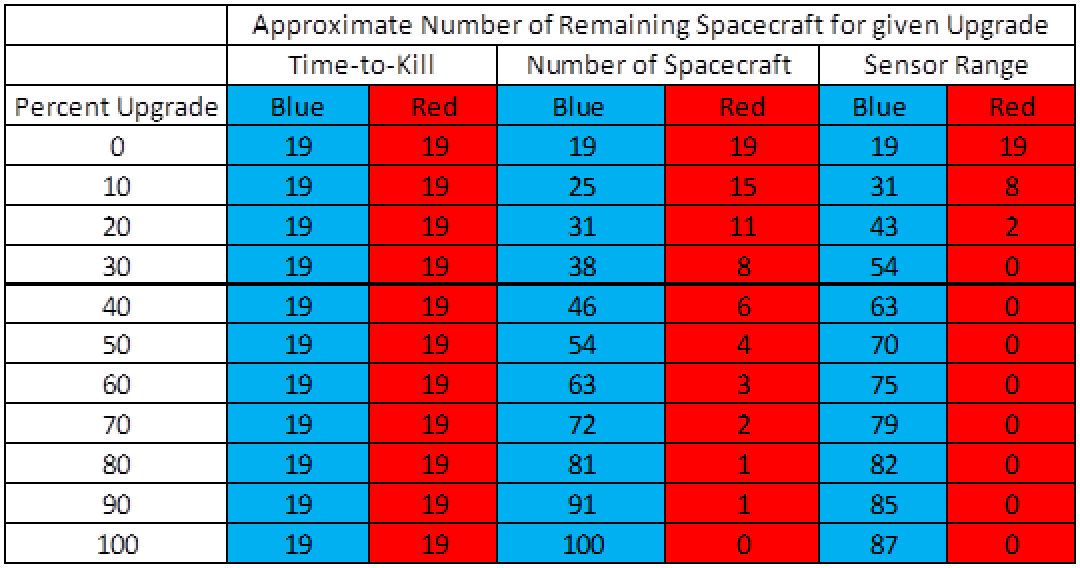


Figure 3.5. Conflict Results After r (Blue Sensor) = 30 km Upgrade

Equipped with this knowledge, blue decision makers choose the sensor upgrade to achieve the greatest improvement in strategic posture. Table 3.1 shows the sensitivity analysis for the various upgrades.

Table 3.1. Upgrade Results Sensitivity Analysis



	Approximate Number of Remaining Spacecraft for given Upgrade					
	Time-to-Kill		Number of Spacecraft		Sensor Range	
Percent Upgrade	Blue	Red	Blue	Red	Blue	Red
0	19	19	19	19	19	19
10	19	19	25	15	31	8
20	19	19	31	11	43	2
30	19	19	38	8	54	0
40	19	19	46	6	63	0
50	19	19	54	4	70	0
60	19	19	63	3	75	0
70	19	19	72	2	79	0
80	19	19	81	1	82	0
90	19	19	91	1	85	0
100	19	19	100	0	87	0

The analysis of this theoretical contest placed a premium on space domain awareness (SDA) and afforded little value to spacecraft survivability because of the vastness of the volume of space in relation to the sensor capabilities and number of spacecraft. Attrition was search-limited from the onset of the conflict. Importantly, different parameter values will yield different results. As more units crowd the battlespace, the sensor capability significance diminishes while survivability grows more important. However, for this volume of conflict, four orders of magnitude in force size would be required for time-to-kill to overtake sensor range in importance.

The Brackney Model sometimes produces interesting results wherein the numerical advantage shifts back and forth between the two forces. In a hypothetical contest using $B = 1200$, $R = 1000$, $r_{Blue\ Sensor} = 30\ km$, $r_{Red\ Sensor} = 20\ km$, $T_B = 1000\ s$, and $T_R = 27500\ s$, blue will begin in the lead, then lag behind numerically, and eventually win the conflict. Reference Figure 3.6 to see the results of this conflict.

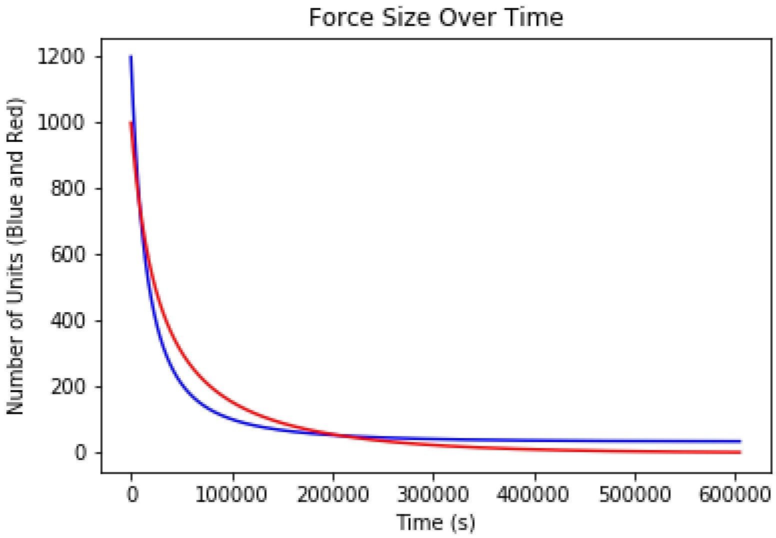


Figure 3.6. Shifting from Time-to-Kill Dominated to Search Dominated Conflict

Historical analogs for this type of conflict exist. A numerically inferior but higher quality force invades a country and defeats massed conventional forces. The defeated force transitions the conflict into an insurgency phase. Though the insurgent force is outnumbered by superior forces, the insurgent force search time is better than the search time of the occupying force. The insurgent force eventually defeats or expels the occupying force. The Brackney Equations are a useful tool for strategic thinking and can be leveraged when making procurement decisions for the USSF.

3.6 Conclusion

We asserted that decision makers could use aggregated modeling to inform procurement strategy in the development of the USSF. In support of this thesis, a potential future conflict environment was suggested, certain aggregated combat models were described, and the use of Brackney Equations was demonstrated. In doing so, we endeavor for the advancement of strategic thinking and modeling to enhance US space power.

Future work will include the integration of game theoretic models to inform the parameters of the aggregated combat models. Aggregated simulation results could be validated by high-fidelity models. Aggregated simulation results may also serve as a performance baseline when evaluating tactical outcomes in high-fidelity environments. This methodology could be integrated into the procurement process of the appropriate System Program Office (SPO) for the enhancement of US space power.

Chapter 4: Survival Analysis for Nanosatellites and Picosatellites

4.1 Abstract

The nascent field of fractionated satellite architectures provides an opportunity to improve spacecraft modularity and afford greater flexibility, adaptability, and upgradeability to spacecraft constellations. Satellite modules within a coherent formation can be replaced without facing the challenges of manufacturing, assembly, or disassembly in the harsh space environment (e.g., satellite modules conducting electromagnetic formation flight (EMFF) are not physically connected such that one module may be replaced with potentially less risk of damaging or degrading the performance of the other modules). Conventionally, the depot for constellation replenishment is located on Earth, however, minor augmentations to spacecraft formations cannot be conducted economically under such a framework. The present research proposes the utilization of proactively launched supply depots to replenish geostationary formations from ultrageostationary orbit (i.e., that volume of space encompassed between the altitude of geostationary orbit and the altitude of the L1 Lagrange point). This work explores reliability factors associated with such a concept by conducting a survival analysis for nanosatellites and picosatellites. Time to failure data is collected for 85 spacecraft in the nano- (1.01 – 10 kg wet mass) and pico- (0.11 – 1 kg wet mass) classes without data censoring. These spacecraft were launched between 2010 and 2019, inclusive, having an internationally diverse set of owners from the sectors of military, government, commercial, and academia. This data is used to build a distribution for the survival analysis of satellites in these classes. JMP Pro 13 is used to conduct a

goodness-of-fit test for multiple distributions. Analysis (using a standard alpha value of 0.05) indicates that the data is from a two-parameter Weibull distribution wherein the spacecraft experience beneficial aging.

4.2 Introduction

The U.S. Air Force 2030 Science and Technology Strategy characterizes five transformational strategic capabilities as integral to the airpower and spacepower of the U.S. including (U.S. Air Force, 2019):

- global persistent awareness
- resilient information sharing
- rapid, effective decision-making
- complexity, unpredictability, and mass
- speed and reach of disruption and lethality

The strategy explicitly ties “global persistent awareness” to the technological opportunity of “small satellites and low-cost launch.” The strategy also explicitly ties “complexity, unpredictability, and mass” to the technological opportunity of “low-cost air and space platforms.” These national security technological opportunities provided an impetus for the development of the Kinetically-Aggregated Infrastructure Revitalization of Spacecraft (KAIROS) concept. KAIROS exists as the replenishment or enhancement of a fractionated spacecraft by a supply depot also located in space. (Hayhurst, Bettinger, & Grandhi, 2021) This current work focuses on the reliability aspects of KAIROS.

In understanding the KAIROS concept, one may consider a simplified use case wherein several spheres flying in formation along the geostationary belt constitute the functional capability of a communications satellite. Approximately homogeneous in mass, these spheres present inertia tensors with no cross-coupling and equal angular inertia values for each axis. A control moment gyroscope mounted internally on each axis provides satellite attitude control and rings embedded along the outer shell of each sphere surge current to create an electromagnetic field in order to generate the force necessary to conduct intra-formation position maneuvers. The spheres can aggregate and use thrusters to perform conventional orbital maneuvers. Power can be distributed wirelessly and computing power can be disaggregated to the different spheres. Supply depots located at higher altitudes in ultrageostationary orbit can send individual spheres to designated formations for the replenishment or enhancement of a particular constellation.

Exploring the reliability factors associated with KAIROS enables an understanding of the failure times for future operational systems. Such knowledge improves the Planning Programming Budgeting and Execution (PPBE) process and affords a more accurate Program Objective Memorandum (POM) for the Future Year Defense Program (FYDP). The subsequent improvements to acquisitions performance in terms of cost, risk, schedule, and system capability ultimately promote the security and prosperity of the U.S..

This article seeks to advance the national security posture of the U.S. through the presentation of research on the reliability factors of an advanced technology conceptual framework. Motivation for the research is contextualized to the acquisitions processes within the U.S. Air Force (USAF) and U.S. Space Force (USSF). Descriptive statistics and data collection of the reliability of 85 satellites is discussed. Finally, analysis and distribution building for the time to failure of these spacecraft is conducted.

4.3 Literature Review

Kong et al. proposed the use of electromagnetic formation flight (EMFF) as a propellant-free alternative to satellite formation flight. (Kong, et al., 2004) Hilton, (Hilton, 2015) Alvisio, (Alvisio, 2015) and many others of the Massachusetts Institute of Technology (MIT) Space Systems Laboratory (SSL) advanced EMFF technology with their work on the Synchronized Position Hold Engage and Reorient Experimental Satellites Resonant Inductive Near-field Generation System (SPHERES-RINGS). The reconstitution of an operational version of such a fractionated spacecraft by a supply depot in space provides an excellent example of the KAIROS concept.

Saleh discussed the application of the Weibull distribution (a more generalized form of the exponential distribution) to spacecraft reliability. (Saleh & Castet, 2010) The U.S. Air Force discussed the potential benefit of using disaggregation to improve the resiliency of spacecraft architectures. (Air Force Space Command, 2016) Cristini, (Cristini, 2010) Mathieu, (Mathieu & Weigel, 2005) Daniels, (Daniels & Pate-Cornell, 2015) and Brown (Brown & Eremenko, 2008) also discussed the benefits of fractionated satellite architectures.

4.4 Analysis

Convenience sampling was used to collect time to failure data for 85 spacecraft in the nano- (1.01 – 10 kg wet mass) and pico- (0.11 – 1 kg wet mass) classes with no censoring. Consistent with an ultraquality framework, reliability was considered only at the system level. (Maier & Eberhardt, 2009) These spacecraft were launched between 2010 and 2019, inclusive, having an internationally diverse set of owners from the sectors of military, government, commercial, and academia. These 85 spacecraft had a mean survival time of 0.513 years (median survival time of 0.186 years) with a standard deviation of 0.961 years and range of 0.003 years to 7.351 years. The failure times for the satellites are plotted in Figure 4.1. These failure times were used to build a distribution for the survival analysis of satellites in these classes.

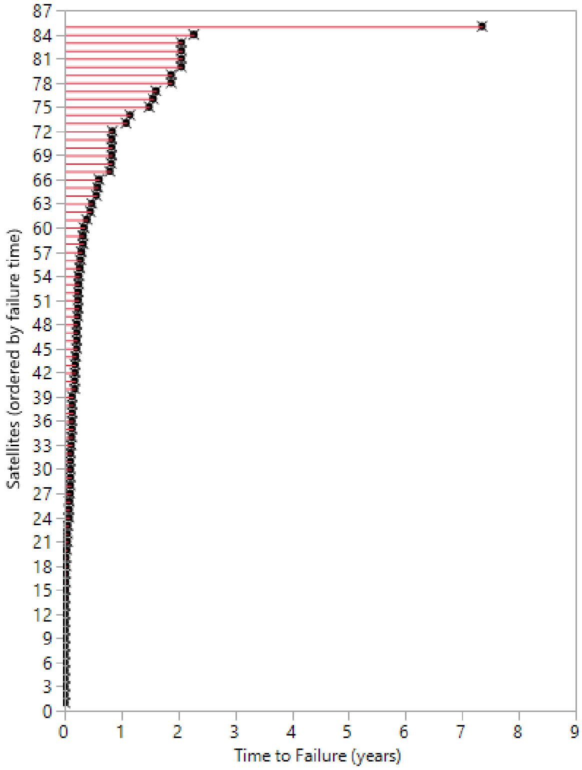


Figure 4.1. Satellite Failure Times

The time to failure data in Figure 4.1 was used to find a probability distribution that could be used to model spacecraft survivability (time until system failure). Spacecraft reliability is sometimes modeled with the exponential distribution, however, the exponential distribution is known to have a memoryless property, which, in this application, would imply that failure at any given time is not dependent on how long the spacecraft has survived already. This is a property in contrast to the beneficial aging that is theorized for this set of satellites. Therefore, two different reliability distributions were considered to model the time until failure: the exponential distribution due to its common application and potential usefulness given the shape of the distribution in Figure 1 and the Weibull distribution which is related to the exponential distribution through a transform of the exponentially distributed random variable yet does not maintain the memoryless property of the exponential distribution (and thus, may better fit the concept of beneficial aging). Specifically, let the time to failure be denoted as random variable X. Then, the exponential distribution for X is expressed as:

$$f(x) = \frac{e^{-\frac{x}{\beta}}}{\beta}$$

with the support of x ranging from zero to infinity. The Weibull distribution is related to the exponential distribution through the random variable transformation:

$$X_{Weibull} = X_{Exponential}^{\frac{1}{\gamma}}$$

to yield a Weibull-distributed random variable Z whose probability density is defined as:

$$f(z) = \frac{e^{-\frac{z^{\gamma}}{\beta}}}{\beta}\gamma z^{\gamma-1}$$

and whose support ranges from zero to infinity. β is the scale parameter (characteristic life span) while γ is the shape parameter.

The Weibull distribution accounts for beneficial or deleterious aging, through its additional parameter, γ, in which $\gamma < 1$ indicates beneficial aging and $\gamma > 1$ indicates deleterious aging. To determine the best distribution for this satellite data, JMP Pro 13 was used to conduct goodness-of-fit testing for both the exponential and Weibull distributions. The Cramer-von Mises W goodness-of-fit test and the Kolmogorov's D goodness-of-fit test were used to formally determine whether or not the Weibull and exponential distributions fit the data, respectively. The Akaike Information Criterion (AIC) goodness-of-fit for the likelihoods of both the exponential and Weibull distribution were compared. Formal statistical testing was conducted using a standard alpha value of 0.05.

Figure 4.2 shows the cumulative distribution function for spacecraft failure with the aforementioned fitted exponential and includes a 95% confidence interval. Ideally, if the data was exponentially distributed, it would follow along the solid line and lie within the 95% confidence bounds. The time to failure data does not follow the expected probability well in Figure 4.2 and via formal testing, failed the Kolmogorov's D goodness-of-fit test, indicating that the data was not from an exponential distribution. Specifically, this test yielded a Kolmogorov's D of 0.250270 and a p-value of 0.01. The AIC value for the best fitting exponential distribution was 58.514460.

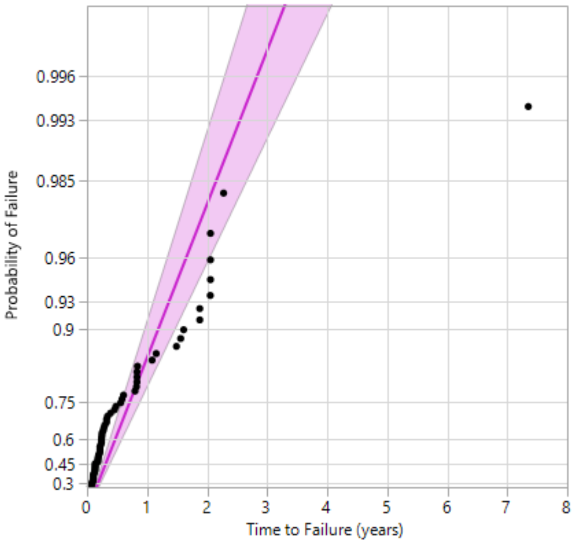


Figure 4.2. Probability of Failure vs. Time to Failure with Fitted Exponential Distribution

The Cramer-von Mises W goodness-of-fit test for a fitted Weibull yielded a Cramer-von Mises W of 0.103840 and a p-value of 0.0907 indicating that the Weibull distribution may be an adequate fit for the data. Fitting the two-parameter Weibull yielded parameter estimates of $\beta = 0.3306607$ and $\gamma = 0.5922925$ which indicates beneficial aging – the expected result in spacecraft reliability. The 95% confidence intervals for these parameter estimates are as follows:

$$0.2240266 \leq \beta \leq 0.4820571$$

$$0.4987221 \leq \gamma \leq 0.693607$$

The AIC value for this best fitting Weibull was 8.831280, indicating a better fit for the Weibull distribution than the exponential distribution (lower AIC value is better).

Figure 4.3 shows the cumulative distribution function for spacecraft failure with the aforementioned fitted Weibull. Figure 3 also encompasses a 95% confidence interval for the Weibull distribution. In general, the data better fits the Weibull distribution as shown in Figure 4.3.

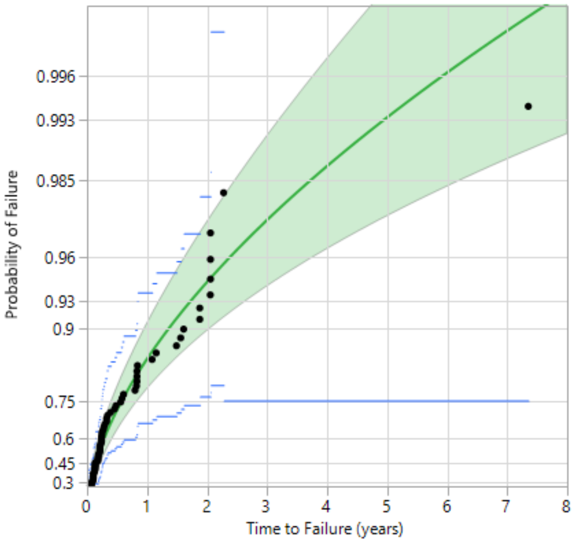


Figure 4.3. Probability of Failure vs. Time to Failure with Fitted Weibull Distribution

4.5 Conclusion

This article created a parametric distribution for a data set encompassing nano- and pico- class satellites to characterize the survival analysis of satellites in these classes. The analysis determined a Weibull distribution parameterized to represent beneficial aging constituted a representation of the data which was both accurate and tractable. Understanding the reliability characteristics of satellites in these classes affords the U.S. Department of Defense the opportunity to increase the efficacy of its acquisition programs. Ultimately, this work strives for the enhancement of the security and prosperity of the U.S. through the advancement of strategic thinking within the space domain.

Future work will integrate this knowledge into a framework which will help guide the acquisition and operational decisions of the USSF. This future framework will integrate parametric distributions such as those discussed in this article with game theoretic models as well as population models including Lanchester, Lotka-Volterra, and Brackney.

Chapter 5: A Game-Theoretic Evaluation of Media Interaction Warfare Theory

5.1 Abstract

Understanding the efficacy of the U.S. military in the domains of land, sea, air, and space provides valuable geopolitical insights which can help guide the policies and actions of the United States. There exists an importance in knowing the effectiveness of the individual elements of the land, sea, air, and space forces as well as the effectiveness of the integrated whole. In conducting such an evaluation, parametric models may afford timely, effective methodologies. Media Interaction Warfare Theory presents a method germane to the field of parametric modeling and asserts the ability to enhance the space warfare posture of the United States. This article investigates the validity of Media Interaction Warfare Theory using the game-theoretic concepts encompassed within the Atomic Competitive Element taxonomy. This work finds a general alignment between the results of Media Interaction Warfare Theory modeling and game-theoretic modeling indicating that Media Interaction Warfare Theory may be a valid tool for determining the efficacy of a fighting force. This work provides some evidence that Media Interaction Warfare Theory and game-theoretic methods validate each other as effective conflict-modeling methods; this work proposes the use of these parametric modeling methodologies in the Department of Defense acquisitions processes.

5.2 Introduction

This article seeks to advance strategic thought on spacepower and enhance the military posture of the U.S. space endeavor. To achieve this goal, this paper will examine the use of parametric modeling in predicting the outcome of multi-domain warfare, especially in the context of an emerging space domain. Specifically, this work will conduct a game theoretic evaluation of Media Interaction Warfare Theory (MIWT) with significant focus on the penultimate (application of matrix theory) and ultimate (calculation of determinants) steps of the MIWT methodology. First, this work presents a literature review which discusses MIWT, game theory, and other historical frameworks for parametric modeling. Second, the methodology of evaluation is expounded using a notional example. Finally, an analysis is conducted on empirical and theoretical historical examples; within this analysis MIWT is evaluated against a game-theoretic framework to discover the presence or absence of a general alignment between the results of MIWT and game-theoretic modeling. Ultimately, this work strives to enrich the sphere of Department of Defense modeling and in doing so promote the peace and prosperity of the United States.

5.3 Literature Review

Scardera and Cesul developed "Media Interaction Warfare Theory: A Novel Analytic Process Supporting Space Warfare Planning Operations" in order to advance and mature the thought and theory associated with space warfare. In their own words:

> With the debate settled over whether space is a war-fighting domain and whether an independent space force should be established, the discussion now shifts toward providing analytic frameworks to answer more strategic questions about space warfare in general.… We describe a novel approach called the "media interaction theory of warfare," which provides a unique and simple way to evaluate

> different integrated force structures, offering a true joint forces perspective to begin addressing these questions while providing a basis for more analytic treatment…. We construct a simple model containing interactions between different domain media. This construct leads to a media interaction matrix mathematical model based on linear algebra. This unique model development separates the analysis from previous work in the area. Based on an order of battle, an integrated force structure matrix can be built, and a determinant taken to provide a single value for the force structure's relative strength. This relative strength may, in turn, be compared to other very diverse force structures to find the dominating integrated armed force. (Scardera & Cesul, 2021)

The nature of the various warfare domains as qualitatively unique spheres of influence is integral to Scardera and Cesul's framework. Scardera and Cesul sought to articulate the qualities of the domains in and of themselves as well as their effects on other dissimilar spheres. In their research, Scardera and Cesul relied on many of the luminaries of military strategic studies throughout history. Principles of land warfare were drawn from Sun Tzu, Carl Von Clausewitz, Antoine-Henri Jomini, and B. H. Liddell Hart. Lessons in naval warfare were taken from Philip H. Colomb, Alfred Thayer Mahan, and Julian S. Corbett. Principles of air warfare were drawn from Hugh M. Trenchard, Giulio Douhet, and William L. Mitchell. The common themes which Scardera and Cesul pulled from these theorists and used to build MIWT include, "interactions within the media dominate … each new medium has a dominating influence over the other media … interactions between media are important … new media greater mobility gives an initiative advantage … offense has a proactive aspect, while defense is retroactive … [and] a geometric or mathematical construct is possible.” (Scardera & Cesul, 2021)

In producing a mathematical construct for parametric modeling according to MIWT, each interaction between each domain must be captured in a matrix for a particular force. Therefore, the size of a given matrix has an n-squared relationship to the number of domains. A matrix which encompasses only land forces presents a one-by-one dimensionality whereas a matrix considering all four domains presents a four-by-four dimensionality with sixteen interactions. An MIWT four-by-four matrix encompasses sixteen scalar values which together characterize the efficacy of a single belligerent in conflict with an opposing belligerent. According to MIWT, the determinant of a given matrix may be calculated to determine the effectiveness of the respective force. The scoring mechanism used to assign a particular number to a cell within a matrix uses scores between zero and one for supremacy conflict cells and offensive cells (i.e., those cells on the diagonal or above the diagonal of the matrix) and uses scores between negative one and zero for defensive cells (i.e., those cells below the diagonal of the matrix). Within MIWT, space forces are considered the most mobile (and therefore most offensive in nature) forces with air, sea, and land forces presenting progressively less mobility – this assumption characterizes the nature of conflict between domains within MIWT. The maximum potential efficacy of a fighting force grows by binary orders of magnitude for each new warfighting domain capability which is added to the matrix. Table 5.1 is adapted from the work of Scardera and Cesul and presented to provide clarity on the development of matrices within MIWT. (Scardera & Cesul, 2021) In application, Table 5.1 would be filled in with sixteen scalar values to show the effectiveness of a single belligerent across sixteen interactions.

Table 5.1. Understanding MIWT

Land to land supremacy efficacy score ranging from zero to one	Sea to land offensive efficacy score ranging from zero to one	Air to land offensive efficacy score ranging from zero to one	Space to land offensive efficacy score ranging from zero to one
Land to sea defensive efficacy score ranging from negative one to zero	Sea to sea supremacy efficacy score ranging from zero to one	Air to sea offensive efficacy score ranging from zero to one	Space to sea offensive efficacy score ranging from zero to one
Land to air defensive efficacy score ranging from negative one to zero	Sea to air defensive efficacy score ranging from negative one to zero	Air to air supremacy efficacy score ranging from zero to one	Space to air offensive efficacy score ranging from zero to one
Land to space defensive efficacy score ranging from negative one to zero	Sea to space defensive efficacy score ranging from negative one to zero	Air to space defensive efficacy score ranging from negative one to zero	Space to space supremacy efficacy score ranging from zero to one

Hayhurst and Colombi developed a taxonomy for categorizing game-theoretic scenarios according to attributes desirable for the stability of an outcome within a game. (Hayhurst & Colombi, Game-Theoretic System Design in the Development of Space Power, 2021) Known as the Atomic Competitive Element (ACE) taxonomy, this construct considers agent utility balance, Nash equilibrium, and Pareto optimality in the illustration and classification of games. Agent utility balance means that the payoff for each player for a given outcome is the same or approximately the same within some margin (epsilon). A Nash equilibrium exists when no agent within a game has the incentive to unilaterally change his or her strategy. A Nash equilibrium may exist in the form of a pure strategy Nash equilibrium (PSNE), a mixed strategy Nash equilibrium (MSNE), or a partially mixed strategy Nash equilibrium (PMSNE). Importantly, a game is not limited to holding one equilibrium and may hold many equilibria of different types. While there exist many types of games, Hayhurst and Colombi considered strategic-normal form games in the development of the ACE taxonomy. In a two-player strategic-normal game-theoretic context, player one chooses from strategies enumerated by the rows of a matrix while player two chooses from strategies enumerated by the columns of a matrix; the intersection of the respective row and column is a cell which displays a payoff for player one and player two. The Pareto front is formed by all non-dominated outcomes of a set according to a specified number of attributes. That is, for any solution on the Pareto front, the user cannot shift to a different solution within the set to improve the score of an attribute without diminishing the score of a separate attribute. Within a two-player game-theoretic context, each agent's payoff exists as an attribute within a set that forms a two-dimensional Pareto front. In the designation of attributes for a particular cell of a matrix within the ACE taxonomy, Hayhurst and Colombi used red to represent agent utility balance, yellow to represent Nash equilibria, and blue to represent the Pareto front. Secondary colors are used to represent the combination of these attributes, white is used to represent the presence of all three attributes, and gray is used to represent the absence of all three attributes. Figure 5.1 is adapted from the work of Hayhurst and

Colombi and presented to provide clarity on the ACE taxonomy. (Hayhurst & Colombi, Game-Theoretic System Design in the Development of Space Power, 2021)

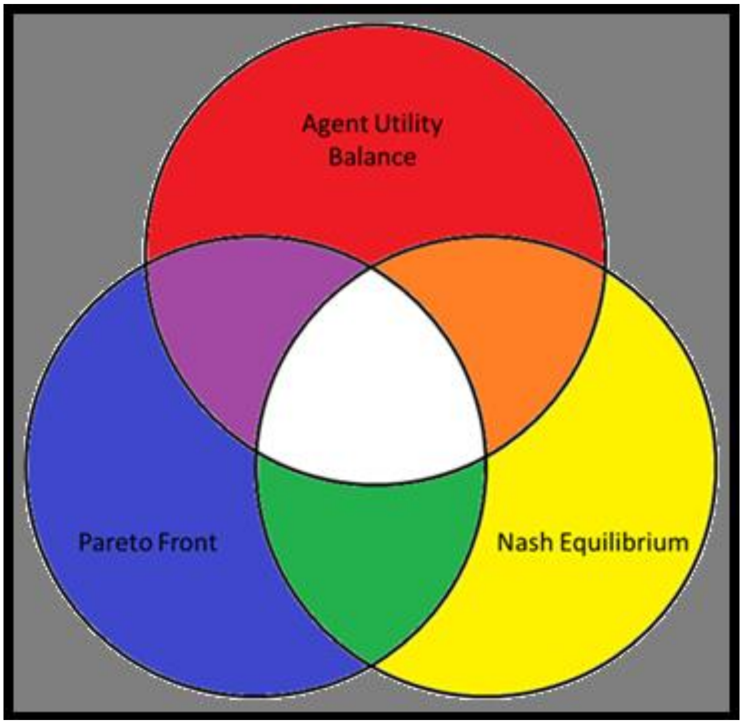


Figure 5.1. Atomic Competitive Element Taxonomy

Parametric combat modeling came of age against the backdrop of World War I when Lanchester developed his square and linear laws of combat. The Lanchester Square Law and Lanchester Linear Law both use a system of differential equations to mathematically model the interaction of two forces engaged in combat; the math shows the induced attrition of each force over time. Encompassed within these equations are the numbers of a particular unit for each belligerent as well as the attrition coefficient (that is, fighting efficacy) of each respective force. (Lanchester, 1916) The Lotka-Volterra Equations follow the general form of the Lanchester Equations in that the Lotka-Volterra Equations present a system of differential equations. Generally considered in a biological context, the Lotka-Volterra Equations are an innovation on the logistic growth model. Whereas logistic growth is a solid representation of intraspecific competition, the Lotka-Volterra innovation enabled the modeling of interspecific competition. (Kingsland, 2015) Brackney, following the same general mathematical form, was able to ground the performance characteristics of a fighting force in tangible, discoverable phenomena. Whereas Lanchester's attrition coefficients were abstract concepts which required empirical data pertinent to the model or educated speculation, the Brackney Equations contain concrete parameters related to the search and destroy endeavors of two belligerents. (Brackney, 1959) A limiting aspect in the usefulness of these models is their assumption of homogeneity with respect to the fighting units within the fighting force of each belligerent. Bonder and Farrell attempted to overcome this limitation with a generalization of the Lanchester Equations. While Bonder and Farrell's contribution to this field is certainly meaningful, their attempt (by their own admission) was not a complete success and encompassed significant methodological gaps. (Bonder & Farrell, 1970)

5.4 Methodology

MIWT and game theory both provide methodologies capable of distilling complex force structures into tractable parametric, quantitative values. The parametric values provided by MIWT and game theory may not be appropriate for use in high-fidelity physics-based simulations where mathematical precision is at a premium (e.g., Systems Tool Kit); similarly, use of these parametric values in agent-based modeling applications (e.g., Advanced Framework for Simulation, Integration, and Modeling) may not be appropriate. However, parametric modeling and the quantitative values encompassed therein may provide useful strategic insight for decision-makers responsible for creating or modifying a force structure. Parametric modeling embodies a complimentary role to high-fidelity physics-based methods as well as agent-based methods. Parametric modeling can be executed relatively quickly and can provide a first-cut approach to the problem which will help guide the problem-solver and save time in the long run. Parametric modeling can provide the intellectual basis for puzzle solving conflict scenarios and may be used to communicate vast, complicated sets of information to a broader audience which may include decision-makers. In considering a parametric framework, the user would be wise to communicate results as approximations instead of mathematically precise answers. Furthermore, the user should consider a stochastic range of outcomes as opposed to a single deterministic answer. Having contextualized the advantages and limitations of parametric conflict modeling, this section will demonstrate the methodological approach of this work – the game-theoretic evaluation of MIWT will be demonstrated using a notional example. In the game-theoretic evaluation of MIWT, the emphasis will be on the discovery of the general alignment or lack thereof of the results of the MIWT methodology and game-theoretic methodology.

The methodology of this evaluation will be demonstrated using the notional MIWT force matrices shown in Table 5.2 and Table 5.3.

Table 5.2. Notional MIWT Blue Force Matrix

0.50	0.00	0.80
-1.00	0.60	0.05
-0.10	-0.90	0.55

Table 5.3. Notional MIWT Red Force Matrix

0.40	0.20	0.85
-0.80	0.30	0.10
-0.25	-0.90	0.50

The determinants of the blue and red force matrices are 0.96 and 0.85, respectively. Therefore, according to MIWT the fighting efficacies (attrition coefficients) of the blue and red forces may be quantified as 0.96 and 0.85, respectively. Dividing the blue attrition coefficient by the red attrition coefficient yields a determinant power ratio (PR) of 1.13 indicating the blue force fields the more effective fighting capability according to MIWT.

To begin the transition of these two matrices to a game-theoretic strategic-normal form game, evaluate the element-wise absolute values of the matrices as shown in Table 5.4 and Table 5.5.

Table 5.4. Notional MIWT Blue Force Matrix with Positive Values

0.50	0.00	0.80
1.00	0.60	0.05
0.10	0.90	0.55

Table 5.5. Notional MIWT Red Force Matrix with Positive Values

0.40	0.20	0.85
0.80	0.30	0.10
0.25	0.90	0.50

Next, transpose the blue force matrix as shown in Table 5.6 to achieve the correct alignment between the offensive and defensive capabilities of the two belligerents within the respective domains.

Table 5.6. Transposed Blue Force Matrix

0.50	1.00	0.10
0.00	0.60	0.90
0.80	0.05	0.55

Next, collate the blue force matrix and red force matrix into a strategic-normal form game as shown in Table 5.7 where p_{B_m} and p_{R_n} represent the strategies which are played with some proportion within the mixed strategy of the game.

Table 5.7. Notional Scenario

Notional Scenario		Red Player		
		p_{R_1}	p_{R_2}	p_{R_3}
Blue Player	p_{B_1}	0.50, 0.40	1.00, 0.20	0.10, 0.85
	p_{B_2}	0.00, 0.80	0.60, 0.30	0.90, 0.10
	p_{B_3}	0.80, 0.25	0.05, 0.90	0.55, 0.50

Apply the ACE taxonomy using 0.10 for agent utility balance as shown in Table 5.8.

Table 5.8. Notional Scenario with ACE Taxonomy

Notional Scenario	Red Player

		p_{R_1}	p_{R_2}	p_{R_3}
Blue Player	p_{B_1}	0.50, 0.40	1.00, 0.20	0.10, 0.85
	p_{B_2}	0.00, 0.80	0.60, 0.30	0.90, 0.10
	p_{B_3}	0.80, 0.25	0.05, 0.90	0.55, 0.50

This game encompasses no pure strategy Nash equilibria (PSNE) and one mixed strategy Nash equilibrium (MSNE) as a population parameter. The blue agent plays strategies p_{B_1}, p_{B_2}, and p_{B_3} with approximate proportions 0.32, 0.33, 0.35, respectively. The red agent plays strategies p_{R_1}, p_{R_2}, and p_{R_3} with approximate proportions 0.35, 0.28, and 0.36, respectively. The aggregate utilities associated with this MSNE for the blue agent and red agent are approximately 0.50 and 0.48, respectively. These payoffs serve as the attrition coefficients in the game-theoretic context and yield an MSNE PR of 1.04 indicating the blue force fields the more effective fighting capability according to game theory. In this notional example, alignment of PR results may be observed between the MIWT approach and the game-theoretic approach.

5.5 Analysis

This analysis uses seven scenarios from Scardera and Cesul’s work including five historical battles between American and Japanese forces in the Pacific Theatre during World War II and two hypothetical scenarios from the Cold War. The first Cold War scenario examines a hypothetical conventional conflict between the United States and the Soviet Union which excludes the use of the space domain. The second scenario also considers a conventional conflict but includes the use of assets in the space domain. (Scardera & Cesul, 2021) Due to the high number of degenerate matrices which exclude the use of an MSNE approach, the PSNE are provided to give the theoretical bounds of an MSNE. An artificial balanced (all strategies are played with equal proportion) mixed strategy PR is provided to enrich the analysis.

Table 5.9 displays the information pertinent to the initial landings. According to MIWT, the attrition coefficients of the blue and red forces are 4.00 and 0.11, respectively, which yields a determinant PR of 36.36. All three PSNE present a power ratio of 2.70. The matrix is degenerate and encompasses infinite PMSNE; the balanced PR is 3.41. While both approaches assert a more effective blue force, the MIWT approach diverges much faster and gives a tremendous advantage to the blue force.

Table 5.9. Initial Landings

Initial Landings		Red Player		
		p_{R_1}	p_{R_2}	p_{R_3}
Blue Player	p_{B_1}	1.00, 0.37	1.00, 0.22	1.00, 0.23
	p_{B_2}	1.00, 0.37	1.00, 0.35	1.00, 0.19
	p_{B_3}	1.00, 0.37	1.00, 0.35	1.00, 0.19

Table 5.10 displays the information pertinent to the battle of Tanaru and East Solomons. According to MIWT, the attrition coefficients of the blue and red forces are 1.90 and 3.24, respectively, which yields a determinant PR of 0.59. Both PSNE present a power

ratio of 1.00. The matrix is degenerate and encompasses infinite PMSNE; the balanced PR is 0.91. While both approaches assert a more effective red force, the MIWT approach diverges faster and gives a greater advantage to the red force.

Table 5.10. Tanaru & East Solomons

Tanaru & East Solomons		Red Player		
		p_{R_1}	p_{R_2}	p_{R_3}
Blue Player	p_{B_1}	1.00, 0.96	1.00, 1.00	1.00, 0.70
	p_{B_2}	0.65, 0.96	0.45, 1.00	0.50, 0.75
	p_{B_3}	1.00, 0.96	1.00, 1.00	1.00, 0.99

Table 5.11 displays the information pertinent to the battle of Henderson Field and Santa Cruz. According to MIWT, the attrition coefficients of the blue and red forces are 2.74 and 3.21, respectively, which yields a determinant PR of 0.85. The PSNE presents a power ratio of 1.00. The balanced PR is 0.94. The disparate approaches are aligned in terms of results.

Table 5.11. Henderson Field & Santa Cruz

	Red Player

Henderson Field & Santa Cruz		p_{R_1}	p_{R_2}	p_{R_3}
Blue Player	p_{B_1}	1.00, 0.96	1.00, 1.00	1.00, 0.83
	p_{B_2}	0.88, 0.96	0.62, 1.00	0.74, 1.00
	p_{B_3}	1.00, 0.96	0.83, 1.00	1.00, 0.84

Table 5.12 displays the information pertinent to the naval battle and Japanese landings. According to MIWT, the attrition coefficients of the blue and red forces are 1.77 and 2.38, respectively, which yields a determinant PR of 0.74. Both PSNE present a power ratio of 1.00. The matrix is degenerate and encompasses infinite PMSNE; the balanced PR is 0.87. The disparate approaches are aligned in terms of results.

Table 5.12. Naval Battle & Japanese Landings

Naval Battle & Japanese Landings		Red Player		
		p_{R_1}	p_{R_2}	p_{R_3}
Blue Player	p_{B_1}	0.73, 1.00	0.73, 1.00	0.73, 0.61
	p_{B_2}	0.60, 1.00	0.52, 1.00	0.69, 0.82
	p_{B_3}	1.00, 1.00	1.00, 1.00	1.00, 0.58

Table 5.13 displays the information pertinent to the Japanese withdrawals. According to MIWT, the attrition coefficients of the blue and red forces are 3.72 and 0.40, respectively, which yields a determinant PR of 9.30. The PSNE present power ratios of

1.47 and 1.85. The matrix is degenerate and encompasses infinite PMSNE; the balanced PR is 1.95. While both approaches assert a more effective blue force, the MIWT approach diverges faster and gives a greater advantage to the blue force.

Table 5.13. Japanese Withdrawals

Japanese Withdrawals		Red Player		
		p_{R_1}	p_{R_2}	p_{R_3}
Blue Player	p_{B_1}	1.00, 0.26	1.00, 0.68	1.00, 0.54
	p_{B_2}	1.00, 0.26	0.86, 1.00	1.00, 0.54
	p_{B_3}	1.00, 0.26	1.00, 0.54	1.00, 0.47

Table 5.14 displays the information pertinent to the Cold War excluding the space domain. According to MIWT, the attrition coefficients of the blue and red forces are 1.92 and 2.65, respectively, which yields a determinant PR of 0.72. All three PSNE present power ratios of 1.00. The matrix is degenerate and encompasses infinite PMSNE; the balanced PR is 1.02. In this instance, the MIWT and game-theoretic approaches do not align in terms of results.

Table 5.14. Cold War

Cold War		Red Player		
		p_{R_1}	p_{R_2}	p_{R_3}

Blue Player	p_{B_1}	0.58, 1.00	0.38, 0.67	0.58, 0.67
	p_{B_2}	1.00, 1.00	1.00, 0.96	1.00, 0.47
	p_{B_3}	1.00, 1.00	1.00, 0.61	1.00, 1.00

Table 5.15 displays the information pertinent to the Cold War including the space domain. According to MIWT, the attrition coefficients of the blue and red forces are 1.79 and 5.59, respectively, which yields a determinant PR of 0.32. All four PSNE present power ratios of 1.00. The matrix is degenerate and encompasses infinite PMSNE; the balanced PR is 0.87. While both approaches assert a more effective red force, the MIWT approach diverges faster and gives a greater advantage to the red force.

Table 5.15. Cold War with Space Domain

Cold War with Space Domain		Red Player			
		p_{R_1}	p_{R_2}	p_{R_3}	p_{R_4}
Blue Player	p_{B_1}	0.58, 1.00	0.38, 0.67	0.58, 0.67	0.58, 1.00
	p_{B_2}	1.00, 1.00	1.00, 0.96	1.00, 0.47	0.50, 1.00
	p_{B_3}	1.00, 1.00	1.00, 0.61	1.00, 1.00	1.00, 1.00
	p_{B_4}	0.55, 1.00	0.78, 1.00	0.80, 0.65	0.39, 1.00

5.6 Conclusion

There exist several key takeaways from the game-theoretic analysis of MIWT. Scardera and Cesul’s normalization mechanism generally leads to degenerate game-theoretic matrices rendering the MSNE approach non-applicable. In the case that a user should want to conduct both MIWT and game-theoretic parametric analysis, the authors recommend using a pairwise scoring scheme which does not cap at 1.00 or any number except in exceptional circumstances. Such an approach will limit the occurrence of weakly dominated solutions, degenerate matrices, and infinitely many PMSNE. Increasing the frequency of non-degenerate matrices will enable the viable implementation of an MSNE approach.

This work presented evidence that the game-theoretic methodology and MIWT methodology provided a limited validation of each other. The game-theoretic results generally concurred with the MIWT results, however, the MIWT results diverged faster. This phenomenon occurred because game-theory is based on agent interaction while MIWT is based on internal synergies of the respective belligerent’s fighting force. The former approach leads to more conservative linear combinations of the data whereas the latter uses divergent non-linear combinations of the data. Which approach is more accurate is likely highly context-specific and the authors recommend using both parametric methods to achieve a robust understanding of the situation. The user can execute both approaches relatively quickly and in doing so may save valuable time in the long term before modeling efforts progress to more complex high-fidelity physics-based or agent-based modeling endeavors.

The United States presently contends with peer adversaries on the world stage; the need to achieve a capable and effective force structure, especially in the context of an emerging space domain, is extremely important. The integration of these MIWT and game-theoretic methodologies into the Joint Capabilities Integration and Development System (JCIDS) process, Materiel Solution Analysis Phase, or Technology Maturation and Risk Reduction Phase of the U.S. Department of Defense acquisitions processes would enhance the U.S. force structure and improve the fighting efficacy of the U.S. military. These parametric modeling methodologies can provide valuable strategic insight to policy-makers and decision-makers and advance the security posture of the United States.

Chapter 6: A Methodological Framework for Parametric Combat Analysis

6.1 Abstract

This work proposes, explains, and demonstrates a methodological framework which affords decision-makers and policy-makers the opportunity to accelerate the tempo and enhance the quality of the U.S. Department of Defense acquisitions community. The effective implementation of this framework will augment the warfighting capability of the U.S. Department of Defense against peer competitors and advance the security posture of the United States. This work provides a theoretical foundation for explaining the distribution of potential outcomes for certain types of warfare while contextualizing the research to the space domain. This framework utilizes reliability models to account for natural attrition while using combat models to account for induced attrition. The use of game-theoretic concepts enables the evaluation of heterogeneous force structures.

6.2 Introduction

In order for the United States to prevail against a peer competitor in a time of conflict, the U.S. Department of Defense (DoD) must develop an acquisitions community which fields new capabilities within an operationally relevant timeframe. Pushing the current acquisitions community to achieve such a goal is a challenging, complex, and multifaceted undertaking which will not be comprehensively addressed within the scope of this article. This work will focus on a specific aspect of the endeavor to accelerate the acquisitions community: how does modeling and simulation inform the acquisitions organizations responsible for the procurement of DoD weapon systems? That is, which systems should the DoD develop and how many of those systems should the DoD produce? The aim of the methodological framework expounded in this paper is to enable the user to conduct parametric combat analysis to project the efficacy of various force compositions – such analysis should support the user in making intelligent and informed procurement decisions.

In order to effectively use the framework discussed in this article, the user must accept the premise that a knowledgeable person could capture the physical phenomena associated with various combat scenarios and express the dynamics of those scenarios in mathematical representations. The user must understand that these mathematical representations encompass parameters which the user may adjust to accurately represent a particular situation in an effort to make an educated prediction concerning the future. The user should not view parametric modeling as a replacement for agent-based modeling (e.g., Advanced Framework for Simulation, Integration, and Modeling) or high-fidelity physics-based modeling (e.g., Systems Tool Kit); parametric modeling is complementary to other modeling methods.

The literature review section of this paper will provide the reader with an academic grounding for the material encompassed with the framework. The methodology section of this paper will present the framework, discuss how various phenomena influence a given instantiation of the framework, and provide an algorithm for a particular instantiation of the framework. The contextualization of framework to the space domain section will discuss potential environments, belligerents, and weapons of a possible space conflict. The numerical demonstration of methodology section of this article will discuss the distribution of potential outcomes for different kinds of conflict and cover multiple notional scenarios between a blue force and a red force. Finally, the conclusion of this article will summarize this work and its recommendations.

6.3 Literature Review

This section reviews literature pertinent to this article, specifically focusing on probability theory, game theory, and combat modeling.

The exponential distribution is a parametric distribution often used in reliability modeling to characterize the time to failure of entities within a data set. The probability density function (PDF) may be expressed as

$$f(t) = \frac{e^{-\frac{t}{\beta}}}{\beta}$$

with the support of t ranging from zero to infinity and β representing the scale parameter which is also known as the characteristic lifespan; β also exists as the expected value of the distribution. (Casella & Berger, 2002) The exponential distribution is memoryless, tractable, and corresponds to the operational phase of life. The cumulative distribution function (CDF) may be presented as

$$F(t) = 1 - e^{-\frac{t}{\beta}}$$

The reliability function may be presented as

$$R(t) = e^{-\frac{t}{\beta}}$$

The hazard function may be presented as

$$h(t) = \frac{f(t)}{R(t)} = \frac{\frac{e^{-\frac{t}{\beta}}}{\beta}}{e^{-\frac{t}{\beta}}} = \frac{1}{\beta}$$

which corresponds to the constant failure rate of

$$\lambda = \frac{1}{\beta}$$

The Weibull distribution is a generalized form of the exponential distribution that can account for beneficial or deleterious aging at the cost of less tractability. (Casella & Berger, 2002) The PDF of the Weibull distribution may be expressed as

$$f(t) = \frac{e^{-\frac{t^\gamma}{\beta}}}{\beta} \gamma t^{\gamma - 1}$$

with the support of t ranging from zero to infinity and γ representing the shape parameter of the distribution. $\gamma < 1$ indicates beneficial aging where failure rate decreases with respect to time; $\gamma = 1$ indicates memoryless aging (i.e., the exponential distribution); $\gamma > 1$ indicates deleterious aging where failure rate increases with respect to time; $\gamma = 2$ indicates the linear increase of failure rate with respect to time (i.e., the Rayleigh distribution). The CDF of the Weibull distribution may be presented as

$$F(t) = 1 - e^{-\frac{t^\gamma}{\beta}}$$

The reliability function may be presented as

$$R(t) = e^{-\frac{t^\gamma}{\beta}}$$

The hazard function may be presented as

$$h(t) = \frac{f(t)}{R(t)} = \frac{\frac{e^{-\frac{t^\gamma}{\beta}}}{\beta} \gamma t^{\gamma - 1}}{e^{-\frac{t^\gamma}{\beta}}} = \frac{\gamma t^{\gamma - 1}}{\beta}$$

The Weibull distribution is related to the exponential distribution through the random variable transformation

$$X_{Weibull} = X_{Exponential}^{\frac{1}{\gamma}}$$

Hayhurst and Colombi developed the Atomic Competitive Element (ACE) taxonomy for categorizing game-theoretic scenarios according to attributes desirable for the stability of an outcome within a game. (Hayhurst & Colombi, Game-Theoretic System Design in the Development of Space Power, 2021) The ACE taxonomy uses red to designate balance optimality, yellow to designate Nash optimality, and blue to designate Pareto optimality; secondary colors are used to represent the combination of these attributes, white is used to represent the presence of all three attributes, and gray is used to represent the absence of all three attributes. Within a given cell of a normal-form game, red indicates the agents' payoffs are approximately the same, yellow indicates the cell is a pure strategy Nash equilibrium (no agent has incentive to unilaterally change their decision), and blue indicates the cell is on the Pareto front (no agent's payoff may be improved without diminishing another agent's payoff). Figure 6.1 is adapted from the work of Hayhurst and Colombi and presented to provide clarity on the ACE taxonomy.

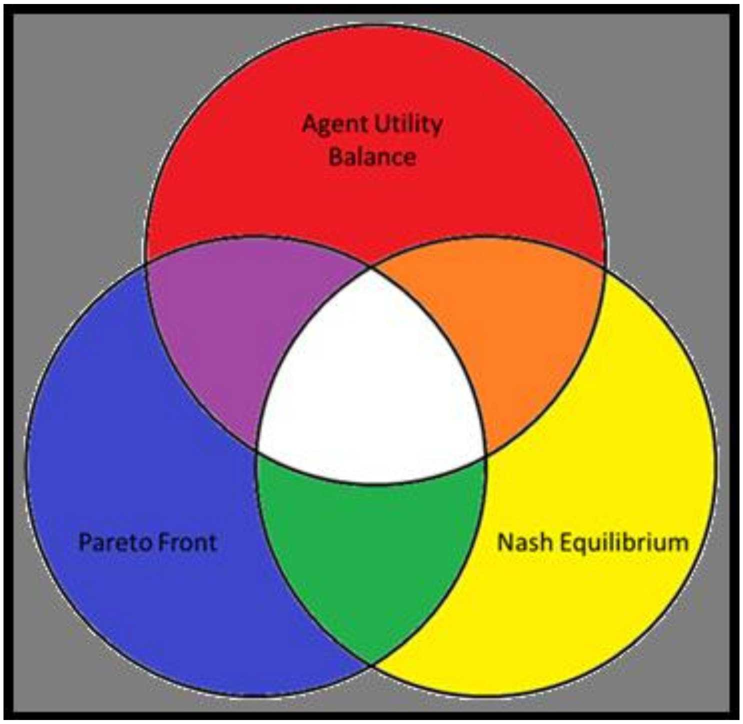


Figure 6.1. Atomic Competitive Element Taxonomy

Hayhurst and Colombi developed and presented eight specific ACEs for eight common games including Deadlock, Pure Coordination, Stag Hunt, Matching Pennies, Stoplight, Prisoner's Dilemma, Take or Share, and Chicken. (Hayhurst & Colombi, Game-Theoretic System Design in the Development of Space Power, 2021)

The Lanchester Linear Law describes the attrition of two belligerents (described here as the blue force and red force or simply blue and red) under simplified engagement assumptions – two belligerents engage in a decisive conflict at a concrete focal point using unaimed fire; this law depends on the homogeneity of units on either side of the conflict and assumes a generally uniform distribution of fire from one side against the other. The Lanchester Linear Law may be presented as

$$\frac{dB}{dt} = -rRB$$

$$\frac{dR}{dt} = -bBR$$

such that

$$b \sim blue\ force\ attrition\ coefficient$$

$$r \sim red\ force\ attrition\ coefficient$$

$$B \sim size\ of\ the\ blue\ force$$

$$R \sim size\ of\ the\ red\ force$$

In the case of the Lanchester Linear Law, the attrition rate for both blue and red is a function of the size of both forces. In this case, the fighting strength of each force may be presented as

$$F_B = bB$$

$$F_R = rR$$

The Lanchester Square Law assumes that each belligerent uses aimed fire and otherwise shares the same assumptions as the Lanchester Linear Law. The Lanchester Square Law may be presented as

$$\frac{dB}{dt} = -rR$$

$$\frac{dR}{dt} = -bB$$

Here, the blue (red) attrition rate is a function of the red's (blue's) force size. In this case, the fighting strength of each force may be presented as

$$F_B = bB^2$$

$$F_R = rR^2$$

For both the Lanchester Linear Law and the Lanchester Square Law, the force with the greater fighting strength will win the conflict. (Lanchester, 1916)

The Bonder Law is a generalized form of the Lanchester Square Law and may be presented as

$$\frac{dB_m}{dt} = -\sum_{n=1}^{N} r_{m,n} R_n \quad for\ m = 1, 2, \dots, M$$

$$\frac{dR_n}{dt} = -\sum_{m=1}^{M} b_{m,n} B_m \quad for\ n = 1, 2, \dots, N$$

The Bonder Law shares the same assumptions as the Lanchester Square Law with the exception that the Bonder Law allows for heterogeneity within each belligerent force structure. (Bonder & Farrell, 1970)

The Lotka-Volterra Interspecific Competition Law may be presented as

$$\frac{dN_1}{dt} = r_1 N_1 \left(\frac{K_1 - N_1 - \alpha N_2}{K_1}\right)$$

$$\frac{dN_2}{dt} = r_2 N_2 \left(\frac{K_2 - N_2 - \beta N_1}{K_2}\right)$$

such that

$r \sim respective\ per\ capita\ growth\ rate$

$N \sim respective\ population\ size$

$K \sim respective\ carrying\ capacity$

$\alpha \sim the\ per\ capita\ effect\ of\ entity\ 2\ on\ carrying\ capacity\ of\ entity\ 1$

$\beta \sim the\ per\ capita\ effect\ of\ entity\ 1\ on\ the\ carrying\ capacity\ of\ entity\ 2$

The Lotka-Volterra Interspecific Competition Law is based on the logistic growth model used to characterize intraspecific competition. The Lotka-Volterra Interspecific Competition Law assumes that two distinct, homogenous species compete over a common set of resources in a mutually deleterious manner to ensure population growth success up to the carrying capacity of the pertinent environment. (Kingsland, 2015) In a conflict scenario in which the belligerents encompass certain characteristics of interspecific competition such as a carrying capacity, logistic growth, and competition over common resources, a Lanchester-Lotka-Volterra Hybrid Law (Hayhurst, Colombi, & Meyer, Aggregated space combat modeling, 2021) may be presented as

$$\frac{dB}{dt} = B\left(\frac{K_B - B - rR}{K_B}\right)$$

$$\frac{dR}{dt} = R\left(\frac{K_R - R - bB}{K_R}\right)$$

where K is still the respective carrying capacity of both entities, α and β have been replaced with the attrition rates of both entities, r and $b,$ and R and B are the force sizes. In this scenario, the goal of conflict is the establishment of a dominant ecological isocline. There exist four isocline scenarios which determine the fate of the conflict. In the case of a dominant blue isocline, the following inequalities hold:

$$K_B > \frac{K_R}{b}$$

$$\frac{K_B}{r} > K_R$$

The blue force will eliminate the red force and grow to its natural carrying capacity. Likewise, a dominant red isocline occurs when:

$$K_R > \frac{K_B}{r}$$

$$\frac{K_R}{b} > K_B$$

The red force will eliminate the blue force and grow to its natural carrying capacity. The isoclines present divergent bifurcation when

$$K_B > \frac{K_R}{b}$$

$$K_R > \frac{K_B}{r}$$

In this scenario, the outcome of the conflict hinges on the parameter values of the respective carrying capacities and attrition coefficients as well as the initial size of each belligerent force. Finally, the isoclines present convergent behavior when

$$\frac{K_B}{r} > K_R$$

$$\frac{K_R}{b} > K_B$$

In this case, the size of each force will converge to a steady equilibrium. (Olson, 2014)

The Brackney Law may be presented as

$$\frac{dB}{dt} = -\frac{RBS_R}{T_B BS_R + V_B}$$

$$\frac{dR}{dt} = -\frac{BRS_B}{T_R RS_B + V_R}$$

such that

$S_B \sim blue\ search\ rate, search\ volume\ per\ unit\ of\ time$

$V_B \sim amount\ of\ volume\ the\ blue\ force\ may\ occupy$

$$T_B \sim amount\ of\ time\ required\ for\ red\ unit\ to\ destroy\ blue\ unit$$

The Brackney Law integrates the fog of war into parametric modeling by describing two distinct, homogenous forces which must find the constituent parts of the adversary force before the destructive engagements may occur; this law assumes a random uniform distribution of each force throughout its respective volume. (Brackney, 1959) The use of the Brackney Law requires knowledge of the specific, tangible capabilities of the fighting units which compose each force. (Washburn & Kress, 2009) Hayhurst et al. adapted the Brackney Law for use in three-dimensional space. (Hayhurst, Colombi, & Meyer, Aggregated space combat modeling, 2021)

6.4 Methodology

This section articulates the methodology of this work by describing the general mathematical construct of the integrated framework. This section also presents a mathematically symbolic demonstration of a specific instantiation of the framework to facilitate understanding. Consider the integrated framework for analysis in its general form presented in Figure 6.2.

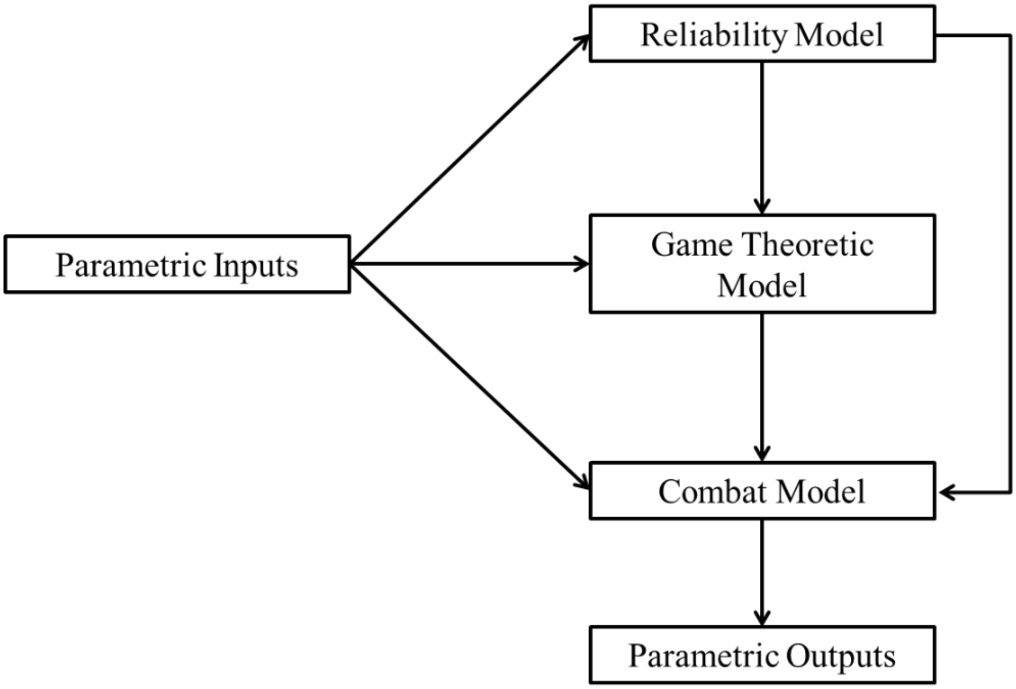


Figure 6.2. Integrated Framework in General Form

This integrated framework encompasses five modules which include the parametric inputs, reliability model, game-theoretic model, combat model, and parametric outputs. The necessary inputs and natural outputs of the framework are based on the specific models chosen by the user to fill in the reliability, game-theoretic, and combat modules; the user should choose models which accurately represent the physical phenomenon under consideration. This modularized approach provides flexibility and adaptability to the user.

The reliability model accounts for the "cold phase" of conflict – a long-duration, low-intensity period characterized by natural attrition (e.g., a spacecraft loses functionality because of battery decay). The reliability model uses a parametric distribution such as (but not limited to) the exponential or Weibull. The reliability model enables the framework to determine the losses associated with the cold phase of conflict for each belligerent. The reliability model receives information from the parametric inputs and provides information to the game-theoretic model and combat model.

The game-theoretic model is an $n \times m$ strategic-normal form game which may manifest or resemble an atomic competitive element such as (but not limited to) Deadlock, Pure Coordination, Stag Hunt, Stoplight, Matching Pennies, Prisoner's Dilemma, Take or Share, or Chicken. The game-theoretic model enables the framework to calculate aggregate attrition coefficients from a larger set of pairwise attrition coefficients. The game-theoretic model receives information from the parametric inputs and reliability model and provides information to the combat model.

The combat model accounts for the "hot phase" of conflict – a short-duration, high-intensity period characterized by combat-induced attrition (e.g., a spacecraft loses functionality because of the hostile use of a directed energy weapon). The combat model uses a system of differential equations such as (but not limited to) the Lanchester Linear Law, Lanchester Square Law, Lotka-Volterra Equations, or Brackney Equations. The combat model enables the framework to determine the losses associated with the hot phase of conflict for each belligerent. The combat model receives information from the parametric inputs, reliability model, and game-theoretic model and provides information to the parametric outputs.

This work will now transition from describing the general form of the framework to articulating a specific instantiation of the framework. The purpose of this illustration is to provide clarity on how the framework might be used; expounding the comprehensive set of uses for the framework in detail would be too expansive. During this symbolic demonstration, the quantity of a given fighting capability is expressed in a fictional currency for three reasons. First, using a fictional currency emphasizes the notional nature of this demonstration – a notional example facilitates public release and broad readership access. Second, while the combat models of this framework traditionally considered the number of fighting units within a capability such an approach might damage the composability of the model whenever a discrepancy in unit cost exists between capabilities. Accounting for the amount of resources invested into a capability instead of the number of fighting units embodied within that capability helps to ensure composability within the framework. Finally, various real-world currencies differ in value and the purchasing power parity from one nation to the next might be starkly different. Essentially, the framework must capture how the purchasing power of a particular belligerent is spent – that is, any amount of purchasing power spent on a particular capability is assumed to be resources not spent on a separate, distinct capability within the same fighting force. The game-theoretic principles are then able to inform the effective allocation of finite resources.

With the groundwork for this example laid, consider two belligerents which act as peer competitors wherein each belligerent simultaneously invests a particular amount of currency into two capabilities each. As peer competitors, each belligerent must strive to match their strength to the weakness of their opponent over the course of combat. The game-theoretic model is, therefore, a two-by-two strategic-normal form matrix which embodies the Matching Pennies game. Consider that each capability is in the operational phase of its lifecycle. Therefore, the reliability model uses the exponential distribution. Consider that each capability does not reproduce, uses aimed fire at a decisive focal point of conflict, and is supported by an effective space domain awareness architecture. Therefore, the combat model uses the Lanchester Square Law. The correct module loadout of the general framework for this scenario is shown in Figure 6.3.

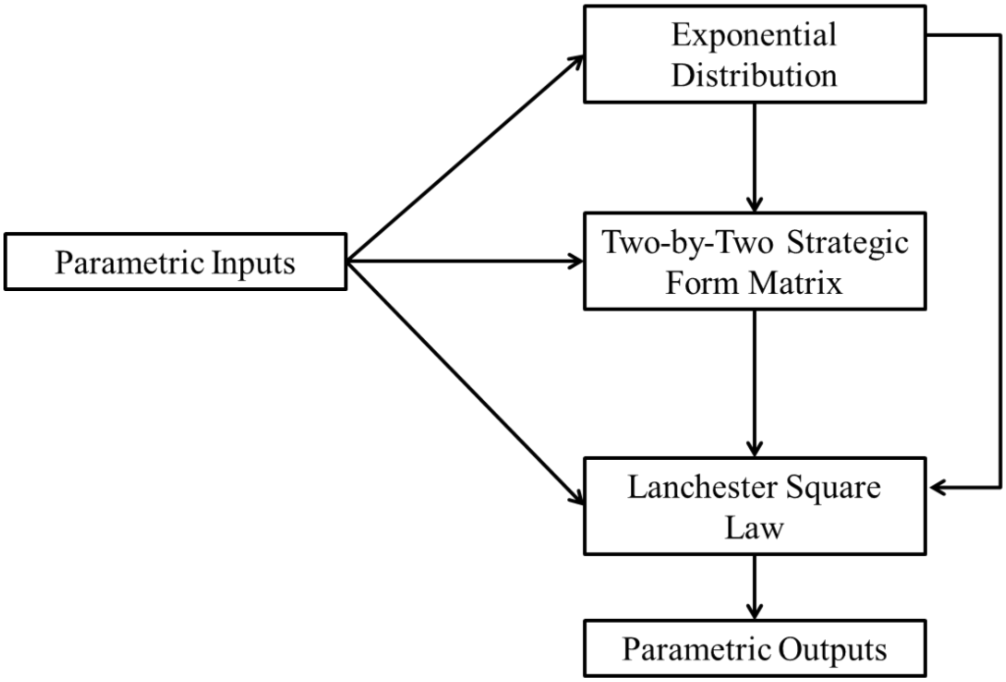


Figure 6.3. Framework with Module Loadout

In order to carry out the analysis of this scenario using the integrated framework, utilize the following process. First, the user supplies 26 parametric inputs including the initial investment in each capability ($B_{1\alpha}$, $B_{2\alpha}$, $R_{1\alpha}$, $R_{2\alpha}$), the characteristic lifespan for the units of currency for each capability ($\beta_{B_{1\alpha}}$, $\beta_{B_{2\alpha}}$, $\beta_{R_{1\alpha}}$, $\beta_{R_{2\alpha}}$), the expected value and variability associated with each pairwise attrition coefficient ($\mu_{b_{1,1}}$, $\mu_{b_{1,2}}$, $\mu_{b_{2,1}}$, $\mu_{b_{2,2}}$, $\mu_{r_{1,1}}$, $\mu_{r_{1,2}}$, $\mu_{r_{2,1}}$, $\mu_{r_{2,2}}$, $[\sigma^2]_{b_{1,1}}$, $[\sigma^2]_{b_{1,2}}$, $[\sigma^2]_{b_{2,1}}$, $[\sigma^2]_{b_{2,2}}$, $[\sigma^2]_{r_{1,1}}$, $[\sigma^2]_{r_{1,2}}$, $[\sigma^2]_{r_{2,1}}$,

$[\sigma^2]_{r_{2,2}}$), and the time spent in the cold and hot phases of the conflict (t_c, t_h). Next, generate a normal distribution for each pairwise attrition coefficient and randomly draw a number from each of those normal distributions to determine the value for each pairwise attrition coefficient.

$$f(b_{1,1}) = \frac{1}{\sqrt{2\pi[\sigma^2]_{b_{1,1}}}} e^{-\frac{\left(b_{1,1}-\mu_{b_{1,1}}\right)^2}{2[\sigma^2]_{b_{1,1}}}}$$

$$\dots$$

$$f(r_{2,2}) = \frac{1}{\sqrt{2\pi[\sigma^2]_{r_{2,2}}}} e^{-\frac{\left(r_{2,2}-\mu_{r_{2,2}}\right)^2}{2[\sigma^2]_{r_{2,2}}}}$$

Calculate the remaining investment after natural attrition during the cold phase by multiplying the initial investment by the exponential reliability function.

$$B_1 = B_{1\alpha} e^{-\frac{t_c}{\beta_{B_{1\alpha}}}}$$

$$B_2 = B_{2\alpha} e^{-\frac{t_c}{\beta_{B_{2\alpha}}}}$$

$$R_1 = R_{1\alpha} e^{-\frac{t_c}{\beta_{R_{1\alpha}}}}$$

$$R_2 = R_{2\alpha} e^{-\frac{t_c}{\beta_{R_{2\alpha}}}}$$

Calculate the game-theoretic mixed strategy as a population parameter for each belligerent. To do this, calculate the proportion of the remaining investment which each capability constitutes. In this game-theoretic context, the mixed strategy exists as a population parameter wherein instead of two decisions being made with certain probabilities there exist two populations with certain proportions of subpopulations.

Table 6.1 provides a symbolic representation of Matching Pennies and affords clarity on the associated calculations.

$$p_{B_1} = \frac{B_1}{B_1 + B_2}$$

$$p_{B_2} = \frac{B_2}{B_1 + B_2}$$

$$p_{R_1} = \frac{R_1}{R_1 + R_2}$$

$$p_{R_2} = \frac{R_2}{R_1 + R_2}$$

Table 6.1. Symbolic Representation of Matching Pennies

Matching Pennies		Red Player	
		p_{R_1}	p_{R_2}
Blue Player	p_{B_1}	$b_{1,1}, r_{1,1}$	$b_{1,2}, r_{1,2}$
	p_{B_2}	$b_{2,1}, r_{2,1}$	$b_{2,2}, r_{2,2}$

Determine the aggregate attrition coefficients by multiplying the probabilities within the three-dimensional probability mass function by the associated payoffs. The payoffs are the pairwise attrition coefficients.

$$b = p_{B_1}p_{R_1}b_{1,1} + p_{B_1}p_{R_2}b_{1,2} + p_{B_2}p_{R_1}b_{2,1} + p_{B_2}p_{R_2}b_{2,2}$$

$$r = p_{B_1}p_{R_1}r_{1,1} + p_{B_1}p_{R_2}r_{1,2} + p_{B_2}p_{R_1}r_{2,1} + p_{B_2}p_{R_2}r_{2,2}$$

Calculate the aggregate remaining investment for each belligerent after natural attrition.

$$B = B_1 + B_2$$

$$R = R_1 + R_2$$

Enable the simulation to run for the hot phase time using the Lanchester Square Law system of differential equations.

$$\frac{dB}{dt} = -rR$$

$$\frac{dR}{dt} = -bB$$

Calculate the margin of victory (or defeat) at the end of the conflict.

$$\mathrm{M} = B|_{t=t_h} - R|_{t=t_h}$$

Since the simulation is stochastic, the user should run the simulation over multiple iterations – such an approach will provide a range of outcomes to articulate the possible results of the physical phenomena.

6.5 Contextualization of Framework to the Space Domain

The universe holds countless numbers of galaxies including the Milky Way which holds countless numbers of stars including Sol. The Sol System presents three regions known as the Inner Solar System, Outer Solar System, and Trans-Neptunian Region. The Inner Solar System encompasses the objects within the orbit of Jupiter. The Outer Solar System encompasses the objects between the orbits of Jupiter and Neptune, inclusive. The Trans-Neptunian Region extends from the orbit of Neptune to the boundary of the Sol System – considered either as the heliopause or the limit of space dominated by Sol's gravity. Together, the Inner Solar System and Outer Solar System constitute the Planetary Region – this work focuses primarily on the Planetary Region. The Trans-Neptunian Region holds the Kuiper Belt, Scattered Disc, heliopause, Detached Objects Region, and the theoretical Oort Cloud which may exist. Many objects move between multiple regions such as comets which transit across most of the Solar System and sednoids which transit between the Scattered Disc and the Detached Objects Region. The Planetary Region encompasses twenty-eight objects approximately rounded by the force of gravity including (listed by proximity to Sol and, in the case of natural satellites, proximity to planet) Sol, Mercury, Venus, Earth, Luna, Mars, Ceres, Jupiter, Io, Europa, Ganymede, Callisto, Saturn, Mimas, Enceladus, Tethys, Dione, Rhea, Titan, Iapetus, Uranus, Miranda, Ariel, Umbriel, Titania, Oberon, Neptune, and Triton. The Planetary Region also encompasses many non-spherical objects such as the asteroids of the Inner Solar System and the centaurs of the Outer Solar System. Any large-scale space conflict in the ten-to-twenty-year timeframe will likely be hosted by one of the twenty-eight Planetary Region objects in approximate hydrostatic equilibrium. An evaluation of the habitability and resources of these celestial bodies as well as the state and trajectory of human technology enables an educated prediction of humanity's exploration (conflict) path. This work predicts that humanity's exploration over the next thirty years will focus on Luna, Mars, Ceres, Callisto, and Titan. The Moon will probably undergo the most robust exploration and resource utilization and as such cislunar warfare constitutes the primary context for this work.

The Moon will likely transform into a vital economic hub for the spacefaring nations. The focal points of this economic center would include resource harvesting, manufacturing, and lunar gateway transit. With respect to resource harvesting, the Moon may yield an abundant supply of rare-earth metals (REM) and Helium-3.

(911Metallurgist, 2015) Even if humanity fails to profitably harvest resources directly from the Moon, Luna could host processing centers built to extract resources from asteroids. Japan already conducted successful sample returns from the asteroids Itokawa (Massachusetts Institute of Technology, 2016) and Ryugu (Shekhtman, 2020) while the United States currently attempts a successful sample return from the asteroid Bennu. (Garner, 2021) Spacefaring nations may one day harvest precious and industrial metals from asteroids such as 16 Psyche, 1986 DA, and 2016 ED85. (Carter, 2021) In the development of the space economy, nations and organizations may determine that transporting asteroids or asteroid fragments to the Moon for extraction yields a greater profit than direct harvesting. Also in consideration of an economic calculus, nations and organizations may decide to manufacture some items on the Moon rather than transport those items from Earth. The upcoming On-Orbit Servicing, Assembly, and Manufacturing 2 (OSAM-2, previously Archinaut One) mission scheduled to launch no earlier than 2022 will strive to mature the technology associated with microgravity manufacturing. (Harbaugh, 2021) Finally, with humanity's desire to visit Mars and beyond, there exists a high probability that manufacturers design some vehicles specialized for Earth-to-Moon transportation and others for transportation beyond the Moon. A lunar gateway would serve as a natural off-loading and on-loading point. The nation that controls Luna in part or in whole would reap tremendous economic benefits from these activities.

A key indicator of a nation's desire and ability to conduct a space conflict is the pace at which that nation develops space weapons and the demonstrable efficacy of those weapons. While the weaponization of space may take many forms, the fielding of effective direct ascent antisatellite (DA-ASAT) weapons provides an important benchmark for a nation’s progress. Perhaps the five most pertinent events related to DA-ASAT weapons include

> the 1985 destruction of the US P78-1 Solwind satellite, using an air-launched ASM-135 (during the era of the Strategic Defense Initiative), the 2007 destruction of the Chinese FY-1C (Fengyun, “Wind and Cloud”) satellite using a ground-launched SC-19, the 2008 destruction of the US USA-193 satellite using a sea-launched Standard Missile-3 (Operation Burnt Frost), [] the 2019 destruction of the Indian Microsat-R satellite using a ground-launched Prithvi Defense Vehicle Mark-II (Mission Shakti, “Power”), (Hayhurst & Colombi, Game-Theoretic System Design in the Development of Space Power, 2021)

and the 2021 destruction of the Russian Kosmos 1408 using a ground-launched A-235 PL-19 Nudol. (Wolfe, 2021) During each test, the satellite was destroyed by its owner; the Russian and (especially) Chinese tests contributed significantly to the formation of geocentric Kessler fields.

The range of space weapon types extends beyond the kinetic DA-ASAT variants and can include dual-use technology. In 2010, China launched the SJ-12 (Shijian, “Practice”) satellite into low Earth orbit (LEO) to conduct a series of rendezvous and proximity operations (RPO) maneuvers with SJ-06F; the two satellites likely made low-speed contact. (Harrison, Johnson, & Roberts, 2018) In 2016, China launched the SJ-17 into geostationary orbit where it conducted proximity operations with multiple Chinese satellites. (Roberts, 2021) The Chinese space endeavor made great strides during 2016, also launching the Aolong-1 ("Proud Dragon") and the Tianyuan-1 ("Fields and Gardens"). Aolong-1 matured robotic arm grappling technology while Tianyuan-1 reportedly demonstrated spacecraft-to-spacecraft refueling. In 2021, China launched the SJ-21 into geostationary orbit where it docked with the defunct Beidou-2 ("Northern Dipper") G2 navigation satellite. (Jones, 2022) In the beginning of 2022, SJ-21 pulled the defunct navigation satellite 3,000 km above the geostationary belt. While not a comprehensive review of space weaponry, these highlights provide insight into the steady transformation of space into a warfighting domain.

Another important indicator of a nation's space capabilities is the ability of that nation to visit celestial bodies. In a joint effort with the European Space Agency (ESA), Russia conducted a crash landing on Mars in 2016 with the ExoMars Schiaparelli EDM Lander – Russia intended to conduct a soft landing. (NASA, n.d.) India conducted two hard landings on the Moon in 2008 and 2019. India planned for the impact probe released from the Chandrayaan-1 ("Moon Craft") to conduct a hard landing in 2008. (Barnett, n.d.) However, India intended the 2019 Chandrayaan-2 mission to conduct a soft landing. (Williams, 2022) China conducted a Martian soft landing in 2021 with the Tianwen-1 ("Heavenly Questions") Lander. (Stein, 2021) China also conducted multiple soft landings on the Moon's surface, the most impressive of which, Chang'e 5 ("Goddess of the Moon"), conducted a robotic sample return. (The Planetary Society, n.d.) While

China challenges the United States as the vanguard of space exploration, the U.S. free-market enterprise continues to experience tremendous and promising growth. Newcomers such as Virgin Galactic, Blue Origin, and especially SpaceX seek to challenge traditional mainstays such as the United Launch Alliance in the space industry sphere. In recent years, U.S. celestial surface exploration efforts focused primarily on Mars with a series of successful soft landings on the Martian surface, with the 2021 Perseverance mission constituting the most recent U.S. success. (NASA, n.d.)

6.6 Numerical Demonstration of Methodology

This section provides a numerical demonstration of the methodological framework using the mathematical symbolic demonstration of the previous section as the foundational algorithm. The Python software package implemented in a Spyder integrated development environment (IDE) provides the computational capability for this demonstration; although a specific language and IDE are used for this demonstration, the methodological framework presented in this work is agnostic to any particular computational implementation. Any parameter values used in this demonstration are purely notional and only for the purpose of demonstrating the potential of the methodology of the previous section of this article. For simplicity and clarity, these scenarios assume each fighting unit of each capability consumes an equal amount of purchasing power; belligerents are not necessarily assumed to hold equal purchasing power.

Given parity, the distribution of outcomes for a Lanchester Square Law scenario under this framework follows a pronounced bimodality as shown in Figure 6.4. Values along the x-axis indicate the numerical discrepancy between the two belligerents at the end of the interaction while the y-axis values indicate the number of times such a discrepancy occurred. This divergence is caused by the bifurcating nature of the Lanchester Square Law which represents the underlying phenomena of aimed combat. That is, unless the dynamics of the conflict change or a latent advantage begins to manifest, small advantages will tend to snowball into increasingly larger advantages. This provides a theoretical foundation for explaining the extraordinary difficulty of predicting the outcome of a decisive peer conflict using aimed fire; approximately mean outcomes of such a conflict are extremely unlikely to occur. Attrition coefficients assume an expected value and variability of one.

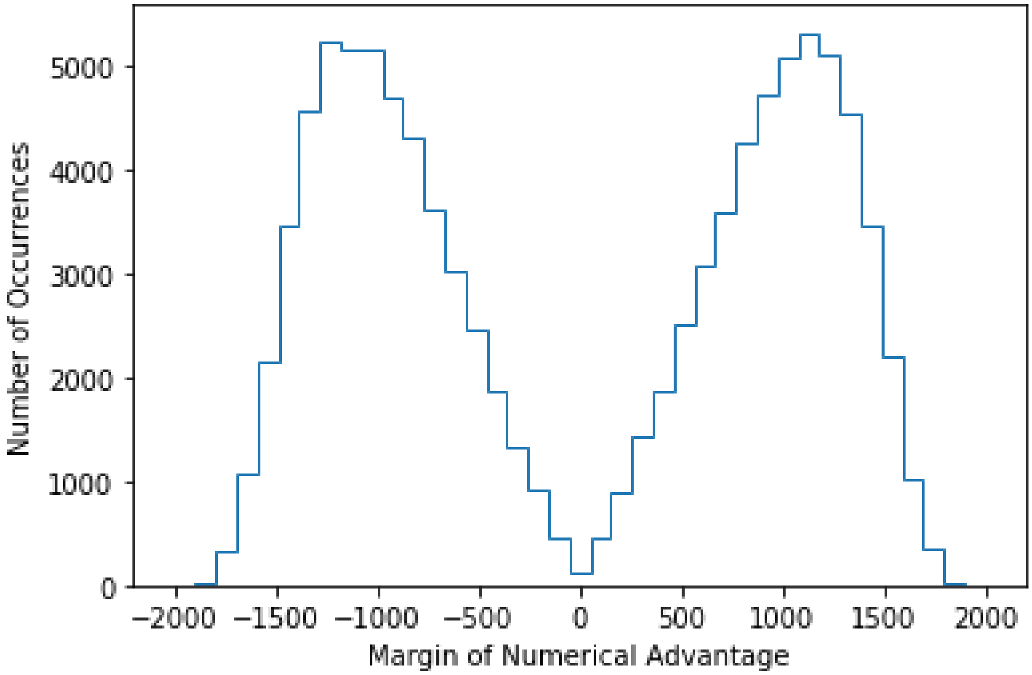


Figure 6.4. Bimodal Lanchester Square Law Outcome Distribution – 100,000 Runs

Given parity, the distribution of outcomes for a Lanchester Linear Law scenario under this framework presents a trapezoidal shape as shown in Figure 6.5. Unaimed combat does not experience the snowball effect of aimed combat; outcomes within a range of numerical parity are approximately equally likely. Attrition coefficients assume an expected value and variability of one.

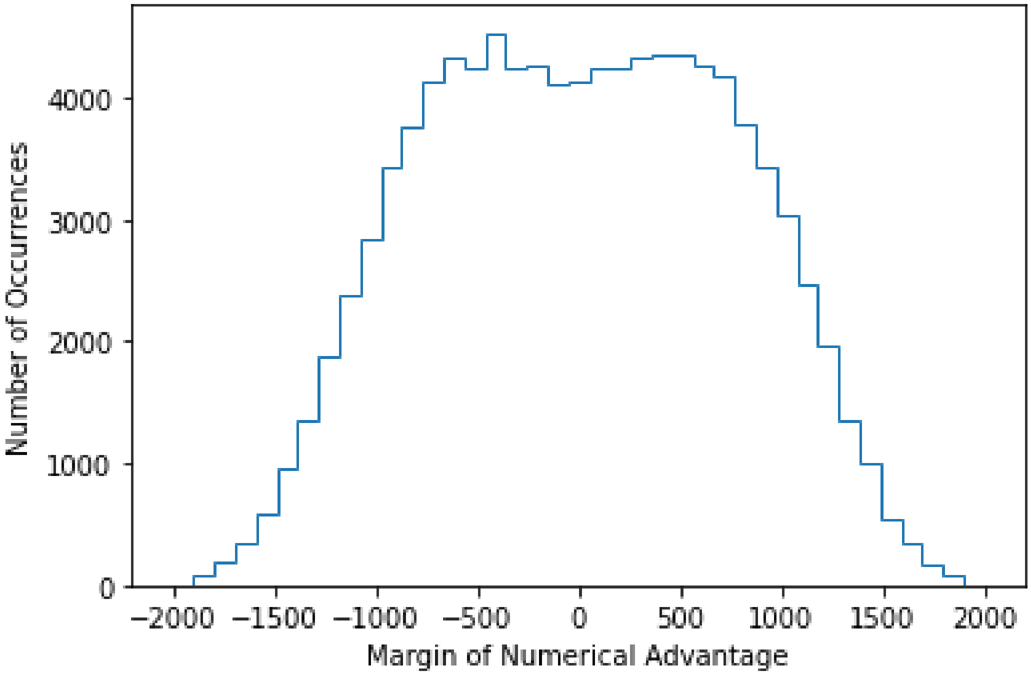


Figure 6.5. Trapezoidal Linear Law Outcome Distribution – 100,000 Runs

Given parity, the distribution of outcomes for a Brackney Law scenario under this framework presents a shape in between the square and linear law distribution shapes as shown in Figure 6.6. This result is expected as the Brackney Law behavior shifts from linear law behavior to square law behavior as the conflict phases from kill-time dominated to search-time dominated. Attrition coefficients assume an expected value and variability of one.

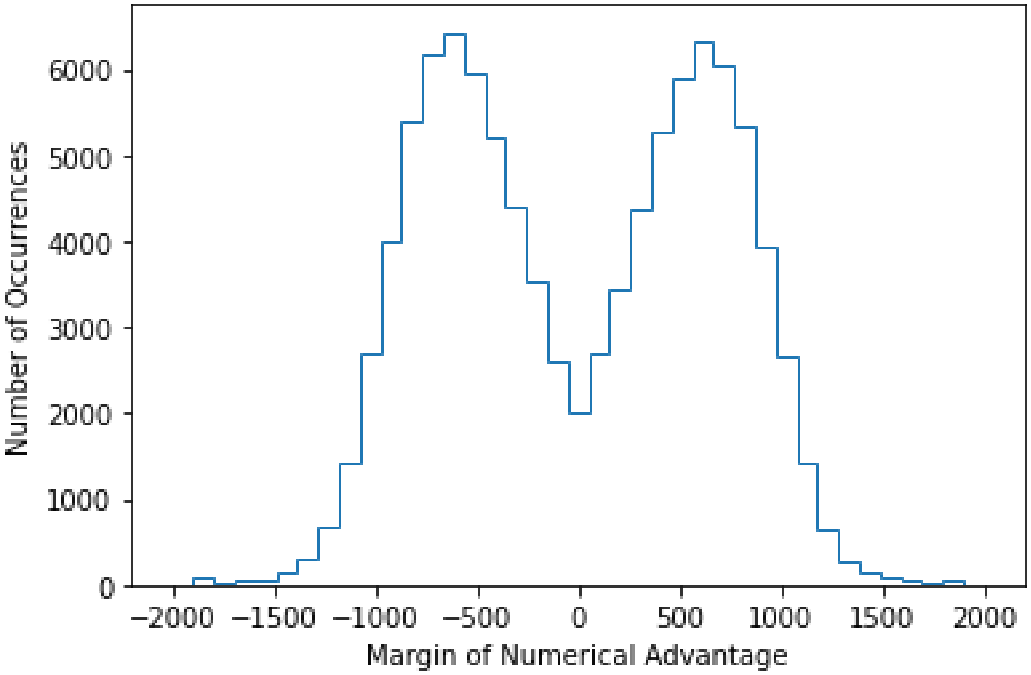


Figure 6.6. Brackney Law Outcome Distribution – 100,000 Runs

Given parity, the distribution of outcomes for a Hybrid Law scenario under this framework presents a uniform shape with two spikes at the extremes as shown in Figure 6.7. The uniform component of the distribution represents stable equilibria while the spikes represent extinction events. Attrition coefficients assume an expected value and variability of one while carrying capacities assume a value of 2,000.

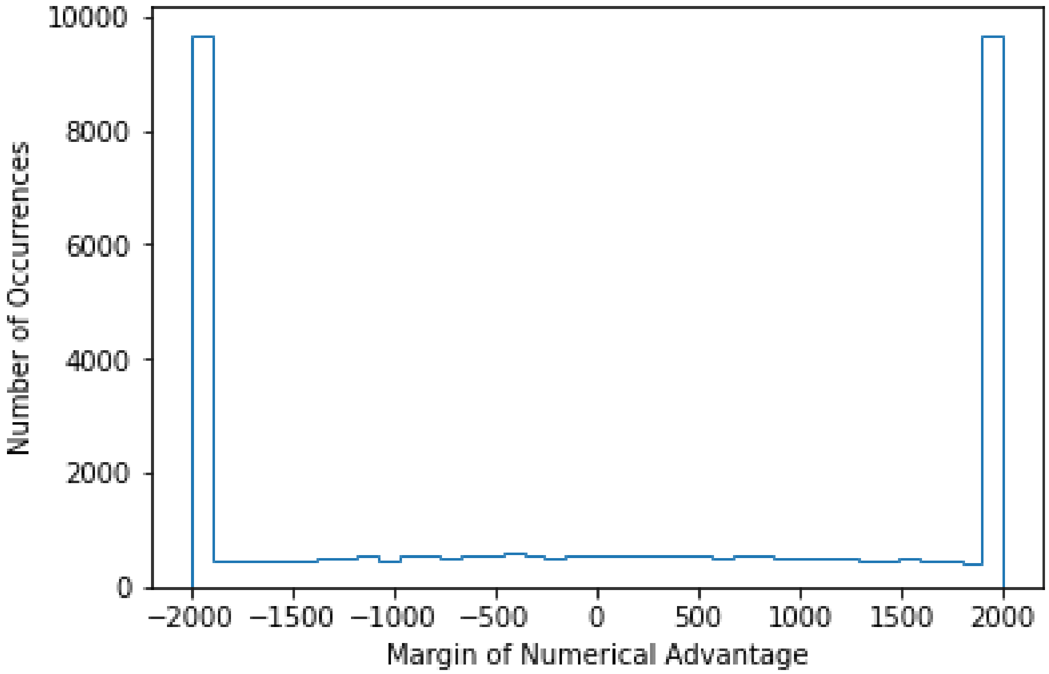


Figure 6.7. Hybrid Law Outcome Distribution – 100,000 Runs

Figures 9 through 11 afford further understanding of how parity shapes outcome distributions – each figure displays the results of a Lanchester Square Law conflict over 10,000 simulation runs in which positive numerical values along the x-axis indicate a blue numerical advantage at the end of the conflict while negative values indicate a red numerical advantage. Figure 6.8 demonstrates the case of qualitative and quantitative parity. Attrition coefficients assume an expected value and variability of one.

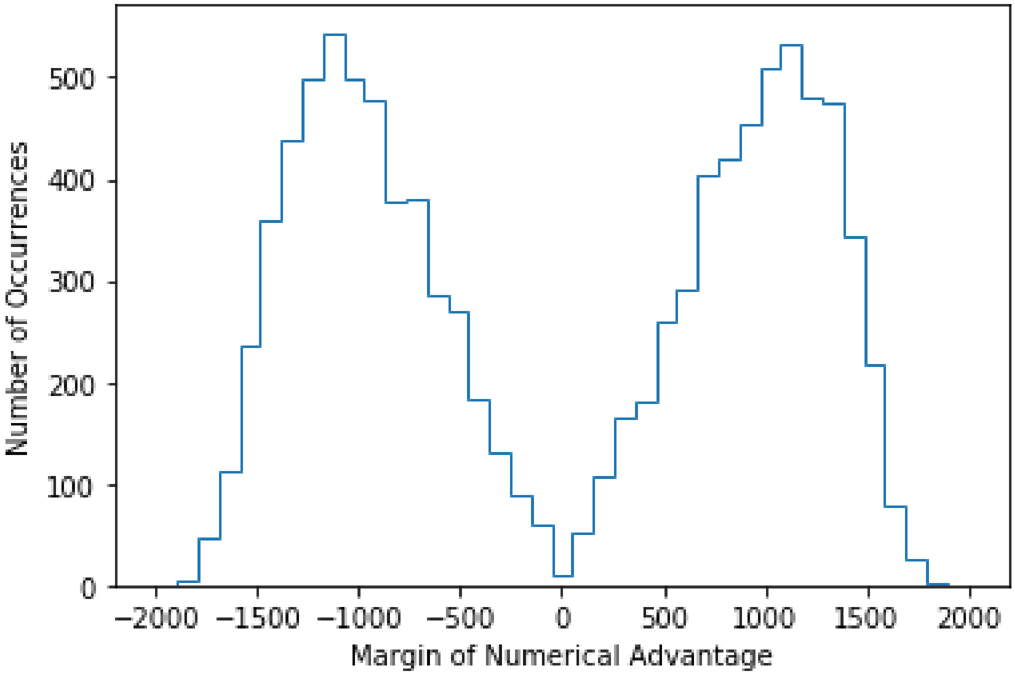


Figure 6.8. Balanced Parity

Figure 6.9 illustrates the case wherein the expected quality of blue forces is four times greater than that of red forces but red presents a force which is quantitatively twice as large as blue forces. Blue boasts 500 fighting units of each capability while red boasts 1,000 fighting units of each capability. All red specific attrition coefficients assume an expected value of one while blue attrition coefficients assume an expected value of four; the variability of all attrition coefficients is one.

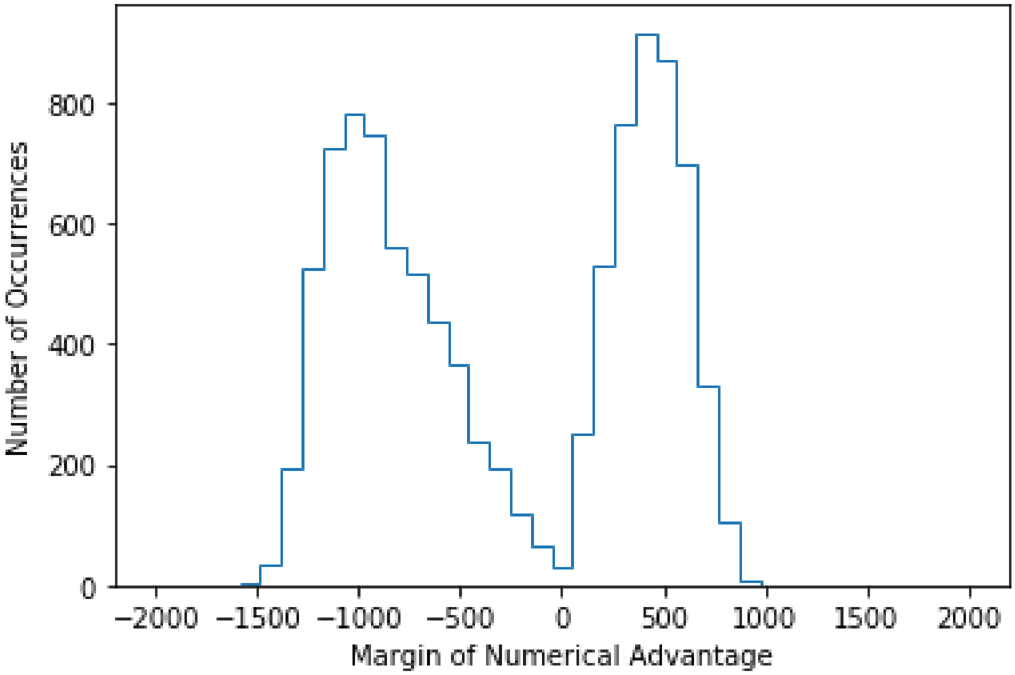


Figure 6.9. Imbalanced Parity

Figure 6.10 illustrates the case wherein red is provided with a ten percent boost to the quantity and quality of its forces relative to blue – conferring such an advantage significantly impacts the distribution of potential conflict outcomes. Blue boasts 500 units of each capability with expected attrition coefficients of one. Red boasts 550 units of each capability with expected attrition coefficients of 1.10.

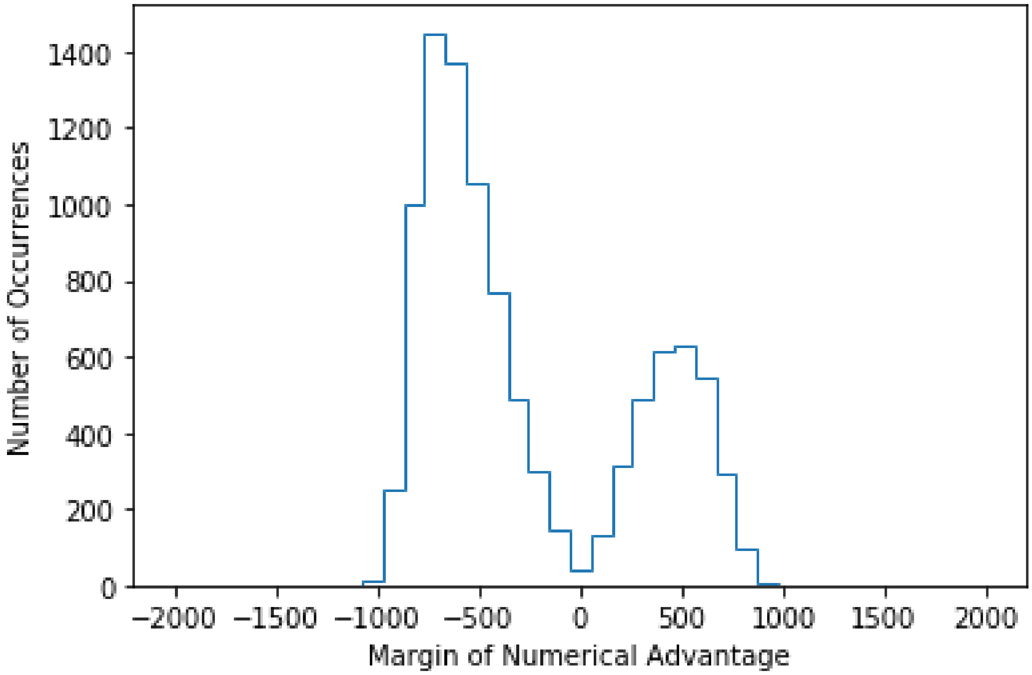
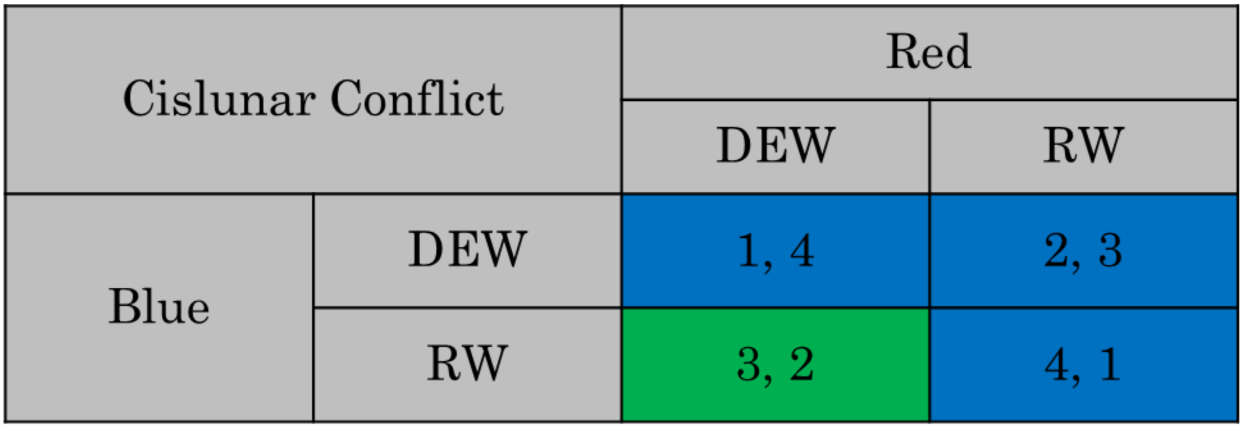


Figure 6.10. Lack of Parity

While Figure 6.10 illustrates a straightforward way for a belligerent to gain a numerical advantage in conflict, more nuanced methods do exist. Consider a space conflict scenario between blue and red using directed energy weapons (DEW) and robotic weapons (RW) in which blue fields capabilities with expected pairwise attrition coefficients of one, two, three, and four while red fields capabilities with coefficients four, three, two, and one as shown in Table 6.2. Consider that blue and red present the same number of total fighting units.

Table 6.2. Cislunar Conflict Game



Cislunar Conflict		Red	
		DEW	RW
Blue	DEW	1, 4	2, 3
	RW	3, 2	4, 1

In such a case, blue DEW perform poorly against red DEW and marginally against red RW; blue RW perform well against red DEW and dominantly against red RW. While such a scenario may seem completely balanced, blue holds an incredibly advantageous position through game-theoretic system design since blue RW versus red DEW exists as a pure strategy Nash equilibrium. However, even in such an advantageous position, blue must still present sufficient competence in force composition to finish the conflict in a numerically advantageous position. For example, if blue invests in 1,500 DEW and 500 RW while red invests in 500 DEW and 1,500 RW, red will likely end the conflict in a numerically advantageous position as shown in Figure 6.11.

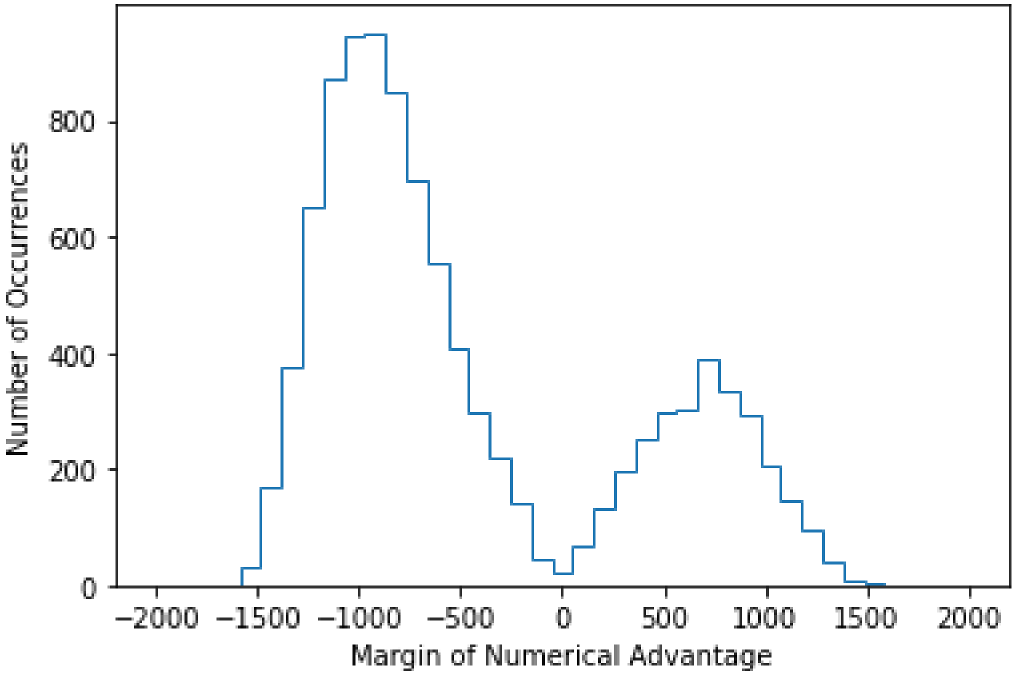


Figure 6.11. Resource Parity, Suboptimal Blue Force Composition

A competent opponent will take advantage of unforced game-theoretic errors.

6.7 Conclusion

This work presented a methodological framework for parametric combat analysis in order to improve the security posture of the United States. After discussing pertinent literature and the methodological framework as well as contextualizing this work to the space domain, this article provided demonstrations of the framework. Though contextualized in this work to the space domain where necessary, this methodological framework could be applied to any domain of combat given that the user characterizes the underlying phenomena with the correct mathematical representations.

The author recommends the U.S. DoD acquisitions community uses this methodological framework to accelerate the capability development tempo to an operationally relevant timeframe. The flexible modularity of this framework enhances the ability of the user to customize the modeling and simulation as necessary. The framework is compatible with game-theoretic concepts which can help the user to optimize force compositions and predict agent behavior. While not a wargame in and of itself, this framework could be used as a wargaming tool to bolster objective analysis. The United States faces multiple peer competitors and is steadily losing military preeminence. Spending significant resources on national defense is no longer sufficient; the United States must invest in capabilities wisely to effectively compete in an increasingly multipolar world.

Chapter 7: Conclusions and Recommendations

7.1 Conclusions and Significance of Research

In answering the question on how game theory can inform spacepower, this dissertation asserted that an agent could play optimally in a strategic-normal form game given knowledge of agent behavior and payoffs. While game theory has long been used for decision-making, the Atomic Competitive Element (ACE) taxonomy articulated the interplay between the nature of the relationship of the agents and the different optimization strategies. Through this process, eight fundamental ACEs were formed and contextualized to the emerging space warfare domain. These ACEs afford game-theoretic heuristics to the game player or designer so as to play or design the game for the benefit of the preferred agent. In doing so, this work created a new taxonomic structure for understanding the tension between factors of stability in game-theoretic outcomes and used that structure to advance strategic thought on spacepower

In answering the question on how aggregated combat models can be applied to space conflict, this dissertation adapted and extended parametric modeling to a three-dimensional cislunar combat environment and used those combat models to advance strategic thought on spacepower. Specifically, this dissertation explored the works of Lanchester, Lotka, Volterra, Brackney, and Bonder while creating the Lanchester-Lotka-Volterra Hybrid Model and adapting Brackney's work to a three-dimensional environment. Such innovations enable the application of these models to the space warfare environment. The application of the three-dimensional Brackney model demonstrated the importance in understanding whether an environment and the occupying belligerents compose a search-time-dominated or kill-time-dominated battlespace.

In answering the question on what similarities exist between the reliability models of large and small spacecraft, this dissertation concluded that the Weibull distribution and at times its more specific form, the exponential distribution, can be useful in characterizing the reliability of both large and small spacecraft. This work applied established reliability modeling methods to a previously unevaluated data set to characterize the reliability behavior of small spacecraft. This process determined the presence of beneficial aging of which the Weibull distribution was able to effectively characterize. In doing so, this work advanced the Kinetically-Aggregated Infrastructure Revitalization of Spacecraft (KAIROS) concept.

In answering the question on how game theory, aggregated combat models, and reliability models can be integrated to provide analysis for space combat, this dissertation created an integrated parametric methodological framework capable of evaluating the efficacy of heterogeneous force compositions in an elegant, flexible manner within an operationally relevant timeframe. Furthermore, this research provided strong evidence for the mutual validation between Media Interaction Warfare Theory and the methodological framework presented in this work. In creating this framework, this research affords decision-makers the opportunity to enhance the U.S. space security posture and promote the safety and security of the United States. This work concludes that decision-makers of the U.S. military should provide an impetus to the force modernization communities (especially those of the U.S. Air Force and U.S. Space Force) to prioritize the development of a force composition capable of deterring or winning a space conflict with a peer competitor. Modeling efforts such as the methodological framework presented in this work can and should be used to prepare for conflict in the emerging space warfare domain which has no precedent of large-scale conflict.

This dissertation addressed the problem statement by demonstrating the effective distillation of heterogeneous force structures into the necessary parameter values for utilization within differential parametric combat modeling. This dissertation achieved the research objective of creating a methodological framework for the analysis of space conflict between two heterogeneous belligerents using differential parametric combat modeling.

7.2 Recommendations for Future Research

Recommendations for future research include

- applying the methodological framework presented in this work and agent-based modeling methods to different initial conditions to understand if and under what circumstances the two methods produce similar or dissimilar results
- contextualizing and applying the methodological framework to other domains of conflict besides the space warfare domain
- applying the novel game-theoretic concepts presented in this work to other spheres such as strategic games, business, or biology
- utilizing this framework to bring increased objectivity to the Department of Defense (DoD) wargaming efforts

- using this methodological framework to inform game-theoretic system design for U.S. space systems
- applying this framework to specific real-world parameters for blue system capabilities and red system capabilities

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14. ABSTRACT

This work presents a taxonomic structure for understanding the tension between certain factors of stability for game-theoretic outcomes such as Nash optimality, Pareto optimality, and balance optimality and then applies such game-theoretic concepts to the advancement of strategic thought on spacepower. This work successfully adapts and applies combat modeling theory to the evaluation of cislunar space conflict. This work provides evidence that the reliability characteristics of small spacecraft share similarities to the reliability characteristics of large spacecraft. Using these novel foundational concepts, this dissertation develops and presents a parametric methodological framework capable of analyzing the efficacy of heterogeneous force compositions in the context of space warfare. This framework is shown to be capable of predicting a stochastic distribution of numerical outcomes associated with various modes of conflict and parameter values. Furthermore, this work demonstrates a general alignment in results between the game-theoretic concepts of the framework and Media Interaction Warfare Theory in terms of evaluating force efficacy, providing strong evidence for the validity of the methodological framework presented in this dissertation.

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