A physically-based analytical model to describe effective excess charge for streaming potential generation in saturated porous media

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Abstract

Different approaches have been proposed to predict streaming potentials in porous media. One approach is based on the excess charge which is effectively dragged in the medium by the water flow. Following the recent theoretical framework of Jougnot et al. (2012), we developed a physically-based analytical model to predict the effective excess charge in saturated porous media. The proposed model allows the determination of the effective excess charge as a function of pore water salinity, fractal dimension and hydraulic parameters like porosity and permeability. This new model has been successfully tested against data from the literature for different porous media. One of the main finding of this study is that it provides a mechanistic explanation to the empirical dependence between the effective excess charge and the permeability that has been found by various researchers. This model is also able to reproduce the evolution of this parameter with respect to the electrolyte concentration.

Introduction

The self-potential (SP) method is one of the few geophysical methods that is directly sensitive to groundwater flow. One source of SP signals, the so-called streaming potential, results from the presence of an electrical double layer at the mineral-pore water interface. When water flows through the pore space, it gives rise to a streaming current and a resulting measurable electrical voltage.

The excess charge distribution in a pore saturated by a binary symmetric 1:1 electrolyte (e.g., NaCl) is given by

$$Q_v(r) = N_A e_0 C^0 \left(e^{-\frac{e_0 \varphi(r)}{k_B T}} - e^{\frac{e_0 \varphi(r)}{k_B T}} \right)$$

where N_A is the Abogadro's number, e_0 the elementary charge, C^0 the ionic concentration far from the mineral surface, φ the local electrical potential, k_B the Boltzman constant, and T is the absolute temperature.
For the thin double layer assumption the local electrical potential can be expressed

$$\varphi(r) = \zeta \, e^{-\frac{r}{l_D}}$$

where ζ is the ζ-potential and l_D the Debye length.

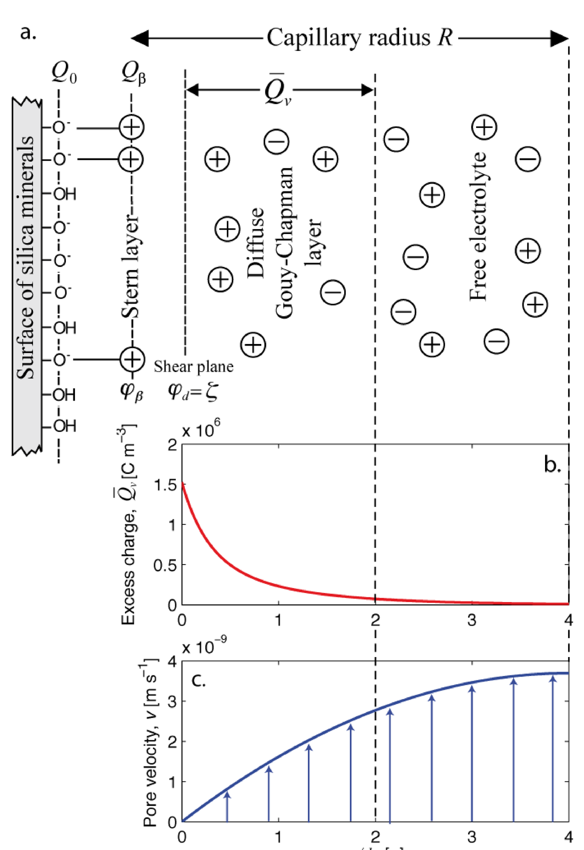

Figure 1: Distribution of charge and hydraulic flow in a capillary saturated with a NaCl electrolyte: (a) sketch of the electrical double layer, (b) excess charge density, and (c) pore velocity distribution. Figure from Jougnot et al (2012).

Constitutive model

The porous medium is conceptualized as an equivalent bundle of capillary tubes with a fractal law distribution of pore sizes.
The cumulative size-distribution of pores whose average radii are greater than or equal to R is assumed to obey the following fractal law:

$$N(R) = \left(\frac{R_{REV}}{R} \right)^D$$

where R_{REV} is the radius of the REV and D ($1 < D < 2$) the fractal dimension.

The resulting porosity of the REV is derived from its definition:

$$\phi = \frac{\tau D}{R_{REV}^{2-D} (2-D)} \left(R_{\max}^{2-D} - R_{\min}^{2-D} \right)$$

where τ is the tortuosity of the capillary tube, $R_{\min}$ and $R_{\max}$ are the minimum and maximum radii, respectively.

The resulting permeability of the REV is obtained by combining the volumetric flow of all individuals with the Darcy law:

$$k = \frac{D}{8 \tau R_{REV}^{2-D} (4-D)} \left(R_{\max}^{4-D} - R_{\min}^{4-D} \right)$$

For $R_{\min} << R_{\max}$ we obtain an expression to estimate permeability from porosity:

$$k = \gamma(R_{REV}, \tau, D) \, \phi^{(4-D)/(2-D)}$$

Note that for $D = 1$ the exponent of the porosity is 3 and the expression is similar to Kozeny-Carman equation.

Effective excess charge in a tube

The effective excess charge carried by the water flow in a single tube of radius R is defined by

$$Q_v^{eff}(R) = \frac{1}{v(R) \pi R^2} \int_A Q_v(r) v(R,r) \, dA$$

where $v(R,r)$ is the water velocity distribution inside the tube (Poiseuille model), $v(R)$ the average velocity, and A the cross-sectional area.

In order to obtain a closed-form analytical expression of $Q_v^{eff}(R)$ we approximate the exponential terms of $Q_v(r)$ by a four-term Taylor series. Then, for the thin double layer assumption (i.e., the thickness of the double layer is small compared to the pore size) we obtain:

$$Q_v^{eff}(R) = \frac{8 N_A e_0 C^0}{(R/l_D)^2} \left[-2 \frac{e_0 \zeta}{k_B T} - \left(\frac{e_0 \zeta}{3 k_B T} \right)^3 \right]$$

To test the validity of the above expression we compare the values of $Q_v^{eff}(R)$ obtain from this analytical solution with numerical estimations obtained using the approach of Jougnot et al. (2012).

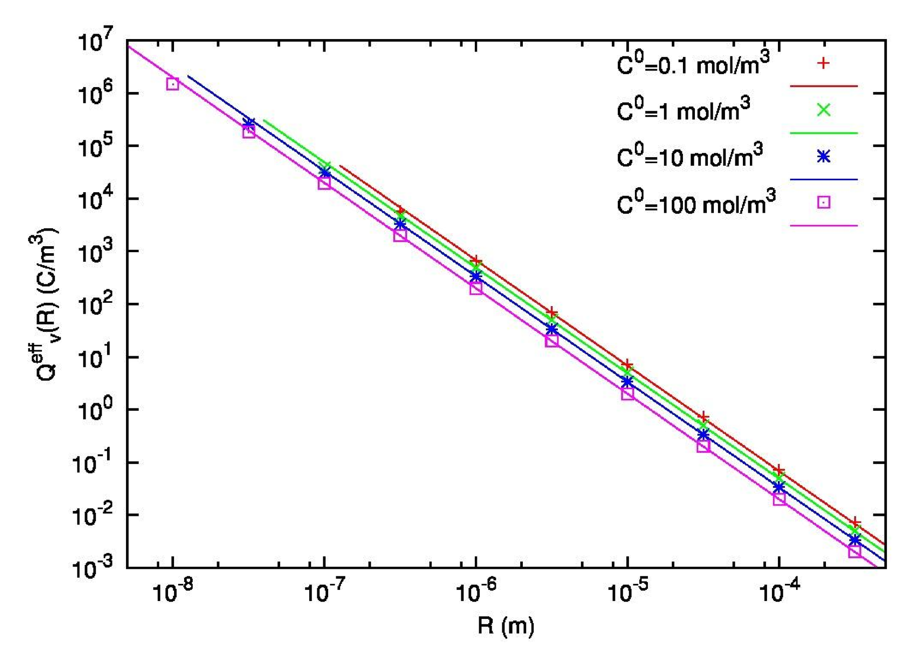

Figure 2: Numerical (points) and analytical (lines) estimates of $Q_v^{eff}(R)$ for different concentration values.

Effective excess charge at REV scale

The effective excess charge carried by the water flow in the REV is defined by

$$Q_v^{eff,REV} = \frac{1}{v_D \pi R_{REV}^2} \int_{R_{\min}}^{R_{\max}} Q_v^{eff}(R) v(R) \pi R^2 \, dN$$

where v_D is the Darcy's flow. Substituting the expression of $Q_v^{eff}(R)$ and assuming $R_{\min} << R_{\max}$ yields:

$$Q_v^{eff,REV} = N_A e_0 C^0 \left[-2 \frac{e_0 \zeta}{k_B T} - \left(\frac{e_0 \zeta}{3 k_B T} \right)^3 \right] \left(\frac{l_D}{\tau} \right)^2 \frac{\phi}{k}$$

The above equation predicts the effective excess charge in terms of both macroscopic hydraulic parameters (porosity, tortuosity and permeability) and electrokinetics parameters (ionic concentration, ζ-potential and Debye lenght).

Comparison with empirical relation

Jardani et al. (2007) proposed the following empirical relationship to estimate $Q_v^{eff,REV}$ from permeability

$$\log_{10}\left(Q_v^{eff,REV} \right) = A_1 + A_2 \log_{10}(k)$$

where $A_1 = -9.2349$ and $A_2 = -0.8219$ are constant values obtained by fitting the proposed expression to a set of experimental data.

This empirical relationship can be derived from our model, with the following constants:

$$A_1 = \log_{10} \left\{ \frac{N_A e_0 C^0}{\gamma^{(2-D)/(4-D)}} \left[-2 \frac{e_0 \zeta}{k_B T} - \left(\frac{e_0 \zeta}{3 k_B T} \right)^3 \right] \left(\frac{l_D}{\tau} \right)^2 \right\}$$

$$A_2 = -\frac{2}{4-D}$$

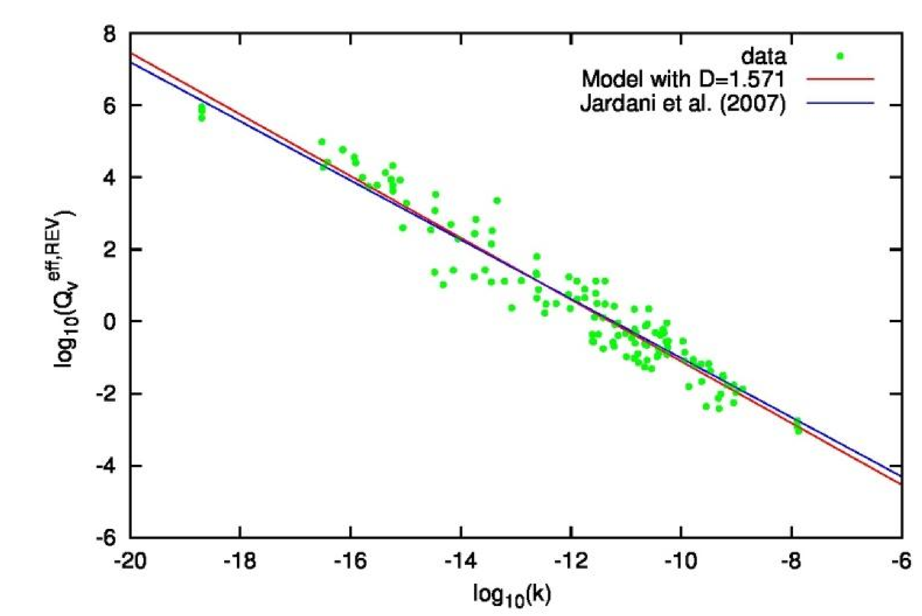

Figure 3: Predicted excess charge using the proposed model in terms of the permeability. As a reference we include in the figure the Jardani et al. (2007) empirical relationship.

Application to laboratory data

The proposed model is applied to the laboratory data of Pengra et al. (1999).
In order to include the effect of salt concentration in ζ-potential we consider the following relation (Pride and Morgan, 1991):

$$\zeta(C^0) = a + b \log_{10}(C^0)$$

being $a = -6.45$ mV and $b = 20.85$ mV (Jaafar et al., 2009).

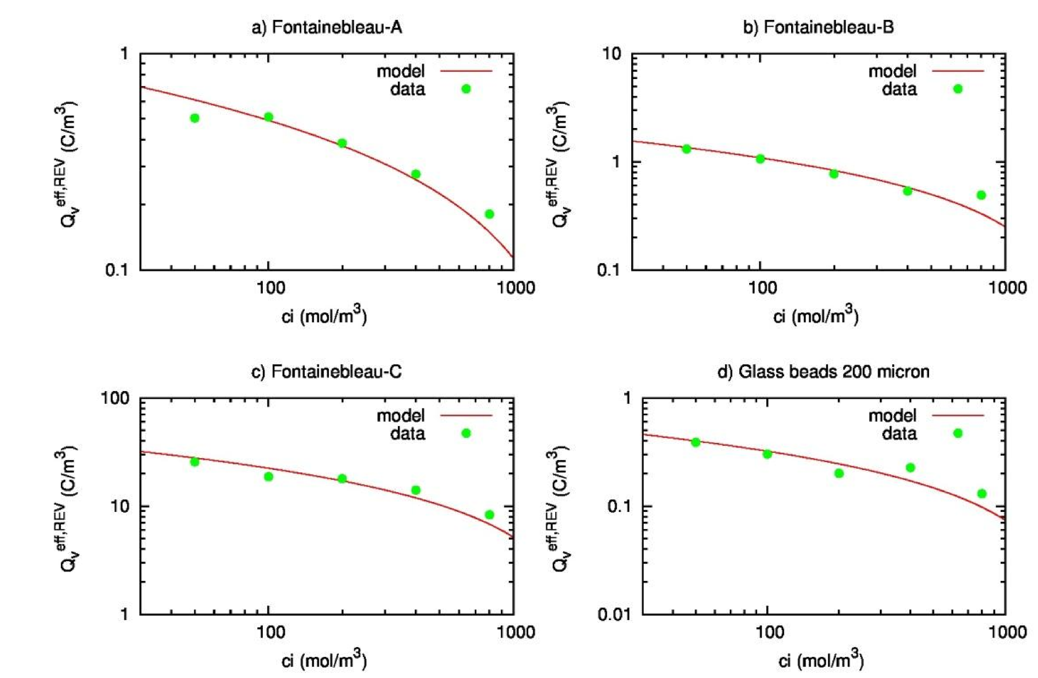

Figure 4: Comparison between measured values of $Q_v^{eff,REV}$ and predicted values using the proposed model for different electrolyte concentrations. Hydraulic parameters of soil samples: a) $\phi = 0.223$, $k = 2.358710^{-12}$m^2, $\tau = 1.95$, b) $\phi = 0.168$, $k = 9.089310^{-13}$m^2, $\tau = 1.83$, c) $\phi = 0.067$, $k = 5.625310^{-15}$m^2, $\tau = 3.24$, d) $\phi = 0.298$, $k = 5.072710^{-12}$ m^2, $\tau = 1.90$. Note that k and ϕ were measured by Pengra et al. (1999).

Finally we compare our model with experimental data of Pengra et al. (1999) for a constant value of ionic concentration and different values of k/ϕ ratios.

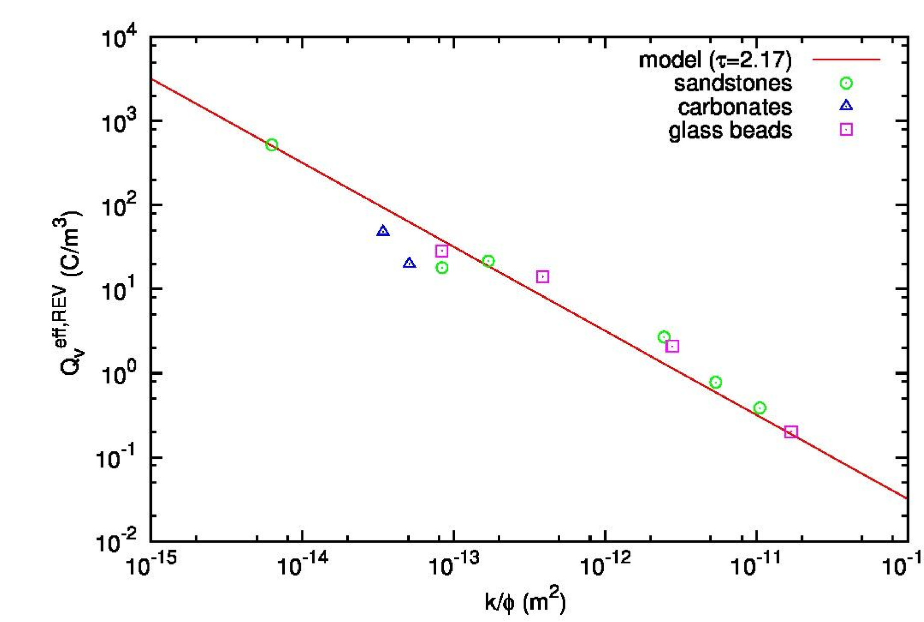

Figure 5: Comparison between measured values of $Q_v^{eff,REV}$ vs k/ϕ and predicted values for $C^0 = 200$ mol/m^3.

Conclusions

In this work we present a physically-based analytical model to describe effective excess charge for streaming potential generation in saturated porous media. The effective excess charge model is explicitly expressed in terms of pore water salinity, fractal dimension, porosity and permeability. The model provides a mechanistic explanation to the empirical dependence between the effective excess charge and the permeability that has been found by various researchers (e.g. Jardani et al., 2007). The proposed petrophysical relationship also contributes to understand the role of porosity and water salinity on effective excess charge and will help to push further the use of streaming potential to monitor groundwater flow.

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