The effect of word-of-mouth marketing strategy on the number of buyers: a mathematical perspective

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Abstract

In this paper we will present a mathematical model for word of mouth marketing strategy by considering proportional recruitment. We divide a population under consideration into four subpopulations: susceptible – those who are the target market or potential buyers (*S*), infected – those who are already active as buyers (*I*), positive – former buyers which have positive comments on the item they purchased (*P*) and negative – former buyers which have negative comments on the item they purchased (*N*). We assume that the rate of new individuals who enter the target market is proportional to the number of *S*, *I*, *P*, and *N* subpopulations. These subpopulations have either a positive or negative contribution to the number of new entry to the susceptible class or the potential buyer. We analyzed the model emphasizing in the effects of the WOM on the number of buyers and its rate of increase.

Keywords: Word-of-mouth; marketing strategy; dynamical system; mathematical model; equilibrium solution

INTRODUCTION

To gain more costumers some marketing strategies are available, such as advertising, display in a shop, distribution channels, promotion, etc. Recently many marketing strategies have included the use of information technology, such as website and social media. Strategies such as the Word-of-Mouth, abreviated as WOMM or WOM marketing, and celebrity endorsment are becoming popular and recognized as effective strategies. The WOM communication is considered to be have advantages over other stratgies for having significantly lower cost and much faster propagation of the messages.

Moreover, Word-of-Mouth marketing is among marketing operation practicized by many companies nowadays.But we can also consider that the Word –of-Mouth marketing operation needs a strategy to leverage the impact. What we mean by strategy here is an action or scenario that may enhance or amplify the result of the the Word-of-Mouth action into the buying behaviour of the commuinity under consideration. The WOM is sometimes called as word of mouth advertising in a form of promotional campaign which acts through an individual's personal recommendations (Chittipaka, 2010). It is now becoming global phenomena since it is inexpensive to operate, especially since the emergimg of some social medias. However, there are only a few of literatures regarding this strategy, e.g. (Chittipaka, 2010; Li et al, 2018; Husniah et al, 2019).

As many other areas of management science and technology, addressing questions in these areas also influenced with the use of quantitative methods such as mathematics, via the process of mathematical modeling. This approach is commonly done in other areas but it is only a beginning in the study of the Word-of-Mouth marketing strategy. In this approach, the practice of the Word-of-Mouth strategy is often looked at by making an analogy to the spread of a disease in a community. Here an individual who has a good/bad experience and opinion regarding a product or service the individual bought/used can pass his/her experience and opinion to others. By this process an individual can be agree to the opinion given by that person who pass the opinion to him/her and consecutively this infected person can infect others. This is the idea behind the process of modeling the propagation of opinion to buy or not to buy a product or a service. The following section discusses briefly an existing model of Word-of-Mouth marketing strategy followed by sections that analyze, explore, and refine the model.

METHOD

As has been explained eralier, many other areas of management science and technology, addressing questions in these areas also influenced with the use of quantitative methods such as mathematics, via the process of mathematical modeling. This approach is commonly done in other areas but it is only a beginning in the study of the Word-of-Mouth marketing strategy. Here we study the mathematical model of the Word-of-Mouth transmission model instead of the Word-of-Mouth process itself as the material of the study.

We follow the model and methodology in (Li et al., 2018). In their paper, they consider that the population under study is divided into four subpopulations, namely susceptible, infected, positive, and negative subpopulations. The susceptible subpopulation consists of potential buyers (with the size at time t is denoted by S(t)), the infected subpopulation consits of active buyers (with the size at time t is denoted by I(t)), the positive subpopulation consists of former buyer who have positive comment regarding the product or other factors related to the buying process (with the size at time t is denoted by P(t)), and the negative subpopulation consists of former buyers with negative comments (with the size at time t is denoted by N(t)), so that the model is given by with the parameters are described in Table 1.

$$\frac{dS(t)}{dt}=\mu-\beta_P P(t)S(t)-\beta_N N(t)S(t)+\gamma_P P(t)+\gamma_I I(t), \qquad (1.a)$$

$$\frac{dI(t)}{dt}=\beta_P P(t)S(t)-\alpha_P I(t)-\alpha_N I(t)-\gamma_I I(t)-\delta_I I(t), \qquad (1.b)$$

$$\frac{dP(t)}{dt}=\alpha_P I(t)-\gamma_P P(t)-\delta_P P(t), \tag{1.c}$$

$$\frac{dN(t)}{dt}=\alpha_N I(t)-\delta_N N(t), \tag{1.d}$$

Table 1. The description of parameters (Li et al., 2018)

Symbol	Definition
μ	the average rate of new individuals enter the target market and become susceptible
β_P	the *P*-infection force or the force or encouragement by the positive comments for a susceptible individual to purchase an item
β_N	the *N*-infection force or discouragement by the negative comments for a susceptible individual to exit from the market
γ_P / γ_N	the *P*-viscosity rate / the I-viscosity rate due to the shopping desire, a positive individual tends to purchase one more item and hence becomes susceptible
α_P	the *P*-comment rate: due to the desire to express the feeling for the recently purchased item, at any time an infected individual makes a positive comment on the item and hence becomes positive
α_N	the *N*-comment rate: due to the desire to express the feeling for the recently purchased item, at any time an infected individual makes a negative comment on the item and hence becomes negative
δ_I / δ_P / δ_N	*I*-exit rate / *P*-exit rate / *N*-exit rate: due to the loss of interest in shopping or other reasons, at any time anindividual exits from the target market

The authors in (Husniah et al., 2019) modify the model in (Li et al., 2018) by assuming there is no demographic factors in the model, since they only give attention on the effect of population sizes of *S*, *I*, *P*, and *N* to the dynamics of the model. Further refinement is made by assuming the rate of new individuals who enter the target market is proportional to the number of each subpopulations, i. e. *S*, *I*, *P*, and *N*. They have either a positive or negative contribution to the number of new entry to the susceptible class or the potential buyer. The model takes form as a system of differential equations shown below,

$$\frac{dS(t)}{dt}=\mu(S+I+P-N)-\beta_P P(t)S(t)-\beta_N N(t)S(t)+\gamma_P P(t), \tag{2.a}$$

$$\frac{dI(t)}{dt}=\beta_P P(t)S(t)-\alpha_P I(t)-\alpha_N I(t), \tag{2.b}$$

$$\frac{dP(t)}{dt}=\alpha_P I(t)-\gamma_P P(t), \tag{2.c}$$

$$\frac{dN(t)}{dt}=\alpha_N I(t)-\delta_N N(t). \tag{2.d}$$

RESULTS

The authors in (Li et al., 2018) show that their model (1.a – 1.d) admits a unique equilibrium, while the model in (Husniah et al., 2019), i.e. (2.a – 2.d), has two equilibria. Different from (Li et al., 2018)in which the non-trivial equilibrium always exists, the authors in (Husniah et al., 2019) prove a rule of thumb that there is a threshold number, called the R_0, and given by

$$R_0=\frac{\mu\gamma_P(\alpha_P+\alpha_N)+(\mu\alpha_P\beta_P(\alpha_N-\gamma_P-\alpha_P))}{\mu\alpha_P\beta_P(\alpha_N-\gamma_P-\alpha_P)+\alpha_N\gamma_P(\alpha_P(\beta_P+\beta_N)+\alpha_N\beta_N)} \tag{3}$$

$$+\frac{\alpha_N\gamma_P(\alpha_P(\beta_P+\beta_N)+\alpha_N\beta_N)}{\mu\alpha_P\beta_P(\alpha_N-\gamma_P-\alpha_P)+\alpha_N\gamma_P(\alpha_P(\beta_P+\beta_N)+\alpha_N\beta_N)}$$

In which if the R0 less than one then the long-term solution is the zero equilibrium (S*=0, I*=0, P*=0, N*=0), while if the R0 more than one then the long-term solution is (S*>0, I*>0, P*>0, N*>0). At first glance, the presence of proportional recruitment rate, theoretically, does affect the structure of the long-term solution.

However, a closer look at the R0, reveals that in fact the magnitude is always larger than one except when the coefficient of the recruitment rate μ is zero then R0 is exactly one. Hence, qualitatively both models in (Li et al., 2018) and (Husniah et al., 2019) has the same structure, that they are both have a unique positive solution for all positive initial values of all the state variables. Fig.1 to Fig.4 show the solutions of the model in (Husniah et al., 2019) for different values of R0, graphed in the form of their respective phase port. All figures show that the number of individuals in subpopulation S never extinct, means there always a group of buyers even when μ=0. This includes the potential buyers (S) and active buyers (I).

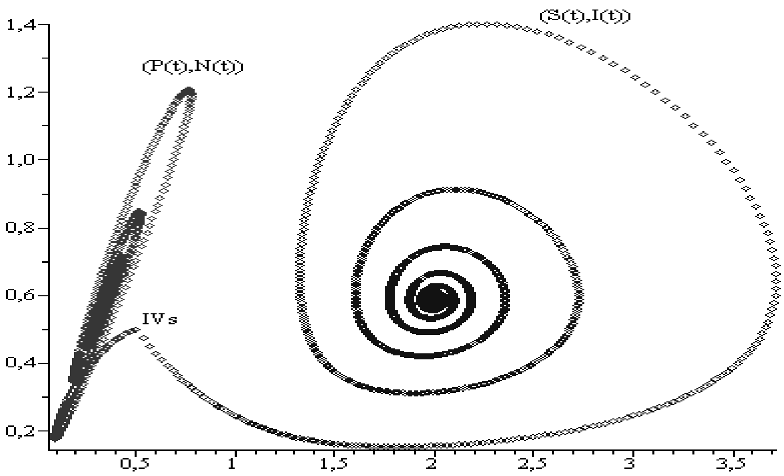

Figure-1. Phase port diagram with parameters as in Figure 1 of [3] with Ro=1.52

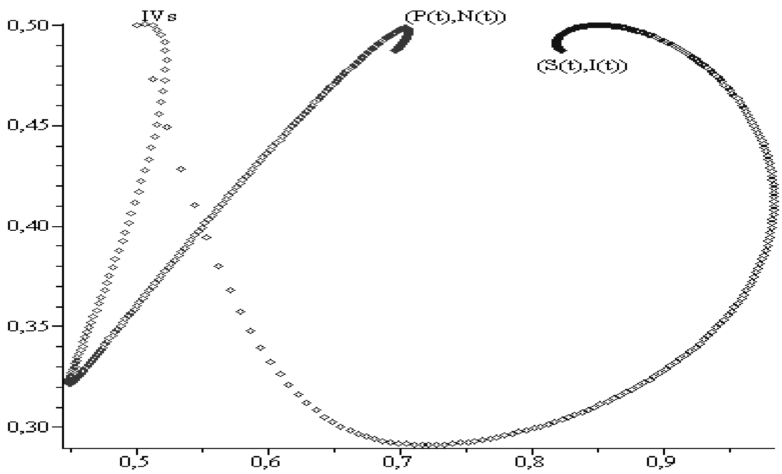

Figure-2. Phase port diagram with parameters as in Figure 2 of [3] with Ro=1.38

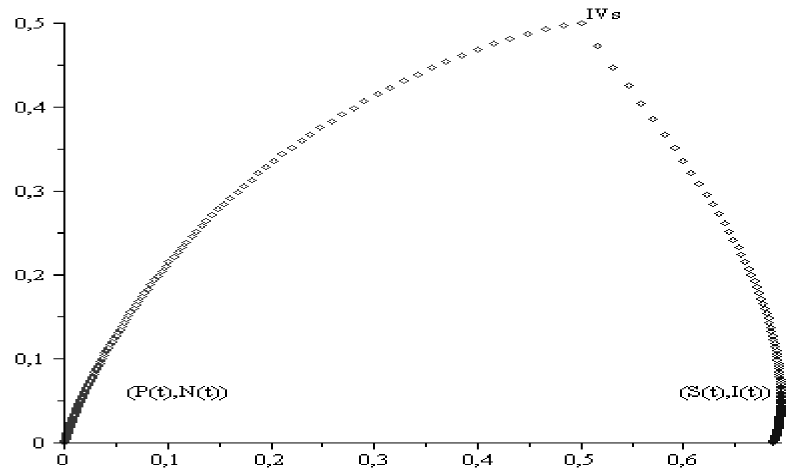

Figure-3. Phase port diagram with parameters as in Figure 1 of [3] except μ=0 with Ro=1.00.

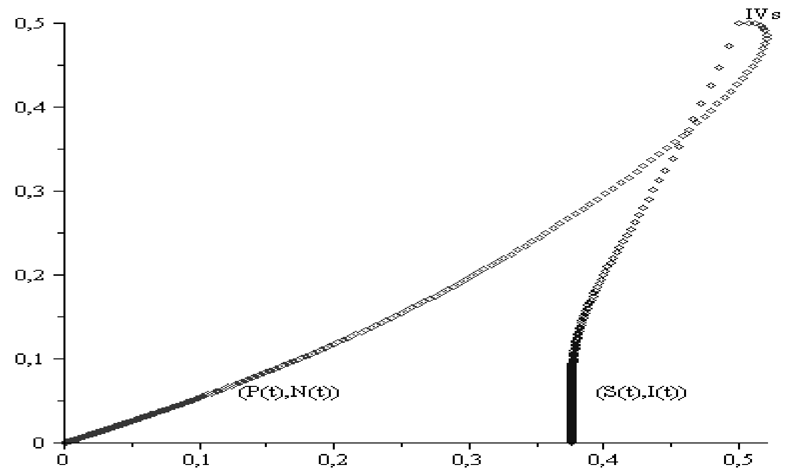

Figure-4. Phase port diagram with parameters as in Figure 2 of [3] except μ=0 with Ro=1.00.

The authors in (Husniah et al., 2019) show that the number of buyers, i.e. the densities of the potential (S*) and the active (I*) buyers, can be written as a linear function of the R0 in equation (3), i.e.

$$S^* = \frac{\gamma_P(\alpha_P+\alpha_N)}{\beta_P\alpha_P}, \tag{4}$$

$$I^* = \gamma_P(R_0 - 1) \tag{5}$$

Note that in (Husniah et al., 2019) the R_0 always greater than one.

It is easy to grasp equation (4). The bigger the encouragement by the positive comments for a susceptible individual to purchase an item, the smaller the number of potential buyers since they now shifted into active buyers. However, the sensitivity of active buyers size to the changes of values of the model's parameters (other than the artificial parameter R0) is not so clear. We plot several sensitivity analysis results in Figures 5 to see the effects of some parameters changes in the values of the equilibrium solution of the active buyers (I*). As expected the plot of R0 and the long-term numbers of active buyers I* are conform. This is easy to understand considering equation (5).

In Figure 5 we see that the effect of P-infection force or the force or encouragement by the positive comments for a susceptible individual to purchase an item is significantly linearly proportional to the equilibrium solution of the active buyers only if the the average rate of new individuals enter the target market and become susceptible (μ) is zero. The contour plots are given in

Figure 6. For non zeo μ the relationship slightly changes with reducing values of the number of active buyers. Other plots of sensitivity analysis can be seen in Figure 7 with the contour plots in Figure 8. The figures show the changes of the number of active buyers as the model's parameters change.

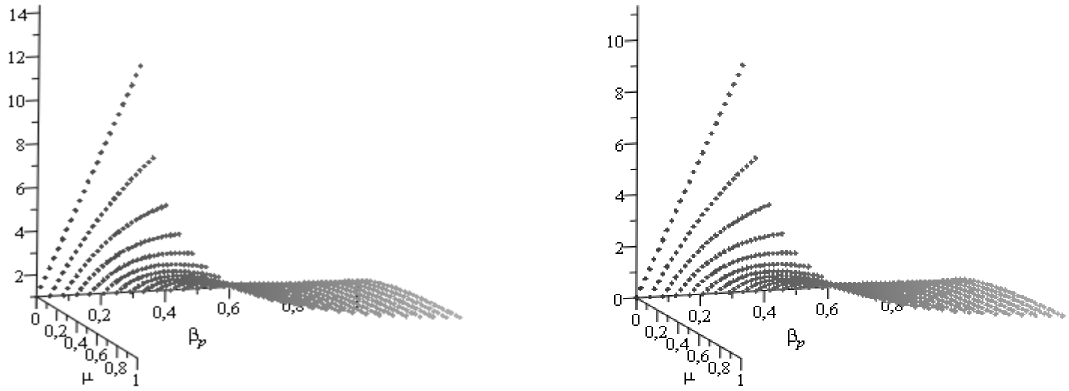

Figure-5. Plots of R_0(left) and I^* (right) as functions of μ and $\beta_P$with all other parameters as in Figure 1.

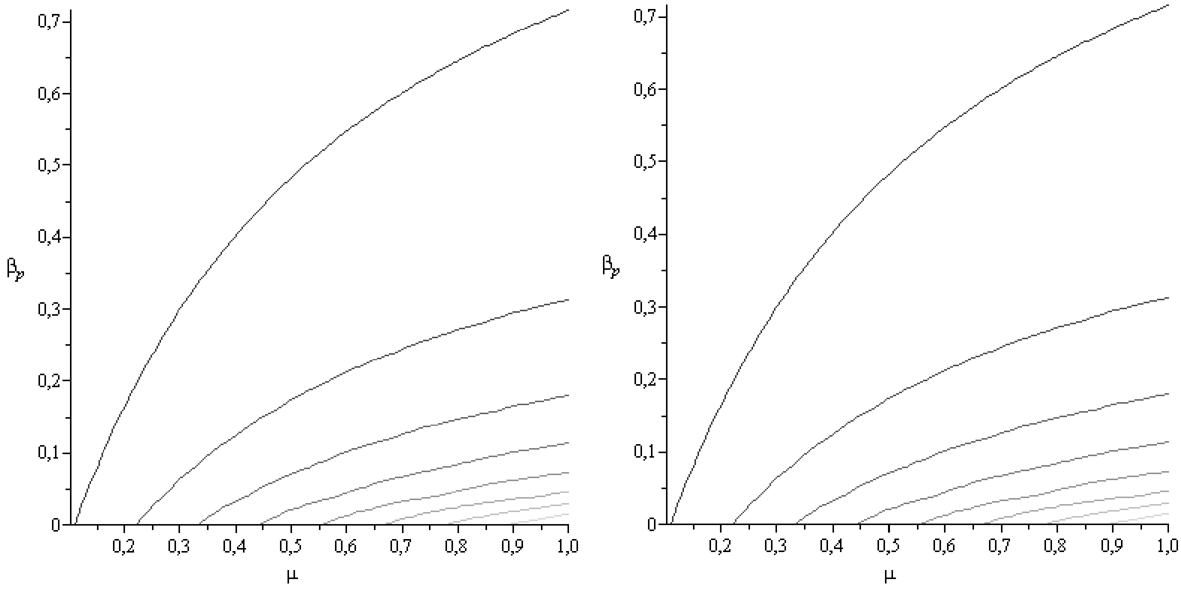

Figure-6. Contour plots of R_0 and I^* in Figure 5.

CONCLUSION

In this paper we have discussed a mathematical model of Word-of-Mouth marketing strategy. The analysis show that the strategy ensures a long-term numbers of active buyers depending on the size of the artificial parameter R_0. We note that the model discussed heredid not consider the natural exit rate from the class S, I, and P. Meanwhile the model in (Li et al., 2018) also did not consider the natural exit rate from the class S. The exit rate in (Li eta al., 2018) only due to the effect of provocation by the buyers who have negative opinian regarding the products they purchased. In reality, exit rate may exist in all classes for whatever the reason. This might be an alternative way for future investigation.

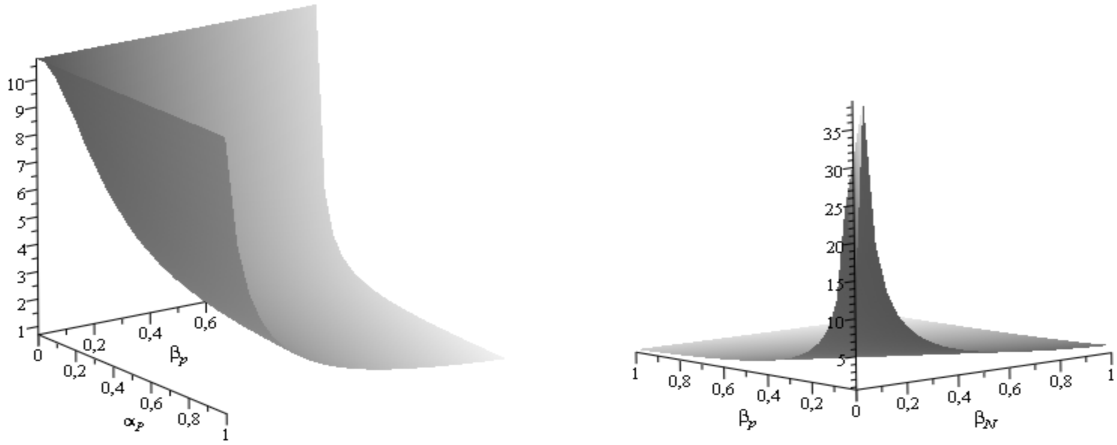

Figure-7. Plots of the active buyersI^* as functions of α_P and β_P(left) and β_P and β_N(right) with all other parameters as in Figure 1.

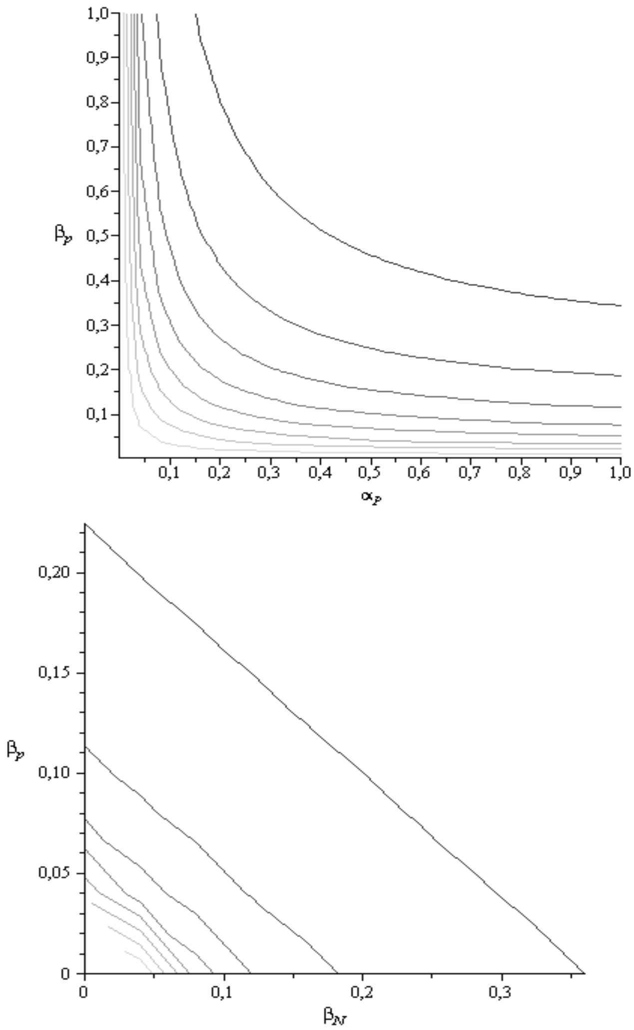

Figure-8. Contour plots of the graphs in Figure 7.

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