SOLVING SYLVESTER MATRIX EQUATIONS WITH LR BIPOLAR TRIANGULAR FUZZY NUMBERS IN ELECTRIC CIRCUITS PROBLEMS

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Abstrak

Nombor rapuh bipolar merujuk kepada dua fungsi dan maklumat yang berbeza dalam sesuatu sistem, iaitu positif dan negatif komponen. Maklumat kebolehjadian dan tak kebolehjadian mampu digambarkan secara serentak dalam nombor rapuh bipolar berbanding dengan nombor rapuh klasik. Walau bagaimanapun, memandangkan nombor rapuh bipolar tidak mampu menangani masalah ketidakpastian, maka nombor kabur bipolar (BFN) digunakan. BFN dalam persamaan matrik Sylvester (SME) memain peranan yang penting dalam sistem kawalan seperti kawalan elektrik. Satu kawalan litar elektrik RLC yang terdiri daripada perintang (R), induktor (L), dan kapasitor (C) digunakan untuk mengawal jumlah arus elektrik sepanjang litar elektrik tersebut. Di samping itu, nombor kompleks yang mengandungi nombor nyata dan nombor khayalan digunakan untuk menyelesaikan litar RLC, dengan nombor nyata melambangkan perintang, manakala nombor khayalan melambangkan induktor atau kapasitor. Berdasarkan pengetahuan kami, pembinaan penyelesaian yang mengintegrasikan SME dengan BFN atau kompleks BFN masih belum diterokai. Oleh yang demikian, kajian ini bertujuan untuk membangunkan pendekatan analitik untuk menyelesaikan persamaan matrik Sylvester kabur bipolar (FSME), kompleks bipolar FSME, persamaan matrik Sylvester kabur penuh bipolar (FFSME), dan persamaan sistem linear kabur kompleks penuh bipolar (FFLS) dalam nombor kabur segi tiga bipolar kiri-kanan (LR). Untuk mendapatkan penyelesaian, bipolar FSME, kompleks bipolar FSME, dan bipolar FFSME perlu ditukarkan kepada sistem linear bipolar dengan menggunakan hasil darab Kronecker dan pengoperasi-*Vec*. Seterusnya, sistem kabur bipolar sepadan (EBLS), sistem kabur kompleks bipolar sepadan (ECBLS), sistem kabur bipolar bersekutu (ABLS), dan sistem kabur kompleks bipolar bersekutu (ACBLS) telah dibangunkan. Kemudian, penyelesaian akhir untuk kaedah ini dapat diperolehi dengan menggunakan kaedah songsang. Maka, empat pendekatan analitik telah dibangunkan dalam untuk menyelesaikan bipolar FSME, kompleks bipolar FSME, bipolar FFSME, dan kompleks bipolar FFLS dalam bentuk LR. Beberapa contoh dikemukakan untuk menggambarkan kaedah yang dibangunkan. Sebagai tambahan, aplikasi litar RLC dengan kompleks bipolar FSME dan kompleks bipolar FFLS juga dilaksanakan. Kesimpulannya, hasil dapatan pendekatan analitik baharu memberikan sumbangan tambahan kepada bidang ilmu persamaan kabur dengan aplikasi bererti dalam kawalan elektrik bipolar.

Kata kunci: Nombor kabur bipolar, Persamaan sistem linear bipolar, Persamaan matrik Sylvester bipolar, Nombor kabur kompleks bipolar, Nombor kabur segi tiga bipolar LR.

Abstract

Bipolar crisp numbers refer to two different functions and information in a given system, namely positive and negative components. Likelihood and unlikelihood information can be simultaneously represented by bipolar crisp numbers rather than classical crisp numbers. However, since bipolar crisp numbers are inadequate in dealing with uncertainty problem, bipolar fuzzy numbers (BFN) are used instead. BFN in Sylvester matrix equations (SME) plays an essential role in the control system such as in electrical controller. An electrical controller of RLC circuit consisting of resistor (R), inductor (L), and capacitor (C), is used to control the amount of electric currents flowing across the electric circuits. Besides, complex numbers which consist of real and imaginary parts are used in solving RLC circuit, where real numbers denote resistance, while imaginary numbers denote inductance or capacitance. To the best of our knowledge, the integration of SME with either BFN or complex BFN is not yet explored. Therefore, this study aims to construct analytical approaches in solving bipolar fuzzy Sylvester matrix equation (FSME), complex bipolar FSME, bipolar fully fuzzy Sylvester matrix equation (FFSME), and complex bipolar fully fuzzy linear system (FFLS) in left-right (LR) bipolar triangular fuzzy numbers. In order to obtain the solutions, bipolar FSME, complex bipolar FSME, and bipolar FFSME are converted into the bipolar linear system by utilizing Kronecker product and *Vec*-operator. Next, an equivalent bipolar linear system (EBLS), equivalent complex bipolar linear system (ECBLS), associated bipolar linear system (ABLS), and associated complex bipolar linear system (ACBLS) are established. Then, the final solutions of the constructed methods are obtained using inverse method. Therefore, four analytical approaches have been constructed in solving bipolar FSME, complex bipolar FSME, bipolar FFSME, and complex bipolar FFLS in LR forms. Several examples are presented to illustrate the constructed methods. Moreover, the application of RLC circuits with complex bipolar FSME and complex bipolar FFLS are also carried out. In conclusion, the new findings of analytical approaches add to the fuzzy equations body of knowledge with significant applications in bipolar electrical controllers.

Keywords: Bipolar fuzzy numbers, Bipolar linear systems, Bipolar Sylvester matrix equations, Complex bipolar fuzzy numbers, LR bipolar triangular fuzzy numbers.

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List of Abbreviations

ALS	Associated Linear System
ABLS	Associated Bipolar Linear System
ACBLS	Associated Complex Bipolar Linear System
BFN	Bipolar Fuzzy Numbers
CxFN	Complex Fuzzy Numbers
CxBFN	Complex Bipolar Fuzzy Numbers
EBLS	Equivalent Bipolar Linear System
$EBLS_1$	Equivalent Bipolar Linear System for Mean Values
$EBLS_2$	Equivalent Bipolar Linear System for Left-Right Spreads
ECBLS	Equivalent Complex Bipolar Linear System
$ECBLS_1$	Equivalent Complex Bipolar Linear System for Mean Values
$ECBLS_2$	Equivalent Complex Bipolar Linear System for Left-Right Spreads
FLS	Fuzzy Linear System
FFLS	Fully Fuzzy Linear System
FSME	Fuzzy Sylvester Matrix Equation
FFSME	Fully Fuzzy Sylvester Matrix Equation
LR	Left-Right
TFN	Triangular Fuzzy Numbers

CHAPTER ONE
INTRODUCTION

This chapter presents the background of the study, problem statement, research objectives, scope of the study, the significance of the study, and organization of the study.

1.1 Background of the Study

A controller is a component of control theory that is used to develop and design a control system to determine an efficient model. It is considered an essential tool and is applied in almost all mechanisms and systems to ensure smoother operations. A controller is located inside a plant and receives input variables before transforming the information into the desired output or commanded variables (Douglas, 2019). For example, an electrical controller of an RLC electric circuit consisting of a resistor (R), an inductor (L), and a capacitor (C) is used to control the amount of electric current flowing across the electric circuit (Scarciotti & Astolfi, 2016). The solution to the RLC electric circuit involves complex numbers in the form of $a + ib$, where a is real number and b is an imaginary number.

In the engineering control system, a controller is commonly used to monitor temperature, pressure, force, flow rate, weight, position, and speed (Asari & Amirfakhrian, 2016). A controller can be expressed mathematically, such as a linear system and a Sylvester matrix equation (SME). Solutions of the linear system and SME can be regarded as the steady-state behaviour of the controller (Scarciotti & Astolfi, 2016). The general form of a linear system and SME are commonly denoted as:

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1. Thape, N. C. S., & Ahmad, N. (2021). Solving bipolar fully fuzzy sylvester matrix equations. *AIP Conference Proceedings*, *2365*(July), 050004. https://doi.org/10.1063/5.0057002

2. Thape, N. C. S., & Ahmad, N. (n.d.). Solving bipolar fully fuzzy sylvester matrix equations with negative fuzzy numbers. *International Journal of Theoretical Physics*. (submitted)

3. Ahmad, N., Thape, N. C. S., Daud, W. S. W., & Ibrahim, H. (n.d.). Solving arbitrary complex fully fuzzy linear systems for a simple electric circuit. *AIP Conference Proceedings*. (accepted)

4. Thape, N. C. S., Ahmad, N., & Daud, W. S. W. (n.d.). Solving complex bipolar fully fuzzy linear systems for a simple electric circuit. *AIP Conference Proceedings*. (submitted)