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# Quasi Spin Pairing and the Structure of the Lipkin Model

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By introducing the concepts of quasi-spin pairing and quasi-spin seniority, the Lipkin model is extended to a variable number of particles. The properties of quasi-spin pairing are seen to be quite similar to those of ordinary pairing. The quasi-spin seniority allows one to obtain a simple classification of excited multiplets. A "pairing plus monopole" model is studied in connection with the Hartree-Fock theory.

## 1. Introduction

The Lipkin Model [1] has proved to be extremely useful in theoretical studies concerning the validity and/or usefulness of diverse theoretical approaches developed in order to investigate the manifold aspects of the nuclear many body problem.
This model is based on the $SU2$ algebra corresponding to the so-called quasi-spin operators, and provides us with readily available exact solutions, which are to be compared with the results obtained by recourse to different types of approximations.
Of the several multiplets assigned to the possible values of the relevant Casimir operator [1], only that corresponding to the ground state of the system is usually dealt with, so that not much attention has been paid to the study and classification of higher multiplets. In the present work, we present a simple and straighforward method to achieve this goal, which allows us to formulate, in quasi-spin language, a $BCS$-like theory which extends the Lipkin model to the case of a variable particle number, without going beyond an $SU2 \times SU2$ algebra.

## 2. The Model

The model deals with $N$ particles, distributed in two $(2\Omega)$-fold degenerate single-particle levels which are separated by the single-particle energy $\varepsilon$.

We characterize the $2\Omega$ lower states by $|p, \mu = -1\rangle$ (for $p = 1, \ldots, 2\Omega$) as Lipkin et al. [1]. In addition to the usual quasi-spin operators,

$$\hat{J}_Z = \tfrac{1}{2} \sum_{p,\mu} \mu C^+_{p,\mu} C_{p,\mu}$$
$$\hat{J}_+ = \hat{J}^+_- = \sum_p C^+_{p,+} C_{p,-}, \tag{1}$$

we introduce the operators,

$$\hat{Q}_+ = \sum_p C^+_{p,+} C^+_{p,-}$$
$$\hat{Q}_- = \sum_p C_{p,-} C_{p,+}$$
$$\hat{Q}_0 = \tfrac{1}{2} \sum_{p,\mu} C^+_{p,\mu} C_{p,\mu} - \Omega, \tag{2}$$

which are easily shown to obey angular-momentum commutation rules. Moreover, any $Q$-operator commutes with all $J$-operators, and vice versa ($SU2 \times SU2$). It is possible then, to form a complete orthonormal basis characterized by the eigenvalues of the operators $\hat{J}^2$, $\hat{Q}^2$, $\hat{J}_Z$, $\hat{Q}_0$, i.e., $|J, Q, J_Z, Q_0\rangle$.
Obviously, $\hat{Q}_+$ creates, and $\hat{Q}_-$ destroys, two particles which yield zero contribution to the $J_Z$-value, and which could then be said to "couple" to $J_Z = 0$. In this spirit, we introduce a quasi-spin "pairing" Hamiltonian

$$\hat{H} = \hat{J}_Z - \frac{|G|}{2} \hat{Q}_+ \hat{Q}_-, \tag{3}$$

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which is seen to commute with the number operator $\hat{N}$, but we shall postpone its study until Section 3.
The minimum possible value of $Q_0$ is attained when $N$, the number of particles, is zero, and then $Q_0 = -\Omega$. The maximum $Q_0$ obtains when $N=4\Omega$, and then $Q_0=\Omega$. In the Lipkin model one always has $N=2\Omega$ and $Q_0=0$. Let us now consider the general multiplet $|Q,Q_0\rangle$. The eigenvalues of $\hat{Q}_0$ depend only upon the particle-number, which in a $Q$-multiplet varies, according to (2), between

$$N_{\min}(Q_0=-Q)=2\Omega-2Q\equiv v, \tag{4}$$

and

$$N_{\max}(Q_0=Q)=2\Omega+2Q=4\Omega-v. \tag{5}$$

In Equation (4) we have introduced the quasi-spin seniority $v$, which indicates the number of particles not "paired" to $J_Z=0(\hat{Q}_-|Q,-Q\rangle=0)$, i.e., $v$ is the number of "unpaired" particles in a $Q$-multiplet. From (4)

$$Q=\tfrac{1}{2}(2\Omega-v). \tag{6}$$

Now, if for a given pair $J$, $Q$ we have $v$ unpaired particles, the minimum possible value for $J_Z$ in this $(J,Q)$-multiplet is equal to $-v/2$.
Consequently, $v$ fixes also the value of $J$

$$J=v/2, \tag{7}$$

$$J+Q=\Omega, \tag{8}$$

so that the above mentioned $SU2\times SU2$ multiplets are characterized just by $v$, $Q_0$, $J_Z$. In the case of the Lipkin model ($N=2\Omega$), the unperturbed ground state (no interaction) has $J=\Omega$, $J_Z=-\Omega$, $Q=Q_0=0$ and attention is generally restricted to the multiplet $J=\Omega$, $Q=Q_0=0$ (only $J_Z$ varies, from $-\Omega$ to $+\Omega$). For this multiplet the quasi-spin seniority is equal to $2\Omega$. The remaining multiplets are those with $Q_0=0$ for which $v$ decreases from the value $2\Omega$ down to zero in steps of two. We have thus a seniority classification of the different multiplets of the Lipkin model. By allowing $Q_0$ to vary we can generalize the model to a variable number of particles.

## 3. Exact Energies

We study now the Hamiltonian of Equation (3) and put $g=|G|$. Its eigenstates are those introduced in the preceding section, i.e., $|J,Q,J_Z,Q_0\rangle$, with eigenvalues

$$E(J,Q,J_Z,Q_0)=J_Z-\frac{g}{2}\{Q(Q+1)-Q_0(Q_0-1)\}, \tag{9}$$

which can be rewritten in terms of $v$ and $N(=2Q_0+2\Omega)$ as follows

$$E(v,J_Z,N)$$
$$=J_Z-\frac{g}{2}\left\{\Omega(N-v)+\frac{v}{2}\left(\frac{v}{2}-1\right)-\frac{N}{2}\left(\frac{N}{2}-1\right)\right\}. \tag{10}$$

The energy of the unperturbed ground state ($v=N$, $Q=Q_0=0$) is

$$E_0=-\Omega. \tag{11}$$

The state of quasi-spin seniority zero, in which all particles are "paired" to $J_Z=0$ can be called the "superconducting" state. It is characterized by $v=0$ and $Q=\Omega$, its energy being

$$E_s=-\frac{g}{2}\{\Omega(\Omega+1)-Q_0(Q_0-1)\}$$
$$=-\frac{g}{2}\left(2\Omega-\frac{N}{2}+1\right)\frac{N}{2}. \tag{12}$$

This state will represent the ground state of the interacting system for

$$g\geqq g_{\text{crit}}=\frac{4\Omega}{N}\frac{1}{\left(2\Omega-\frac{N}{2}+1\right)}. \tag{13}$$

In Lipkin's case $g_{\text{crit}}=2/(\Omega+1)$.
One readily sees that the energy $E_{N-2}$ of the state characterized by $v=N-2$ and $J_Z=-v/2$ is given by

$$E_{N-2}=-\frac{N}{2}+1-g\left(\Omega-\frac{N}{2}+1\right). \tag{14}$$

This energy will be lower than $E_0$ for $g>1$ (independent of $N$). On the other hand, $g_{\text{crit}}$ is a minimum for $N=2\Omega$ and reaches a maximum value, equal to unity, for $N=4\Omega$ or $N=2$. In the latter case, the state with $v=N-2$ is the superconducting one, while in the former one there is just *one* possible state for the system. Consequently, as the strength $g$ of the interaction grows, the system suffers a "phase transition" from the state of maximum quasi-spin seniority to the superconducting one, at $g=g_{\text{crit}}$. This behaviour is depicted in Fig. 1 for the case $\Omega=3$.

## 4. The "BCS" Solution

Following the standard BCS treatment [2], we represent the ground state by the trial wave function

$$|\text{BCS}\rangle=\prod_{p=1}^{2\Omega}(u_p+v_p\,C^+_{p,+}\,C^+_{p,-})|0\rangle. \tag{15}$$

The Bogoliubov-Valatin coefficients [2] do not depend, in the present situation, upon the quantum number $p$. As a consequence, the quasi-particle operators $a_{p,\mu}$ take the form

$$a^+_{i,\mu} = \cos\beta\, C^+_{i,\mu} - \mu \sin\beta\, C_{i,-\mu}$$
$$a_{i,-\mu} = \cos\beta\, C_{i,-\mu} + \mu \sin\beta\, C^+_{i,\mu} \tag{16}$$

so that

$$a_{i,\mu}|\mathrm{BCS}\rangle = 0 \quad \text{for all } i,\mu. \tag{17}$$

The angle $\beta$ is chosen so as to minimize the expectation value

$$E_p = \langle \mathrm{BCS}|\hat{H}|\mathrm{BCS}\rangle$$
$$= -g\Omega\sin^2\beta(\sin^2\beta + 2\Omega\cos^2\beta), \tag{18}$$

with the restriction

$$N \overset{!}{=} \langle \mathrm{BCS}|\hat{N}|\mathrm{BCS}\rangle = 4\Omega\sin^2\beta. \tag{19}$$

The conservation condition (19) is usually tackled by introducing a Lagrange multiplier $\lambda$, which leads to the minimization problem [2]

$$\delta\{\langle \mathrm{BCS}|\hat{H} - \lambda\hat{N}|\mathrm{BCS}\rangle\} = 0. \tag{20}$$

Within the present context, however, Equation (19) fixes $\beta$, and we obtain

$$E_p = -\frac{g}{2}\left(2\Omega - \frac{N}{2} + \frac{N}{4\Omega}\right)\frac{N}{2}. \tag{21}$$

As a check, we verify that (21) coincides with the exact result (12) for the trivial case $N = 4\Omega$. For $N = 2\Omega$ (Lipkin Model) one has

$$E_p = -\frac{g}{2}\Omega(\Omega + \tfrac{1}{2}). \tag{22}$$

Should one procceed according to the prescription (20), one would obtain for $\lambda$ the value

$$g\{N(2\Omega - 1)/8\Omega - \Omega/4\}.$$

If one now considers the non-negative quantity

$$D = E_p - E_s = \frac{g}{2}\left(1 - \frac{N}{4\Omega}\right)\frac{N}{2}, \tag{23}$$

which measures the difference between the exact superconducting solution and the BCS one, it is possible to appreciate the fact that $D$ is a maximum for $N = 2\Omega$, i.e., for the Lipkin case.
One can also evaluate

$$\langle \mathrm{BCS}|\hat{N}^2|\mathrm{BCS}\rangle = N^2 + 2N - \frac{N^2}{4\Omega}, \tag{24}$$

in order to obtain

$$\frac{\langle \mathrm{BCS}|\hat{N}^2 - (\hat{N})^2|\mathrm{BCS}\rangle}{\langle \mathrm{BCS}|(\hat{N})^2|\mathrm{BCS}\rangle} \approx \frac{1}{N}, \tag{25}$$

as expected.
The number-projected BCS energy is given by

$$E_N = \frac{\langle \mathrm{BCS}|\hat{H}P_N|\mathrm{BCS}\rangle}{\langle \mathrm{BCS}|P_N|\mathrm{BCS}\rangle}. \tag{26}$$

The number projection (or $Q_0$-projection operator) is

$$P_N = \sum_{J,Q,J_Z} |J,Q,J_Z,Q_0(N)\rangle\langle Q_0(N),J_Z,Q,J|. \tag{27}$$

On the other hand, we can write the $|\mathrm{BCS}\rangle$ wave function as a superposition of states of good particle number.

$$|\mathrm{BCS}\rangle = \sum_{Q_0} \mathscr{C}(Q_0')|J=0,Q=\Omega,J_Z=0,Q_0'\rangle. \tag{28}$$

Insertion of (28) in (26) yields, taking into account the fact that $\hat{H}$ preserves $Q_0$

$$\langle \mathrm{BCS}|\hat{H}P_N|\mathrm{BCS}\rangle$$
$$= |\mathscr{C}(Q_0)|^2\langle 0,\Omega,0,Q_0|\hat{H}|0,\Omega,0,Q_0\rangle. \tag{29}$$

Moreover

$$\langle \mathrm{BCS}|P_N|\mathrm{BCS}\rangle = |\mathscr{C}(Q_0)|^2. \tag{30}$$

Equations (29) and (30) assure one that $E_N$ coincides with the exact superconducting energy.
It may be of interest, in order to compare quasi-spin pairing with "ordinary" pairing, to express the Hamiltonian (3) in terms of the quasi-particle operators $a$. In the usual notation we write [2]

$$\hat{H} = U + \hat{H}_{20} + \hat{H}_{11} + \hat{H}_{\mathrm{int}}, \tag{31}$$

with

$$U = -g\Omega\sin^2\beta(\sin^2\beta + 2\Omega\cos^2\beta), \tag{32}$$

$$\hat{H}_{20} = \{-g\Omega\sin\beta\cos\beta(\cos^2\beta - \sin^2\beta)$$
$$-g\sin^3\beta\cos\beta\} \times \sum_k (a^+_{k,+}a^+_{k,-} + a_{k,-}a_{k,+}) \tag{33}$$

$$\hat{H}_{11} = \hat{J}_Z + \frac{g}{2}\{\sin^4\beta + \sin^2\beta\cos^2\beta(4\Omega - 1)\}$$
$$\times \sum_k (a^+_{k,+}a_{k,+} + a^+_{k,-}a_{k,-}). \tag{34}$$

$\hat{H}_{\mathrm{int}}$ contains products of four $a$'s and is dropped in the quasiparticle approximation. Further, we define

$$\hat{H}' = \hat{H} - \lambda\hat{N} \tag{35}$$

and, as expected, we now obtain for $\beta$ and $\lambda$ the same values as reported above if we demand that

$$\hat{H}'_{20}=0, \tag{36}$$

in conjuction with the condition (19).
We see that (31 – 34) quite resemble the expressions used in nuclear pairing theory for the so-called degenerate model [2]. The only difference to be found between that case and ours arises in connection with the one-body part of the corresponding Hamiltonians. In the degenerate model the single-particle energy can be taken to be zero, so that, for states of $J_Z=0$, our expressions are identical with those of the referred model. This is not so, of course, for other values of $J_Z$.
The ground state energy of the degenerate model is [2]

$$E=-\frac{g}{8}N(4\Omega-N+2), \tag{37}$$

which agrees with our exact energy for

$$g>g_{\text{crit}}(J_Z=0)$$

The main *formal* difference between quasi-spin pairing and ordinary pairing lies in the fact that the former "pairs" particles of different s.p. energy, while the latter couples those of the same s.p. energy.

## 5. Excited States

As an application, we shall briefly discuss two widely applied theoretical approximations (and compare them with the exact solution), in connection with the excited states of our system. For the sake of simplicity we shall restrict ourselves to the $J_Z=0$ multiplet, in the Lipkin case ($N=2\Omega$), for $g>g_{\text{crit}}$.
The exact energies are given by

$$E=-\frac{g}{2}Q(Q+1), \tag{38}$$

where $Q$ takes the values $\Omega$ (ground state), $\Omega-1$, ..., 0. We have then $\Omega+1$ non-degenerate states. Notice that (38) just gives the energy-spectrum corresponding to the degenerate model for $N$ particles [2], i.e.,

$$E(N,v)=-\frac{g}{8}(N-v)(4\Omega-N-v+2), \tag{39}$$

where the seniority $v$ takes the values 0, 2, 4, ...$N$.
In the *quasi-particle approximation*, obtained by dropping $\hat{H}_{\text{int}}$ in (31), the excited energies are the so-called two quasi-particle energies [2]. The q.p. energies $e_k$ are, according to (34), given by

$$e_k=\frac{g}{2}\{\sin^4\beta+\sin^2\beta\cos^2\beta(4\Omega-1)\}=\frac{gN}{4}. \tag{40}$$

In this approximation, then, the excited states are degenerate and lie at an energy of $gN/2$, which is that corresponding, in the exact treatment, to the first excited state. Since we have $N$ q.p. states, there are $N(N-1)/2$ degenerate excited states.
Next, we consider the *quasi-boson approximation*, [2], and make the definitions

$$\begin{aligned} A_k^+&=a_{k,+}^+ a_{k,-}^+, \\ \hat{n}_k&=a_{k,+}^+ a_{k,+}+a_{k,-}^+ a_{k,-}. \end{aligned} \tag{43}$$

The Hamiltonian of Eq. (31) may be written in terms of the $A$'s. It contains terms of zero, first, and second order in the operators $A$ and $\hat{n}$. In the quasi-particle approximation, the second-order term is ignored. It is the interaction between quasi-particles and may be written

$$\begin{aligned} &\hat{H}_{\text{int}}=\hat{H}_c+\hat{H}_{\text{res}}, \\ &\hat{H}_c=-\frac{g}{2}\sum_{k,m}(\cos^2\beta A_k^+-\sin^2\beta A_k) \\ &\cdot(\cos^2\beta A_m-\sin^2\beta A_m^+). \end{aligned} \tag{44}$$

In the quasi-boson approximation [2] one drops $\hat{H}_{\text{res}}$, and, further, neglects $\hat{n}_k$ in the commutator

$$[A_k, A_i^+]=1-\hat{n}_k\delta_{k,i}. \tag{45}$$

If we write now $\hat{H}'=\hat{H}-\lambda\hat{N}$ we have, in this approximation (remember that we restrict our attention to the multiplet $J_Z=0$), in view of (33, 34)

$$\hat{H}'=\frac{g}{8}N(1-N)+\frac{g}{4}N\sum_i\hat{n}_i-\frac{g}{8}\sum_{i,j}(A_i^+-A_i)(A_j-A_j^+) \tag{46}$$

In order to write (46) one must take into account that here $\sin^2\beta=\frac{1}{2}$ and $\lambda=-g/4$. (Obviously, $\hat{H}'_{20}=0$).
Defining now

$$q_i=(A_i+A_i^+)/\sqrt{2}\,; \quad p_i=-i(A_i-A_i^+)/\sqrt{2}, \tag{47}$$

one obtains

$$\begin{aligned} &[\hat{H}', q_m]=-i\sum_k X_{mk}p_m, \\ &[\hat{H}', p_m]=i\sum_k y_{mk}q_k, \end{aligned} \tag{48}$$

with

$$x_{mk}=2e_m\delta_{mk}-\frac{g}{2},$$
$$y_{mk}=2e_m\delta_{mk}. \tag{49}$$

Consequently, if we find a transformation

$$Q_l=\sum_m \lambda_{lm}q_m;\quad P_l=\sum_m \mu_{lm}p_m, \tag{50}$$

so that

$$[Q_l,P_k]=\delta_{lk};\quad [\hat{H}',Q_l]=-iB_lP_l;$$
$$[\hat{H}',P_l]=i\,C_lQ_l \tag{51}$$

it follows [2] that the operator

$$O_l=\sqrt{C_l}\,Q_l-i\sqrt{B_l}\,P_l \tag{52}$$

is such that $O_l|\mathrm{BCS}\rangle=0$ and that the eigenstate $O_l^+|\mathrm{BCS}\rangle$ has an energy equal to $W_l=\sqrt{B_lC_l}$. The required transformation satisfies the matrix equations

$$[x][\lambda]_l=B_l[\mu]_l$$
$$[y][\mu]_l=C_l[\lambda]_l, \tag{53}$$

in rather obvious notation.
By solving (53) one gets $(N-1)$ degenerate states at an excitation energy of $gN/2$. We see that the spectrum looks like the one obtained with the quasi-particle approximation, but at least we have here the correct order of magnitude for the number of states.

## 6. Pairing plus Monopole Interaction

We add now to our pairing Hamiltonian the monopole force introduced by Lipkin et al. [1]. Our new Hamiltonian reads

$$\hat{H}=\hat{J}_Z+\frac{v}{2}(\hat{J}_+^2+\hat{J}_-^2)-\frac{g}{2}\hat{Q}_+\hat{Q}_-. \tag{54}$$

In (54) we have two competing interactions. On the one hand, the quasi-spin pairing, which favors states with $J=0$. On the other one, the monopole force, under whose action the system tries to attain the maximum possible value of $J$. We are thus reminded of the competition between pairing and deformation [3] in atomic nuclei, case in which, however, no exact treatment is available for the quadrupole force [3]. Thus, the present model allows one to perform detailed studies concerning the reliability of different approximations, specially for those values of $v$ and $g$ for which phase transitions occur.

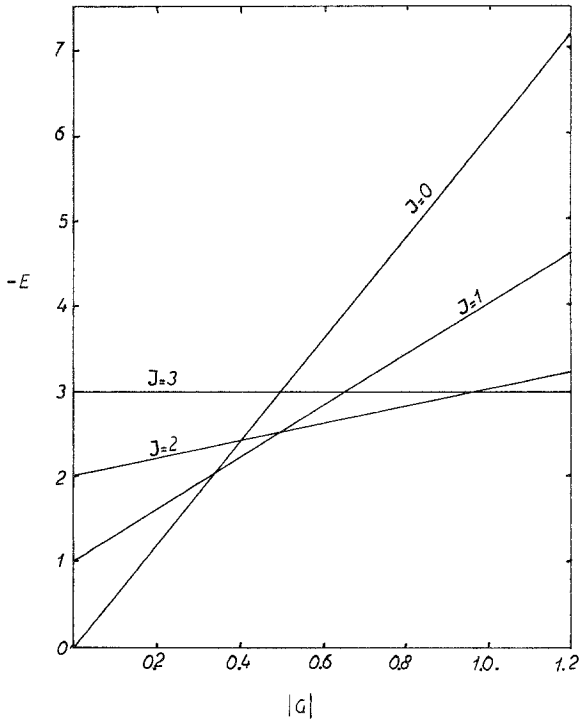


**Fig. 1.** Exact energies of the lowest states in each $J$-multiplet (for $N=6$) versus coupling constant $g$

The exact solution is found by diagonalizing $\hat{H}$ in the basis $|J,J_Z,Q,Q_0=0\rangle$. The corresponding matrix elements are

$$\langle J,J_Z',Q,0|\hat{H}|J,J_Z,Q,0\rangle$$
$$=J_Z\delta_{J_ZJ_Z'}+\frac{v}{2}\delta_{J_Z',J_Z+2}\{(J-J_Z)(J_Z+J+1)$$
$$\cdot(J-J_Z-1)(J_Z+J+2)\}^{\frac{1}{2}}$$
$$+\frac{v}{2}\delta_{J_Z',J_Z-2}\{(J-J_Z+1)(J_Z+J)$$
$$\cdot(J-J_Z+2)(J_Z+J-1)\}^{\frac{1}{2}}$$
$$-\frac{g}{2}(\Omega-J)(\Omega-J+1)\,\delta_{J_ZJ_Z'}. \tag{55}$$

Let us consider, for a fixed value of $v$, the exact energies corresponding to the lowest state in each $J$-multiplet, as a function of g. The corresponding behaviour is that depicted in Figure 1 for $N=6$. As $v$ increases, the results for $J=0$ remain unchanged, while the energies corresponding to the other $J$ values become smaller. As a consequence, the ground state of the system undergoes the transition from $J=3$ to $J=0$ for larger values of $g$ when $v$ is different from zero than for $v=0$.

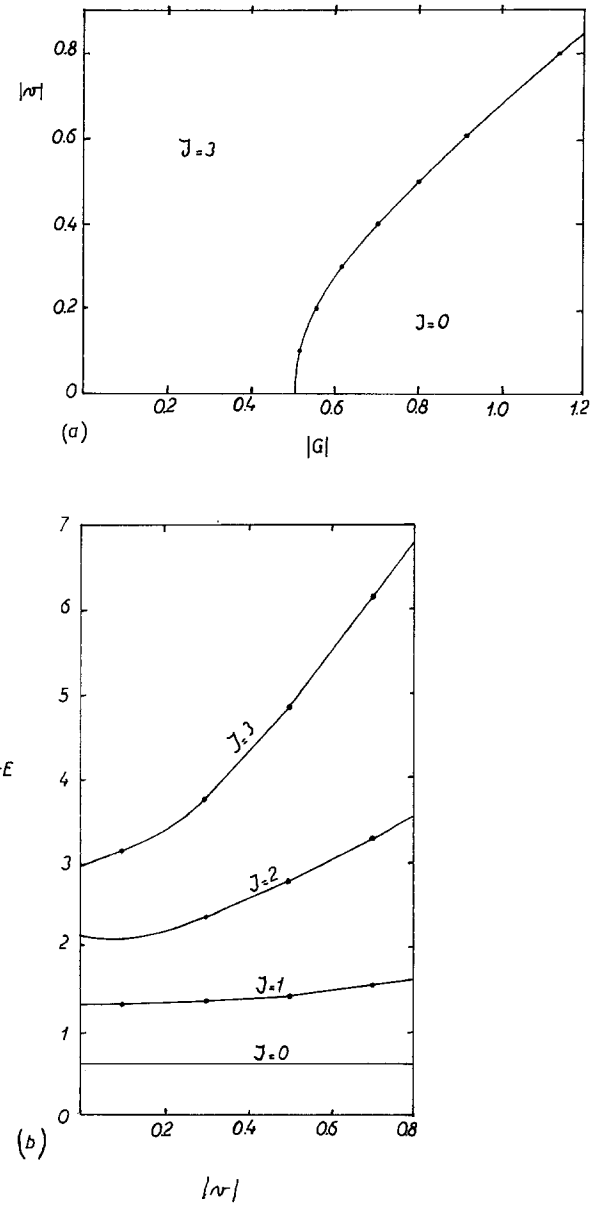


**Fig. 2. a** Values of $g$ and $v$ defining the phase transition between the superconducting state and the $J=J_{max}$ state. **b** Exact energies of the lowest states in each $J$-multiplet (for $N=6$) versus coupling constant $v$. $g$ is kept fixed at the value 0.1

Figure 2a depicts, for each $v$, the corresponding value of $g$ for which this transition occurs ($N=6$).
In Figure 2b we show the above mentioned energies, as a function of $v$ for small values of $g(\leqq 0.3$ in this case, as $N=6$). As $g$ grows, the curve for $J=3$ is not affected, while the others move upwards. The straight line corresponding to $J=0$ will thus succesively intersect the curves corresponding to $J=1$, 2, 3.
As an application let us consider the present model from the point of view of the Hartree-Fock (HF)

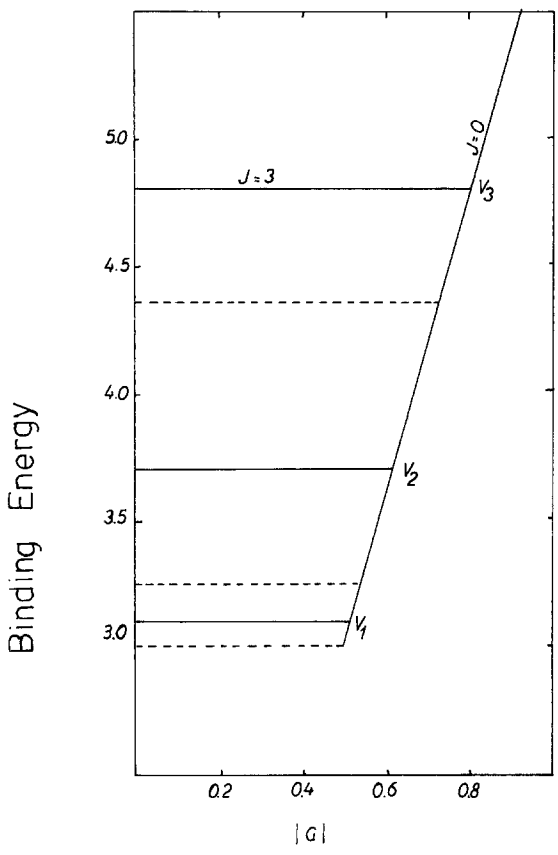


**Fig. 3.** Binding energy of the interacting ground state versus coupling constant $g$ for different values of $v$: $v_1=0.1$, $v_2=0.3$, $v_3=0.5$. The solid line corresponds to the exact treatment, the dashed one to the HF approach

approach. Within this context, the HF transformation reads [4]

$$\begin{pmatrix} b_{p,-} \\ b_{p,+} \end{pmatrix} = \begin{pmatrix} \cos\beta/2 & -i\sin\beta/2 \\ -i\sin\beta/2 & \cos\beta/2 \end{pmatrix} \begin{pmatrix} C_{p,-} \\ C_{p,+} \end{pmatrix} \tag{56}$$

and the corresponding trial states are obtained by building up in the new basis $b$ the trial states.

$$|HF\rangle = |J, J_Z = -J, Q = \Omega - J, 0\rangle. \tag{57}$$

Notice that the operators $\hat{Q}_+$ and $\hat{Q}_-$ are invariant under the transformation (56). We write now the Hamiltonian (54) in terms of the $b$-operators and evaluate its expectation values with respect to the states (57)

$$\langle \mathrm{HF}|\,\hat{H}(b)\,|\mathrm{HF}\rangle = -J\cos\beta$$
$$+\frac{v}{2}J(1-2J)\sin^2\beta - \frac{g}{2}(\Omega-J)(\Omega-J+1). \tag{58}$$

This is to be minimized with respect to $\beta$. Two solutions exist. If $v \leqq 1/(2J-1)$ we have $\beta=0$ and

$$E_{\mathrm{HF}} = -J - \frac{g}{2}(\Omega - J)(\Omega - J + 1). \tag{59}$$

For larger values of the monopole coupling constant one gets

$$\cos\beta = \frac{1}{v(2J-1)} = \frac{1}{b}; \qquad v > 1/(2J-1) \tag{60}$$

and

$$E_{\mathrm{HF}} = -\frac{J}{b} - \frac{bJ}{2}\left(1 - \frac{1}{b^2}\right) - \frac{g}{2}(\Omega - J)(\Omega - J + 1). \tag{61}$$

As in the exact case, the pairing force does not contribute to the HF energy for $J = \Omega$. On the other hand, for $J = 0$, the monopole HF contribution is null.

Ground state energies for the $N = 6$ case are depicted in Figure 3 as a function of $g$, for several values of $v$. Solid lines are exact results and dashed ones the HF energies. Horizontal lines correspond to $J = 3$. For $J = 0$ the HF energy is identical with the exact one.

Although there is a noticeable difference between the exact and the approximate values for $J = 3$, it must be pointed out that the HF theory predicts fairly well the values of $g$ for which the phase transition occurs.

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