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Mathematical Justification Research in Mathematics Education Across Grades: A Systematic Literature Review

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Abstract

Justification is a set of responses or responses that a person gives when asked to provide mathematical reasons for the results he makes. Justification can be used as a social process in which mathematical knowledge is explained, and systematically verified based on ideas, definitions, and properties that apply in mathematics such as representations used to display concepts. Research on justification in mathematics education has been carried out for a long time. This study aims to evaluate mathematics justification studies from articles published after 2016 and before April 2022 from ERIC and Google Scholar Databases. The key questions in this study are how these articles are distributed based on year publication, country, participant, mathematics subject, method, and how to support mathematical justification. This study used a Systematic Literature Review (SLR) by Preferred Reporting Items for Systematic Reviews and Meta-Analysis (PRISMA) guidelines including inclusion and exclusion criteria and then the data were extracted, resulting in 30 studies to be reviewed. The result shows that by year of publication the articles fluctuation distributed, based on country no data from Africa and Australia was found in the databases, the participant is mostly preservice mathematics teachers, whereas only one publication studied in high school students, geometry is the most mathematics subject used in research since every grade contain this material, the method is frequently given by qualitative research with various approach, how to supporting mathematical justification is also discussed.

***Keywords**: mathematical justification, mathematical communication, mathematical argumentation, systematic literature review*

INTRODUCTION

Mathematics as a universal science underlies the development of modern technology because various disciplines depend on mathematics. In the field of mathematics education, it can be useful to train structured knowledge, develop critical thinking, objective, and open-minded thinking skills, this is certainly a concern so that students can face the development of science and novelty in society. More generally, there are several objectives in learning mathematics proposed by NCTM including Problem Solving, Argumentation and Reasoning, Communication, Connection, and Representation. Argumentation and reasoning are of particular concern because students can be said to understand a concept if they can argue the concept in their own language. Therefore, To equip students with such abilities, today's mathematics learning should be focused on efforts to train students to use their thinking potential so that teaching mathematics today requires a strong understanding and ability to argue mathematically (Makowski, 2020; Rott, 2021). In an effort to create mathematics learning that can hone students' reasoning and help teachers monitor the extent of students' understanding, it is necessary to have a meaningful process in a mathematical discussion. This process is known as justification.

Justification is one of the arts of communication. Students' mathematical knowledge can be analyzed through what is said (oral) or other things put forward, for example, written arguments (Ayala-Altamirano & Molina, 2021). Adding that opinion, students' shrewdness in arguing should be seen both sequentially so that a clear description of the argumentation and compatibility with the written argument can be seen. Justification is a set of responses or responses that a person gives when asked to provide mathematical reasons for the results he makes (Bieda et al., 2022). They mention that justification in learning can be in the form of justification for student answers, work results, methods used, justifying why something is right or wrong, as well as justification for the thoughts expressed and the reasons. In educational research, there have been many definitions of justification including: (Ayala-Altamirano & Molina, 2021) that defined justification as a social process in which mathematical knowledge is explained, and systematically verified based on ideas, definitions, and properties that apply in mathematics such as representations used to display concepts. Whereas (Boon Liang, 2016; Hamidy, 2016) define it as the way someone determines and explains whether a

statement is true or false. Furthermore, the role of the existing definitions provides space for readers or educators to make justification as something that is associated with the learning process. Mathematical justification can help students express themselves clearly and it can help educators understand what students are thinking and obtain the necessary pedagogical informed decisions (Boon Liang, 2016; Ingram et al., 2019). A person can be said to understand a mathematical concept if he can rephrase the concept correctly and can also convince others that the concept or idea is true, as (Lo et al., 2008; Weber et al., 2020) stated that mathematical justification can be used as a tool to explain and convince others about ideas. Because of that, the ability to justify is very important for students and educators, someone can be trusted to understand some mathematical concepts if he is able to explain a reason for "why is that true?" or "Why is it possible to solve the problem?" (Lithner, 2008), so, mathematical justification makes something meaningful to improve the ability to speak and use mathematical language (Yilmaz et al., 2019).

Mathematical justification has different levels depending on a person's level of education, the justification given by elementary school students is basically different from the justification by high school students, especially in mathematics education, as well as at other levels (Bieda et al., 2022). The above concept was also put forward by (Staples et al., 2012) that the role of justification includes: verification (related to the truth of a statement), explanation (providing an explanation of why the statement is true), systematization (organizing various answers deductively into a system of axioms, concepts, and theorems), discovery (finding or creating new answers), communication (spreading knowledge of Mathematics), and incorporation (using a fact for a new frame of mind). Meanwhile, according to (Back et al., 2009) explain some of the objectives of justification in learning mathematics including; (1) supporting conceptual understanding, (2) encouraging long-term mathematical ability and disposition, (3) assessment and assessment for evaluation, and (4) manage differences.

Several studies have attempted to describe and categorize the characteristics of justification and arrange its levels, such as (Back et al., 2009) found five types of justification on student answers, namely assumptions, vague/broad statements, rules, procedural descriptions, and own explanations. Among them, those with the lowest level are assumptions which are reasons without a clear basis, rule, or definition. While the highest level of justification is own explanation which is a reason with a clear basis using

self-constructed language. In addition, the level of justification ability was also described by (Simon & Blume, 1996) that are; (1). Primitive, student answers do not lead to justification, (2) Nave empiricism, justification based on a few examples only, (3) Crucial experiment, justification based on testing an example that is not specific, (4) Generic examples, justification is based on certain cases but is still included in general examples, (5) Thought experiment, justification does not use examples but uses conceptual proof. Moreover, (Lo et al., 2008) classifying the degree of student justification into five levels based on the results of student answers in writing such as: (1) Level 0 which means there is no answer or the answer does not contain a valid reasoning strategy; (2) Level 1, that justification is only descriptive or simply explains the steps of completion; (3) Level 2, some of the justifications contain incorrect mathematical concepts or do not contain clear enough details; (4) Level 3, justification is mostly clear and conceptually correct, but slightly omits some important aspects; (5) Level 4, the overall justification is clear, complete, and conceptually correct. The justification ability of students who study mathematics at school really needs to be investigated so that meaning can be drawn on whether learning has achieved its goals and as an evaluation for educators to continue to put forward the argue mathematical aspect in learning at school.

Research on justification has been carried out by several researchers around the world, and studies on mathematical justification have been evaluated in the decade before 2016 with several limitations, in order to refine studies on justification, it is necessary to conduct a more comprehensive systematic literature review on mathematical justification in studies published after 2016. The study was conducted as a means to report the mathematical justification ability of students in learning mathematics, and provide suggestions as an evaluation for teachers, and or researchers who are interested in studying the justification of mathematics at all levels of education and mathematics teachers, especially in the search for research gaps.

This study uses a systematic literature review method based on the Preferred Reporting Items for Systematic Reviews and Meta-Analysis (PRISMA) guidelines. The purpose of this study is to describe the results of research on justification in mathematics education in articles published in 2016-2022 contained in the ERIC and Google Scholar databases. To achieve this goal, several relevant research questions are: (1) how is the distribution of articles in terms of the year of publication? (2) how is the distribution of

articles in terms of the country? (3) how is the distribution of articles in terms of research subjects? (3) how is the distribution of articles based on the research method used? (4) how is the distribution of articles based on the mathematical material used? and (5) how to support mathematical justification at all levels of education?

METHOD

This study protocol was reported following the Preferred Reporting Items for Systematic Reviews and Meta-Analysis (PRISMA) guidelines. PRISMA can provide syntheses of the state of knowledge in a field, from which future research priorities can be identified; address questions that otherwise could not be answered by individual studies; identify problems in primary research that should be rectified in future studies, and generate or evaluate theories about how or why phenomena occur. The guidelines of PRISMA include eligibility criteria, information sources, search strategy, selection process, data collection process, and data items (Page et al., 2021).

Eligibility criteria

We included all published empirical studies with available abstract and full-text articles that study mathematical justification in mathematics education classrooms at elementary, secondary, high school, and university students. By considering the year of publication, we only review articles published from 2016 since there was an available review about this topic below 2016. Articles were excluded from the review if the study design was a literature review, not in English or Indonesian language, not fulfill the included criteria e.g not in mathematics education and others.

Information sources

The literature search was conducted in the ERIC Institute of Education Sciences and Google Scholar databases. According to previous research done in January 2016. We used a limit to searching the literature, the limit is publication from 2016 to 2022 with a databases filter and advanced keyword search. The literature search was conducted in April 2022. Thus, the studies after 2022 were not included in this review.

Search

We used a combination of keywords justify, justification, students, teacher, and preservice to find the appropriate articles. The last three indicate the subject in an article that wants to be reviewed. Boolean operators were used to combine the keywords. In the ERIC database we used the keyword “("justification" OR "justify") AND ("student" OR

"preservice")" to find articles related to our study and by adding pubyearmin:2016 pubyearmax:2022 in front of the keyword, the results were filtered to limitation of year publication. In the Google Scholar database, we used the keyword "allintitle: justification OR justify AND student OR teacher OR preservice OR mathematics" in English, and "allintitle: justifikasi AND siswa OR guru OR calon guru OR matematika" in Indonesia and used advanced search filter added year publication shows us the eligible articles.

Study selection

The electronic search was conducted by authors and did the critical reading title and abstract selection to identify studies that potentially met the inclusion criteria explained above. Duplicates were removed through Mendeley software after the title and abstract were downloaded from the databases. Studies that did not include justif* were removed by the automatic search feature in Mendeley.

Data collection process

Data collection was designed in Mendeley, we used the group feature to collect articles from ERIC and Google Scholar, then created a "joined" folder to identify the duplicates. Therefore, the automatic tools being used here were checking duplicates then removing them manually and title search then removing articles that were not included justif* in the title.

Data items

The data extracted from each study included the author, the study period, the study design, the number of participants, how the test was being conducted, and the country.

RESULTS AND DISCUSSION

Study selection

Our strategy of searching by combining any keyword extracted 3988 published articles from any journal (3759 from ERIC, and 229 from Google Scholar). 3796 articles from ERIC and 156 articles from Google Scholar have been removed by year criteria (automatically by a tool), and the remaining articles were removed by their duplicates in the folder "join" (20 articles were excluded). After applying the study selection, we found only 30 articles met the inclusion criteria and were included in the present review (see Figure 1).

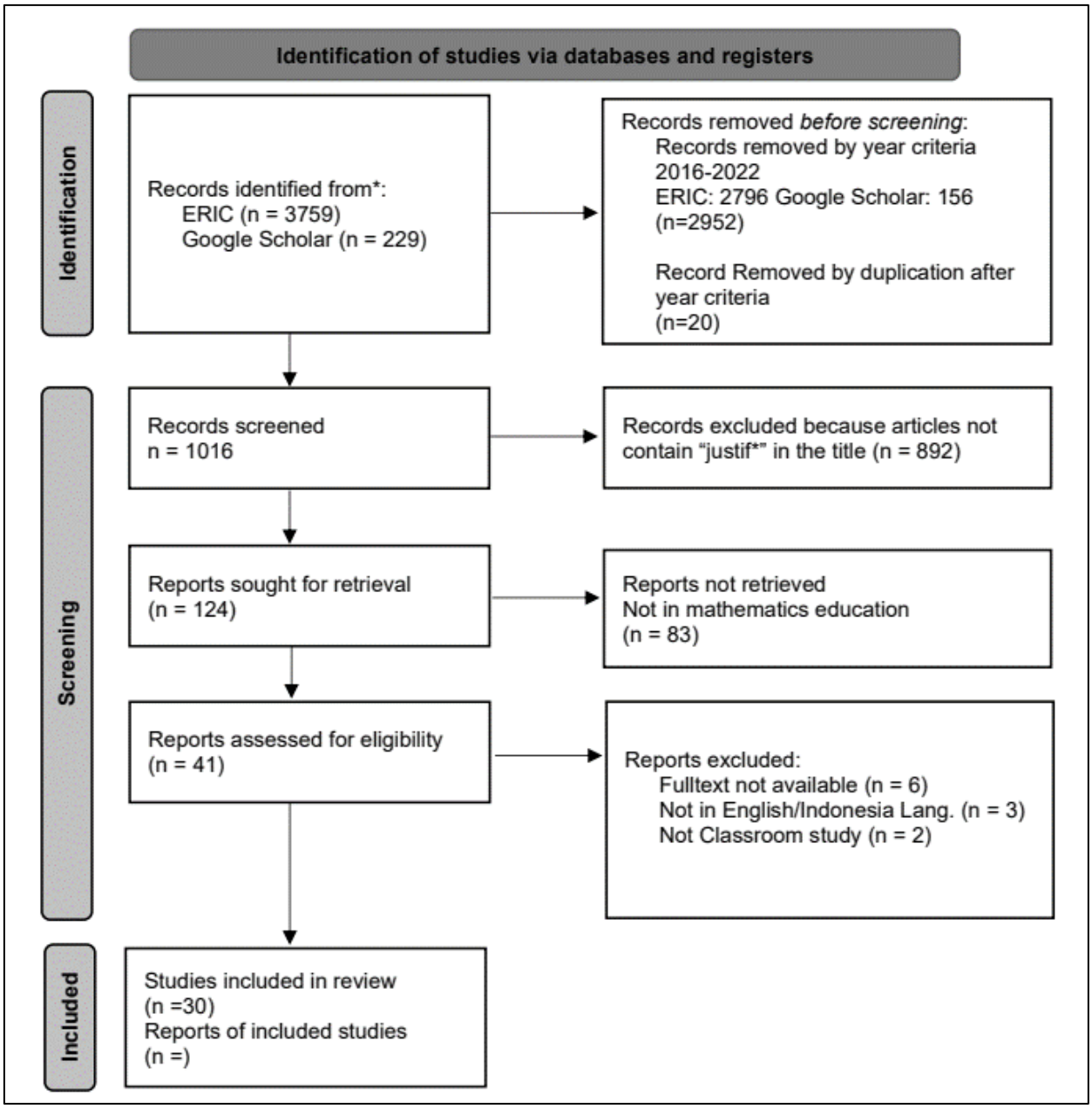


Figure 1. PRISMA's flow map

Studies' characteristics

Of 30 published articles included in this review, Brazil, Germany, Ireland, and Taiwan has one article, two from Portugal, three from Singapore, five from Turkey, and eight from each Indonesia and America. All studies were published between 2016 and 2022 without gaps. The studies included 2 to 581 participants. Elementary, secondary, high school, college students, and teachers were included in the study. Based on the study design, most of them used a qualitative design, and some also used quantitative and mixed methods.

Synthesis of results

Charts, tables, and diagrams will be used to present the data for answering the research question. A narrative synthesis will be provided to explain the characteristics and findings of the data.

Figure 2. The distribution of articles by year of publication

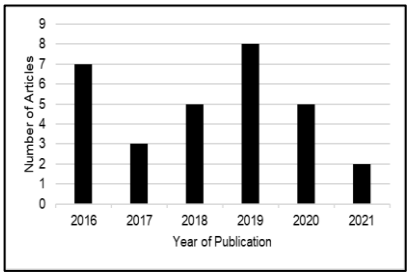


Figure 3. The distribution of articles by country

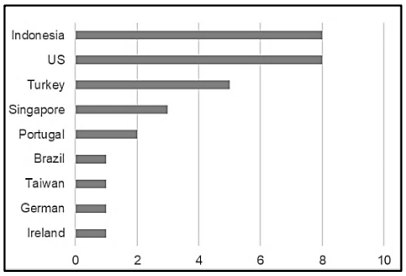
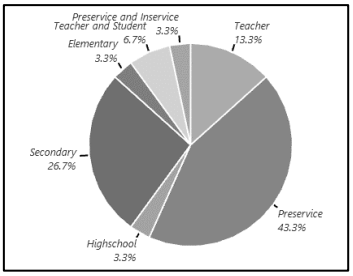


Figure 4. The distribution of articles by subject.



When figure 2 is examined, mathematical justification studies in mathematics education were carried out mostly in 2016 and 2019. The number of studies in these years presents half of the total, where 10 studies were done in 2018 and 2020 with the same number of studies, also three and two studies were done in 2017 and 2021 respectively. On the other hand, no study in this year was found in the database until April 2022. Studies from 2019 are decreasing.

According to figure 3, studies about justification in mathematics education were done in Asia, America, and Europe. The majority of the empirical research comes from Indonesia and the US then followed by Turkey, Singapore, and Portugal, the remaining countries have one article in each. In this field, Indonesia and US have contributed 53% of the total research reviewed. On the other hand, five and three studies have been done in Turkey and Singapore respectively. In contrast, there no studies in Australia and Africa were found in the databases ERIC and Google Scholar.

As shown in figure 4. The study group of mathematical justification in mathematics education is mostly composed of preservice mathematics teachers i.e undergraduate students which perform 13 of 30 studies, that is six from the US, five from Turkey, and the rest from Brazil and Indonesia. Students in secondary school have also been researched in eight studies followed by teachers in four studies. In contrast, only one study has been done in high school and elementary school of grade 2. Some studies included two groups of participants i.e observing teacher and student, and preservice and in-service teacher which give a contribution

by two and one studies respectively. The fact that only a smaller number of mathematical justification research in high school provides us some consideration to conduct mathematical justification understanding.

Table 1. The distribution of articles by methods

Method	Actual	Relative
Qualitative	23	77%
Quantitative	3	10%
Mix Methods	3	10%
RnD	1	3%

The method related to mathematical justification was more frequently given by qualitative research with 77% of the total 30 articles. The types of qualitative used in these articles are case study, action research, design experiment, design-based research, descriptive, explorative, content-analysis, survey, and phenomenological research. The RnD research was done in Indonesia to develop learning tools in secondary student's classrooms while mixed methods were done in Singapore and Turkey with treatment and control groups and collected by pre-test and post-test.

Table 2. The distribution of articles by mathematics subject.

Level	Learning Area	Actual
Elementary	Geometry	1
Secondary	Geometry	1
	Number Theory	2
	Algebra	1
High School	Trigonometry	1
College and	Number Theory	3
Teacher	Geometry	7
	Graf	1
	Pattern	2
	Statistic	1
	Calculus	1
	Linear Algebra	1
	Algebra	2
Student and Teacher	Number, Algebra, Geometry and Statistics	2

Top mathematics subject conducted by mathematical justification researchers above 2016 are presented in Table 2. From the table, we can see that geometry take place at least in 11 articles, and more frequently in college student and teacher. All mathematics subjects in elementary, secondary, and high school are included in college students and teachers by

considering trigonometry as a calculus subject. The distribution of mathematics subjects is mostly spread among college students and teachers, in contrast, high school grade only has one. The distributions of geometry subjects are volume in elementary, circle, triangle, and quadrilateral in the secondary, triangle, and Pythagorean theorem in college and teacher.

At the preservice level, the results showed that most students had not been able to provide correct justification and rationalization, difficulty in justifying even though they gave the correct answer (Amorim et al., 2019; Aziz, 2021; Dündar & Gündüz, 2017; Prasad & Barron, 2019). Especially in geometry, this is because students have low-level geometric thinking skills (Bozkurt, 2018). The same thing also happened at the secondary level, where students had not been able to answer the question above "why?" in the concept of geometry (Hamidy, 2016; Pamungkas et al., 2018; Perdanawati et al., 2018). Several studies have tried to develop justification skills and stated that by using scaffolding students can justify the circle area that previously had not been able to provide justification (O et al., 2016), the same thing was also conveyed by (Duffy & Heinz, 2019) in the discussion of the tower problem stating that their use of Socratic questioning could challenged them to think critically, experiment, justify and generalize and also interested in the ideas of their peers. At the high school level, only 7% of students can give a clear justification for trigonometry concepts, the rest can answer correctly but with the wrong justification (Eko et al., 2018). At the teacher level, the same thing was conveyed by (Boon Liang et al., 2019; CHUA, 2016) that the teacher could show correct results but did not provide a clear justification for the answer, this was because the teacher seldom had to work out mathematical solutions in process form so had struggled with the justification task (Boon Liang et al., 2019).

The ability to justify is very important for students and educators (Lesseig, 2016), someone can be trusted to understand mathematical concepts if he is able to explain a reason for "*why?"* which in this case is known as justification, a set of responses that are offered when students are in a situation to justify (Bieda et al., 2022). To support mathematical justification, teachers must be able to provide a challenging learning environment that means not only lesson solving by procedure but also taking the whole class into mathematics discussion (Mata-Pereira & da Ponte, 2013). Several things that can support mathematical justification abilities are classroom atmosphere and teacher strategies in learning. As stated by (Bozkurt & Koc, 2020) that the use of inquiry-based activities in dynamic geometry and physical instructional material environments might have helped learners improve their level of justification in geometry. Manipulative strategies can also help students improve their justification skills, preservice

teacher successfully justified their arguments using manipulatives and evidence indicated that using manipulatives during instruction with proof-related tasks was associated with positive perceptions of proof-related instruction (Bostic, 2016). In addition to strategy, the classroom atmosphere also plays an important role in this, as stated by (Mata-Pereira & Pedro da Ponte, 2017) paths of teacher's actions that rely on the design principles enable students to present rather complete justifications based on logical coherence and on mathematical aspects of the situation. Whilst (Martin, 2019) stated that learning with the homework-based task gives undergraduate students time and space to struggle with theoretical concepts in calculus and solidify their own understanding and they develop good mathematical communication through written justification. In an effort to evaluate the mathematics justification ability of high school or college students, and mathematics teachers, researchers can take a gap research from several studies that have been reviewed previously. The gap-research in question is to lead mathematical justification research on the Australian or African continents, conduct research to evaluate the justification ability of mathematics education students in Indonesia because there is still very little research on this, the justification of students and students on calculus concepts including derivatives and integrals must be known immediately , apart from the cognitive aspect regarding the justification task, the researcher assumes that the affective aspect also needs to be known about the characteristics of the mathematical justification ability.

CONCLUSIONS

There have been many researchers from various parts of the world who have studied the justification abilities of school students. There are 30 studies on mathematical justification in the ERIC and Google Scholar databases that match the inclusion criteria. Of all these studies, there is no research published in 2022, in the 2016-2021 range research on mathematical justification has fluctuated and decreased from 2019. In fact, there are still many problems regarding this mathematical justification, this has created an awareness among scholars. researchers to identify abilities, develop strategies or methods, and learning atmosphere related to mathematical justification. Especially in the distribution of countries, Asia, America, and Europe have contributed to justification research, it is hoped that researchers from Australia and Africa can conduct research on this matter in order to expand the knowledge base. Most of the methods used are qualitative, which is 77%, but quantitative methods, mixed methods, and development also contribute to the research being reviewed. Based on the mathematical material, there is an imbalance between the material used, namely studying a lot of geometry, this can be a gap for researchers to expand the study of aspects of the mathematical material,

and from the subjects given there are gaps at the elementary and high school students. For teachers, the supporting mathematical justification section can be used as a reference for making a strategy or method in teaching in order to improve the students' justification abilities. The use of two databases in this study also can be considered as a limitation.

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