DYNAMIC HYBRID PRICING FORMULATION FOR EQUITY WARRANTS

SITI ZULAIHA BINTI IBRAHIM

MASTER

UNIVERSITI UTARA MALAYSIA

2022

Awang Had Salleh
Graduate School
of Arts And Sciences

Universiti Utara Malaysia

PERAKUAN KERJA TESIS / DISERTASI
(Certification of thesis / dissertation)

Kami, yang bertandatangan, memperakukan bahawa
(We, the undersigned, certify that)

SITI ZULAIHA IBRAHIM

calon untuk Ijazah
(candidate for the degree of) MASTER OF SCIENCE (MATHEMATICS)

telah mengemukakan tesis / disertasi yang bertajuk:
(has presented his/her thesis / dissertation of the following title):

"DYNAMIC HYBRID PRICING FORMULATION FOR EQUITY WARRANTS"

seperti yang tercatat di muka surat tajuk dan kulit tesis / disertasi.
(as it appears on the title page and front cover of the thesis / dissertation).

Bahawa tesis/disertasi tersebut boleh diterima dari segi bentuk serta kandungan dan meliputi bidang ilmu dengan memuaskan, sebagaimana yang ditunjukkan oleh calon dalam ujian lisan yang diadakan pada : ***09 November 2021.***
That the said thesis/dissertation is acceptable in form and content and displays a satisfactory knowledge of the field of study as demonstrated by the candidate through an oral examination held on: ***09 November 2021.***

Pengerusi Viva: *(Chairman for VIVA)*	**Assoc. Prof. Dr. Hasimah Sapiri**	Tandatangan *(Signature)*
Pemeriksa Luar: *(External Examiner)*	**Dr. Norizarina Ishak**	Tandatangan *(Signature)*
Pemeriksa Dalam: *(Internal Examiner)*	**Assoc. Prof. Dr. Masnita Misiran @ Bakun**	Tandatangan *(Signature)*
Nama Penyelia/Penyelia-penyelia: *(Name of Supervisor/Supervisors)*	**Dr. Teh Raihana Nazirah Roslan**	Tandatangan *(Signature)*
Nama Penyelia/Penyelia-penyelia: *(Name of Supervisor/Supervisors)*	**Dr. Ali Fareed Jameel**	Tandatangan *(Signature)*

Tarikh:
(Date) ***09 November 2021***

Permission to Use

In presenting this thesis in fulfillment of the requirements for a postgraduate degree from Universiti Utara Malaysia, I agree that the Universiti Library may make it freely available for inspection. I further agree that permission for the copying of this thesis in any manner, in whole or in part, for scholarly purpose may be granted by my supervisor(s) or, in their absence, by the Dean of Awang Had Salleh Graduate School of Arts and Sciences. It is understood that any copying or publication or use of this thesis or parts thereof for financial gain shall not be allowed without my written permission. It is also understood that due recognition shall be given to me and to Universiti Utara Malaysia for any scholarly use which may be made of any material from my thesis.

Requests for permission to copy or to make other use of materials in this thesis, in whole or in part, should be addressed to:

Dean of Awang Had Salleh Graduate School of Arts and Sciences

UUM College of Arts and Sciences

Universiti Utara Malaysia

06010 UUM Sintok

Abstrak

Waran ekuiti adalah instrumen yang dikeluarkan oleh syarikat yang memberi pemegang saham keistimewaan untuk membeli saham pada harga pelaksanaan tertentu dalam jangka masa tertentu. Berdasarkan kepada kajian empirikal, model penentuan harga opsyen Black-Scholes tidak sesuai dalam menentukan harga waran kerana kedua-dua andaian ruapan tetap dan kadar faedah tetap dalam model adalah tidak serasi. Kajian ini mencadangkan model hibrid Heston-Cox-Ingersoll-Ross (Heston-CIR) untuk mengenal pasti kesan ruapan stokastik dan kadar faedah stokastik dalam penentuan harga waran ekuiti. Kajian ini membangunkan formula baru bagi penentuan harga analitikal untuk waran ekuiti dengan menggunakan pendekatan transformasi Cauchy dan persamaan pembezaan separa. Kaedah pengoptimuman setempat digunakan untuk mencari anggaran nilai parameter dengan menentukur model Heston-CIR. Keberkesanan model dinilai melalui kajian empirik menggunakan data Bursa Malaysia. Model cadangan menunjukkan penambahbaikan yang bererti pada masa pengiraan anggaran sembilan parameter model, antara 38.12 hingga 62.62 saat berbanding model sedia ada. Selanjutnya, kajian empirikal menunjukkan model cadangan adalah tepat jika dibandingkan dengan pasaran sebenar sepanjang tempoh lima tahun. Model ini menunjukkan ralat penentuan harga terkecil berbanding model sedia ada. Hasil dapatan juga menunjukkan peluang *moneyness* bagi waran ekuiti, 88.75% daripada waran adalah menguntungkan. Kesimpulannya, model cadangan menunjukkan prestasi terbaik dalam mengenal pasti kesan ruapan stokastik dan kadar faedah stokastik dalam penentuan harga waran ekuiti.

Keywords: Waran ekuiti, Ruapan stokastik, Kadar faedah stokastik, Model Heston-CIR.

Abstract

Equity warrants are instruments issued by a company that give the stockholder the privilege of buying a stock at a certain strike price within a particular timeframe. Motivated by empirical studies, the Black-Scholes option pricing model is not suitable to price a warrant since both assumptions of constant volatility and constant interest rates in the model are incompatible. This study proposed the Heston-Cox-Ingersoll-Ross (Heston-CIR) hybrid model to identify the effects of stochastic volatility and stochastic interest rates in pricing equity warrants. The study constructed new analytical pricing formulas for equity warrants by using Cauchy transformation and partial differential equation approaches. The local optimization method is employed to obtain the estimated parameter values by calibrating the Heston-CIR model. The effectiveness of the proposed model is investigated through the empirical study using the data from Bursa Malaysia. The proposed model shows significant improvement on the computation time in estimating nine model parameters, ranging from 38.12 to 62.62 seconds compared to the existing models. Moreover, the empirical study suggested that the proposed model is accurate when compared to the real market over five years period. This model also produced smallest pricing errors among the existing models. The finding also suggested equity warrants in moneyness opportunity, 88.75% of the warrants are profitable. In conclusion, the proposed model performs the best in identifying the effects of stochastic volatility and stochastic interest rates in pricing equity warrants.

Keywords: Equity warrants, Stochastic volatility, Stochastic interest rates, Heston-CIR model.

Acknowledgment

In the name of Allah, the Most Gracious and the Most Merciful.

All praises to Allah SWT and His blessing for the completion of this thesis. I am grateful to Allah SWT for all the opportunities, trials and strength that have been showered on me to finish writing the thesis. During these phases, I learned so much, not only from the academic aspect but also from the aspect of personality. I would like to express my humblest gratitude to the holy Prophet Muhammad (Peace be upon him) whose way of life has been a continuous guidance for me.

First and foremost, I would like to express my gratitude to my supervisor, Dr Teh Raihana Nazirah bt Roslan, my lecturer, Dr Sharmila bt Karim and my co-supervisor, Dr Ali Fareed Jameel, who has given me a lot of advice and extensive guidance in writing this thesis. Their expertise and research experience have been a source of inspiration for me throughout the writing of this master's thesis and confirmed my desire to dive further into academic research.

I also dedicate these acknowledgements to Mrs. Zakiah bt Hashim and Dr Azizah bt Mohd Rohni for their motivation, immense knowledge and keep supporting me spiritually throughout this master's journey. They have all provided a huge amount of their precious time and effort for me.

Last but not least, I am truly grateful to my beloved family members and colleagues who have always inspired me to work on this thesis throughout the entire period. I would also like to thank them for their endless support, understanding and love.

Table of Contents

List of Tables

List of Figures

List of Abbreviations

ATM	At-the-money
CEV	Constant Elasticity Variance
CDF	Cumulative distribution function
CIR	Cox-Ingersoll-Ross
ESO	Executive stock options
FBM	Fractional Brownian motion
GARCH	Generalized AutoRegressive Conditional Heteroskedasticity
GBM	Geometric Brownian motion
HJM	Heath-Jarrow-Morton
HW	Hull-White
ITM	In-the-money
KLSE	Kuala Lumpur Stock Exchange
LIBOR	London Interbank Offered Rates
LSM	Least square method
MAE	Mean absolute error
MAPE	Mean absolute percentage error
MSE	Mean square error
OTM	Out-of-the-money
PDE	Partial differential equation
RMSE	Root mean square error
RVMSE	Relative value of the mean squared error
SABR	Stochastic Alpha Beta Rho
SDE	Stochastic differential equation

List of Symbols

μ	Stock price return
σ	Stock price volatility
$w(t)$	Brownian motion or Wiener process
$S(t)$	Stock price of an asset at time t
G	Exercise price or strike price
$\phi(\cdot)$	Standard normal cumulative distribution function
$C(S(t),t)$	Call option value
W_{BS}	Warrant value for the Black-Scholes model
W_{NW}	Warrant value for the Noreen Wolfson model
$T-t$	Time to expiration
σ_S	Standard deviation of the log returns
$r(t)$	Instantaneous interest rate
ϑ	Elasticity of variance
$v(t)$	Stochastic volatility parameter
κ	Mean reversion speed for instantaneous variance
θ	Long-term mean for instantaneous variance
σ	Volatility of the $v(t)$
ρ	Correlation coefficient between stock price and volatility
F	Forward price of an asset
α	Mean reversion speed for instantaneous rate
β	Long-term mean for instantaneous rate
η	Volatility of the interest rate
$P(r,t,T)$	Zero-coupon bond price at time t with maturity T
$c(t)$	Time-dependent of drift term
$df(t,T)$	Instantaneous forward interest rate of zero-coupon bond at time T
$L_j(t)$	Forward rate at time $[T_j, T_{j+1}]$
$w(t)^d$	Brownian motion with d-dimension
Ω	Sample space
ω	All possible outcomes
$\mathcal{F}$	Event space
$\mathbb{P}$	Probability measure

γ	Random variable
$X(t)$	Stochastic process
$g(y)$	*Borel*-measurable function
N	Number of shares
M	Number of warrants outstanding
k	Share per warrant
$+$	Warrant's role as a call or put option
n	Set of warrants
W_i	Market price
W_i^{Ω}	Model price
$v(0)$	Initial volatility
$r(0)$	Initial interest rate
$F(t)$	Future price

CHAPTER ONE

INTRODUCTION

This chapter aims to provide a comprehensive overview and pertinent information linking to this research. Section 1.1 and Section 1.2 are devoted to the background of the study and information about warrants in Malaysia. Section 1.3 stipulates the stochastic elements with focus on the stochastic volatility and stochastic interest rates, followed by Section 1.4 which discusses the problem statement. Section 1.5 and Section 1.6 present the research questions and research objectives, respectively. Next is Section 1.7 which highlights the limitation of the study, whereas Section 1.8 underlines the significance of the research. Finally, Section 1.9 provides the outline of the thesis.

1.1 Background of the Study

A derivative is a financial instrument that procures value from an underlying asset. This underlying asset might be any currency, equity, interest rate or commodity. Moreover, derivatives play an essential role in the financial system and are also found important in risk management. In Malaysia, several risk management products are options or embedded options, for example, equity warrants and call warrants (Qizam, Irdiansyah, & Haron, 2015). In agreement with Gunawan, Ibrahim and Rahim (2017), warrants have been considered as a leveraging instrument for investment that have the potential to generate greater capital gains.

Being a derivative, warrant is a type of contract that confers upon the contract holder the right, but without necessity to purchase or sell the underlying asset at a predetermined price until the fixed expiration date. Here, the predetermined price is also known as the exercise price, which is fixed. Furthermore, the expiration date refers to the date when the privilege to purchase or sell warrants become obsolete. There are

REFERENCES

Abbasi, W. (2015). Pricing warrants models: an empirical study of the Indonesian market. *Research Journal of Economics, Business and ICT*, *10*(1).

Abdoh, W., Aziz, K., Daud, W., & Mustafa, N. (April, 2019). *Pricing Call Warrant by Using Binomial Model and Historical Volatility*. In Proceedings of the Second International Conference on the Future of ASEAN (ICoFA), Singapore.

Abínzano, I., & Navas, J. (2013). Pricing levered warrants with dilution using observable variables. *Quantitative Finance*, *13*(8), 1199-1209. doi: 10.1080/14697688.2013.771280

Abudy, M., & Izhakian, Y. (2013). Pricing stock options with stochastic interest rate. *International Journal Portfolio Analysis and Management*, *1*(3), 250-277. doi: 10.1504/IJPAM.2013.054408

Ahn, D., & Gao, B. (1999). A parametric nonlinear model of term structure dynamics. *The Review of Financial Studies*, *12*(4), 721-762. doi: 10.1093/rfs/12.4.721

Al-Hagyan, M. (2018). *Modeling financial environments using geometric fractional Brownian motion model with long memory stochastic volatility.* (Unpublished Doctoral dissertation). Sintok: Universiti Utara Malaysia.

Alobaidi, G. (June, 2017). *A note on bond prices in the Vasicek model*. The 2017 WEI International Academic Conference Proceedings, Vienna, Austria.

Amar, S., Sudiarso, A., & Herliansyah , M. (2019). The accuracy measurement of stock price numerical prediction. *Journal of Physics*, *1569*(3), 032027. doi: 10.1088/1742-6596/1569/3/032027

Andersen, T., & Benzoni, L. (2010). *Stochastic volatility.* (Working Paper No. 2009-04). Federal Reserve Bank of Chicago.

Arulanandam, D. B., Sin, K. W., & Muita, M. A. (2017). Relevance of black scholes model on Malaysia KLCI options: an empirical study. *Intercontinental Journal of Finance Research Review, 5*(2), 1-33.

Atlan, M., & Leblanc, B. (2005). Hybrid equity-credit modelling. *Risk*, *8*(8), 18

Aziz, K. A., Idris, M. F., Saian, R., & Daud, W. S. (May, 2015). *Adaptation of warrant price with Black Scholes model and historical volatility.* AIP Conference Proceedings.

Bacha, O. (2012). *Financial Derivatives: Markets and Applications in Malaysia* (4th edition). Kuala Lumpur: McGraw Hill Education.

Baile, W. (1987). An empirical investigation of the market for comex gold future options. *Journal of Finance* , *42*(5), 1187-1194. doi: 10.2307/2328521

Bakshi, G., Cao, C., & Chen, Z. (1997). Empirical performance of alternative option pricing models. *Journal of Finance*, *52*(5), 2003–2049. doi: 10.1111/j.1540-6261.1997.tb02749.x

Baldeaux, J., & Badran, A. (2013). Consistent modelling of VIX and equity derivatives using a 3/2 plus jumps model. *Applied Mathematical Finance*, *21*(4), 299-312. doi: 10.1080/1350486X.2013.868631

Bendob, A., & Bentouir, N. (2019). Options pricing by Monte Carlo simulation, binomial tree and BMS model: a comparative study of Nifty50 options index. *Journal of Banking and Financial Economics*, *1*(4), 79-95. doi: 10.7172/2353-6845.jbfe.2019.1.4

Benth, F., & Koekebakker, S. (2008). Stochastic modeling of financial electricity contracts. *Energy Economics*, *30*(3), 116-1157. doi: 10.1016/j.eneco.2007.06.005

Beytollahi, A., & Zeinali, H. (2020). Comparing prediction power of artificial neural networks compound models in predicting credit default swap prices through Black–Scholes–Merton model. *Iranian Journal of Management Studies*, *8*(3),69-93. doi: 10.22059/ijms.2019.276260.673534

Bhat, A., & Arekar, K. (2016). Empirical performance of Black-Scholes and GARCH option pricing models during turbulent times: the Indian evidence. *International Journal of Economics and Finance*, *8*(3), 123-136. doi: 10.5539/ijef.v8n3p123

Bi, Z., Yousuf, A., & Dash, M. (2014). A Study on Options Pricing Using GARCH and Black-Scholes-Merton Model. *Asian Journal of Finance & Accounting*, *6*(1), 423-439. doi: 10.5296/ajfa.v6i1.4830

Black, F. (1975). Fact and fantasy in the use of options. *Financial Analysts Journal*, *31*(4), 36-41. doi: 10.2469/faj.v31.n4.36

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *The Journal of Political Economy*, *81*(3), 637-654. doi: 10.1086/260062

Blattberg, R., & Gonedes, N. (1974). A comparison of the stable and student distributions as statistical models for stock prices. *The Journal of Business, 47*(2) 244-280. doi: 10.1086/295634

Blomeyer, E., & Boyd, J. (1988). Empirical test of boundary conditions for options on treasury bond futures. *The Journal of Futures Markets*, *8*(2), 185. doi: 10.1002/fut.3990080206

Brigo, D., & Mercurio, F. (2007). *Interest rate models-theory and practice: with smile, inflation and credit* (2nd edition). Milano: Springer Science & Business Media.

Burtnyak, I., & Malytska, A. (2018). Spectral study of options based on CEV model with multidimensional volatility. *Investment Management and Financial Innovations*, *15*(1), 18-25. doi: 10.21511/imfi.15(1).2018.03

Bursa Malaysia. (2021). The Derivative Markets: Bursa Malaysia. Available at http://www.bursamalaysia.com

Cai, N., Song, Y., & Chen, N. (2017). Exact simulation of the SABR model. *Operations Research*, *65*(4), 931-951. doi: 10.1287/opre.2017.1617

Cannon, J., R. (1984). *The One-Dimensional Heat Equation* (1st edition). Florida: Cambridge University Press.

Cao, J., Lian, G., & Roslan, T. R. N. (2016). Pricing variance swaps using stochastic volatility and stochastic interest rate. *Applied Mathematics and Computation, 277*, 72-81. doi: 10.2139/ssrn.2485692

Carmichael, D., & Bustamante, B. (2014). Interest rate uncertainty and investment value: a second order moment approach. *International Journal of Engineering Management and Economics*, *4*(2), 176-189. doi: 10.1504/IJEME.2014.066588

Carr, P., & Sun, J. (2007). A new approach for option pricing under stochastic volatility. *Review of Derivatives Research*, *10*(2), 87-150. doi: 10.1007/s11147-007-9014-6

Chang, J., & Liao, S. (2010). Warrant introduction effects on stock return processes. *Applied financial economics*, *20*(17), 1377-1395. doi: 10.1080/09603107.2010.491440

Chang, Y., Wang, Y., & Zhang, S. (2021). Option pricing under double Heston model with approximative fractional stochastic volatility. *Mathematical Problems in Engineering*, *2021*(1). doi: 10.1155/2021/6634779

Chen, B., Grzelak, L., & Oosterlee, C. (2012). Calibration and Monte Carlo pricing of the SABR-Hull-White model for long-maturity equity derivatives. *Journal of Computational Finance*, *15*(4), 1–35. doi: 10.21314/JCF.2012.237

Christoph, C., & Schachermayer, W. (2017). Portfolio optimisation beyond semimartingales: shadow prices and fractional Brownian motion. *The Annals of Applied Probability*, *27*(3), 1414-1451. doi: 10.1214/16-AAP1234

Cox, J., Ingersoll , J., & Ross, S. (1985). A theory of the term tructure of interest rates. *Econometrica: Journal of the Econometric Society*, *53*(2), 385-407. doi: 10.2307/1911242

Cozma , A., Mariapragassam, M., & Reisinger, C. (2018). Convergence of an Euler scheme for a hybrid stochastic-local volatility model with stochastic rates in foreign exchange markets. *SIAM Journal on Financial Mathematics*, *9*(1), 127-170. doi: 10.1137/17M1114569

Cozma, A., Mariapragassam, M., & Reisinger, C. (2019). Calibration of a hybrid local-stochastic volatility stochastic rates model with a control variate particle method. *SIAM Journal on Financial Mathematics*, *10*(1), 181-213. doi: 10.1017/CBO9781139020534

Crisóstomo, R. (2014). *An Analyisis of the Heston Stochastic Volatility Model: Implementation and Calibration Using Matlab*. (Working Paper No. 58).

Crouhy, M., & Galai , D. (1991). Common errors in the valuation of warrants. *Financial Analysts Journal*, 47(5), 89-90. doi: 10.2469/faj.v47.n5.89

Cuadro-Sáez, L., Peña, J., & Romo, J. (2003). *Detecting abnormal market behavior using sample resampling techniques.* (Working Paper No. 03-13). University Carlos III of Madrid.

Duffie, D., & Stein, J. (2015). Reforming LIBOR and other financial market benchmarks. *Journal of Economic Perspectives*, *29*(1), 191-212. doi: 10.1257/jep.29.2.191

Dunbar, S. (2016). *Mathematical Modeling in Economics and Finance: Probability and Stochastic Processes.* Lincoln: American Mathematical Society.

Edeki, S. O., & Owoloko, A. E. (February, 2016). *The modified blackscholes model via constant elasticity of variance for stock options valuation.* AIP Conference Proceedings.

Edeki, S. O., Ugbebor, O. O., & Owoloko, E. A. (2015). Analytical solutions of the Black–Scholes pricing model for European option valuation via a Pprojected differential transformation method. *Entropy*, *17*(11), 7510-7521. doi: 10.3390/e17117510

Edeki, S., Ugbebor, O., Akinlabi, G., & González-Gaxiola, O. (October, 2017). *Approximate solutions of a variable volatility driven Black-Scholes option pricing model.* 2nd International Conference on Knowledge Engineering and Applications, London.

Eisenberg, J. (2015). Optimal dividends under a stochastic interest rate. *Insurance: Mathematics and Economics*, *65*, 259-266. doi: 10.1016/j.insmatheco.2015.10.007

Elliot, W. (2015). Analyzing options market toxicity and the Black-Scholes formula in the presence of jump diffusion as simulated with agent-based modeling. *Undergraduate Economic Review*, *11*(1), 14.

Fan, C., Luo, X., & Wu, Q. (2017). Stochastic volatility vs. jump diffusions: Evidence from the Chinese convertible bond market. *International Review of Economics & Finance*, *49*, 1-16. doi: 10.1016/j.iref.2016.04.009

Feller, W. (1954). Diffusion processes in one dimension. *Transactions of the American Mathematical Society*, *77*(1), 1-31. doi: 10.1090/S0002-9947-1954-0063607-6

Ferri, M., Kremer, J., & Oberhelman, H. (1986). An analysis of the pricing of corporate warrants. *Advances in Futures and Options Research*, *1*, 201-226.

Flore, F., & Nappo, G. (2019). *A Feynman-Kac type formula for a fixed delay CIR model*. Retrieved from https://arxiv.org/abs/1806.00997v1

Foad, S. (2017). *Valuation of equity warrants for uncertain financial market.* Retrieved from https://arxiv.org/abs/1711.08356v2

Galai , D., & Schneller, M. (1978). Pricing of warrants and the value of the firm. *The Journal of Finance*, *33*(5), 1333-1342. doi: 10.2307/2327269

Galai, D. (1989). A note on equilibrium warrant pricing models and accounting for executive stock options. *Journal of Accounting Research*, *27*(2), 313-315. doi: 10.2307/2491238

Gatheral, J. (2011). *The Volatility Surface: A Practitioner's Guide.* New York: John Wiley & Sons.

Geske, R., & Roll, R. (1983). The fiscal and monetary linkage between stock returns and inflation. *The Journal of Finance*, *38*(1), 1-33. doi: 10.2307/2327635

Geske, R., & Roll, R. (1984). On valuing American call options with the Black-Scholes European formula. *The Journal of Finance*, *39*(2) 443-455. doi: 10.2307/2327870

Goudenege, L., Molent, A., & Zanette, A. (2019). Pricing and hedging GMWB in the Heston and in the Black–Scholes with stochastic interest rate models. *Computational Management Science*, *16*(1), 217-248. doi: 10.1007/s10287-018-0304-2

Grasselli, M. (2017). The 4/2 stochastic volatility model: a unified approach for the Heston and the 3/2 model. *Mathematical Finance*, *27*(4), 1013-1034. doi: 10.1111/mafi.12124

Grzelak, L., A., & Oorsterlee, C., W. (2011). On the Heston model with stochastic interest rates. *SIAM Journal of Financial Mathematics*, 1, 255-286.

Gray, D. (2019). *Doing research in the real world.* New Delhi: Sage Publications Limited.

Gunawan, N. I., Ibrahim, S. N., & Rahim, N. A. (January, 2017). *A review on black-scholes model in pricing warrants in Bursa Malaysia*. AIP Conference Proceedings.

Hagan, P., Kumar, D., Lesniewski, A., & Woodward, D. (2002). Managing smile risk. *The Best of Wilmott*, 249-296.

Hamid, S., & Ismail, R. (June, 2018). *Evaluation of the prices of Shariah-compliant company warrants in liquid and illiquid markets by using modified Black-Scholes model.* AIP Conference Proceedings.

Handley, J. (2002). On the valuation of warrants. *Journal Futures Markets*, *22*(8), 517-534. doi: 10.1002/fut.10032

Haron, R. (2014). Derivatives, Pricing efficiency and gharar: evidence on embedded options in Malaysia. *Journal of Islamic Finance*, *3*(2), 039-048. doi: 10.12816/0025104

Harrison, J., & David , M. (1979). Martingales and arbitrage in multiperiod. *Journal of Economic Theory*, *20*(3), 381–408. doi: 10.1016/0022-0531(79)90043-7

Harrison, J., & Pliska, S. (1981). Martingales and stochastic integrals in the theory of continuous trading. *Stochastic processes and their applications*, *11*(3), 215-260. doi: 10.1016/0304-4149(81)90026-0

He, X. (2017). *A 360◦ comprehensive study of quantitative study pricing of options, ranging theory and computation to model calibration and empirical studies* (Unpublished Doctoral Dissertation). University of Wollongong.

He, X. J., & Lin, S. (2019). A semi-analytical pricing formula for European options under the rough Heston-CIR model. *The ANZIAM Journal, 61*(4), 431-445 doi: 10.1017/S1446181120000024

He, X., & Zhu, S. (2018). A closed-form pricing formula for European options under the Heston model with stochastic interest rate. *Journal of Computational and Applied Mathematics*, *335*, 323-333. doi: 10.1016/j.cam.2017.12.011

Heath , D., Jarrow, R., & Morton, A. (1992). Bond pricing and the term structure of interest rates: A new methodology for contingent claims valuation. *Econometrica: Journal of the Econometric Society*, *60*(1), 77-105. doi: 10.2307/2951677

Heston, S. (1993). A closed-form solution for options with stochastic volatility with applications to bond and currency options. *The review of financial studies*, *6*(2), 327-343. doi: 10.1093/rfs/6.2.327

Hilber, N., Matache, A., M., & Schwab, C. (2005). Sparse wavelet methods for option pricing under stochastic volatility. *Journal of Computational Finance, 8*(4), doi: 10.21314/JCF.2005.131

Ho, T., & Lee, S. (1986). Term structure movements and pricing interest rate contingent claims. *The Journal of Finance*, *41*(5), 1011–1029. doi: 10.2307/2328161

Hordahl, P., & Packer, F. (2007). *Understanding asset prices: an overview.* Basel: Bank for International Settlements, Monetary and Economic Department.

Huang, Y., & Chen, S. (2002). Warrants pricing: Stochastic volatility vs. Black–Scholes. *Pacific Basin Finance Journal*, *10*(4), 394-409. doi: 10.1016/S0927-538X(02)00066-5

Hull, J. (2006). *Options, futures and other derivatives* (6th edition). Upper Saddle River*:* Pearson Prentice Hall.

Hull, J. (2009). *Option, futures and other derivatives* (7th edition). Upper Saddle River*:* Pearson Prentice Hall.

Hull, J., & White, A. (1987). The pricing of options on assets with stochastic volatilities. *Journal of Finance*, *42*(2), 281-300. doi: 10.1111/j.1540-6261.1987.tb02568.x

Hull, J., & White, A. (1990). Pricing interest-rate derivative securities. *Review of Finance Studies*, *3*(4), 573–592. doi: 10.1093/rfs/3.4.573

Hull, J., & White, A. (1993). One-factor interest-rate models and the valuation of interest-rate derivative securities. *Journal of Financial and Quantitative Analysis*, *28*(2), 235-254. doi: 10.2307/2331288

Iacus, S. (2008). *Simulation and inference for stochastic differential equations: with R examples* (1st edition). New York: Springer-Verlag.

Iacus, S. (2011). *Option pricing and estimation of financial models with R* (1st edition). John Wiley & Sons.

Iwuoha, O. (2020). Insight: how the exit of libor will impact the nigerian financial market. *The Gravitas Review of Business & Property Law*, *11*(4), 77-87.

Janková, Z. (2018). Drawbacks and limitations of black-scholes model for options pricing. *Journal of Financial Studies & Research*, *2018*, 1-7. doi: 10.5171/2018.179814

Jankowska, M. A., Karageorghis, A., & Chen, C. S. (2018). Kansa RBF method for nonlinear problems. *International Journal Computer Methods and Exp Measures*, *6*(6), 1000-1007. doi: 10.2495/CMEM-V6-N6-1000-1007

Jarrow, R., & Van Deventer, D. (2015). Simulating and validating a multi-factor Heath, Jarrow and Morton model with negative interest rates. *Journal of Risk Management in Financial Institutions*, *8*(4), 332-346.

Jin, B., van der Wel, M., & Oorschot, J. (2019). *Pricing Derivatives in Periods of Low or Negative Interest Rates* (Unpublished Master's thesis). Erasmus School of Economics, Roterdam.

Jordan, J. V., & William, E. S. (1986). Transaction data tests of minimum prices and put call parity for treasury bond future options. *Advances in Futures and Options Research*, *4*(2-3), 63-87. doi: 10.1016/1057-5219(95)90014-4

Kim, J., Yoon, J., & Yu, S. (2014). Multiscale stochastic volatility with the Hull–White rate of interest. *Journal of Futures Markets*, *34*(9), 819-837. doi: 10.1002/fut.21625

Kim, Y. (2002). Option pricing under stochastic interest rates: an empirical investigation. *Asia-Pacific Financial Markets*, *9*(1), 23-44. doi: 10.1023/A:1021155301176

Klinpratoom, A. (2010). *An Analysis of the Covered Warrants Market in the UK* (Unpublished Doctoral dissertation). University of Exeter Business School, Europe.

Kou, S. (2007). Jump-diffusion models for asset pricing in financial engineering. *Handbooks in operations research and management science*, *15*, 73-116. doi: 10.1016/S0927-0507(07)15002-7

Krznaric, M. (2016). *Comparison of option price from Black-Scholes model to actual values* (Unpublished Bachelor Degree's thesis). University of Akron, Ohio.

Kuchuk-Iatsenko, S., & Mishura, Y. (2015). Pricing the European call option in the model with stochastic volatility driven by Ornstein–Uhlenbeck process. Exact formulas. *Modern Stochastics: Theory and Application*, *2*(3), 233-249. doi: 10.15559/15-VMSTA36CNF

Kuchuk-Iatsenko, S., & Munchak, Y. (2016). *Application of Malliavin calculus to exact and approximate option pricing under stochastic volatility.* Retrieved from https://arxiv.org/abs/1608.00230v1

Kumar, R., & Agrawal, R. (2017). An empirical investigation of the Black-Scholes call option pricing model with reference to NSE. *International Journal of BRIC Business Research*, *6*(2), 1-11. doi: 10.14810/ijbbr.2017.6201

Kyun, H. (2004). *Empirical study of Black-Scholes warrant pricing model on the stock exchange of Malaysia* (Unpublished Master's thesis). Universiti Sains Malaysia, Pulau Pinang.

Lauterbach, B., & Schultz, P. (1990). Pricing warrants: an empirical study of the black-scholes and its alternatives. *The Journal of Finance, 45*(4), 1181-1209. doi: 10.1111/j.1540-6261.1990.tb02432.x

Lawson, B. (2015). *An Exposition and Calibration of the Ho-Lee Model of Interest Rates* (Unpublished Undergraduate Honors thesis). Claremont McKenna College, United States.

Lellisa, T. (2018). *The impact of industry concetration on performance, exploring a comprehensive bank performance model: the case of the Ethiopian bank sector* (Unpublished Doctoral thesis). University of South Africa, Africa.

Leonard, D., & Solt, M. (1990). On using the Black-Scholes model to value warrants. *Journal of Financial Research*, *13*(2), 81-92. doi: 10.1111/j.1475-6803.1990.tb00539.x

Li, D., Rong, X., Zhao, H., & Yi, B. (2017). Equilibrium investment strategy for DC pension plan with default risk and return of premiums clauses under CEV model. *Mathematics and Economics*, *72*, 6-20. doi: 10.1016/j.insmatheco.2016.10.007

Liao, L., Li, Z., Zhang, W., & Zhu, N. (2014). Exercise to lose money? Irrational exercise behavior from the Chinese warrants market. *Journal of Futures Markets*, *34*(5), 399-419. doi: 10.1002/fut.21608

Lim, K., & Terry, E. (2003). The valuation of multiple stock warrants. *Journal of Future Markets: Futures, Options, and Other Derivative Products*, *23*(6), 517-534. doi: 10.1002/fut.10079

Liu, B. (2009). Some research problems in uncertainty theory. *Journal of Uncertain systems*, *3*(1), 3-10.

Liu, S., Borovykh, A., Grzelak, L., & Oosterlee, C. (2019). A neural network-based framework for financial. *Journal of Mathematics in Industry*, *9*(9), 1-28. doi: 10.1186/s13362-019-0066-7

Loerx. A, & Sachs, E., W. (2013). Model calibration in option pricing. *University Journal for Science [SQUJS]*, *17*(1), 84-102. doi: 10.24200/squjs.vol17iss1pp84-102

Londani, M. (August, 2013). *Influence of mathematical models on warrant pricing with fractional Brownian motion as numerical method.* Proceedings 59th ISI World Statistics Congress, Hong Kong.

Lu, X. (2014). *An extension of the Hull White model for interest rate modeling* (Master's thesis). Retrieved from ProQuest.

Mansor, N. J., & Jaffar, M. M. (July, 2014). *Black-Scholes finite difference modeling in forecasting of call warrant prices in Bursa Malaysia.* AIP Conference Proceedings.

Melek, A., & Sakarya, S. (2018). Pricing of covered warrants: an analysis on Borsa Istanbul. *Sosyoekonomi*, *26*(37), 201-218. doi: 10.17233/sosyoekonomi.2018.03.11

Merton, R. (1976). Option pricing when underlying stock returns are discontinuous. *Journal of Financial Economics*, *3*(1-2), 125-144. doi: 10.1016/0304-405X(76)90022-2

Mikosch, T. (1998). *Elementary stochastic calculus with finance in view* (1st edition). Singapore: World scientific.

Miller, W. (1977). *Symmetry and Separation of Variables* (1st edition). United States: Addison Wesley Publishing Company.

Misiran M. (2016). Maximum likelihood estimation with dynamic measurement errors and application to interest rate modeling. *International Journal of Pure and Applied Mathematics*, *110*(3), 433-446. doi: 10.12732/ijpam.v110i3.5

Mrázek, M., & Pospíšil, J. (2017). Calibration and simulation of Heston model. *Open Mathematics*, *15*(1), 679-704. doi: 10.1515/math-2017-0058

Mwamba, J., Thabo, L., & Uwilingiye, J. (2014). Modeling short term interest rate with stochastic differential equation in continuous time: linear and nonlinear models. MPRA Paper No. 64386. Retrieved from https://mpra.ub.uni-muenchen.de/64386/

Nagendran, R., & Venkateswar, S. (2014). Black-Scholes model in pricing Indian stock call options. *Journal of Applied Finance and Banking*, *4*(3), 89.

Najafi, A., & Mehrdoust, F. (2017). Bond pricing under mixed generalized CIR model with mixed Wishart volatility process. *Journal of Computational and Applied Mathematics*, *39*(1), 108-116. doi: 10.1016/j.cam.2016.12.039

Njogu, S. (2009). *Heat Equations and Their Applications* (Unpulished Master's thesis). University of Nairobi, Kenya.

Nohrouzian, H. (2015). *An Introduction to Modern Pricing of Interest Rate Derivatives* (Unpublished Master's thesis). Mälardalen University, Sweden.

Noreen, E., & Wolfson, M. (1981). Equilibrium warrant pricing models and accounting for executive stock options. *Journal of accounting research*, *19*(2), 384-398. doi: 10.2307/2490872

Okaro, C., Okeke, C., & Uzondu, C. (2018). Efficiency of the Black-Scholes option pricing model in the pricing of palm-oil futures in Nigeria's physical market. *International Journal of Academic Management Science*, *2*(9), 83-92.

Øksendal, B. (2003). *Stochastic differential equations* (5th edition). Berlin: Springer.

Orlando, G., Mininni, R., & Bufalo, M. (2020). Forecasting interest rates through Vasicek and CIR models: a partitioning approach. *Journal of Forecasting*, *39*(4), 569-579. doi: 10.1002/for.2642

Patel, J. (2013). *Pricing and Hedging in a GBM market with Markov switching: a survey* (Unpublished Doctoral dissertation). Indian Institute of Science Education and Research, Pune.

Paulin, C., & Lindström, M. (2020). *Option pricing models: a comparison between models with constant and stochastic volatilities as well as discontinuity jumps* (Bachelor Degree's thesis). Umea Unversity, Sweden.

Peng, J., & Yao, K. (2011). A new option pricing model for stocks in uncertainty markets. *International Journal of Operations Research*, *8*(2), 18-26.

Platen, E. (1997). *A non-linear stochastic volatility model.* (Working Paper). Australian National University.

Pola, G., Bujorianu, M., Lygeros, J., & Di Benedetto, M. (June, 2003). *Stochastic hybrid models: an overview*. IFAC Proceedings.

Polyanin, A. (2001). *Handbook of linear partial differential equations for engineers and scientists* (1st edition). London: Chapman and Hall/CRC.

Poularikas, A. (2018). *Transforms and Applications Handbook* (3rd edition). London: CRC Press.

Qizam, I., Irdiansyah, M., & Haron, R. (2015). Shariah issue of warrant contract: empirical evidence from warrant mispricing in Malaysia market. *Global Review of Islamic Economics and Business*, *1*(1), 041-055. doi: 10.14421/grieb.2013.011-04

Queirós, A., Faria, D., & Almeida, F. (2017). Strengths and limitations of qualitative and quantitative research methods. *European Journal of Education Studies*. *3*(9), 369-387. doi: 10.5281/zenodo.887089

Rahman, M. (2016). The advantages and disadvantages of using qualitative and quantitative approaches and methods in language "testing and assessment" research: a literature review. *Journal of Education and Learning*, *6*(1), 102-112. doi: 10.5539/jel.v6n1p102

Rogers, L., & Williams, D. (2000). *Diffusions, Markov processes and martingales* (2ns edition). United Kingdom: Cambridge University Press.

Roslan, T.R.N., Ibrahim, S., & Karim, S. (2020). Hybrid equity warrants pricing formulation under stochastic dynamics. *International Scholarly and Scientific Research & Innovation*, *14*(11), 133-136.

Rostek, S. (2009). *Option pricing in fractional Brownian markets* (1st edition). Switzerland: Springer-Verlag Berlin Heidelberg.

Rujivan, S., Hongsai, P., Maleewong, M., Rakwongwan, U., & Boonchot, C. (2020). Stochastic modeling for SET50 Index in Thailand using the Black-Scholes model with a time-dependent drift parameter and its application to SET50 futures pricing. *Thai Journal of Operations Research: TJOR*, *8*(2), 42-50.

Safaei, M., Neisy, A., & Nematollahi, N. (2019). Generalized componentwise splitting scheme for option pricing under the Heston-Cox-Ingersoll-Ross model. *Journal of Statistical Theory and Applications*, *18*(4), 425-438. doi: 10.2991/jsta.d.191209.001

Sakti, M., & Qoyum, A. (2017). Testing the warrants mispricing and their determinants: the panel data models. *Global Review of Islamic Economics and Business*, *5*(2), 118-129. doi: 10.14421/grieb.2017.052-05

Schulz, G., & Trautmann, S. (1994). Robustness of option-like warrant valuation. *Journal of Banking & Finance*, *18*(5), 841-859. doi: 10.1016/0378-4266(94)00030-1

Schwartz, E. (1977). The valuation of warrants: Implementing a new approach. *Journal of Financial Economic*, *4*(1), 79-93. doi: 10.1016/0304-405X(77)90037-X

Scott, L. (1987). Option pricing when the variance changes randomly: Theory, estimation, and an application. *Journal of Financial and Quantitative analysis*, *22*(4), 419-438. doi: 10.2307/2330793

Shastri, K., & Sirodom, K. (1995). An empirical test of the BS and CSR valuation models for warrants listed in Thailand. *Pacific-Basin Finance Journal*, *3*(4), 465-483. doi: 0.1016/0927-538x(95)00018-g

Shastri, K., & Tandon, K. (1986). An empirical test of a valuation model for American options on futures contracts. *Journal of Financial and Quantitative Analysis*, *21*(4), 377-392. doi: 10.2307/2330687

Shoarinezhad, V., Wieprecht, S., & Haun, S. (2020). Comparison of local and global optimization methods for calibration of a 3D morphodynamic model of a curved channel. *Water*, *12*(5) doi: 10.3390/w12051333

Shreve, S. (2004). *Stochastic calculus for finance II: Continuous-time models* (1st edition). Pittsburgh, USA: Springer Science & Business Media.

Sidenius, J. (1996). Warrant pricing—is dilution a delusion? *Financial Analysts Journal*, *52*(5), 77-80. doi: 10.2469/faj.v52.n5.2027

Sørensen, H. (2000). *Inference for diffusion processes and stochastic volatility models* (Unpublished Doctoral dissertation). University of Copenhagen, Denmark.

Srivastava, A., & Shastri, M. (2018). A study of relevance of Black-Scholes model on option prices of Indian stock market. *International Journal of Governance and Financial Intermediation*, *1*(1), 82-104. doi: 10.1504/IJGFI.2018.091495

Su, X., Wang, W., & Wang, W. (2018). Pricing warrant bonds with credit risk under a jump diffusion process. *Discrete Dynamics in Nature and Society*, *2018*(2). doi: 10.1155/2018/4601395

Tian, M., Yang, X., & Kar, S. (2019). Equity warrants pricing problem of mean-reverting model in uncertain environment. *Physica A: Statistical Mechanics and its Applications*, *531*(1), 121593. doi: 10.1016/j.physa.2019.121593

Tong, Z., & Liu, A. (2018). Analytical pricing of discrete arithmetic Asian options under generalized CIR process with time change. *International Journal of Financial Engineering*, *5*(1), 1850002. doi: 10.1142/S2424786318500020

Ukhov, A. (2004). Warrant pricing using observable variables. *Journal of Financial Research*, *27*(3), 329-339. doi: 10.1111/j.1475-6803.2004.00100.x

Vasicek, O. (1977). An equilibrium characterization of the term structure. *Journal of Financial Economics*, *5*(2), 177-188. doi: 10.1016/0304-405X(77)90016-2

Veld, C. (2003). Warrants pricing: a review of empirical research. *European Journal of Finance*, *9*(1), 1-25. doi: 10.1080/13518470110047648

Wang, X., Wu, M., Zhou , Z., & Jing, W. (2012). Pricing European option with transaction costs under the fractional long memory stochastic volatility model. *Physica A: Statistical Mechanics and its Applications*, *391*(4), 1469-1480. doi: 10.1016/j.physa.2011.11.014

Wong, W., K., Lean, H., McAleer, M., & Tsai, F., T. (2018). Why are warrant markets sustained in Taiwan but not in China? *Sustainability*, *10*(10), 3748. doi: 10.3390/su10103748

Wu, X. Y., Ma, C. Q., & Wang, S. Y. (2012). Warrant pricing under GARCH diffusion model. *Economic Modelling*, 29(6), 2237-2244. doi: 10.1016/j.econmod.2012.06.020

Xiao, W., & Zhang, X. (2016). Pricing equity warrants with a promised lowest price in Merton's jump–diffusion model. *Physica A: Statistical Mechanics and its Applications*, *458*, 219-238. doi: 10.1016/j.physa.2016.03.100

Xiao, W., Zhang, W., Xu, W., & Zhang, X. (2014). The valuation of equity warrants under the fractional Vasicek processof the short-term interest rate. *Physica A:*

Statistical Mechanics and its Applications, *394*, 320-337. doi: 10.1016/j.physa.2013.09.033

Yang, Y., Liu, S., Wu, Y., & Wiwatanapataphee, B. (2021). Pricing of volatility derivatives in a Heston–CIR model with Markov-modulated jump diffusion. *Journal of Computational and Applied Mathematics*, *393,* 113277. doi: 10.1016/j.cam.2020.113277

Yerlikaya-Özkurt, F., Vardar-Acar, C., Yolcu-Okur, Y., & Weber, G. (2014). Estimation of the Hurst parameter for fractional Brownian. *Journal of Computational and Applied Mathematics*, *259*, 843-850. doi: 10.1016/j.cam.2013.08.001

Zhang, T., Li, L., Lin , Y., Xue, W., Xie, F., Xu, H., & Huang, X. (2015). An automatic and effective parameter optimization method for model tuning. *Geoscientific Model Development*, *8*(11), 3579-3591. doi: 10.5194/gmd-8-3579-2015

Zhang, W., Xiao, W., & He, C. (2009). Equity warrants pricing model under Fractional Brownian motion and an empirical study. *Expert Systems with Applications*, *36*(2), 3056-3065. doi: 10.1016/j.eswa.2008.01.056

Zhang, X., Shu, H., Kan, X., Fang, Y., & Zheng, Z. (2018). The call option pricing based on investment strategy with stochastic interest rate. *Journal of Mathematical Finance*, *8*(1), 43-57. doi: 10.4236/jmf.2018.81004

Zhou, Q., & Zhang, X. (2020). Pricing equity warrants in Merton jump diffusion model with credit risk. *Physica A: Statistical Mechanics and its Applications*, *557*, 124883. doi: 10.1016/j.physa.2020.124883

Zhu, X. (2015). *Performance of Dynamic Hedging Strategies for Cash Balance Pension Plans* (Unpublished Master's thesis). University of Waterloo, Ontario, Canada.

Zili, M. (2006). On the mixed fractional Brownian motion. *International Journal of Stochastic Analysis*, *2006*, 1-9, doi: 10.1155/JAMSA/2006/32435