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IMPLEMENTATION OF MONTE CARLO MOMENT MATCHING METHOD FOR PRICING LOOKBACK FLOATING STRIKE OPTION

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Abstract. *The Monte Carlo method was a numerical method that was popular in finance. This method had disadvantages at convergences, so the moment matching was used to improve the efficiency from the Monte Carlo method. The research has discussed the pricing of the lookback floating strike option using the Monte Carlo moment matching method. The monthly stock price of PT TELKOM from 2004 to 2021 that used in this research. The results were obtained by adding variance reduction moment matching in the Monte Carlo method, which produces a relatively smaller error when compared to the relative error of the standard Monte Carlo method. The orders of convergence from the Monte Carlo method with variance reduction moment matching for the call and put options are about 1.1 and 1.4. The conclusion is that the addition of the moment matching can increase the efficiency of the Monte Carlo method in determining the price of the lookback floating strike option.*

Keywords: *lookback option, moment matching, Monte Carlo method, variance reduction.*

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1. INTRODUCTION

Investment is an activity that is expected to provide benefits in the future. Every investment is always accompanied by risk because of the uncertainty that occurs and rapidly changing economic conditions, so to minimize the risk we need to have a strategy [1]. One of the strategies that can be done is using derivative products in investment. Derivative products are actively traded because they can anticipate market conditions by transferring various risks in the economy from one entity to another [2]. One of the popular derivative products is the option.

An option is a contract that gives the right, not the obligation, to buy or sell an underlying asset at a specific price known as a strike price (K) at or before maturity (T) [3]. Based on the rights, options are divided into the call and put options. The option owner does not have to exercise if it is not profitable [4]. When viewed from the determination of the payoff, the options are divided into vanilla and exotic options. Exotic options have a more complex structure than vanilla options. In addition, exotic options can also provide investors with a much more view of the future behavior of the market so that these options continue to develop in financial markets [5].

The lookback option is one of the exotic options that depend on the path of the price of the underlying assets called the path-dependent option [6], [7]. In determining the payoff of this option, it depends on the maximum and minimum prices apart from the stock price at maturity and the strike price. Lookback options are divided into the fixed strike and the floating strike lookbacks. The main characteristic of this option is to buy at the lowest price and sell at the highest price [8]. The advantage of this option is that it can minimize the regret of the owner and can provide important information about the behavior of the stock over time [9].

The model for determining the price of European options was introduced by Black and Scholes in 1973 which is known as the Black Scholes model. Option pricing using the Black Scholes model at the beginning has been applied to European-type options [10]. This model was used to obtain an analytical solution to the option price [11]. However, most exotic options do not have analytical solutions, so a numerical method is needed to approach the analytical solution of the option price. One of the numerical methods that can be used is the Monte Carlo method.

The Monte Carlo method, also known as statistical simulation and random sampling, is a stochastic simulation based on probability theory and statistics by using random numbers to solve computational problems [12]. This method has an algorithm that is easy to understand and flexible to use so it is popularly used in the financial sector, especially in determining the price of complex derivative products such as derivative products that depend on the path [13]. However, this method has a disadvantage in convergence because it takes a lot of iterations. So, it is necessary to develop this method by adding a variance reduction technique to increase the efficiency of the Monte Carlo method.

One of the variance reductions currently being developed is the moment matching method. The idea of the moment matching method is to match the moments of the sample with the population so that the moments of the sample can match the statistical properties of the population. The moment matching method uses a sequence of random numbers that are consistent with the first, second, or more moments. In addition to the use of random numbers, the moment matching method can be used to match the moment of the population stock price based on risk-neutral valuation with a sample of simulated stock prices. The addition of moment matching to the Monte Carlo method is carried out by Syata (2021) [14] to determine the price of the cash-or-nothing option. The results obtained by adding moment matching are that it can increase the efficiency of Monte Carlo. In this research, the pricing of the lookback floating strike option was determined using the Monte Carlo Moment Matching method.

2. RESEARCH METHODS

The methods used in this research are as follows:

2.1. Model of Stock Price

Uncertain stock price changes can be modeled by stochastic differential equations. The stochastic differential equation with a coefficient that depends on the time interval Δt based on neutral risk valuation is defined as follows:

$$dS(t) = r\,S(t)dt + \sigma S(t)\,dW_t \tag{1}$$

where r is risk-free interest rate, σ is volatility for the underlying asset that represents the standard deviation of the log return and W_t is Brownian motion [15].

The solution of Equation (1) at interval of time t_{i-1} to t_i is given as follows:

$$S(t_i) = S(t_{i-1})\exp\left(\left(r - \frac{\sigma^2}{2}\right)\Delta t + \sigma \Delta W(t_i)\right); i = 1,2,3,\dots,n \tag{2}$$

where $S(t_i)$ is stock price at t_i, interval of time $\Delta t = t_i - t_{i-1}$, n is the number of time subintervals during the option's life period and W is Wiener process.

Hence, the equation of the stock price model used in the Monte Carlo method is given as follows:

$$S(t_i) = S(t_{i-1})\exp\left(\left(r - \frac{\sigma^2}{2}\right)\Delta t + \sigma\varepsilon(t_i)\sqrt{\Delta \mathrm{t}}\right); i = 1,2,3,\dots.n \tag{3}$$

where $\varepsilon(t_i)$ is a random number which is normally distributed, $\varepsilon(t_i) \sim N(0,1)$ [4].

2.2. Monte Carlo Method for the Option Price

The Monte Carlo method is a numerical method that is suitable for random phenomena such as financial problems [16]. The need for an accurate method of solving financial problems makes this method increasingly popular for applications, such as in option pricing. The algorithm in the process of determining the price of lookback options using the Monte Carlo method is listed as follows [17]:

1. Determination of stock prices using the following stochastic differential equation:

$$S(t_i) = S(t_{i-1})\exp\left(\left(r - \frac{\sigma^2}{2}\right)\Delta t + \sigma\varepsilon(t_i)\sqrt{\Delta \mathrm{t}}\right) \tag{4}$$

2. Calculation of the payoff value of the floating strike lookback option on simulation i [18]

$$c_{float}^i = \left(S(T)^i - m_0^T\right) \tag{5}$$
$$p_{float}^i = \left(M_0^T - S(T)^i\right)$$

where M_0^T is the maximum value of the asset price and m_0^T is the minimum value of the asset price and $S(T)$is the stock price at time T.

3. Determination of the present value of the option price sample on simulation i

$$V_{MC}(call)^i = e^{-rt}\,c_{float}^i \tag{6}$$
$$V_{MC}(put)^i = e^{-rt}\,p_{float}^i$$

4. Determination of the Monte Carlo estimator

$$\hat{V}_{MC} = \frac{1}{n}\sum_{i=1}^{n} V_{MC}{}^i \tag{7}$$

where n is the number of samples.

The price of the lookback floating strike option for calls and puts can be estimated using the Monte Carlo estimator as follows:

$$V_{call} \approx \hat{V}_{MC}$$
$$V_{put} \approx \hat{V}_{MC}$$

2.3. Moment Matching

The moment matching method can be applied to a single underlying asset $S(t)$ which is simulated based on a neutral risk valuation with a risk-free interest rate r. If it is assumed that there is no dividend rate, then the mean of the stock price population is $\mu_S = e^{rt}S(0)$. Suppose that n stock prices are simulated as sample data, $S(t_1), \dots, S(t_n)$ at time t_i. Hence, the mean is calculated as follows:

$$\bar{S}(t) = \frac{1}{n}\sum_{i=1}^{n} S(t_i) \qquad ; i = 1, \dots, n \tag{8}$$

The transformation of stock price S_t from simulated paths using the first moment defined as follows [19]:

$$\tilde{S}(t) = S(t_i) - \bar{S}(t) + \mu_S \qquad ; i = 1, \dots, n \tag{9}$$

In addition to the transformation using mean, the stock price transformation can also be carried out by adding the standard deviation of the population using Equation (10) below.

$$\sigma_S = S_0\sqrt{e^{2rt}\left(e^{\sigma^2 t} - 1\right)} \tag{10}$$

and for the standard deviation (s_S) of the sample data using Equation (11) as follows.

$$s_S = \sqrt{\frac{\sum_{i=0}^{n}\left(S(t_i) - \bar{S}(t)\right)^2}{n-1}} \tag{11}$$

The stock price transformation using two moments of the stock price is defined as follows:

$$\tilde{S}(t) = \left(S(t_i) - \bar{S}(t)\right)\frac{\sigma_S}{s_S} + \mu_S \tag{12}$$

where s_S is standar deviation of sample $S(t)$[20].

2.4. Data Source

This research used data monthly stock price from PT Telekomunikasi Indonesia (TELKOM) from October 2004 to December 2021 which was obtained from website https://finance.yahoo.com/. The data has log return that is normally distributed. The stock price data is depicted in following Figure 1.

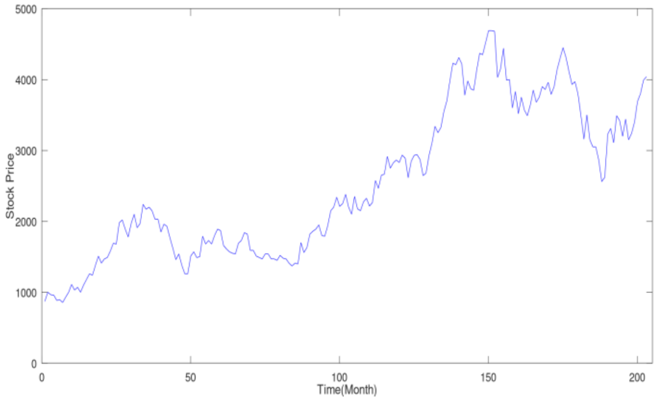

Figure 1. Data Stock Price

2.5. Steps of research

The steps of research are carried out as follows:

1. Determine the value of input such as initial stock price $(\boldsymbol{S(0)})$, time maturity $(\boldsymbol{T})$, and risk-free interest rate $(\boldsymbol{r})$.
2. Calculate the log return of the stock price.
3. Test normal distribution of log return.
4. Calculate the volatility $(\boldsymbol{\sigma})$ of the stock price.
5. Determine stock price from the Monte Carlo method.
6. Transform the value stock price with moment matching.
7. Calculate the price of the lookback floating strike option.

3. RESULTS AND DISCUSSION

3.1. Log Return of Data

Using the stock price data of PT TELKOM from October 2004 to December 2021, the log return is calculated Equation (13).

$$R(t) = \ln\left(\frac{S(t)}{S(t-1)}\right) \quad ; t = 1,2,3, \dots n \tag{13}$$

where

$R(t)$: log return value of stock price at time t

$S(t)$: stock price at time t

$S(t-1)$: stock price at time $t-1$

The result is depicted in the following Figure 2.

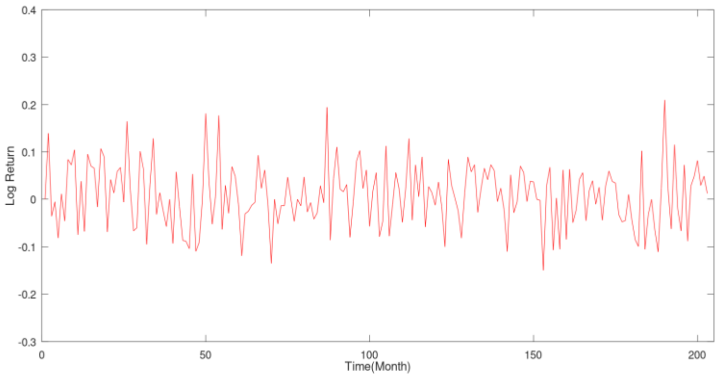

Figure 2. Log Return

Furthermore, the normality test process was carried out using the Kolmogorov-Smirnov test with the help of SPSS 28 software with the following hypothesis:

H_0 = sample comes from population that is normally distributed

H_1 = sample does not come from population that is normally distributed

Based on the test results obtained p-value = 0.200 > α = 0.05

H_0 is accepted, so that the log returns of PT Telkom's data are normally distributed and this data can be used in this research.

3.2. Volatility

Volatility (σ) is a measure that shows how much asset prices fluctuate in a period and is calculated using this formula:

$$u(i) = \ln\left(\frac{S(i)}{S(i-1)}\right) \quad ; i = 1,2,\dots,m \tag{14}$$

$$s = \sqrt{\frac{1}{m-1}\sum_{i=1}^{m}(u(i)-\bar{u})^2} \tag{15}$$

$$\sigma = \frac{s}{\sqrt{\Delta t}} \tag{16}$$

where m is the amount of data and s is standard deviation of u [4].

Based on Equations (14) – (16) using historical data of PT Telkom, the volatility value is 22.95% per year.

3.3. The Monte Carlo Method with Moment Matching

In the numerical simulation, the parameters used are initial stock price $S_0 = \text{Rp } 4040$, risk-free interest $r = 3.5\%$, option expiration time $T = 1$, volatility $\sigma = 22.95\%$, and the length of sub intervals $\Delta t = \frac{1}{12}$.

The calculation of the relative error is used to see the error resulting from the numerical process carried out on the value of the analytical process. The relative error value of the numerical process can be calculated by Equation (17) below.

$$\text{error} = \frac{|y - y_0|}{y_0} \tag{17}$$

where y is approximate value from numerical process and y_0 is the analytical solution as given by Kwok (2008) [21]. The exact value for call and put options are respectively 748.61 and 714.21. The convergence rate of the method is calculated using the error ratio. The error ratio can be determined with Equation (18) [22]:

$$\text{Ratio(Error)} = \frac{\text{error}(\text{simulation}(t))}{\text{error}(\text{simulation}(t+1))} \tag{18}$$

The numerical results of the simulation process using the Standard Monte Carlo method are given in Table 1. The price of lookback floating strike option has a smaller error with the increasing number of simulations. From Table 1, it is shown that the error ratio for call and put option are about 1.1 and 1. Therefore, the convergence rate of the standard Monte Carlo method for call and put option are about 1.1 and 1.

Table 1. Option prices using the standard Monte Carlo method

Number of simulations ($\boldsymbol{M}$)	Call option	Error of call option	Error Ratio of call option	Put option	Error of put option	Error Ratio of put option
100	566.92	0.2427	-	510.95	0.2846	-
1000	591.46	0.2099	1.1562	521.09	0.2704	1.0525
10000	592.26	0.2089	1.0051	524.13	0.2661	1.0160
100000	598.93	0.1999	1.0446	527.68	0.2612	1.0190

The numerical results shown in Table 2 and Table 3 are the price of lookback floating strike option using Monte Carlo method with variance reduction moment matching. The advantage from using variance reduction is to increase the efficiency of Monte Carlo method. So that the results of Monte Carlo method with variance reduction moment matching have smaller errors than using the standard Monte Carlo method for pricing lookback floating strike option.

Table 2 The Price and relative error of call options from moment matching stock prices

Number of simulations (M)	Call option MCMM1	Call option MCMM2	Error MCMM1	Error Ratio of MCMM1	Error MCMM2	Error Ratio of MCMM2
100	654.75	838.16	0.1254	-	0.1196	-
1000	660.74	835.38	0.1174	1.0682	0.1159	1.0320
10000	664.15	828.16	0.1128	1.0404	0.1063	1.0908
100000	671.67	824.37	0.1028	1.0977	0.1012	1.0500

The numerical result of call option with variance reduction moment matching for the Monte Carlo method with the first moment and the first two moments are shown in Table 2. The relative error in the first two moments has a better relative error than that of the first one. The convergence rate for call option with the first moment and the first two moments is about 1.1.

Table 3 The Price and relative error of put options from moment matching stock prices

Number of simulations (M)	Put option MCMM 1	Put option MCMM2	Error MCMM1	Error Ratio of MCMM1	Error MCMM2	Error Ratio of MCMM2
100	505.74	730.34	0.2919	-	0.0226	-
1000	510.56	726.92	0.2851	1.0237	0.0178	1.2691
10000	515.02	721.02	0.2789	1.0224	0.0095	1.8668
100000	518.07	720.24	0.2746	1.0251	0.0084	1.1294

The numerical result of the put option with variance reduction moment matching for the Monte Carlo method with the first moment and the first two moments is shown in Table 3. The relative error in the first two moments has a better relative error than that of the first one and the option price converges to the analytical solution as the number of iteration increase. The convergence rate for put option with the first moment and the first two moments are about 1 and 1.4.

Based on the results above, adding the reduction of variance can increase the efficiency of the standard Monte Carlo method. With respect to the moment matching process, the use of the first two moments has better error values than the first moment, both in calculating call and put options from lookback floating strike option. The numerical results using the Monte Carlo method with the moment matching technique have values that are close to the analytical results as the number of simulations increases.

4. CONCLUSIONS

Based on the discussion above, it can be concluded that adding a moment matching variance reduction technique to the Monte Carlo method can increase its efficiency in calculating the lookback floating strike option. It is shown by relative error value which is getting smaller as the number of iterations increases. The rate of convergence from the Monte Carlo Method with reduction variance moment matching for the call and put options are about 1.1 and 1.4. For further research, it is possible to add the moment used for moment matching and use variant reduction techniques other than moment matching to increase the efficiency of the Monte Carlo method in determining the price of the lookback floating strike option.

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