5-1991

An examination of the validity of hypotheses of two theories of soybean response to phosphorus and potassium

Edwin Lewis Anderson

To the Graduate Council:

I am submitting herewith a dissertation written by Edwin Lewis Anderson entitled "An examination of the validity of hypotheses of two theories of soybean response to phosphorus and potassium." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Agricultural Economics.

Roland K. Roberts, Major Professor

We have read this dissertation and recommend its acceptance:

S.D. Mundy, W.L. Sanders, D. Howard, L.H. Keller

Accepted for the Council:

Carolyn R. Hodges

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

To the Graduate Council:

I am submitting herewith a dissertation written by Edwin Lewis Anderson entitled "An Examination of the Validity of Hypotheses of Two Theories of Soybean Response to Phosphorus and Potassium." I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Agricultural Economics.

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Luther H. Keller

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AN EXAMINATION OF THE VALIDITY OF HYPOTHESES
OF TWO THEORIES OF SOYBEAN RESPONSE
TO PHOSPHORUS AND POTASSIUM

A Dissertation
Presented for the
Doctor of Philosophy
Degree
The University of Tennessee, Knoxville

Edwin Lewis Anderson

May 1991

ACKNOWLEDGEMENTS

I have many people to thank so without further written comment; Thank you kindly to:

J.M. and D.M. Anderson for unending encouragement and mental and financial support and for your willingness to sacrifice on my behalf.

J.M. Jr., C.T., and T.W. Anderson for encouragement to get the job done and all the fun times together through the years.

R.W. Anderson for encouragement, his confidence, agreeable nature, and recreational financing.

T.C. Anderson for his care free approach that was always a refreshing change from graduate school and for recreational financing.

M.A. Willhite for his friendship, for all that fun we had together, and for all the loans.

S.A. Coady for his friendship, for all that fun we had together, and for all the loans.

Committee Members:

R.K. Roberts for ceaseless patience with me and my faults and mistakes of omission and commission.

S.D. Mundy for the way he always found something good and positive to say. This quality was also recognized by many of my contemporaries.

W.L. Sanders for his time in consultation and for the valuable time saving statistical procedure he allowed me to use.

D. Howard for his willingness to lend me any or all of his data. To me, he is truly a cooperative scientist. And for his time reviewing my manuscript.

L.H. Keller for his agreement to serve on my committee at the last minute and for his time spent reviewing and making suggestions for improvement to my thesis.

Others:

M.D. Gray for his encouragement and personal confidence as well as his valuable time working out my problems. He was very helpful to me in completing my work.

B.R. McManus for his absolutely positive outlook on all people and their circumstances. He affected my thinking enormously.

J. Martin and H. Williamson for the opportunity they gave me when many others also deserved the opportunity.

Department of Plant and Soil Science (G.M. Lessman, J. Graveel, J. Jared, S. Logan and D. Tyler)

ABSTRACT

The theoretical hypotheses concerning crop response to nutrients must be reconciled with observable facts. This reconciliation is important to scientists, crop managers, and to those who assist crop managers. In this study the theories of Liebig and Mitscherlich are examined with respect to soybean response to phosphorus and potassium on the Memphis and Henry soil types of Western Tennessee. The theories are evaluated with respect to data coherency, data admissibility, valid conditioning, and encompassment. Because a complete specification of Mitscherlich's theory results in a nonlinear statistical specification, the logarithmic function was substituted throughout the study.

The results suggest that the logarithmic equation performed well with respect to data coherency; that is, its errors were more often normally distributed and homoskedastic than those associated with the Liebig equation. However, Liebig's equation was found to be data admissible, validly conditioned, and encompassing.

The results of this research supported the general conclusion that soybeans respond to potassium more so than to phosphorus, at least on the Memphis and Henry soil types. The results also supported Liebig's hypotheses that a single ratio of phosphorus to potassium be used no matter what yield level is desired and that phosphorus and potassium do

not substitute for each other in maintaining a given yield level.

Fertilizer recommendations derived from the Liebig model were very conservative. Only prices that have never been observed for soybeans resulted in fertilizer being recommended. Fertilizer recommendations derived from the logarithmic equation were much more liberal. By comparison the University of Tennessee personnel recommendations were in between recommendations of the Liebig and logarithmic models.

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INTRODUCTION

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Tennessee soybean producers respond to phosphorus and potassium fertilizer prices (Roberts and Anderson 1989). Hence, they apply more phosphorus and potassium fertilizer on a given soybean acre when fertilizer prices are low. Similarly, they apply less phosphorus and potassium fertilizer on a given soybean acre when fertilizer prices are high. This response to phosphorus and potassium fertilizer prices by Tennessee soybean producers suggests that there are perceived economic benefits that can be reaped by adjusting the amount of phosphorus and potassium made available to soybean plants. However, soybean yield response to larger amounts of phosphorus and potassium has not been consistently found on the Vicksburg and Henry silt loam soils of West Tennessee (Howard et al. 1982). If soybean yields and quality do not respond to a higher total amount of phosphorus and potassium, then certainly there is little economic benefit in increasing the total amount of phosphorus and potassium made available to soybeans.

A goal of soil test programs is to provide farmers a fertilizer recommendation that is economically sound (Evans 1987; Eckert 1987; Tisdale et al. 1985; Whitney et al. 1985;

Olson et al. 1982). To achieve this goal, administrators of soil test programs need research results that relate crop yields and quality to the total amount of plant nutrients in the soil. They also need research results that relate the total amount of a plant nutrient in the soil to different quantities of applied fertilizer. When the natures of these two relationships are known, a fertilizer recommendation based on sound economic principles is possible.

The focus of this research is to examine the validity of hypotheses of two theories of soybean yield response to the total amount of phosphorus and potassium, ceteris paribus. The relationship between the total amount of phosphorus and potassium in the soil and quantities of applied fertilizer is not examined.

The latest in agronomic thinking is that any crop will respond to a larger total amount of plant nutrients in an exponential or logarithmic fashion (Tisdale et al. 1985; McLean and Watson 1985; Eckert 1987). This school of thought can be traced to the German scientist Zvi Mitscherlich. While this nonlinear response may be logically valid, other scientists consider linear crop response to a larger total amount of plant nutrients to be theoretically sound from a biological standpoint and computationally easier (Lanzer and Paris 1981; Sanchez and Salinas 1981; Perrin 1976; Paris and Paris 1985; Ackello-Ogutu et al. 1985; Paris and Knapp 1989; Boyd 1970; Bondorff

1924; Plessing; Boresch; Gurnow 1973; Waggner and Norvell 1979). This school of thought can be traced to another German scientist named Julius von Liebig (1855).

The methodological approach of this research is to first study some rudiments of soil fertility theory and then examine the application of these principles in the literature. Secondly, the assertions made by Liebig and Mitscherlich with respect to crop response to nutrients are examined. Then mathematical models consistent with Liebig's and Mitscherlich's assertions can be specified and estimated with a view toward hypothesis testing.

Objectives

1. Examine the validity of hypotheses of Liebig's and Mitscherlich's theories of soybean yield response to an estimate of available soil phosphorus and potassium.

2. Make soil test recommendations for phosphorus and potassium based on an empirical estimate of the theoretical model selected as having valid hypotheses, and, assumptions concerning the relationship between applied fertilizer and its transmission to soil test levels.

CHAPTER II

LITERATURE REVIEW

One of the most important problems in biometric research is to verify the appropriateness of hypotheses of competing theories that attempt to explain some phenomena (Kmenta 1986). The problem of theoretical validation is evidenced in the literature devoted to crop response to plant nutrients. Until recently few scientist concerned themselves with the theoretical validity of their models.

A perspective of crop response literature and its relationship to this research is available by studying Figure 2.01. Box A in Figure 2.01 indicates the set of literature devoted to estimating crop response functions with empirical models. Box B indicates the set of literature devoted to estimating crop response functions with biological models. Box C indicates the literature devoted to estimating crop response functions with both empirical and biological models and then evaluating model performance based on residual sum of squares. Box D indicates the set of literature devoted to estimating crop response functions with both empirical and biological models and then evaluating the validity of the models with statistical tests.

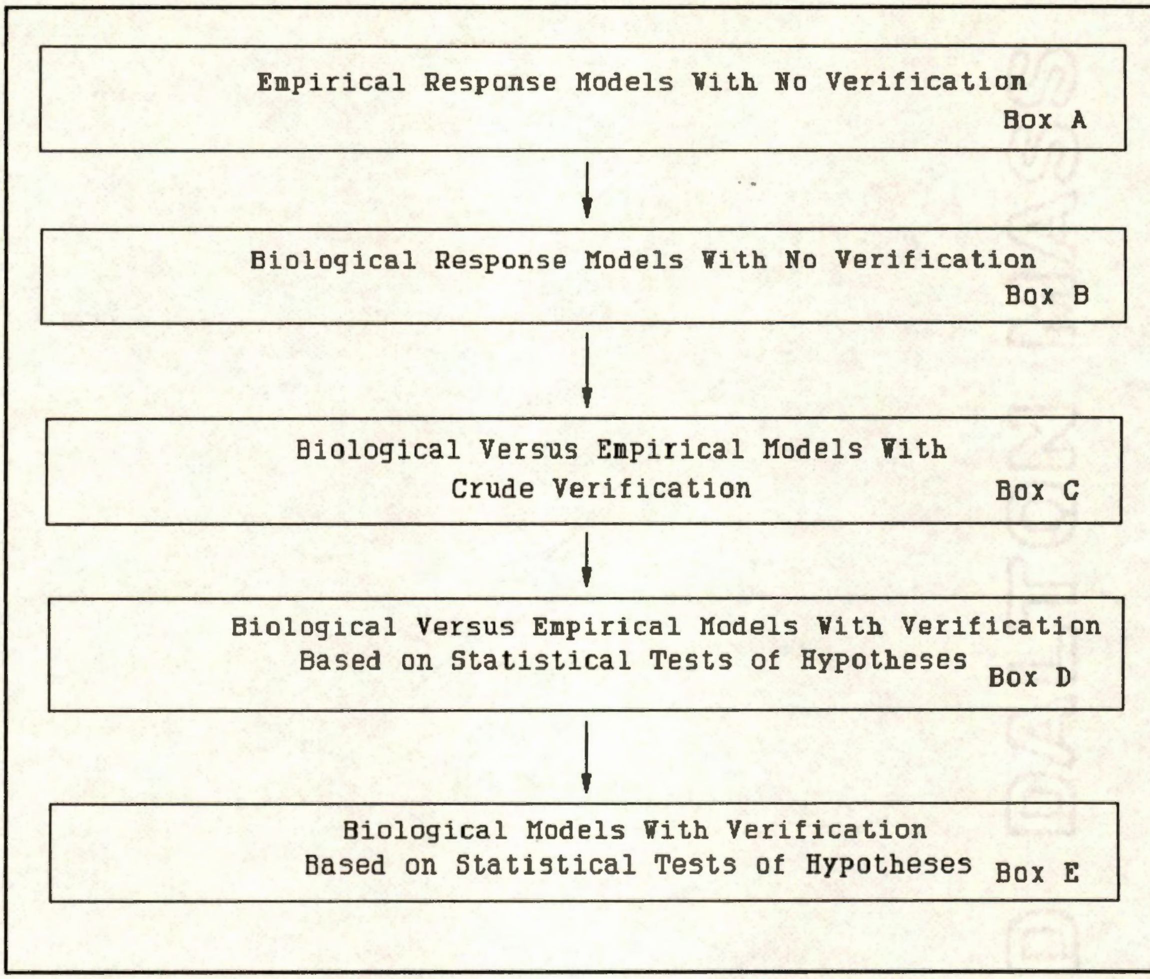

Figure 2.01 Literature Devoted To Crop Response

A neglected area of crop response-model verification literature is a comparison of the two most popular theories of crop response to plant nutrients. Box E in Figure 2.01 represents the extension of the literature attempted in this research which examines the validity of hypotheses made by Liebig and Mitscherlich with respect to crop response.

The next section provides a brief review of some important features of soil fertility theory. The purpose of this review is to assist the reader's understanding of the biological basis for the models used in the literature and in this research.

Soil Fertility Theory

A basic review of soil fertility theory should include discussions of plant growth and the factors affecting growth, plant nutrition, basic soil-plant relationships and basic soil-fertilizer relationships (Tisdale et al. 1985). This section is devoted to a rudimentary review of these topics with the exception of soil-plant relationships which will be covered later. This review of soil fertility theory will be useful in reviewing the literature represented in Figure 2.01, in specifying models, and in interpreting the results of this study.

Growth is defined as the progressive development of an organism (Tisdale et al. 1985). The progressive development of the soybean plant is important to soybean farmers and to those who recommend soybean cultural practices. The marketable portion of growth (and its quality) is important in placing value on the product; therefore, growth is often defined in terms of bushels of soybeans per acre.

Soybean growth is affected by many factors. The soybean production function given by Tisdale et al. (1985) is represented as:

$$y_s = f(GENETICS, ENVIRONMENT) \qquad 2.01$$

where y_s = growth of soybeans in terms of bushels per acre. Tisdale et al. (1985), have identified nine important environmental factors in soybean production: (1) temperature, (2) moisture supply, (3) radiant energy, (4) composition of the atmosphere, (5) soil structure and composition of soil air, (6) soil reaction, (7) biotic factors, (8) amount of plant nutrients, and (9) absence of growth restricting substances.

Farmers with limited resources are not able to control most genetic and environmental factors that contribute to soybean growth. However, a soybean production function which is subject to farmer control is written as;

$$y_s = f(\text{ Soil reaction, Amount of plant nutrients} \mid G, E) \qquad 2.02$$

where y_s is the yield of soybeans in bushels per acre, G is genetic factors, E is environmental factors other than soil reaction and amount of plant nutrients, f is the rule that transforms soil reaction and the amount of plant nutrients per acre into bushels of soybeans per acre holding genetics and other environmental factors constant, and | means that factors to its right are held constant.

There are six basic plant nutrients: carbon, hydrogen, oxygen, nitrogen, phosphorus, and sulfur (Tisdale et al. 1985; Brady 1984). In addition to these six, there are 14 other nutrients which are essential to the growth of plants including soybeans: calcium, magnesium, potassium, iron, manganese, molybdenum, copper, boron, zinc, chlorine, sodium, cobalt, vanadium, and silicon (Tisdale et al. 1985).

For soil nutrients to be useful to plants, they must come in contact with a plant root. There are three ways in which nutrients in soils can reach the root surface: (1) root interception, (2) diffusion of nutrients in the soil solution, and (3) movement of nutrients by mass movement with the soil solution (Tisdale et al. 1985; Brady 1984). Diffusion is the most important mechanism in potassium and phosphorus movement to roots (Barber and Olson 1968; Tisdale et al. 1985; Moody et al. 1988).

The diffusion of a nutrient in the soil is influenced by three principle factors: (1) volumetric water percentage, (2) tortuosity, and (3) buffering capacity. Tisdale et al. 1985, predict that diffusion will be increased by high soil moisture content because this leads to a larger volumetric water percentage and lower tortuosity. Diffusion is also increased by decreasing the buffering capacity of a soil by adding commercial fertilizers. The diffusion coefficient is also very sensitive to temperature (Tisdale et al. 1985).

Phosphorus and potassium fertilizers are often applied to soils used for soybean production. The reaction of soils to additional phosphorus and potassium is complex and dependent on many factors. Basically, the fertilizer is converted to various forms, some of which are available to plants and others which are not available to plants.

"Available" nutrients are nutrient ions or compounds in forms which plants can absorb and utilize in growth (Glossary of Soil Science Terms 1984). Figure 2.02 is a schematic illustration of the various forms of phosphorus and their exchange relationships (Chauhan et al. 1981; Sample et al. 1980). The total amount of phosphorus on a given acre of soil is the sum of the individual components represented by boxes A,B,C,D,E,F,G, and H in Figure 2.02.

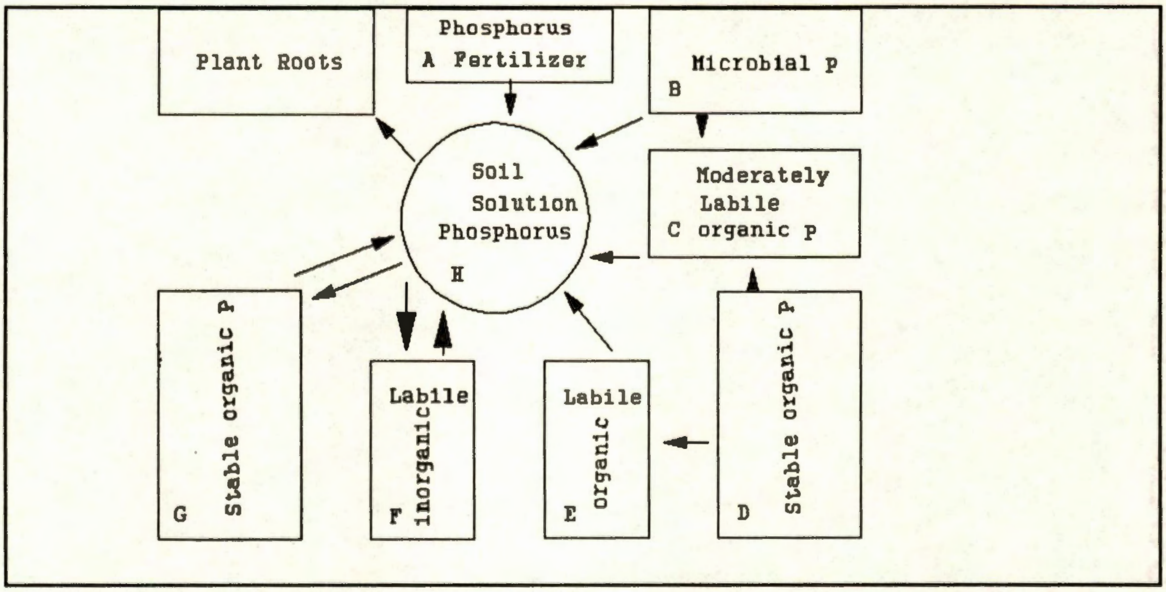

Figure 2.02 Forms of Soil Phosphorus and Their Exchange Relationships

Figure 2.03 is a schematic illustration of the forms of potassium in the soil and their exchange relationships (Tisdale et al. 1985; van Diest 1978). The total amount of potassium on a given acre of soil is the sum of the individual components represented by boxes A,B,C,D,E,G,H minus F in Figure 2.03.

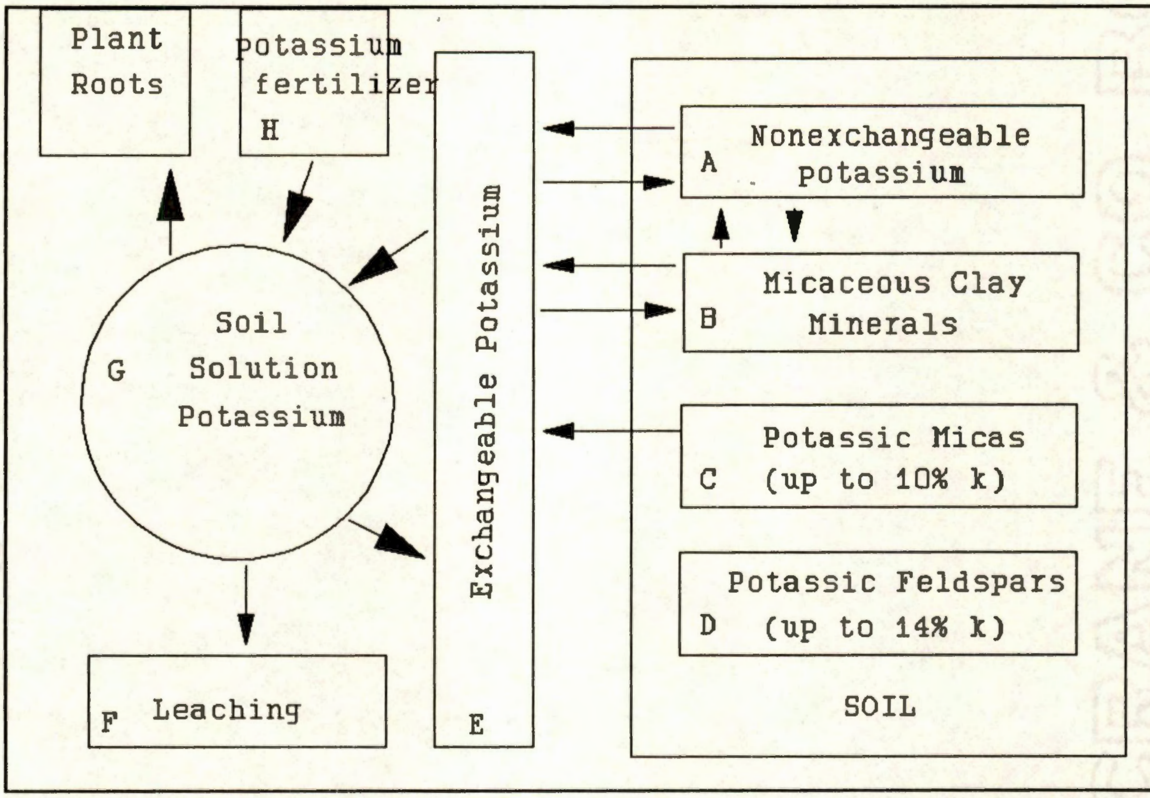

Figure 2.03 Forms of Soil Potassium and Their Exchange Relationship

A soil test is a chemical, physical, or biological procedure which estimates a property of the soil pertinent to the suitability of the soil to support plant growth (Glossary of Soil Science Terms 1984). The "property" which soil tests estimate is the amount of inorganic phosphorus and the amount of exchangeable potassium. For example, a chemical soil test for phosphorus extracts the form of phosphorus represented by boxes G plus F in Figure 2.02, while a chemical soil test for potassium extracts the form of potassium represented by box E in Figure 2.03. The amount of phosphorus and potassium extracted depends on which chemical procedure is used by the laboratory and the amount of phosphorus and potassium present in the soil sample.

Extracting phosphorus and potassium from soil samples provides quantitative data which can then be related to yield response. The yield relationships developed can then be used to assist farmers in planning management practices.

Review of Literature

The literature underlying Figure 2.01 is extensive, especially the sets of literature represented in boxes A and B. By comparison, the literature devoted to comparing different models, represented by boxes C, D and E, is small.

Each box of Figure 2.01 will be reviewed with respect to theoretical consistency, data coherency, valid conditioning, data admissibility and encompassment. Theoretical consistency is achieved when crop response model specifications conform to the precepts of soil fertility theory. Theoretical consistency ensures that estimated parameters carry specific interpretations. Data coherency is achieved when the error component of the model specification is homoskedatic, nonautocorrelated, and normal. Valid conditioning is concerned with whether the explanatory variables are correlated with the error term. Data admissibly is concerned with whether the data could have possibly been generated by the model in question. The concept of encompassment is concerned with how well given models predict the performance of alternative models (McAleer 1987; Gujarati 1988).

Box A (Figure 2.01) represents literature devoted to estimating empirical response models (ERM). An ERM is one for which the parameters have no biological interpretation, although some scientist have shown the possibility for interpretation (Niklas and Miller 1927; Fisher 1983). The most frequently used ERM is the second-degree polynomial. The review of literature represented by box A included 25 second-degree polynomial response models. The popularity of this model is attributable to several factors: (1) estimation is easy; (2) generalizations are easy; (3) a maximum is reached; and (4) the response pattern can be adjusted by making appropriate transformations (Mead and Pike 1975). The disadvantages of the second-degree polynomial as a response function include: (1) using a smoothing function with no biological justification; (2) extrapolation is tricky; (3) it is symmetric about the maximum; and (4) no asymptotic form is possible. In general, the literature in box A can be summarized as the estimation of nontheoretical crop response functions to applied fertilizers with no concern for data coherency, valid conditioning or data admissibility. The nature of analysis ruled out concern for encompassment. If the final objective of crop response research is to make economic fertilizer recommendations, then these response functions are practically worthless to scientists responsible for making general recommendations.

While the majority of research in box A considered only applied levels of fertilizer, some researchers recognized the importance of initial soil test levels. Two University of Tennessee agronomists, O.H. Long and W.M. Walker (1965), estimated soybean yield response to initial phosphorus and potassium soil test levels on Loring, Calloway, and Hatchie soil types. However, they did not present their model specification and estimating procedure; hence, evaluation of the theoretical consistency, data coherency, valid conditioning and data admissibility characteristics of their model was difficult. One important conclusion from their research was that the same soybean yield level could be obtained from different combinations of phosphorus and potassium soil test levels.

Peevy et al. (1972) estimated soybean yield response to soil test phosphorus, applied phosphorus, and pH on a Oliver silt loam soil. They concluded that soybean yields at a soil pH of 5.1 would be low at any soil phosphorus level and with or without any additional application of phosphorus. Increasing the pH to 5.6 improved yields by 6 to 8 bushels with or without additional applications of phosphorus. Increasing the pH to 6.4 resulted in even higher yields than those associated with pH levels of 5.6 or 5.1, even without phosphorus fertilizer. Finally, the highest yields were produced at a pH of 7.0. The yields were higher at pH 7.0 without fertilizer than they were with phosphorus fertilizer applied at any of the lower pH levels. There was no response to fertilizer at pH 7.0. Furthermore, Peevy et al. suggested that soil phosphorus should be kept at 70 to 80 pounds per acre for good soybean yields.

Howard et al. (1982) reported the results of soybean yield response to phosphorus and potassium fertilizer applications on the Vicksburg and Henry silt loam soils of West Tennessee. In addition, they compared the effects of one small annual application versus a large single application of phosphorus and potassium on soybean yields. They concluded that the two soil types differed in their response to fertilization. Yields on the Vicksburg soil were not increased with fertilization, while yields on the Henry soil did respond to potassium fertilization. The large single fertilization rates did not increase yields above those produced by the small annual rate. Yields were not increased with phosphate fertilization on either soil even when the soil tested low for phosphorus. They concluded that 9 to 11 and 88 to 100 pounds per acre of phosphorus and potassium, respectively, were sufficient for soybean production on the Vicksburg soil. Also, 9 to 12 and 73 to 105 pounds per acre of phosphorus and potassium, respectively, were sufficient for soybean production on the Henry soil.

Box B in Figure 2.01 represents literature devoted to estimating biological response models (BRM). Two popular biological response models were developed by Liebig and Mitscherlich. In the literature review under Box B, five papers used Liebig's theory and nine papers used Mitscherlich's theory.

One advantage of BRM's is their theoretical consistency and interpretable parameters. However, the scientists in this set of literature do not concern themselves with data coherency, valid conditioning or data admissibility. The nature of analysis ruled out interest in encompassment.

The majority of contributors to the research in Box B include soil test values in crop response functions. Rouse (1965, 1968) estimated a modified Mitscherlich soybean response equation to soil test values of phosphorus and potassium on three groupings of soil type. The first soil type group included the soils of the Sandy Coastal Plain. The second soil type group included soils of the Clay Loam Coastal Plain, the Piedmont, the Appalachian Plateau, the Highland Rim, and the Black Belt. The third soil type category included the red soils of the Limestone Valley and acid soils of the Black Belt. His results suggested 40 pounds of phosphorus per acre were sufficient to produce 100 percent of relative yield on soil groups 1 and 2 and 20 pounds of phosphorus per acre on soil group 3. His results also suggested that 70, 100 and 140 pounds of potassium per acre were sufficient to produce 100 percent of relative yield on soil groups 1, 2, and 3, respectively.

Lanzer et al. (1987, 1981, 1981) considered soybean response to phosphorus and potassium soil test levels and applied fertilizers in a dynamic setting using a modification of Liebig's theory which allowed for diminishing marginal returns. Their results indicated that 19 pounds of phosphorus per acre and 139 pounds of potassium per acre were optimal for soybeans grown in Brazil. Furthermore, initial soil fertility conditions did not affect the computed optimal soil fertility targets for phosphorus and potassium.

Box C in Figure 2.01 represents literature devoted to estimating BRM's and ERM's and subsequently comparing the models based on the residual sums of squares. Heady and Pesek (1954) estimated corn response to applied nitrogen and phosphorus using 35 different response functions. The response functions included both single input regressions and two input regressions. They found that the two-input quadratic square root response function provided the best fit but that the square root function seemed the best all around equation for predicting single input relationships. The only biological function estimated was of the Mitscherlich type. However, it was not selected as best and, therefore, not used in subsequent analyses.

Cate and Nelson (1971) described a method for establishing soil test ratings as high, medium, or low for the purpose of making cotton fertilizer recommendations. Their procedure was to split the data into two groups using successive tentative critical levels to ascertain the particular critical level that maximized overall predictive ability (R^2) with the means of the two groups as the predictor values. They compared their procedure with several continuous correlation models such as the Mitscherlich, linear, quadratic, logarithmic, and reciprocal. Their results suggested that none of the continuous correlation models gave as high an R^2 as their procedure.

Anderson and Nelson (1975) compared the optimal nitrogen recommendation for corn on the basis of the quadratic, square root, and linear plateau models. Their results indicated that the quadratic and square root models had a upward bias and did not fit the data well. The linear plateau model was advocated as the appropriate model from the standpoint of estimation, recommendations, and minimization of ill effects on the environment.

Perrin (1976) suggested an approach to determine the value of alternative corn response models. He estimated a quadratic and a linear plateau model and concluded that the linear plateau model could provide fertilizer recommendations as valuable to the farmer as those from the quadratic model. He also concluded that soil test information was valuable regardless of which model it was incorporated into.

Waggoner and Norvell (1978) applied Liebig's Law of the Minimum in fitting the yield of corn to applied nitrogen and phosphorus fertilizer. They used a methodology proposed by Cate and Hsu (1978) to fit the model using ordinary least squares. Essentially, the procedure allowed for assigning values to the parameters and then evaluating which assignment maximized R^2. They compared this maximized R^2 with the R^2 obtained from the quadratic, logarithmic and square root specifications and found that the Law of the Minimum (Tisdale et al. 1985) produced the highest R^2.

Johnson (1953) estimated corn response to applied nitrogen using a quadratic, logarithmic and Mitscherlich model. He found that the polynomial fit the observations much more closely than either the logarithmic or the Mitscherlich models and, therefore, suggested it be used for interpolation. He also suggested that if extrapolation was desired the Mitscherlich equation would be more appropriate because of its biological basis.

Hall (1983) estimated soybean response to lime using the square root, logarithmic, linear plateau and quadratic spline models. He found that the linear plateau model had the highest R^2 of all the models.

Neeteson and Wadman (1987) estimated sugar beet and potato response to applied nitrogen with quadratic and Mitscherlich type models. He found that for both sugar beets and potatoes the Mitscherlich model was much better than the quadratic model on the basis of the residual sum of squares.

Grove et al. (1987) evaluated the soil potassium-soybean response relationship on three experimental sites where significant responses to applied potassium were observed. They evaluated the quadratic, logarithmic and Mitscherlich models on Belknap, Maury and Tilsit soils. The Mitscherlich model provided the best fit for soil potassium on the Belknap and Maury soils while the quadratic provided the best fit for the Tilsit soil. Their results indicated that 95 percent of relative yield was attainable with a potassium soil test of 55, 95, and 73 mg kg^{-1}, on the Belknap, Maury and Tilsit soils, respectively. Another conclusion was that once the solution phase of potassium increases to a certain level relative to manganese, potassium had little or no impact on soybean yields.

Box D in Figure 2.01 represents the literature where a BRM was forced to compete with a ERM in statistical tests. There are two pieces of literature in this box. Ackello-Ogutu et al. (1985) estimated corn response to soil phosphorus and potassium plus applied fertilizer. They were forced to estimate soil phosphorus and potassium levels by specifying and estimating phosphorus and potassium carryover functions since soil test data were not taken on a yearly basis. The sum of soil phosphorus and potassium plus that applied in the fertilizers was then used to estimate square root, quadratic and Law of the Minimum response models for each year of corn response data.

Ackello-Ogutu et al. (1985) tested each of the hypotheses of Law of the Minimum versus square root, square root versus Law of the Minimum, Law of the Minimum versus quadratic and quadratic versus Law of the Minimum with two nonnested tests proposed by Cox (1961) and MacKinnon et al. (1983). Ackello-Ogutu et al. (1985) found that, for each hypothesis tested the Law of the Minimum failed to be rejected whereas the square root and quadratic models were rejected.

Grimm et al. (1987) tested the Law of the Minimum against quadratic, square root and three-halves response functions for corn, silage, wheat, cotton and sugar beets responding to nitrogen and water. Like Ackello-Ogutu et al. (1985), they used Cox's test and a variant of this test proposed by Godfrey and Pesaran (1983) to accommodate small sample sizes. Grimm et al. (1987) failed to reject the Law of the Minimum for three of the five crops using Cox's method and they failed to reject the Law of the Minimum for all five crops using Godfrey and Pesaran's method.

Finally, there is no research that this scientist has located that could be placed in Box E. Hopefully, this research will provide the extension into Box E. Like some of the current literature, this research estimates soybean response to an estimate of the available forms of phosphorus and potassium. Unlike the current literature, this research is concerned with the hypothesis that a given BRM is more appropriate than another BRM. Unlike the current literature, this research is concerned with data coherency, data admissibility and the specification errors of omitted variables, incorrect functional form and simultaneous equations.

Theories of Liebig and Mitscherlich

Liebig's theory, called the Law of the Minimum, was the first attempt to define the fundamental relationship between nutrient availabilities and crop yield. Liebig (1855) stated the law of the minimum in these words;

> Every field contains a maximum of one or more and a minimum of one or more nutrients. With this minimum, be it lime, potash, nitrogen, phosphoric acid, magnesia or any other nutrient, the yields stand in direct relation. It is the factor that governs and controls yields. Should this minimum be lime, yield will remain the same and be no greater even though the amount of potash, silica, phosphoric acid, etc. be increased a hundred fold.

The assumptions on which Liebig's theory rest are:

1. The relationship between minimum nutrients and crop yields is that of constant productivity. In other

words, each unit increase in the amount of the minimum nutrient relative to the fixed factor land, adds an equal amount to total yield up to some maximum. Technically, this assumption is referred to as linearly homogeneousness. After the maximum yield is reached, increasing the ratio of this nutrient to the fixed factor land will add nothing to yield.

2. The "best" ratio of two nutrients like phosphorus to potassium given the fixed factor land, does not vary with the absolute level of yield.
3. Any two nutrients are technically independent, unless both nutrients are in the minimum. If two nutrients are in the minimum they are technically complementary.
4. By virtue of 3, the marginal rate of substitution between any two nutrients is zero, hence the elasticity of substitution is zero.

Graphically, the first assumption is depicted in Figure 2.04. Consider a unit of land where all nutrients but one are sufficiently high such that phosphorus is the minimum nutrient. Carefully notice that if phosphorus is not only the minimum nutrient but its total amount on the unit of land is zero, no yield is forthcoming per Liebig's theory. If the amount of phosphorus per acre is increased from O to A, Liebig's theory suggest that yield will

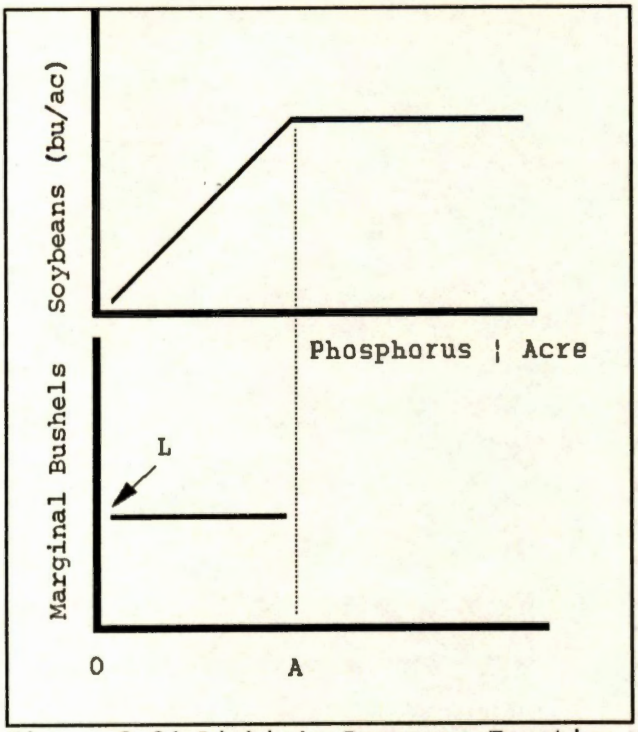

Figure 2.04 Liebig's Response Function

increase in a linear fashion up to the amount of phosphorus at point A. Increasing the amount of phosphorus per acre beyond point A will have no effect on yield. Stated in an equivalent fashion by the bottom part of the Figure 2.04, each unit increment of phosphorus in the range OA adds a constant amount to yield. Beyond point A, each unit increment of phosphorus adds nothing to soybean yield.

"Best" in Liebig's second assumption means that a given soybean yield per acre could not be obtained by a smaller quantity of any one minimum nutrient given the level of all others. This second assumption implies that the response function is homothetic, in other words, that the MRS along a given ray out of the origin, does not change as yield increases. Liebig's theory suggests that no matter what yield level is desired, the best ratio of phosphorus to potassium is fixed.

Liebig's third assumption of technical independence is related to the first assumption. This third assumption means that the marginal productivity of minimum nutrients is not affected by increasing the amount of nonminimum nutrients.

The lack of nutrient interaction in Liebig's third assumption gives rise to the fourth assumption. Liebig's theory suggests that any attempt to increase soybean yield by increasing the amount of phosphorus without concomitantly increasing the amount of potassium per acre will meet with failure. To increase soybean yield per acre, combinations of phosphorus and potassium along a single ray out of the origin must be selected. In other words, each nutrient plays a unique role in soybean yield formation and no other nutrient can be substituted into this role.

In summary, Liebig's theory supposes that plant nutrients must be combined in fixed proportion and that this fixed proportion does not vary with the yield desired. Secondly, plant nutrients do not substitute for one another, hence no additional crop yield will be forthcoming if the amount of one nutrient is increased relative to another. Finally, the marginal product schedules of plant nutrients are independent of one another unless both are at the minimum.

Many scientists have provided model specifications, methods of estimation, and empirical support for Liebig's theory (Lanzer and Paris 1981; Sanchez and Salinas 1981; Perrin 1976; Paris and Paris 1985; Ackello-Ogutu et al.

1985; Paris and Knapp 1989; Boyd 1970; Bondorff 1924; Plessing; Boresch; Gurnow 1973; Waggner and Norvell 1979). Others have demonstrated the superiority of Liebig's theory of crop response over polynomial specifications (Anderson and Nelson 1975; Lanzer and Paris 1981; Ackello-Ogutu et al. 1985; Grimm et al. 1987; Sanchez and Salinas 1981; Boyd 1970). Hence, Liebig's theory has been cited as being more appropriate for making recommendations to farmers (Anderson and Nelson 1975; Lanzer and Paris 1981; Ackello-Ogutu et al. 1985).

Mitscherlich's theory, called the "Law of Physiological Relationships" states (Tisdale et al. 1985):

> Yield can be increased by each single growth factor even when it is not present in the minimum as long as it is not present in the optimum.

Mitscherlich's theory is based on the assumptions that:

1. Response to the amount of any nutrient is negatively exponential hence marginal response to any nutrient will diminish but, always remain positive.
2. Proportionate increases in all nutrients do not give proportionate increases in yield, i.e., decreasing returns to scale hold.
3. The "best" ratio of any two nutrients like phosphorus and potassium can vary with the absolute level of yield.

4. Two nutrients are considered to be technically complementary.

5. By virtue of (4), the marginal rate of substitution between any two nutrients diminishes, hence the elasticity of substitution is greater than zero.

Graphically, Mitscherlich's first assumption is depicted in Figure 2.05 which illustrates Mitscherlich's belief that crop response to nutrients is negatively exponential. Hence, Mitscherlich's response function approaches, but never reaches, the maximum attainable yield level. Stated equivalently in the lower graph of Figure 2.05, each successive increase in phosphorus and potassium will add a positive amount to yield, but less than its immediate predecessor. Mitscherlich and Spillman (1923) both independently suggested that crop response data exhibited this diminishing marginal yield. Some scientists objected to the implication that marginal response to

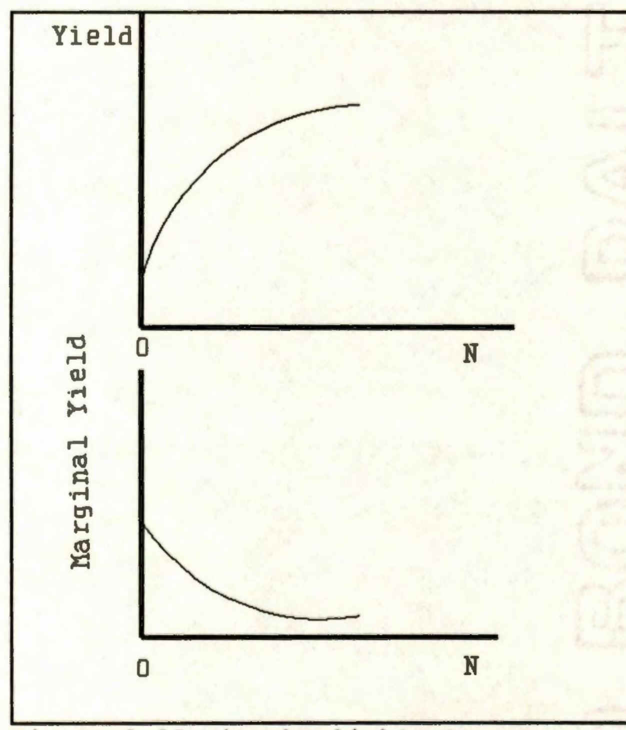

Figure 2.05 Mitscherlich's Response Function

nutrients is always positive, hence Mitscherlich respecified his response function to include a "damage factor" due to excessive amounts of nutrients (Briggs; Mitscherlich).

Mitscherlich's second assumption essentially means that the response function is not linearly homogenous.

Mitscherlich's third assumption means that the response function is not homothetic. This assumption maintains the possibility that the slopes of all isoquants along a single ray out of the origin can be different.

Mitscherlich's fourth assumption was an outgrowth of his belief that the parameter that controls the curvature of the yield response function was constant for widely varying soil and climatic conditions. Given a constant value for this parameter, logically a deficient amount of a nutrient always produces the same percentage of the maximum attainable yield. Hence, Baule (1936) concluded that when more than one nutrient was deficient, the final percentage of the maximum yield attainable was the product of the individual sufficiencies. For example, if the amount of phosphorus is good for producing 90 percent of the maximum yield and the amount of potassium is good for producing 60 percent of the maximum yield, then the final yield is 90 percent of a 60-percent yield or 54-percent of maximum yield.

Mitscherlich's fifth assumption suggests that one nutrient may be substituted for another nutrient in producing a given number of bushels per acre. "Substitution" in this context should be taken to mean that; (a) as the composition or maturity of the plant changes, it substitutes some less needed nutrients for others, or (b) the availability of one nutrient in the soil is altered as the amount of the other is increased. Neither Liebig or Mitscherlich believed that two nutrients could substitute for one another in the physiological processes of plants.

Mitscherlich's theory also has the support of many scientists and is the basis for most recommendations issued to farmers in the United States (Eckert 1987; Tisdale et al. 1985; Rouse 1968; Sabbe and Breland 1974; Bray (1954); Bray (1958); Balba and Bray 1956; Grove 1987; Hanaway and Dumenil 1953; McLean and Watson 1985; Olson et al. 1982; Paul 1986; Marking 1989; Mombiela et al. 1981; Neeteson and Wadman 1987; Benbi et al 1988; Lessman et al. 1986). Like Liebig's theory Mitscherlich's theory has been shown to be superior to polynomial specifications (Benbi et al. 1988).

The differences between Liebig's and Mitscherlich's theories that are obvious from the "Law of the Minimum" and the "Law of Physiological Relationships" include the relationship between soybeans and the amount of nutrients and the relationships between nutrients and other nutrients.

First, Liebig asserts that the yield of soybeans will increase in constant proportion to the amount of the limiting factor whereas Mitscherlich asserts that yield will increase, but at a decreasing rate, to the amount of any nutrient. Second, Liebig asserts that any two nutrients at the minimum are considered technical complements, otherwise they are technically independent, and from Mitscherlich's theory that any two nutrients are considered technical complements. Third, Liebig asserts that only one combination of any two nutrients is needed to produce a given yield whereas Mitscherlich asserts that a range of combinations of two nutrients can be used to produce a given yield. Fourth, Liebig asserts that there is one "best" ratio of minimum nutrients and that this ratio does not change with the level of yield, whereas Mitscherlich asserts that the "best" ratio depends on the level of yield.

As yet no researcher has attempted to examine the data coherency, data admissibility, valid conditioning and encompassment characteristics of Liebig's or Mitscherlich's theory within the framework of a single, continuous, fertility response experiment. However, several researchers have expressed the need for this type of research (Swanson 1971; Redman and Allen 1954; Johnson 1953; Bray 1954, 1958). This void in the literature may result from the statistical difficulties involved in directly comparing the two theories because models which incorporate the characteristics of the two theories are not linearly separable. This difficulty can be overcome by reducing the problem to a manageable level. For example, substituting a linear in the parameters function for Mitscherlich's nonlinear function.

Some scientists, that have either been frustrated by the statistical complexity of a direct comparison, or, have been unwilling to reduce the problem to a manageable level, have sought to reconcile the two theories (Bray 1954; Lanzer et al. 1981, 1987; Lanzer and Paris 1981; Swanson 1971). Bray (1954) observed that relatively mobile nutrients like nitrogen tend to follow Liebig's "Law of the Minimum" whereas immobile nutrients like phosphorus and potassium tend to follow Mitscherlich's "Law of Physiological Relationships." While this philosophy may be true, it has not been adhered to in the literature. On one hand there are estimates of nitrogen response via Mitscherlich's theory (Fuller 1965; Neeteson and Wadman 1987; Reid 1972) and on the other hand there are estimates of phosphorus and potassium response via Liebig's theory (Paris and Knapp 1989; Perrin 1976; Paris and Paris 1985; Ackello-Ogutu et al. 1985; Benbi et al. 1988; Colwell 1979). Lanzer et al. (1987) have attempted to redress the two theories by allowing Liebig's assumption of no nutrient substitution and Mitscherlich's assumption of diminishing marginal returns to both be adopted in crop response analyses. They hoped that this compromise would open a dialogue between agronomists and economists. Furthermore, data to test the imperfect nutrient substitution hypothesis were reportedly not available from agronomists because as Lanzer et al. (1987) state, "because agronomists strongly believe in Liebig's nonsubstitution hypothesis, . . . [they seldom attempt to] prove the obvious." While agronomists' fertility trials often involve only one or two nutrients, the assertion that agronomists strongly believe in Liebig's no nutrient substitution hypothesis is presumptuous. In fact, a recent survey of extension agronomists nationwide revealed that a majority of respondents adhere to Mitscherlich's hypothesis of imperfect nutrient substitution when making interpretations and recommendations (Eckert 1987).

Efforts to reconcile Liebig's and Mitscherlich's theories have added to the richness and potential explanation of crop response but judging from the following comments, these efforts skirt the fundamental issue of imperfect nutrient substitution: "The hypothesis of nutrient nonsubstitution needs to undergo a rigorous statistical test (Lanzer et al. 1987)"; "Further information is badly needed on the ranges within which physical substitution is possible and the rates of substitution within these ranges (Johnson 1953)"; "One of the relationships [that] appears to be in need of exploring pertains to the possibility of substitution among growth factors which includes nutrient substitution (Redman and Allen 1954)"; "Study of the logical implications of various concepts of plant growth is important in terms of . . . choosing appropriate procedures for disseminating results (Swanson 1971)."

Nonnested Hypothesis Tests and Methods of Model Discrimination

The theories of Liebig and Mitscherlich give rise to crop response functions which are not nested. The two response functions are nonnested because an arbitrary simple hypothesis in one cannot be obtained as a limit of simple hypotheses in the other (Cox 1961). This means that a model consistent with Liebig's theory cannot be derived by restricting the parameters of a model consistent with Mitscherlich's theory nor visa versa. Many researchers have developed and evaluated statistical tests for comparing nonnested regression models, both in the case of two linear models or the case of two nonlinear models (Godfrey and Pesaran 1983; Hall 1983; Fisher 1983; Cox 1961; Pesaran 1981, 1978, 1982a,b; Ericsson 1983; Fisher and McAleer 1981; Bernanke 1988; Gourieroux et al. 1983; Dastoor 1985; White 1982; Dastoor 1983; Mizon and Richard 1986; Godfrey 1983; Efron 1984). Much less work has been done in the case of a nonnested linear versus nonlinear regression model.

There are three methods of testing nonnested linear regression models (Godfrey and Pesaran 1983; Pesaran 1982b; Pesaran 1974); (a) the use of specification error tests, (b) the orthodox or comprehensive model method in which nonnested equations are embedded in a general specification, and (c) the use of procedures based upon the seminal papers by Cox (1961) and Atkinson (1970). In the case of a linear versus log-linear regression model, Godfrey et al. (1988) have categorized tests into three categories; (i) tests that exploit the fact that the one model is tested against a specific nonnested alternative, (ii) tests based on the Box-Cox transformation, and (iii) diagnostic tests of functional form misspecification against an unspecified alternative. Godfrey et al. (1988) examined several tests in each category for their power and their robustness to nonnormality of errors. One of their conclusions was that tests in category (i) were no more powerful in rejecting false models than those in category (iii). Also, several of the tests considered were found to be robust to nonnormality.

Economic Literature

Spillman (1923) was the first economist to examine yield-fertilizer relationships in an economic light by using a production function equivalent to Mitscherlich's specification. Spillman specified a profit per acre formula as;

$$\pi = P_y Y - P_k K - C$$

where π = profit per acre, P_y = price per unit of yield, P_k = price per unit of nutrient applied, K = the applied nutrient per acre, and C = the fixed cost of raising an acre of a crop. Spillman states that the amount of K that gives π its maximum value, i.e., the application of K that gives the greatest net return per acre, is found by differentiating π with respect to K, setting the resultant derivative equal to zero and solving for K.

Fuller (1965) estimated corn response to nitrogen via Mitscherlich's theory. Fuller also considered the role of risk and uncertainty in the decision making process of a farmer. He felt that a notable case of exposure to risk was demonstrated in fertilizer response trials where the response varies with weather conditions. He noted that in such a situation a single production function does not exist but rather each combination of inputs results in a frequency distribution of outputs, hence, he considered the parameters of a corn-nitrogen production process to be random variables.

Since Spillman's and Fuller's time, economists have seldom used Mitscherlich's theory, their preference for the polynomial specification being obvious (Heady and Pesek 1954; Heady et al. 1963; Heady et al. 1960; Hildreth 1957; Knetsch 1959; Rosegrant and Roumasset 1985; Smith and Parks 1967; Sundquist (1957); Swanson and Tyner 1965). Nevertheless, some economists have continued to present the theoretical aspects of Mitscherlich's theory (Johnson 1953; Swanson 1965; Redman and Allen 1954; Lanzer et al. 1981).

Recently, some scientists have advocated abandoning polynomial specifications in favor of Liebig's theory (Anderson and Nelson 1975; Lanzer and Paris 1981; Ackello-Ogutu et al. 1985; Grimm et al. 1987; Sanchez and Salinas 1981). Many of these scientists have shown the economics of Liebig's theory (Redman and Allen 1954; Swanson 1965; Paris and Paris 1985).

The next chapter presents the procedure for comparing models based on the crop-response theories of Liebig and Mitscherlich. The statistical tests and their calculations are also reviewed.

CHAPTER III

PROCEDURE

Model Specification

The first step in meeting Objective 1 was to estimate the parameters of Liebig's and Mitscherlich's response functions. The model specification consistent with the assumptions and characteristics that Liebig presupposed would exist in soybean response to the amount of phosphorus and potassium is given by:

$$y_{ij} = \min\ [(f_p(TP,\beta_p) \mid W_i,S_j),\ (f_k(TK,\beta_k) \mid W_i,S_j),\ (m \mid W_i,S_j)] + e_{ij} \qquad 3.01$$

where y_{ij} = bushels of soybeans per acre in the i^{th} year on the j^{th} soil type, min is an operator that selects the minimum value from within the brackets, TP = total amount of phosphorus, TK = total amount of potassium, β_p, β_k = parameters that indicate the contribution to yield from root interception, diffusion, and mass movement of the total amount of phosphorus and potassium (to be estimated), W_i = year effects on yield from factors like disease, pest, volumetric water percentage and tortuosity, S_j = soil effects from factors like percent clay, sand and silt, water retention capacity and clay morphology, m is the potential maximum bushels of soybeans per acre when factors other than root interception, diffusion and mass movement of phosphorus and potassium limit yield, f_p is a function that multiplies TP by β_p, f_k is a function that multiplies TK by β_k, | means that everything to its right is fixed, and ϵ_{ij} is the error which is a sum of negligible factors and is assumed to be identically and independently distributed around a mean of zero with a constant variance, σ^2. Equation 3.01 has all the characteristics that Liebig assumed would be present in crop response, namely, every nutrient is necessary to produce yield (if one nutrient is limiting $y_{ij} = 0$), constant marginal productivity (i.e., successive unit increases in the minimum nutrient lead to increases in soybean yields by a constant amount), technical independence, no factor substitutability between nutrients, a MRS equal to zero, one "best" ratio, and an elasticity of substitution equal to zero.

Liebig's hypothesis of proportional soybean response to the total amount of a minimum nutrient can be expressed mathematically for phosphorus as;

$$y_{ij} = f_p(TP, \beta_p | W_i, S_j) = \beta_p TP \qquad 3.02$$

where y_{ij} is bushels of soybeans per acre, β_p is the parameter that indicates the contribution to yield per pound of phosphorus, and TP is the total amount of phosphorus. Unfortunately, the quantity TP is never known. As portrayed in Figure 2.02 the total amount of phosphorus can be expressed as;

$$TP = P_A + P_B + P_C + P_D + P_E + P_F + P_G + P_H \qquad 3.03$$

where P_A is the amount of phosphorus contained in box A in Figure 2.02, etc. However, a soil test only estimates the component P_G plus P_F. Obviously if P_G plus P_F equals 0, TP is not necessarily equal to 0, hence y_{ij} will not necessarily equal zero. Properly interpreted, Liebig's proportionality hypothesis means that if the sum of all the P's on the right hand-side of equation 3.03 is zero, yield of soybeans will be zero. Compressing all components of soil phosphorus for which there are no routine estimates, a summary equation may be written as;

$$TP \doteq \alpha_p + (P_G + P_F) \qquad 3.04$$

where α_p is the sum of all unknown, unestimated components that make up the total amount of soil phosphorus. Now Liebig's response function can be rewritten as;

$$y_{ij} = a + \beta_p P^s \qquad 3.05$$

after substituting equation 3.04 into equation 3.02 and defining a as the product of β_p and α_p and where P^s is the value of the soil test that estimates P_G plus P_F. A similar logic can be followed when potassium is the limiting nutrient. The total amount of potassium is theoretically expressed as;

$$TK = K_A + K_B + K_C + K_D + K_E - K_F + K_G + K_H \qquad 3.06$$

where K_A is nonexchangeable potassium in Box A in Figure 2.03, etc. However, a soil test only estimates the component K_E. If K_E equals 0 this does not imply TK = 0, hence y_{ij} will not necessarily equal 0. Compressing all components of potassium for which there are no routine estimates, the summary equation is written as;

$$TK \doteq \alpha_k + K_E \qquad 3.07$$

Substituting and rewriting the response function gives;

$$y_{ij} = c + \beta_k K^s \qquad 3.08$$

where c equals β_k times α_k and K^s is the value of the soil test that estimates K_E.

The interpretation of a and c deserves some clarification as the interpretation presented here is slightly different from the interpretation presented in other studies. The difference in interpretations arises because the right-hand side variable in most studies is the applied level of the nutrient rather than the soil test level. When the applied level interpretation is used, a and c represent the respective proportional contribution to bushels of soybeans per acre from the inorganic source of phosphorus and the exchangeable source of potassium as estimated by soil tests (Paris and Paris 1985; Paris and

Knapp 1989; Lanzer et al. 1987; Lanzer and Paris 1981; Lanzer et al. 1981; Ackello-Ogutu et al. 1985; Anderson and Nelson 1975; Waggner and Norvell 1979; Cate and Nelson 1971; Grimm et al. 1987; Gurnow 1973). However, if there is no application of commercial fertilizer, some yield may be forthcoming from the inorganic and exchangeable forms of phosphorus and potassium. Hence, this interpretation allows for a nonzero intercept and is consistent with soil fertility theory.

In this study, applied fertilizer was not considered; therefore, the inorganic form of phosphorus and the exchangeable form of potassium, as measured by soil tests, were used as explanatory variables. Hence, a and c in this research represent the yield forthcoming from the total amount of phosphorus and potassium represented in boxes A,B,C,D,E and H and A,B,C,D,G and H minus F in Figures 2.02 and 2.03, respectively. The amount of phosphorus and potassium recovered from these boxes is largely governed by root interception, therefore a and c have a clear biological interpretation.

The interpretation of β_p and β_k is similar for other research and this research. Namely, β_p and β_k measure the importance of diffusion and mass movement of phosphorus and potassium to the plant both in the case that soil test values or applied fertilizers are used as explanatory variables.

The interpretations adopted in this study are also consistent with soil fertility theory and soil fertility research. They also allow for nonzero intercepts.

Mitscherlich's response function given by,

$$y_{ij} = A \prod \ [1 - \exp(\rho_p \ TP)] \ [1 - \exp(\rho_k \ TK)] \qquad 3.09$$

is not linearly separable and because it is nonlinear, it has a less developed statistical theory. Hence, to facilitate comparison with the model specification proposed for Liebig's theory, a surrogate function must be specified. The surrogate function should possess as many characteristics of Mitscherlich's theory as possible and still be amendable to ordinary least squares (OLS) estimation. Consider the specification;

$$y_{ij} = [\alpha TP^{\rho_p} TK^{\rho_k} e^{\epsilon_{ij}} | W_i, S_j] \qquad 3.10$$

where y_{ij} = bushels of soybeans per acre, TP = the total amount of phosphorus, TK = the total amount of potassium, ρ_p and ρ_k are production parameters to be estimated, W_i = year effects on yield from factors like disease, pest, volumetric water percentage and tortuosity, S_j = soil effects from factors like percent clay, sand and silt, water retention capacity and clay morphology, and ϵ_{ij} is the error term.

The difference between Mitscherlich's specification (3.09) and the logarithmic specification (3.10) is four-fold: (a) Mitscherlich's equation is not homothetic whereas the logarithmic specification is homothetic, (b) Mitscherlich's equation has isoquants that do not exhibit constant elasticity of substitution whereas the logarithmic specification exhibits an elasticity of substitution equal to 1 over the entire range of data, (c) Mitscherlich's equation is not linearly homogeneous whereas the logarithmic specification is potentially linearly homogenous, and (d) Mitscherlich's equation is not linearly separable whereas the logarithmic equation is linearly separable, and (e) Mitscherlich's specification approaches an asymptotic maximum whereas the logarithmic specification does not (Beattie and Taylor 1985; Griffin et al. 1984).

However, the logarithmic equation is amendable to OLS estimation and simultaneously maintains the essential features of diminishing yields, decreasing returns to scale, diminishing marginal rate of substitution, nutrient complementarity, multiple "best" ratios, and an elasticity of substitution greater than zero (specifically equal to 1) espoused by Mitscherlich.

Estimation Procedures

Liebig's function is not linearly separable, therefore it is supposedly not amendable to OLS estimation. However, Cate and Hsu (1978) have developed a technique to allow OLS to estimate the parameters. Paris and Knapp (1989) extended the concepts developed by Hsu and Cate. The following is a development of the extended methodology but using the interpretations embraced in this research.

Liebig's yield response function can be estimated by OLS or maximum likelihood procedures (Paris and Knapp 1989, Hsu and Cate, Waggoner and Norvell 1979, Kmenta 1986, Poirier 1976). The fundamental concepts for applying OLS to Liebig's function are these: Let subfunctions f_p and f_k of equation 3.01 be stated as;

$$f_p = a + \beta_p P^s \qquad 3.11a$$
$$f_k = c + \beta_k K^s \qquad 3.11b$$

where β_p and β_k are interpreted as a combination of diffusion and mass movement parameters. The intercept terms a and c measure the yield per acre forthcoming from the set of fixed factors, including all sources of phosphorus and potassium not extracted by the chemical soil test but received by plants from root interception. The intercept terms indicate the yield produced per acre when the extracted phosphorus and potassium approach zero. From soil fertility theory, a, β_p, c and β_k are all expected to be greater than zero. The parameters a, β_p, c and β_k are also expected to be larger for years with greater rainfall because of a higher volumetric water percentage and a lower tortuosity.

Given yield observations and corresponding soil test levels, the object is to determine which yield observations are limited by phosphorus, which by potassium and which by m.[1] The estimation of a, β_p, c, β_k and m proceeded in the following fashion. The maximum yield attainable is assumed to correspond to some initial extractable levels of phosphorus and potassium. From these initial levels, the yield-extractable phosphorus-extractable potassium grid can be separated into 4 sectors. Sectors 1, 2, and 3 can be defined by the equations:

$$\text{Sector 1: } y_{ij} = m + \sum \gamma_i W_i + \sum \delta_j S_j + e_{ij} \qquad 3.12a$$
$$\text{Sector 2: } y_{ij} = a + \beta_p P^* + \sum \gamma_i W_i + \sum \delta_j S_j + e_{ij} \qquad 3.12b$$
$$\text{Sector 3: } y_{ij} = c + \beta_k K^* + \sum \gamma_i W_i + \sum \delta_j S_j + e_{ij} \qquad 3.12c$$

where γ_i are intercept shifters for each sector for each year of data, W_i. The estimates of δ_j, also shift the intercepts m, a, and c depending on the soil type, S_j. Because observations in sector 4 correspond to extractable levels of phosphorus and potassium less than that needed to generate maximum yield they are ambiguous with respect to which nutrient is limiting. Hence, to classify the observations in sector 4, OLS regressions are run for the second and third sectors to obtain:

---

[1]The subscript indexing the correspondence of yield to a given soil test in a specific year is suppressed throughout all equations.

$$\hat{y}_{ij}=\hat{a}+\beta_p P^s+\sum \gamma_i W_i+\sum \delta_j S_j \qquad 3.13a$$
$$\hat{y}_{ij}=\hat{c}+\beta_k K^s+\sum \gamma_i W_i+\sum \delta_j S_j \qquad 3.13b$$

where the hats indicate OLS estimates. The estimates obtained from the above equations are used to classify each observation in sector 4, y_{ij4}, by the following criterion:

$$IF\ (\hat{a}+\beta_p P^s+\sum \gamma_i W_i+\sum \delta_j S_j) \le (\hat{c}+\beta_k K^s+\sum \gamma_i W_i+\sum \delta_j S_j) \qquad 3.14a$$
$$THEN\ y_{ij4}\ belongs\ with\ sector\ 2$$

$$IF\ (\hat{a}+\beta_p P^s+\sum \gamma_i W_i+\sum \delta_j S_j) > (\hat{c}+\beta_k K^s+\sum \gamma_i W_i+\sum \delta_j S_j) \qquad 3.14b$$
$$THEN\ y_{ij4}\ belongs\ with\ sector\ 3$$

With observations from sector 4 classified the following equations are estimated:

$$Sector\ 1:\ \hat{y}_{ij}=\hat{m}+\sum \gamma_i W_i+\sum \delta_j S_j \qquad 3.15a$$
$$Sector\ 2:\ \hat{y}_{ij}=\hat{a}+\beta_p P^s+\sum \gamma_i W_i+\sum \delta_j S_j \qquad 3.15b$$
$$Sector\ 3:\ \hat{y}_{ij}=\hat{c}+\beta_k K^s+\sum \gamma_i W_i+\sum \delta_j S_j \qquad 3.15c$$

Equations 3.15a, b, and c are obtained from the initial assumption concerning the extractable levels needed to produce maximum yield. However, this initial assumption may or may not correspond to the minimum sum of squares for error. Hence, the yield-extractable phosphorus-extractable potassium grid must be searched for the extractable levels that result in the minimum sum of squares error.

One disadvantage of the search procedure is that the standard errors of the coefficients are no longer valid (Paris and Paris 1985). However, the asymptotic standard errors for the coefficients and for the extractable levels corresponding to minimum sum of squares for error are obtainable using a nonlinear least squares procedure developed by Sanders of the University of Tennessee.

The estimating procedure for the logarithmic specification:

$$\ln y_{ij} = \ln \alpha + \rho_p \ln P^s + \rho_k \ln K^s + \sum \gamma_i W_i + \sum \delta_j S_j + e_{ij} \qquad 3.16$$

is straightforward in OLS.

The error structure assumed in equation 3.16 implies that the variance of the errors is proportional to the mean level of y_{ij}. Equation 3.16 becomes $y=\alpha(P^s)^{\rho}(K^s)^{\rho}e^{e}$ after taking the anti-logarithm. The purpose behind making a logarithmic transformation is to render heteroskedastic errors homoskedastic. In contrast, the specification for the Liebig model in equation 3.01 does not imply a relationship between the mean level of y_{ij} and the variance of the errors. Therefore, if the true error structure is proportional to the mean of y_{ij}, the error in equation 3.16 would be homoskedastic, while the error in equation 3.01 would likely be heteroskedastic. Similarly, if the error in equation 3.01 is homoskedastic, the error in equation 3.16 would likely be heteroskedastic. Therefore, because of the different error structures assumed to hold for Liebig's and the logarithmic models, heteroskedasticity is likely to exist in one equation or the other.

Data

Meeting Objective 1 requires an accurate data set. Howard has compiled fertility trial data for soybeans grown on West Tennessee soils which is summarized in Table 3.01 with respect to factors, measured factors, and soil type.

Table 3.01. Summary of Soybean Fertility Trials Used in Research

Crop/Years	Factors	Measured Variables	Soil Type
Soybeans/1985 to 1987	RF	P^s, K^s, pH, Rn	Henry silt loam
Soybeans/1985 to 1987	RF	P^s, K^s, pH, Rn	Memphis silt loam

RF = Residual fertility, P^s = Phosphorus soil test, K^s = Potassium soil test, pH = measure of reaction, Rn = rainfall

Both experiments were initialized in 1984 at the Ames Plantation as factorial arrangements in a randomized complete block design. In these experiments, twelve treatments of phosphorus and potassium were randomly assigned to each of four replications. The treatments were (in pounds per acre) 0-0, 0-30, 0-60, 0-90, 30-0, 30-30, 30-60, 30-90, 60-0, 60-30, 60-60, and 60-90.

In 1985 one of the replications was not fertilized, hence, it represented a one year in two frequency of fertilizer application. In 1986 two of the replications were not fertilized, hence they represented a one year in three and a two years in three frequency of fertilizer application. In 1987 none of the three replications received fertilizer, hence they represented a one year in four, a two years in four, and a three years in four frequency of fertilizer application.

The data used in this research were the residual phosphorus and potassium fertility levels, as extracted by the Melich I procedure, from the one year in four, the two years in four, and the three years in four frequency of fertilizer application.

Table 3.02 is a schematic of the experimental layout

Table 3.02 Experimental Layout for Data Used in Research

	1	2	3	4	5	6	7	8	9	10	11	12
A												
B												
C												

1 through 12 indicate plots where soil tests were taken, A = 1 year in 4 frequency, B = 2 years in 4 frequency, and C = 3 years in 4 frequency

except that treatments were randomly assigned to each plot rather than as shown in Table 3.02. In any given row, the soil test levels theoretically increase from left to right and in any given column, the soil test levels theoretically increase from top to bottom.

The experiments were executed on a Henry silt loam and a Memphis silt loam. The following two paragraphs summarize the nature of these two soils as given in the Soil Survey - Fayette County, Tennessee, 1960.

The Henry soil series are deep, poorly drained, acid soils on uplands. These soils generally have a fragipan at a depth of about 20 inches, but in a few places the pan is only faint or is missing. These soils are generally level to gently sloping and, in some places, are in depressions. Henry soils have a brown or grayish-brown, silty surface layer, about 6 inches thick. The subsoil is dominantly gray and overlies sandy or clayey material of the Costal Plain. These soils are low in plant nutrients, particularly potassium. Except in areas where material has washed in from other soils, the content of organic matter is very low. Runoff is slow to very slow, and some areas are ponded. Permeability is moderate to slow in the surface soil and is very slow in the subsoil. The available water is low in summer and is excessive in winter and spring. The natural fertility is low.

The Memphis series consist of well-drained, level to moderately steep, silty soils on broad ridgetops and sideslopes. These soils are medium acid or strongly acid.

The surface layer is brown silt loam, and the subsoil is brown to reddish-brown silt loam or silty clay loam. Memphis soils contain a moderate amount of plant nutrients, particularly potassium. They respond well to additions of fertilizer. They have developed under a hardwood forest, but most areas have been cleared and cultivated. These soils are suited to all crops common in the area.

Statement of Research Hypotheses

Neither Liebig nor Mitscherlich specified an error structure to complement their structural specification of crop response. Hence, the first set of hypotheses to be tested concerns the validity of the assumptions made for the error term of the classical linear regression model. The assumptions invoked in classical linear regression are: (1) the errors have a distribution that is normal, (2) the mean of the error distribution is zero, (3) the variance of this distribution is the same no matter what level the independent variable(s) are at, (4) that every error is independent of all other errors, and (5) that the independent variables are all nonstochastic variables with values fixed in repeated samples such that not all values of the independent variables are the same.

A large battery of hypotheses will be tested post regression. For certain statistical tests to have validity, the errors of Liebig's and the logarithmic equations must be serially uncorrelated, homoskedastic and obey weak moment conditions. Hence, the null hypothesis of serial uncorrelatedness and the alternative:

$$H_o: \rho=0$$

$$H_a: \rho\neq0$$

where ρ is the value of the test statistic which will be compared to Durbin-Watson values.

The presence of heteroskedasticity in the errors of an otherwise properly specified linear model leads to unbiased and consistent but inefficient parameter estimates and inconsistent covariance matrix estimates (Kmenta 1986; White 1980). Hence, the conventionally calculated confidence intervals and tests of significance are invalid. Therefore, the null hypothesis of homoskedasticity and the alternative:

$$H_o: \sigma_1^2 = \sigma_2^2 = ... = \sigma_m^2$$

$$H_a: \sigma_i^2 = g(\omega_o + \omega_1 z_{i1} + ... + \omega_p z_{ip})$$

will be tested using the White (1980) test. The z_{ij}'s are second order products and cross-products of the original regressors and the omega's are parameters to be estimated. The White test is based on comparing the sample variances of the least squares estimators under homoskedasticity and under heteroskedasticity. If the null hypothesis of homoskedasticity is true, the two estimated variances should differ because of sampling fluctuations and not because of heteroskedasticity. White's test does not require a specification of the form of heteroskedasticity and is not dependent on normality. Under the null hypothesis of homoskedasticity, n times R^2 is asymptotically distributed as χ^2 with p degrees of freedom.

Specific statistical tests used in this research require that regression disturbances be normal. Hence the hypothesis of normality and the alternative:

$$H_o: \beta_1 = 0 \text{ and } \beta_2 = 3$$

$$H_a: \beta_1 \neq 0 \text{ and } \beta_2 \neq 3$$

will be tested using the statistic, $n[\beta_1/6 + (\beta_2 - 3)^2/24]$ which is distributed as chi square with 2 degrees of freedom (Bowman and Shenton 1975). The above statistic tests for the values of the moments corresponding to the shape of the distribution in question. The null hypothesis above relates to the normal distribution, β_1 measuring skewness and β_2 measuring kurtosis.

Extended P Test

MacKinnon et al. (1983) have developed the P_e test where the null and the alternative hypothesis can be expressed, for the purposes of this research, as:

$$H_0: \ln y_{ij} = \ln \alpha + \rho_p \ln P^s + \rho_k \ln K^s + \sum \gamma_i W_i + \sum \delta_j S_j + e_{ij} \qquad 3.17a$$

$$H_1 : y_{ij} = \min \begin{bmatrix} (m+\sum \gamma_i W_i + \sum \delta_j S_j) ; \\ (a+\beta_p \ P^s+\sum \gamma_i W_i+\sum \delta_j S_j) ; \\ (c+\beta_k \ K^s+\sum \gamma_i W_i+\sum \delta_j S_j) \end{bmatrix} + e_{ij} \qquad 3.17b$$

where y_{ij} is bushels of soybeans per acre during the i^{th} year on the j^{th} soil type, P^s is the extractable level of phosphorus, K^s is the extractable level of potassium, α, ρ_p, ρ_k, δ_j, m, β_p, β_k, a, and c are parameters to be estimated, and the ϵ_{ij}'s are errors that theoretically satisfy all classical conditions. P_e is calculated in the following manner. Denoting the OLS estimates of the log-linear model by hats (ˆ) and OLS estimates of the linear model by tildes (˜), the OLS predicted values for both models are:

$$\ln \hat{y}_{ij} = \ln \hat{\alpha} + \hat{\beta}_p \ln P^s + \hat{\rho}_k \ln K^s + \sum \hat{\gamma}_i W_i + \sum \hat{\delta}_j S_j \qquad 3.18a$$

$$\tilde{y}_{ij} = \min \left[\begin{array}{c} (\tilde{m} + \sum \tilde{\gamma}_i W_i + \sum \tilde{\delta}_j S_j) ; \\ (\tilde{a} + \tilde{\beta}_p P^s + \sum \tilde{\gamma}_i W_i + \sum \tilde{\delta}_j S_j) ; \\ (\tilde{c} + \tilde{\beta}_k K^s + \sum \tilde{\gamma}_i W_i + \sum \tilde{\delta}_j S_j) \end{array} \right] \qquad 3.18b$$

The P_e test is equivalent to testing the hypotheses that Θ_0 equals zero and that Θ_{a1}, Θ_{b1}, and Θ_{c1} jointly equal zero in the following,

$$\ln y_{ij} = \ln \alpha + \rho_p \ln P^s + \rho_k \ln K^s + \sum \delta_j S_j + \Theta_0 [\tilde{y}_{ij} - \exp(\ln \hat{y}_{ij})] + \epsilon_{ij} \qquad 3.19a$$

$$y_{ij} = \min \left[\begin{array}{c} (m + \sum \delta_j S_j + \Theta_{a1} [\ln \hat{y}_{ij} - \ln \tilde{y}_{ij}]) ; \\ (a + \beta_p P^s + \sum \delta_j S_j + \Theta_{b1} [\ln \hat{y}_{ij} - \ln \tilde{y}_{ij}]) ; \\ (c + \beta_k K^s + \sum \delta_j S_j + \Theta_{c1} [\ln \hat{y}_{ij} - \ln \tilde{y}_{ij}]) \end{array} \right] + \epsilon_{ij} \qquad 3.19b$$

where θ_0 and θ_{a1}, θ_{b1}, and θ_{c1} are the coefficients of the additional variables that are formed by subtracting the predicted yield per acre of each model from the yield per acre predicted by the other model after the indicated transformation. If H_o (equation 3.17a) were true, θ_0 would not be significantly different from zero and θ_{a1}, θ_{b1}, and θ_{c1} would jointly be significantly different from zero. If H_o were false, θ_0 would be significantly different from zero and θ_{a1}, θ_{b1}, and θ_{c1} would jointly not be significantly different from zero.

The two sets of equations (3.18a and b) and (3.19a and b) are identical except for the additional variables associated with the θ's. The logarithmic equations 3.18a and 3.19a can be written:

$$SST = SSR_{3 \cdot 18_a} + SSE_{3 \cdot 18_a} \qquad 3.20a$$

$$SST = SSR_{3 \cdot 19_a} + SSE_{3 \cdot 19_a} \qquad 3.20b$$

where $SSR_{3.18a}$ is the sum of squares regression of equation 3.18a, $SSE_{3.18a}$ is the sum of squares error of equation 3.18a, $SSR_{3.19a}$ is the sum of squares regression of equation 3.19a, and $SSE_{3.19a}$ is the sum of squares error of equation 3.19a. The total sum of squares (SST) is the same for both equations. Now, if the additional explanatory variable, θ_0, is not relevant in explaining the variation of ln y_{ij}, then, in the population, $SSR_{3.18a}$ and $SSR_{3.19a}$ would be the same and the observed difference between them would be entirely due to sampling error. Hence, the hypothesis that $\theta_0 = 0$ can be tested by calculating:

$$\frac{(SSR_{3\cdot 19_a} - SSR_{3\cdot 18_a})/(1)}{SSE_{3\cdot 19_a}/(n-(i+1))} \quad 3.21$$

which is distributed as F with 1 degree of freedom in the numerator and [n-(i+1)] degrees of freedom in the denominator. The significance testing of θ_{a1}, θ_{b1}, and θ_{c1} in equation 3.19b required only a slight modification in the above test statistic. Liebig's function will be estimated in three separate regressions, namely,

$$\text{Sector 1: } \hat{y}_{ij}=\hat{m} + \sum \hat{\gamma}_i W_i + \sum \delta_j S_j \qquad 3.22a$$
$$\text{Sector 2: } \hat{y}_{ij}=\hat{a}+\beta_p P^{s} + \sum \hat{\gamma}_i W_i + \sum \delta_j S_j \qquad 3.22b$$
$$\text{Sector 3: } \hat{y}_{ij}=\hat{c}+\beta_k K^{s} + \sum \hat{\gamma}_i W_i + \sum \delta_j S_j \qquad 3.22c$$

hence there are three sums of squares for regression and error. Now the two equations 3.18b and 3.19b provide six sums of squares for regression and error. However, since each of these sums is a random variable and distributed as chi-square, then the distribution of the sum of the variables is also distributed as chi-square. Hence, the joint significance of θ_{a1}, θ_{b1}, and θ_{c1} in equation 3.19b is calculated with the following F statistic;

$$\frac{(SSR_{sector_1}+SSR_{sector_2}+SSR_{sector_3})_{3\cdot 19_b}-(SSR_{sector_1}+SSR_{sector_2}+SSR_{sector_3})_{3\cdot 18_b}/(3)}{(SSE_{sector_1}+SSE_{sector_2}+SSE_{sector_3})_{3\cdot 19_b}/(n-(i+1))}$$

The philosophy behind the P_e test is to test whether the null hypothesis can predict the performance of the alternative in a significant fashion. It is not a test of any specific hypothesis like "no nutrient substitution" or any other maintained hypothesis. However, it can be viewed as a composite test of all maintained hypotheses because a model that maintains incorrect hypotheses is not likely to predict the performance of an alternative model with correctly maintained hypotheses.

MacKinnon et al. (1983) have shown that the P_e test is applicable if the errors are serially uncorrelated and homoskedastic. Godfrey et al. (1988) found that the P_e test was robust to nonnormal distributions that are either highly skewed or have thick tails and that the test also had high power.

The RESET Specification Error Test

The RESET test is a general check of the structural part of a regression model where information regarding a specific alternative is not used (Ramsey 1972). RESET is a specification error test for the errors of omitted variables, incorrect functional form, and simultaneous equations (Ramsey 1972). This development of the RESET test proceeds by assuming the particular alternative hypothesis is that of omitted variables. Assume that the true model is given by:

$$y = X_1\beta_1 + X_2\beta_2 + \epsilon \qquad 3.24$$

where X_1 is nonstochastic and observable but X_2 is unobservable and ϵ satisfies all basic assumptions. The hypothesis of no omitted variables is:

$$H_o: \beta_2 = 0$$
$$H_1: \beta_2 \neq 0$$

Since X_2 is unobservable, the product $X_2\beta_2$ has to be approximated by Zv, where Z is a set of observable, nonstochastic test variables, and v is a vector of coefficients. Ordinary least squares is then applied to the model:

$$y = X_1\beta_1 + Zv + \epsilon \qquad 3.25$$

The OLS estimator of v in matrix notation is:

$$\hat{v} = (Z'M_1Z)^{-1}\,(Z'M_1y) \qquad 3.26$$

where $M_1 = I - X_1(X_1'X_1)^{-1}$. Substituting 3.24 for y in 3.26 gives:

$$\hat{v} = (Z'M_1Z)^{-1}Z'M_1(X_1\beta_1 + X_2\beta_2 + \epsilon) \qquad 3.27$$

Under the null hypothesis, $\beta_2 = 0$, the expectation of the OLS estimator is zero but under the alternative hypothesis the expectation of the estimator is not equal to zero. For RESET to be a valid test, the alternative hypothesis must be different from zero. This means that $Z'M_1X_2\beta_2$ must not equal zero which implies that Z and X_2 must be correlated. Traditionally, the test variables, Z, have been powers of the explanatory variables, X_1, or powers of the predicted values from the equation:

$$y = X_1 \beta_1 + \epsilon$$

(Ramsey and Gilbert 1972; Ramsey 1974; Kmenta 1986; McAleer 1987).

Godfrey et al. (1988) found that of the several variable addition tests they considered, RESET appeared to be the most useful in combining relatively good power with simplicity of computation and therefore recommended it to potential users.

The null and alternative hypotheses associated with the RESET test are:

$$H_0\text{: } \ln y_{ij} = \ln \alpha + \rho_p \ln P^s + \rho_k \ln K^s + \sum \gamma_i W_i + \sum \delta_j S_j + e_{ij}$$
$$H_1\text{: } \ln y_{ij} = \ln \alpha + \rho_p \ln P^s + \rho_k \ln K^s + \sum \gamma_i W_i + \sum \delta_j S_j + \lambda_1 (\ln \hat{y}_s)^2 + \lambda_2 (\ln \hat{y}_s)^3 + e_{ij}$$

and

$$H_0:\ y_{ij} = \min\begin{bmatrix} (m + \sum\gamma_i W_i + \sum\delta_j S_j)\ ; \\ (a + \beta_p P^s + \sum\gamma_i W_i + \sum\delta_j S_j)\ ; \\ (c + \beta_k K^s + \sum\gamma_i W_i + \sum\delta_j S_j) \end{bmatrix} + e_{ij}$$

$$H_1:\ y_{ij} = \min\begin{bmatrix} (m + \sum\gamma_i W_i + \sum\delta_j S_j + \lambda_3\hat{y}_{ij}^2 + \lambda_4\hat{y}_{ij}^3)\ ; \\ (a + \beta_p P^s + \sum\gamma_i W_i + \sum\delta_j S_j + \lambda_5\hat{y}_{ij}^2 + \lambda_6\hat{y}_{ij}^3)\ ; \\ (c + \beta_k K^s + \sum\gamma_i W_i + \sum\delta_j S_j + \lambda_7\hat{y}_{ij}^2 + \lambda_8\hat{y}_{ij}^3) \end{bmatrix} + e_{ij}$$

The RESET test is a test of significance of the additional coefficients, λ_1 through λ_8. The calculation of the statistic is very similar to the calculation for the P_e test in that the F distribution is used. In fact, the test is identical to equations 3.21 and 3.23 except for the adjustment to degrees of freedom.

CHAPTER IV

RESULTS

In pursuing the testing of variance specification and structural specification of Liebig's and the logarithmic equations, two covariance models were specified. These models were:

$$y_{ij} = \min[(a + \beta_p P^s \mid W_i, S_j); (c + \beta_k K^s \mid W_i, S_j); (m \mid W_i, S_j)] + \epsilon_{ij} \quad 4.01$$

and

$$\ln y_{ij} = \ln \alpha + \rho_p \ln P^s + \rho_k \ln K^s + \sum \gamma_i W_i + \sum \delta_j S_j + \epsilon_{ij} \quad 4.02$$

where y_{ij} = bushels of soybeans per acre in the i^{th} year on the j^{th} soil type, P^s = extractable phosphorus, K^s = extractable potassium, β_p, β_k, ρ_p, and ρ_k = parameters that transform the extractable phosphorus and potassium into bushels of soybeans per acre and are to be estimated, | means that everything to its right is fixed, W_i = year effects (i = 1985, 1986, and 1987), S_j = soil effects (j = Memphis and Henry), and m is the potential maximum bushels of soybeans per acre when factors other than phosphorus and potassium are limiting, min is an operator that selects the minimum value from within the brackets, and ϵ_{ij} is the error which is a sum of negligible factors and is identically and independently distributed around a mean of zero and a constant variance, σ^2.

The estimation of models 4.01 and 4.02 proved difficult. Heteroskedasticity was present in both equations and efforts to correct the equations were futile. In view of this fact, the advice of Steele and Torrie (1980), Fuller (1965), Redman and Allen (1954), Anderson (1968; 1971) and Hutton (1955) and the example of Ackello-Ogutu et al. (1985), and Paris and Paris (1985) was followed. Hence, the two models were estimated for each individual year separately.

One advantage in separating the data by years is that it will help farmers to evaluate the risk and uncertainty associated with the parameters of soybean response to phosphorus and potassium. In making soybean yield response estimates, experimental data for specific years will be more useful than averages compiled from experiments conducted over a period of years (Hutton 1955). For the reader's convenience, some weather data for the sample period 1985, 1986, and 1987 are reported in Table 4.01.

The results of estimating equation 4.01 for 1985 are presented in Table 4.02. The values of P^s and K^s that minimized the residual sum of squares (maximized yield) for 1985 were 14 pounds per acre of phosphorus and 85 pounds per acre of potassium. The two slope coefficients for P^s and K^s both had expected positive signs and were significant. The

Table 4.01 Precipitation and Temperature Records for Soybean Fertility Trials Conducted at Ames Plantation

	1985	1986	1987
Precipitation in inches			
June	7.29	5.81	4.74
July	4.27	2.11	2.43
August	5.78	2.16	1.31
Total	17.34	10.08	8.48
Temperature in Fahrenheit (Average Max,Min)			
June	(86.4,64.3)	(88.0,67.1)	(87.9,65.1)
July	(88.9,67.1)	(94.0,69.8)	(89.4,67.3)
August	(86.6,67.0)	(88.3,64.0)	(93.3,67.1)

Table 4.02 Results of Estimating Liebig's Equation for 1985 Soybean Fertility Trial Data Collected at Ames Plantation.

	Sector 1	Sector 2	Sector 3
m	41.65* (.55)		
$P^{\#}$	14* (4.3)		
$K^{\#}$	85* (4.8)		
a		37.82* (.50)	
β_p		.12* (.03)	
c			27.40* (4.0)
β_k			.16* (.05)
δ_{Henry}	-1.44 (.75)	.55 (.81)	-2.74 (1.5)
n	86	169	33
R^2	.04	.01	.15
WT		2.92	3.02
N	31.10*	9.04*	1.67

$P^{\#}$,$K^{\#}$ = values of P^s and K^s that maximized yield; n number of observations; R^2 measure of goodness of fit; WT value of White's test statistic for heteroskedastic errors; N = value of statistic testing for normal errors; * significant at the 5 percent level. Asymptotic standard errors are in parentheses under coefficients.

null hypothesis of homoskedasticity failed to be rejected in either the P^s or K^s subfunctions at the 5 percent level, however, the null hypothesis of normal errors was rejected for the yield plateau subfunction and the phosphorus subfunction but not for the potassium subfunction. Also, the mean level of the response function was not significantly different across soil types.

The results of estimating equation 4.02 for 1985 are given in Table 4.03. All variables had an appropriate sign and all were significant at the 5 percent level. The null

Table 4.03 Results of Estimating the Logarithmic Equation for 1985 Soybean Fertility Trial Data Collected at Ames Plantation

Inter	ρ_p	ρ_k	δ_{Henry}	n	R^2	WT	N
3.06* (.16)	.03* (.01)	.12* (.04)	-.03* (.01)	288	.09	6.88	51.90*

n number of observations; R^2 measure of goodness of fit; WT value of White's test statistic for heteroskedastic errors; N = value of statistic testing for normal errors; * significant at the 5 percent level.

hypothesis of homoskedasticity failed to be rejected for the logarithmic equation, however, the null hypothesis of normality was rejected. Also, the mean level of response was higher for the Memphis silt loam than for the Henry silt loam.

The results of the P_e and RESET statistical tests for 1985 are given in Table 4.04. The null hypothesis that the

Table 4.04. Results of P_e and RESET Test for 1985 Soybean Fertility Trial Data Collected at Ames Plantation

P_e		RESET	
Liebig	Logarithmic	Liebig	Logarithmic
.46	8.04*	.34	.07

* significant at the 5 percent level

logarithmic equation encompasses the characteristics of Liebig's function was rejected and the alternative hypothesis that Liebig's equation encompasses the logarithmic equation failed to be rejected. The hypothesis that no misspecification occurred failed to be rejected for both equations.

The results for 1985 suggest that the maintained hypotheses made by Liebig with respect to soybean response encompassed the maintained hypotheses associated with the logarithmic equation. This suggests that Liebig's maintained hypotheses would have been more appropriate for establishing soil test recommendations for farmers in 1985.

The results of estimating equation 4.01 for the year 1986 are given in Table 4.05. The values of P^s and K^s that minimized the residual sum of squares for 1986 were 4 and

Table 4.05. Results of Estimating Liebig's Equation for 1986 Soybean Fertility Trial Data Collected at Ames Plantation

	Sector 1	Sector 2	Sector 3	Corrected Sector 3
m	20.56* (.65)			
$P^{\#}$	4.00* (.66)			
$K^{\#}$	125.00* (16.00)			
a		19.30* (.66)		
β_P		-.07 (.09)		
c			3.05	.27 (.95)
β_k			.11	.14* (.01)
δ_{Henry}	-7.31* (2.30)	-9.34* (.83)	-4.07	-3.36* (.34)
n	24	58	350	350
R^2	.32	.70	.41	.96
WT	-	4.14	38.61*	2.63
N	2.13	31.20*	106.00*	133.00*

$P^{\#}, K^{\#}$ = values of P^s and K^s that maximized yield; n = number of observations; R^2 = measure of goodness of fit; WT = value of White's test statistic for heteroskedastic errors; N value of statistic testing for normality of errors;
* significant at the 5 percent level; asymptotic standard errors are in parentheses below coefficients.

125 pounds per acre of phosphorus and potassium, respectively. The slope coefficient for K^s had the expected positive sign and was significant at the 5 percent level however, the coefficient for P^s was unexpectedly negative but not significant. The null hypothesis of homoskedasticity failed to be rejected for the phosphorus subfunction, however, it was rejected for the potassium subfunction. The null hypothesis of normality of residuals was rejected for both the phosphorus and potassium subfunctions.

For the P_e and RESET tests to be valid, homoskedastic residuals are required. Since the estimation of the potassium subfunction was based on several observations of soybean yields for each soil test level of potassium, the variance of the disturbances could be estimated for each level of potassium. The following procedure was used to estimate the variance at each level of K^s by the formula;

$$s_i^2 = \sum \frac{(y_{is} - \bar{y})}{n - 1}$$

where s_i^2 is the estimate of the variance of yield and y_{ij} is the yield of soybeans from the i^{th} level of potassium on the j^{th} soil type. These estimates were then used to deflate the yield observations for the potassium subfunction;

$$Sector3: \frac{y_{ij}}{s_{ij}} = c\,\frac{1}{s_{ij}} + B_k\,\frac{K^s}{s_{ij}} + \frac{1}{s_{ij}}\sum \gamma_i W_i + \frac{1}{s_{ij}}\sum \delta_j S_j + \frac{e_{ij}}{s_{ij}}$$

Thus, applying this weighted least squares model gave the results for the corrected sector 3 in Table 4.05. The corrected sector 3 residuals are now homoskedastic and therefore the standard errors are presented.

The results of estimating equation 4.02 for 1986 are given in Table 4.06. All variables had appropriate signs

Table 4.06. Results of Estimating the Logarithmic Equation for 1986 Soybean Fertility Trial Data Collected at Ames Plantation

Inter.	ρ_p	ρ_k	δ_{Henry}	n	R^2	WT	N
-.61* (.26)	.08* (.02)	.66* (.06)	-.43* (.03)	432	.51	7.04	37.10*

n = number of observations; R^2 = measure of goodness of fit;
WT = value of White's test statistic for heteroskedastic errors;
N value of statistic testing for normality of errors;
* significant at the 5 percent level.

and all were significant at the 5 percent level. The hypothesis of homoskedasticity failed to be rejected for the logarithmic equation, however, the hypothesis of normality was rejected. The results of the P_e and RESET statistical tests for 1986 are given in Table 4.07. The null hypothesis that the logarithmic equation encompasses Liebig's equation was rejected and the alternative hypothesis that Liebig's function encompasses the logarithmic equation failed to be rejected. Also, the hypothesis of no misspecification

Table 4.07. Results of P_e and RESET Test for 1986 Soybean Fertility Data Collected at Ames Plantation

P_e		RESET	
Liebig	Logarithmic	Liebig	Logarithmic
.40	13.82*	.13	4.09*
* significant at the 5 percent level			

failed to be rejected in Liebig's case but was rejected in the logarithmic case.

The results for 1986 suggest that the Liebig equation encompassed the logarithmic equation and therefore, Liebig's equation may contain the more appropriately maintained hypotheses for soybean response. The results also suggest that the logarithmic equation had structural problems such as omitted variables, incorrect functional form, and/or simultaneous equations. Hence, Liebig's equation would have been more appropriate for establishing farmer fertilizer recommendations based on extractable levels of phosphorus and potassium in 1986.

The results of estimating equation 4.01 for the year 1987 are given in Table 4.08. The values of P^s and K^s that minimized the residual sum of squares (maximized yield) for 1987 were 10 and 95 pounds per acre of phosphorus and potassium respectively. The slope coefficient for P^s had the expected positive sign and was significant at the 5 percent level. The slope coefficient for K^s had the

Table 4.08. Results of Estimating Liebig's Equation for 1987 Soybean Fertility Trial Data Collected at Ames Plantation

	Sector 1	Sector 2	Sector 3	Corrected Sector 3
m	26.16* (.62)			
$P^{\#}$	10.00* (2.00)			
$K^{\#}$	95.00* (6.10)			
a		14.55* (1.01)		
β_p		.66* (.15)		
c			18.87	18.94* (3.51)
β_k			.07	.07 (.04)
δ_{Henry}	-8.95* (1.40)	-1.35 (2.20)	-9.68	-9.69* (.87)
n	81	145	206	206
R^2	.36	.05	.47	.95
WT	-	3.61	48.45*	1.15
N	.37	55.30*	44.10*	226.00*

$P^{\#}$,$K^{\#}$ = values of P^s and K^s that maximized yield; n = number of observations; R^2 = measure of goodness of fit; WT = value of White's test statistic for heteroskedastic errors; N value of statistic testing for normality of errors; * significant at the 5 percent level; asymptotic standard errors are in parentheses below coefficients.

expected positive sign but was not significantly different from zero. The null hypothesis of homoskedasticity failed to be rejected for the phosphorus subfunction but was rejected for the potassium subfunction. The hypothesis of normality was rejected for both the phosphorus and potassium subfunctions.

The heteroskedasticity in the potassium subfunction was corrected by using weighted least squares where the weights were the estimated standard deviation of soybean yield by soil type. Using soil type as the category variable instead of the K^s variable gave the best results. The estimates for the corrected sector 3 were consistent with theory and all were significant at the 5 percent level. The hypothesis of normality was still rejected.

The results of estimating equation 4.02 for 1987 are given in Table 4.09. All of the coefficients had the expected signs and were significant at the 5 percent level. The hypotheses of homoskedasticity and normality both failed to be rejected.

The results of the P_e and RESET tests for 1987 are given in Table 4.10. Again, Liebig's function demonstrated that it could predict the performance of the logarithmic function. Furthermore, Liebig's function was not misspecified according to the RESET test but the logarithmic function was misspecified.

Table 4.09. Results of Estimating the Logarithmic's Equation for 1987 Soybean Fertility Trial Data Collected at Ames Plantation

Inter.	ρ_p	ρ_k	δ_{Henry}	n	R^2	WT	N
1.34* (.35)	.19* (.03)	.28* (.08)	-.41* (.03)	432	.35	13.95	.006

n = number of observations; R^2 = measure of goodness of fit; WT = value of White's test statistic for heteroskedastic errors; N value of statistic testing for normality of errors; * significant at the 5 percent level

Table 4.10. Results of P_e and RESET Test for 1987 Soybean Fertility Trial Data Collected at Ames Plantation

P_e		RESET	
Liebig	Logarithmic	Liebig	Logarithmic
.64	21.49*	.07	6.33*

* significant at the 5 percent level

The results for 1987 suggest that the Liebig equation would have been more appropriate for establishing farmer fertilizer recommendations based on extractable levels of phosphorus and potassium in 1987.

Table 4.11 illustrates a summary of the minimum conditions suggested by the results of estimating equations 4.01 and 4.02. The logarithmic equation performed well with respect to data coherency; that is, its errors were more often normal and homoskedastic than those associated with

Table 4.11 An Examination of the Minimum Conditions

	Liebig	Log
Data Coherency		+
Data Admissibility	+	
Parameter Constancy	?	?
Valid Conditioning	+	
Parsimonious Parameters		+
Interpretable Parameters	yes	yes
Encompassing	+	

the Liebig equation. Hence, the logarithmic equation received a plus mark. Liebig's equation received a plus mark for data admissibility because of its performance with respect to the RESET test. The parameter constancy category was not subjected to a test in this research, therefore question marks are used in those boxes. The valid conditioning category went to the Liebig equation for its performance in the RESET test. The parsimonious parameters category went to the logarithmic equation where only four parameters had to be estimated. Both of the models have parameters that are interpretable. The Liebig equation was given the plus mark in the encompassing category for its performance in the P_e test.

The examination of these results is intended to fulfill Objective One. Although the results are not unanimous, serious consideration of the hypotheses associated with Liebig's theory seems warranted with respect to these response data.

Fertilizer Recommendations

The results obtained thus far may be used to offer guidance on adjusting soil test levels on soils used to produce soybeans. The guidance offered in this research depends on the quality of assumptions concerning the relationship between applied fertilizer and soil test values. Other important factors include the expected prices that farmers will receive for soybeans and pay for fertilizers on fertilizer budgets, and on the type and quantity of soil used to produce soybeans.

One characteristic of Liebig's theoretical function is constant transformation. Constant transformation holds true if each equal increment in the quantity of phosphorus or potassium on an acre of land results in equal additions to the total bushels of soybeans per acre. In this research, the premise of constant transformation has not been refuted statistically.

Liebig's response function results in economic recommendations that are ordered. In a world of perfect knowledge and foresight, profit-oriented soybean producers will apply fertilizers to an acre of soil to the point where the increase in soybeans just pays for the additional fertilizer. In the Liebig scheme, this point will be in one of two places, either at the intercept or the maximum. Therefore, one question soybean producers will desire answered is; "what soils can be expected to yield enough soybeans to warrant investing in fertilizer?" The soybean producer would allocate fertilizer towards the soil on which the return is expected to be greatest until the response function is at a maximum for each acre of that soil or until the fertilizer budget is exhausted. If the fertilizer budget is not exhausted the next most productive soil would be fertilized. This process would continue until either the next most productive soil does not economically warrant fertilizer or until the fertilizer budget is exhausted.

The quantity of phosphorus or potassium in a given fertilizer is nearly always expressed in percentages of P_2O_5 and K_2O. These quantities are different from those measured in soil tests, hence, they should be converted to percentages of elemental phosphorus (P) and potassium (K) for economic analysis. Also, each of the fertilizers dealt with in this analysis is assumed to have a 100-percent availability to plants. John Jared, Professor of Agronomy at the University of Tennessee, has suggested that to increase the phosphorus soil test by 1 pound, application of 5 to 15 pounds of elemental phosphorus is required.

Similarly, he suggests that application of 10 pounds of elemental potassium is required to change the potassium soil test by 1 pound.

Table 4.12 illustrates the fertilizer recommendations made by the University of Tennessee for soybeans, the comparison with the results of estimating equation 4.01 for 1985, and the assumptions concerning the conversion of elemental P and K into soil test phosphorus, P^s and potassium, K^s. The rows "Phosphorus[a]" and "Phosphorus[b]" correspond to the maintained hypotheses that 15 pounds and 5 pounds of elemental P are required to raise the soil test by 1 pound, respectively. The row "Potassium[c]" corresponds to the maintained hypothesis that 10 pounds of elemental K are required to raise the soil test by 1 pound. University of Tennessee personnel recommend 34.4 pounds of elemental phosphorus be applied on soils having extractable phosphorus ranging from 0 to 14 pounds per acre. Under the assumption that 15 pounds are needed to raise the soil test by 1 pound, results from the current research suggest 15 pounds of elemental P for each pound difference in the soil test value and the maximum yield soil test of 14 pounds per acre. If the 15 to 1 ratio is true, the University of Tennessee recommendation would only raise the extractable phosphorus a little more than 2 pounds. Hence, the recommendation may call for too little phosphorus in the extractable phosphorus range 0 to 11, just the right amount at a soil test value of

Table 4.12. A Comparison of the University of Tennessee Soybean Fertilizer Recommendations With the Recommendations Suggested by the Current Research for 1985 Data.

	Extractable Amount (pounds per acre)	U.T. Fertilizer Recommendation (pounds per acre)	Current Research Recommendation (pounds per acre)
Phosphorus[a]	0 to 14	34.4	Soil Test 0? 210 Soil Test 14? 0
	15 to 18	34.4	0
	19 to 30	17.2	0
	31 to 120	0	0
	120 plus	0	0
Phosphorus[b]	0 to 14	34.4	Soil Test 0? 70 Soil Test 14? 0
	15 to 18	34.4	0
	19 to 30	17.2	0
	31 to 120	0	0
	120 plus	0	0
Potassium[c]	0 to 85	66.4	Soil Test 0? 850 Soil Test 85? 0
	86 to 90	66.4	0
	91 to 160	33.2	0
	161 to 320	0	0
	320 plus	0	0

[a] Assumes 15 pounds of applied elemental phosphorus converts to 1 pound of soil test phosphorus; [b] Assumes 5 pounds of applied elemental phosphorus converts to 1 pound of soil test phosphorus; [c] Assumes 10 pounds of applied potassium converts to 1 pound of soil test potassium.

12, and too much in the soil test range 12 to 30. This analysis also depends on economics. For example, the response function for 1985 suggests that .12 bushels of soybeans will be forthcoming from each additional pound of elemental phosphorus less than or equal to 14 pounds. Assuming the farmer can expect to receive $6.50 per bushel and that he/she expects to pay $.20 per pound of P_2O_5, the expected benefit is $.78 ($6.5*.12) and the cost is $6.87 ($.20*2.29*15), where 2.29*15 is the amount of P_2O_5 that is equivalent to 15 pounds of elemental phosphorus (Tisdale et al. 1985). This example implies that applying P_2O_5 is not profitable under these assumptions. Given $.20 per pound of P_2O_5 the soybean price would have to be $57.25 per bushel to make phosphorus application profitable.

Under the assumption that 5 pounds are needed to raise the soil test by 1 pound, the University of Tennessee recommendation would raise the soil test by about 7 pounds. The current research suggests that 5 pounds of elemental phosphorus be added for each pound difference in the soil test and the maximum yield soil test of 14 pounds per acre. If the 5 to 1 ratio is true, University personnel may recommend too little fertilizer in the soil test range 0 to 6, just the right amount at the soil test value of 7, and too much in the range 7 to 31. Under the assumption of a 5 to 1 ratio, $6.50 soybeans, and $.20 per pound of P_2O_5 the expected benefit is $.78 and the cost would be $2.29

(\$.20*2.29*5). This example implies that applying P_2O_5 is not profitable under these assumptions. Given a price of \$.20 per pound of P_2O_5, the soybean price would have to be \$19.08 per bushel to be profitable.

University personnel recommend 66.4 pounds of elemental potassium for a soil test reading from 0 to 85. Under the assumption that a 10 to 1 ratio is true the current research suggests that 10 pounds of potassium be applied for each pound difference in the soil test and the maximum yield soil test of 85. University personnel may recommend too little fertilizer in the soil test range 0 to 77, just the right amount at a soil test of 78, and too much at soil test values between 78 and 161. For example, the response function for 1985 suggests that .16 bushels of soybeans will be forthcoming from each additional pound of elemental potassium less than or equal to 85 pounds. Assuming the farmer can expect to receive \$6.50 per bushel and that he/she expects to pay \$.15 per pound of K_2O, the expected benefit is \$1.04 and the cost is \$1.80 (\$.15*1.2*10) where 1.2*10 is the amount of K_2O that is equivalent to 10 pounds of elemental potassium (Tisdale et al. 1985). This example implies that applying K_2O is not profitable under these assumptions. Given \$.15 per pound of K_2O, the soybean price would have to be \$11.25 to make potassium application profitable.

The levels of phosphorus and potassium that maximize profit for the 1985 logarithmic function have also been calculated for comparison. Assuming P_2O_5 is $.20 per pound, K_2O is $.15 per pound, and soybeans are $6.50 per bushel, the logarithmic equation suggest a phosphorus and potassium soil test of 2.3 and 26.85 pounds per acre, respectively. These quantities are low, lower than University of Tennessee personnel recommendations but not as low as recommendations associated with the Liebig function.

Table 4.13 illustrates the fertilizer recommendations made by University of Tennessee personnel for soybeans and the comparison with the results of estimating equation 4.01 for 1986. Under the assumption of a 15 to 1 ratio, the current research suggests that 15 pounds of elemental P be applied for each pound difference in the soil test and the maximum yield soil test of 4 pounds per acre. If the 15 to 1 ratio is true, University personnel may recommend too little phosphorus in the soil test range 0 to 1, just the right amount at a soil test of 2, and too much in the soil test range from 2 to 30. Under the assumption that a 5 to 1 ratio is true, University personnel may recommend too much phosphorus in all soil test ranges up to 31.

The University personnel recommendation for potassium may be too little in the range 0 to 121, just right at 122, and too much in the range 122 to 161. The response function for 1986 had an unexpected negative coefficient; therefore, no economic analysis was attempted.

Table 4.13. A Comparison of the University of Tennessee's Soybean Fertilizer Recommendation With the Recommendation Suggested by the Current Research for 1986 Data.

	Extractable Amount (pounds per acre)	U.T. Fertilizer Recommendation (pounds per acre)	Current Research Recommendation (pounds per acre)
Phosphorus[a]	0 to 4	34.4	Soil Test 0? 60 Soil Test 4? 0
	5 to 18	34.4	0
	19 to 30	17.2	0
	31 to 120	0	0
	120 plus	0	0
Phosphorus[b]	0 to 4	34.4	Soil Test 0? 20 Soil Test 4? 0
	5 to 18	34.4	0
	19 to 30	17.2	0
	31 to 120	0	0
	120 plus	0	0
Potassium[c]	0 to 90	66.4	Soil Test 0? 900 Soil Test 90? 350
	91 to 125	33.2	Soil Test 91? 340 Soil Test 125? 0
	161 to 320	0	0
	320 plus	0	0

[a] Assumes 15 pounds of applied phosphorus converts to 1 pound of soil test phosphorus; [b] Assumes 5 pounds of applied phosphorus converts to 1 pound of soil test phosphorus; [c] Assumes 10 pounds of applied potassium converts to 1 pound of soil test potassium.

The response function for 1986 suggests that .14 bushels of soybeans will be forthcoming from each additional pound of elemental potassium less than or equal to 125 pounds. Assuming the farmer can expect to receive $6.50 per bushel and that he/she expects to pay $.40 per pound of K_2O, the expected benefit is $.91 and the cost is $4.80 ($.40*1.2*10) where 1.2*10 is the amount of K_2O that is equivalent to 10 pounds of elemental potassium (Tisdale et al. 1985). This example implies that applying K_2O is not profitable under these assumptions. Given $.40 per pound of K_2O, the soybean price would have to be $34.28 to make potassium application profitable.

The levels of phosphorus and potassium that maximize profit for the 1986 logarithmic function have also been calculated for comparison. Assuming P_2O_5 is $.20 per pound, K_2O is $.40 per pound, and soybeans are $6.50 per bushel, the logarithmic equation suggest a phosphorus and potassium soil test of 25 and 196 pounds per acre, respectively. These recommendations are vastly different from that of the Liebig function, however, they are somewhat similar to University of Tennessee personnel recommendations.

Table 4.14 illustrates the fertilizer recommendations made by University of Tennessee personnel for soybeans and the comparison with the results of estimating equation 4.01 for 1987.

Table 4.14. A Comparison of the University of Tennessee's Soybean Fertilizer Recommendation With the Recommendation Suggested by the Current Research for 1987 Data.

	Extractable Amount (pounds per acre)	U.T. Fertilizer Recommendation (pounds per acre)	Current Research Recommendation (pounds per acre)
Phosphorus[a]	0 to 10	34.4	Soil Test 0? 150 Soil Test 10? 0
	11 to 18	34.4	0
	19 to 30	17.2	0
	31 to 120	0	0
	120 plus	0	0
Phosphorus[b]	0 to 10	34.4	Soil Test 0? 50 Soil Test 10? 0
	11 to 18	34.4	0
	19 to 30	17.2	0
	31 to 120	0	0
	120 plus	0	0
Potassium[c]	0 to 90	66.4	Soil Test 0? 900 Soil Test 90? 50
	91 to 95	33.2	Soil Test 91? 40 Soil Test 95? 0
	96 to 160	33.2	0
	161 to 320	0	0
	320 plus	0	0

[a] Assumes 15 pounds of applied phosphorus converts to 1 pound of soil test phosphorus; [b] Assumes 5 pounds of applied phosphorus converts to 1 pound of soil test phosphorus; [c] Assumes 10 pounds of applied potassium converts to 1 pound of soil test potassium.

If the 15 to 1 ratio is true, University personnel may recommend too little phosphorus in the soil test range 0 to 7, just the right amount at a soil test of 8, and too much in the soil test range from 8 to 31. Under the assumption for phosphorus, University personnel may recommend too little phosphorus in the 0 to 3 range, just the right amount at a soil test of 4, and too much in the range 4 to 31. For example, the response function for 1987 suggests that .66 bushels of soybeans will be forthcoming from each additional pound of elemental phosphorus less than or equal to 10 pounds. Assuming the farmer can expect to receive $6.50 per bushel and that he/she expects to pay $.20 per pound of P_2O_5, the expected benefit is $4.29 ($6.5x.66) and the cost is $6.87 ($.20*2.29*15), where 2.29x15 is the amount of P_2O_5 that is equivalent to 15 pounds of elemental phosphorus (Tisdale et al. 1985). This example implies that applying P_2O_5 is not profitable under these assumptions. Given $.20 per pound of P_2O_5, the soybean price would have to be $10.41 per bushel to make phosphorus application profitable.

Under the assumption of a 5 to 1 ratio, $6.50 soybeans, and $.20 per pound of P_2O_5 the expected benefit is $4.29 and the cost would be $2.29 ($.20*2.29*5). This example implies that applying P_2O_5 is profitable.

The response function for 1987 suggests that .07 bushels of soybeans will be forthcoming from each additional pound of elemental potassium less than or equal to 95 pounds. Assuming the farmer can expect to receive $6.50 per bushel and that he/she expects to pay $.40 per pound of K_2O, the expected benefit is $.45 and the cost is $4.80 ($.40*1.2*10) where 1.2*10 is the amount of K_2O that is equivalent to 10 pounds of elemental potassium (Tisdale et al. 1985). This example implies that applying K_2O is not profitable under these assumptions. Given $.40 per pound of K_2O, the soybean price would have to be $68.57 to make potassium application profitable.

The levels of phosphorus and potassium that maximize profit for the 1987 logarithmic function have also been calculated for comparison. Assuming P_2O_5 is $.20 per pound, K_2O is $.40 per pound, and soybeans are $6.50 per bushel, the logarithmic equation suggest a phosphorus and potassium soil test of 28.5 and 39.9 pounds per acre, respectively. These quantities are low, lower than University of Tennessee personnel recommendations but not as low as recommendations associated with the Liebig function.

CHAPTER V

CONCLUSIONS, LIMITATIONS, AND IMPLICATIONS FOR FUTURE RESEARCH

One objective of this research was to examine Liebig's and Mitscherlich's theories of soybean response to the total amount of phosphorus and potassium in the soil. The theories were examined with respect to data coherency, data admissibility, parsimonious parameters, valid conditioning, and encompassment. Because Mitscherlich's theory leads to a nonlinear crop response model specification, a logarithmic model specification was used.

The results suggested that the logarithmic specification was data coherent and parsimonious but failed in every other aspect of the examination. These results lend support to the assumptions made by Liebig with respect to soybean response to phosphorus and potassium. The economic consequence of this conclusion is that the "best" ratio of phosphorus to potassium does not vary with the absolute level of soybean yield; however, this ratio is dependent on environmental conditions in a given year. Also, phosphorus and potassium are either applied to the maximum or not at all, depending on the ratio of fertilizer prices to soybean price.

The results of this research lend support to the University of Tennessee's emphasis on potassium before phosphorus in soybean production. In two out of three cases this conclusion was confirmed by the Law of the Minimum and in all cases this conclusion was confirmed by the logarithmic specification. The results of this research also lend support to the University of Tennessee's emphasis on a single ratio of phosphorus to potassium in soybean production. The current ratio suggested by the University of Tennessee is about .5. The ratios obtained from the Law of the Minimum for each year were the following: 1985 (.1647), 1986 (.0320), and 1987 (.1053). These ratios suggest that University of Tennessee personnel could possibly adjust the recommended ratio downward. The calculated ratios are in keeping with the supposition that the phosphorus requirement of plants is one-tenth that of potassium (Follet et al. 1981).

The results of this research, taken together with the maintained hypotheses concerning the relationship between applied fertilizer and soil test levels, suggest that the quantity of fertilizer recommended by University of Tennessee personnel may be conservative in the lower range of phosphorus and potassium soil tests and excessive in the upper ranges of soil tests.

In all but one case, the economic analyses suggested that price conditions would have to be extremely unusual for fertilization to be warranted. In general, the Liebig response function suggested less than the logarithmic model and the logarithmic model suggested less than University of Tennessee personnel.

The response of soybeans to phosphorus and potassium is very complex. A realistic model that accounted for all the genetic and environmental aspects of the problem as well as their interactions would be expensive. Because the models used in this research were relatively inexpensive, they have limitations.

One limitation of this research is that neither model was able to account for year effects. This conclusion has been pointed to by several other scientists using various models (Steele and Torrie 1980; Fuller 1965; Redman and Allen 1954; Anderson 1968, 1971; Hutton 1955; Ackello-Ogutu et al. 1985; Paris and Paris 1985). This problem of inadequate modeling of year effects and their interactions may be solved in the future but it seems that most scientists have ignored the problem or have attempted to adjust their estimates for year effects without addressing the root of the problem. For future research, models that have the explicit capability to account for year effects will yield much more economic insights and should be used where possible.

Another limitation to this study involved the narrowness of the data. The conclusions reached in this research apply to a very small geographic area and to only three years. In addition, the range of soil test levels used in this research may have been too narrow. Many years of data covering a wide range of soil test levels would improve the reliability of the results. Also, a wider range of pH measurements would allow more general conclusions.

Finally, the RESET statistical test used in this research may be misleading. It is design to bring attention to the statistical problems of omitted variables, simultaneous equations, and incorrect functional form. This research assumed that the problem with the logarithmic equation was that of incorrect functional form; however, the problem may lie in omitted variables or simultaneous equations.

The economic consequences of this research depend on how it, along with similar research, changes the philosophy of soybean fertilization. Currently, the philosophy of sufficiency is dominant in the United States (Eckert 1985). Sufficiency advocates fertilizing to the point where almost 100-percent of the maximum yield is possible. Another philosophy is that of maintenance. This philosophy advocates fertilizing soils to the point where soil tests indicate high levels of nutrients and then, in addition, to add the amount of nutrients the soybean crop is expected to remove. Both of these philosophies have been cited as excessive (Eckert 1985; Grove 1987).

The philosophy of this research has been that of fertilizing the crop and not the soil. If the conclusions are closer to the truth than conclusions reached using other philosophies and if these conclusions change the way fertilizer recommendations are made, then the economic consequences for the short run may include:

1. Fewer fertilizer resources can likely be used to produce the same amount of soybeans with possibly no increase in production risk. This research suggest that soybeans can be produced with less fertilizer than previously thought, at least on the Memphis and Henry soil types.
2. If fertilizer rates are reduced, variable cost savings in soybean production are possible.
3. Reduced demand for fertilizer should have a positive impact on U.S. international trade balance with Canada if everything else remains the same.
4. If this research leads to reduced fertilization of soybeans, domestic fertilizer industries can expect lower sales volume.
5. A shift in fertilizer demand would also theoretically lead to lower per unit fertilizer prices.

The economic consequences in the long run may include:

1. More crop acres can be fertilized with a given fertilizer budget if less fertilizer is required for each individual acre.
2. Farmers will have an enhanced net income potential, all other things being equal.
3. A possible improvement in water quality around the farms and areas with high soybean acreages.
4. If farmers reduce their long-run demand, there will likely be fewer fertilizer firms or lower sales volume for the same number of firms.
5. If reductions in fertilizer are actually the more efficient way of producing soybeans, there will be a move towards the production possibilities curve. This would suggest that more goods and services are being produced with the same bundle of resources. Assuming the money supply to be constant, this movement towards the production possibilities curve could reduce inflationary pressure.
6. An improved international competitive position assuming other nations do not follow a "fertilize the crop" philosophy.

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