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Radion stabilization in the presence of a Wilson line phase

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We study the stabilization of an extra-dimensional radius in the presence of a Wilson line phase of an extra $U(1)$ gauge symmetry on a 5D space-time, using the effective potential relating both the radion and the Wilson line phase at the one-loop level. We find that the radion can be stabilized by the introduction of a small number of fermions.

Subject Index B41, B43

1. Introduction

Superstring theory is a powerful candidate for particle physics including quantum gravity. It is defined on the 10D space-time, and necessarily possesses extra dimensions from the viewpoint of our 4D world. Many scalar fields and extra gauge symmetries generally appear after compactification. Scalar fields such as moduli and the dilaton are massless at the tree level, and how those fields are stabilized with suitable masses is a big issue. Interesting solutions have been proposed in type IIB string theory [1,2].

The study of effective field theories on a higher-dimensional space-time can provide useful hints to the stabilization of massless scalar fields.

In higher-dimensional gravity theory, a scalar field called the "radion" appears in the extra-dimensional components of the graviton, and its vacuum expectation value is related to the size of the extra space. The stabilization of the radius is crucial for the solution to the hierarchy problem in the Randall–Sundrum model [3]. It is also vital for the realization of inflation based on the radion [4]. The radius is fixed through the interactions with bulk scalars in the Randall–Sundrum background based on an orbifold S^1/Z_2 [5,6]. The stabilization is realized by the quantum effects of the graviton and fermions in a 5D model with an extra space S^1 [7].

In higher-dimensional gauge theory, the extra-dimensional components of gauge bosons are massless at the tree level due to the gauge invariance; their zero modes become dynamical degrees of freedom called Wilson line phases and are stabilized by quantum corrections [8]. There is a possibility that realistic gauge symmetries including the standard model ones survive after the stabilization of the Wilson line phases. As a solution to the gauge hierarchy problem, it has been pointed out that the Wilson line phase receives finite radiative corrections to its mass and can play the role of the

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Higgs boson [9]. An inflation model has been proposed based on the idea that the Wilson line phase becomes the inflaton [10].

Hence, it is interesting to study whether the radion is stabilized in the presence of Wilson line phases or not. This question has been examined in a specific gauge-Higgs unification model [11,12]. Their studies are based on warped space-time with the Randall–Sundrum metric and branes on the fixed points of S^1/Z_2. In addition to radiative corrections from the bulk gauge bosons, their brane-localized kinetic terms are crucial in stabilizing the radion.

In this paper, we investigate how a Wilson line phase and the Casimir energy from various bulk fields involve the stabilization of the radion in a different setup. Concretely, we study the stabilization of an extra-dimensional radius of S^1 in the presence of a Wilson line phase of an extra $U(1)$ gauge symmetry on a 5D space-time with a flat background metric and without branes, using the effective potential relating both the radion and the Wilson line phase at the one-loop level.

The content of our paper is as follows. In the next section, we derive the effective potential including the radion and the Wilson line phase at the one-loop level. In Sect. 3, we study the asymptotic behaviors of the effective potential, and obtain the conditions to stabilize the radion. It will be shown that incorporating neutral fermions is crucial to achieve the stability of the potential. In the last section, we give conclusions and discussions.

2. One-loop effective potential

We derive the effective potential concerning both the radion and the Wilson line phase at the one-loop level, in the framework of gravity theory coupled to a $U(1)$ gauge theory defined on a 5D space-time including S^1 as an extra space.

2.1. *Warm-up*

As a warm-up, we obtain the effective potential evaluating contributions from a massive complex scalar field φ coupled to an extra $U(1)$ gauge boson B_M in a 5D gravity theory. The action of φ is given by

$$S_\varphi = -\int \sqrt{-\hat{g}_5}\left[\hat{g}^{MN}(D_M\varphi)^*(D_N\varphi) + m_\varphi^2|\varphi|^2\right] d^5x, \tag{1}$$

where $\hat{g}_5 = \det\hat{g}_{MN}$, $\hat{g}_{MN}$ $(M, N = 0, 1, 2, 3, 5)$ is the metric of 5D space-time, $D_M = \partial_M - ig'_5 q'_\varphi B_M$, and m_φ is the mass of φ. g'_5 is the 5D gauge coupling constant and q'_φ is the $U(1)$ charge of φ.

We set $\hat{g}_{MN}$ as

$$\hat{g}_{MN} = \Phi^{-1/3}\begin{pmatrix} g_{\mu\nu} + A_\mu A_\nu \Phi & A_\mu \Phi \\ A_\nu \Phi & \Phi \end{pmatrix}, \tag{2}$$

then $\sqrt{-\hat{g}_5} = \Phi^{-1/3}\sqrt{-g_4}$.

We compactify the extra space on S^1 with the circumference $L = 2\pi R$ and impose periodic boundary conditions on every field. Then, the fields are given by the Fourier expansions

$$\hat{g}_{MN}\left(x^\mu, y\right) = \sum_{n=-\infty}^{\infty} \hat{g}_{MN}^{(n)}\left(x^\mu\right) e^{i\frac{2\pi n}{L}y}, \tag{3}$$

$$B_M\left(x^\mu, y\right) = \frac{1}{\sqrt{L}} \sum_{n=-\infty}^{\infty} B_M^{(n)}\left(x^\mu\right) e^{i\frac{2\pi n}{L}y}, \tag{4}$$

$$\varphi\left(x^\mu, y\right) = \frac{1}{\sqrt{L}} \sum_{n=-\infty}^{\infty} \varphi^{(n)}\left(x^\mu\right) e^{i\frac{2\pi n}{L}y}, \tag{5}$$

where x^μ $(\mu = 0, \ldots, 3)$ and y denote 4D coordinates and the S^1 one, respectively. We have two massless 4D scalar fields $\Phi^{(0)}$ and $B_5^{(0)}$ besides $\varphi^{(0)}$.

We take the Minkowski metric $\eta_{\mu\nu}$ as the background value of the 4D metric $g_{\mu\nu}$, i.e., $\left\langle g_{\mu\nu}^{(0)}\right\rangle = \eta_{\mu\nu}$. Other zero modes are assumed to have the following classical values:

$$\left\langle A_\mu^{(0)}\right\rangle = 0,\ \left\langle \Phi^{(0)}\right\rangle = \phi,\ \left\langle B_\mu^{(0)}\right\rangle = 0,\ \left\langle B_5^{(0)}\right\rangle = \frac{\theta}{g_4' L}, \tag{6}$$

where ϕ is the radion and θ is the Wilson line phase given by $\theta = g_4' \int_0^L dy \left\langle B_5^{(0)}\right\rangle$ and $g_4' = g_5'/\sqrt{L}$. The massive modes called Kaluza–Klein (KK) modes are assumed to have zero classical values.

To compute the one-loop potential, it suffices to know the quadratic terms of $\varphi^{(n)}$ in the 4D action. These are given by

$$S_\varphi = \sum_{n=-\infty}^{\infty} \int \left[\varphi^{(n)*}\left\{-\Box + \left(\frac{2\pi n + q_\varphi' \theta}{\phi^{1/2} L}\right)^2 + \frac{m_\varphi^2}{\phi^{1/3}}\right\}\varphi^{(n)}\right] d^4x. \tag{7}$$

Taking the standard procedure, the effective potential at the one-loop level is given by

$$V_\varphi^{\text{eff}} = 2V^{\text{eff}}(m_\varphi, q_\varphi'),\quad V^{\text{eff}}(m, q') \equiv \frac{1}{2}\sum_{n=-\infty}^{\infty} \int \frac{d^4 p_{\text{E}}}{(2\pi)^4} \ln\left[p_{\text{E}}^2 + \left(\frac{2\pi n + q'\theta}{\phi^{1/2} L}\right)^2 + \frac{m^2}{\phi^{1/3}}\right]. \tag{8}$$

After the integration of the Euclidean momentum p_{E}, $V^{\text{eff}}(m, q')$ is calculated as

$$\begin{aligned} V^{\text{eff}}(m, q') = -\frac{3}{4\pi^2}\frac{1}{\phi^2 L^4}\text{Re}\Big[&\text{Li}_5\left(e^{-Lm\phi^{1/3}} e^{iq'\theta}\right) + Lm\phi^{1/3}\text{Li}_4\left(e^{-Lm\phi^{1/3}} e^{iq'\theta}\right) \\ &+ \frac{1}{3}L^2 m^2 \phi^{2/3}\text{Li}_3\left(e^{-Lm\phi^{1/3}} e^{iq'\theta}\right)\Big], \end{aligned} \tag{9}$$

where $\text{Li}_n(x)$ are the polylogarithm functions

$$\text{Li}_n(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^n}. \tag{10}$$

2.2. *Our model*

Our model consists of the 5D graviton $\hat{g}_{MN}$, a $U(1)$ gauge boson B_M, c_1 charged fermions ψ_i $(i = 1, \ldots, c_1)$, and c_2 $U(1)$ neutral fermions η_l $(l = 1, \ldots, c_2)$. We take $M^4 \times S^1$ as a background 5D space-time and impose periodic boundary conditions on every field. Here, M^4 is the Minkowski space.

Because of the Lorentz covariance and the gauge invariance and consideration of the statistics for particles, the effective potential due to the one-loop effect of the X field is written as

$$V_X^{\text{eff}} = f_X V^{\text{eff}}(m_X, q_X'), \tag{11}$$

Table 1. The physical degrees of freedom, mass, and $U(1)$ charge for each field.

X	f_X	m_X	q'_X
$\hat{g}_{MN}$	5	0	0
B_M	3	0	0
ψ_i	−4	m_i	q'_i
η_l	−4	μ_l	0
φ	2	m_φ	q'_φ

where f_X, m_X, and q'_X are the physical degrees of freedom, the mass, and the $U(1)$ charge of X, respectively. The values of f_X, m_X, and q'_X for each field are given in Table 1. Note that the graviton and the $U(1)$ gauge boson have no $U(1)$ charge and are massless. Unphysical modes of $\hat{g}_{MN}$ and B_M are eliminated by the contributions from their ghosts.

Hence, we obtain the full effective potential at the one-loop level [13–15]:

$$\begin{aligned} V(\phi,\theta) &= \sum_X V_X^{\text{eff}} = 8V^{\text{eff}}(0,0) - 4\sum_i V^{\text{eff}}(m_i, q'_i) - 4\sum_l V^{\text{eff}}(\mu_l, 0) \\ &= -\frac{6}{\pi^2}\frac{1}{\phi^2 L^4}\zeta(5) + \sum_i \frac{3}{\pi^2}\frac{1}{\phi^2 L^4}\mathrm{Re}\Big[\mathrm{Li}_5\Big(e^{-Lm_i\phi^{1/3}}e^{iq'_i\theta}\Big) \\ &\quad + Lm_i\phi^{1/3}\mathrm{Li}_4\Big(e^{-Lm_i\phi^{1/3}}e^{iq'_i\theta}\Big) + \frac{1}{3}L^2m_i^2\phi^{2/3}\mathrm{Li}_3\Big(e^{-Lm_i\phi^{1/3}}e^{iq'_i\theta}\Big)\Big] \\ &\quad + \sum_l \frac{3}{\pi^2}\frac{1}{\phi^2L^4}\Big[\mathrm{Li}_5(e^{-L\mu_l\phi^{1/3}}) + L\mu_l\phi^{1/3}\mathrm{Li}_4\Big(e^{-L\mu_l\phi^{1/3}}\Big) \\ &\quad + \frac{1}{3}L^2\mu_l^2\phi^{2/3}\mathrm{Li}_3\Big(e^{-L\mu_l\phi^{1/3}}\Big)\Big], \end{aligned} \tag{12}$$

where we use the formula relating the Riemann zeta function $\zeta(n)$:

$$\zeta(n) = \sum_{k=1}^{\infty}\frac{1}{k^n} = \mathrm{Li}_n(1). \tag{13}$$

The one-loop diagrams due to the 5D graviton and its ghosts (c_μ and c), 5D gauge boson and its ghost (C), charged fermions, and neutral ones are shown in Figs. 1(a), (b), (c), and (d), respectively.

3. Stabilization of the radion

We study the asymptotic behaviors of the effective potential and its stability, in the simple case that ψ_i and η_l have common masses m and μ, respectively, and ψ_i has a common $U(1)$ charge $q'_i = 1$. Then, the potential is written as

$$\begin{aligned} V(\phi,\theta) = &-\frac{6}{\pi^2}\frac{1}{\phi^2L^4}\zeta(5) + c_1\frac{3}{\pi^2}\frac{1}{\phi^2L^4}\mathrm{Re}\Big[\mathrm{Li}_5\Big(e^{-Lm\phi^{1/3}}e^{i\theta}\Big) + Lm\phi^{1/3}\mathrm{Li}_4\Big(e^{-Lm\phi^{1/3}}e^{i\theta}\Big) \\ &+ \frac{1}{3}L^2m^2\phi^{2/3}\mathrm{Li}_3\Big(e^{-Lm\phi^{1/3}}e^{i\theta}\Big)\Big] + c_2\frac{3}{\pi^2}\frac{1}{\phi^2L^4} \\ &\Big[\mathrm{Li}_5\Big(e^{-L\mu\phi^{1/3}}\Big) + L\mu\phi^{1/3}\mathrm{Li}_4\Big(e^{-L\mu\phi^{1/3}}\Big) + \frac{1}{3}L^2\mu^2\phi^{2/3}\mathrm{Li}_3\Big(e^{-L\mu\phi^{1/3}}\Big)\Big]. \end{aligned} \tag{14}$$

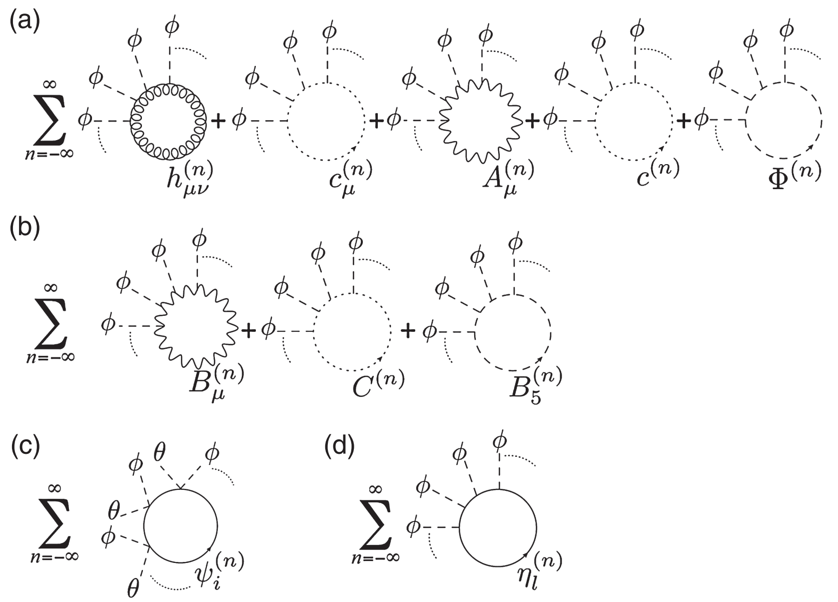

Fig. 1. One-loop diagrams due to the 5D graviton and its ghosts (c_μ and c) (a), the 5D gauge boson and its ghost (C) (b), charged fermions (c), and neutral ones (d).

3.1. *Asymptotic behaviors*

First, we investigate the behavior of the potential for large values of ϕ. For the region of ϕ satisfying

$$e^{-Lm\phi^{1/3}},\ e^{-L\mu\phi^{1/3}},\ \mathrm{Li}_n\left(e^{-Lm\phi^{1/3}}\right),\ \mathrm{Li}_n\left(e^{-L\mu\phi^{1/3}}\right) \ll 1, \tag{15}$$

the potential can be approximated as

$$V(\phi,\theta) \simeq -\frac{6}{\pi^2}\frac{1}{\phi^2 L^4}\zeta(5). \tag{16}$$

It approaches zero from below as ϕ goes to infinity independently of θ and other parameters. This is a natural consequence of the fact that the physical circumference of S^1 is given by $L_{\mathrm{phys}} = \phi^{1/3}L$ and the effective potential should vanish for $L_{\mathrm{phys}} \to \infty$ due to the 5D gauge invariance.

Next, we study the behavior of the potential for small values of ϕ. The polylogarithm functions can then be approximated as

$$\begin{aligned}\mathrm{Li}_n\left[\left(1-Lm\phi^{1/3}\right)e^{i\theta}\right] &\simeq \sum_{k=1}^{\infty}\frac{\left(1-kLm\phi^{1/3}\right)}{k^n}e^{ik\theta}\\ &= \mathrm{Li}_n\left(e^{i\theta}\right) - Lm\phi^{1/3}\mathrm{Li}_{n-1}\left(e^{i\theta}\right)\end{aligned} \tag{17}$$

and

$$\mathrm{Li}_n\left(1-L\mu\phi^{1/3}\right) \simeq \zeta(n) - L\mu\phi^{1/3}\zeta(n-1). \tag{18}$$

Then, the potential takes the form

$$\begin{aligned}V(\phi,\theta) \simeq \frac{3}{\pi^2}\frac{1}{\phi^2L^4}\Bigg[&(c_2-2)\,\zeta(5) - \frac{2c_2}{3}L^2\mu^2\phi^{2/3}\zeta(3) - \frac{c_2}{3}L^3\mu^3\phi\zeta(2)\\ &+\mathrm{Re}\left\{c_1\mathrm{Li}_5\left(e^{i\theta}\right) - \frac{2c_1}{3}L^2m^2\phi^{2/3}\mathrm{Li}_3\left(e^{i\theta}\right) - \frac{c_1}{3}L^3m^3\phi\mathrm{Li}_2\left(e^{i\theta}\right)\right\}\Bigg].\end{aligned} \tag{19}$$

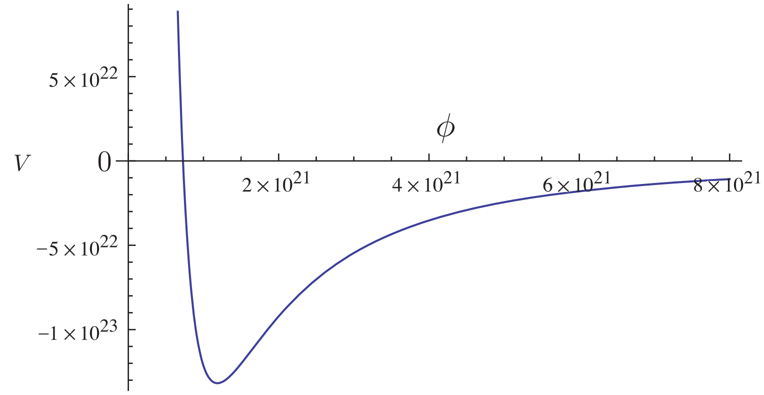

Fig. 2. The one-loop effective potential $V(\phi, \theta)$ for $\theta = 0$ with $c_1 = 3$, $c_2 = 3$, $m = 1 \times 10^{10}$ GeV, $\mu = 1 \times 10^{10}$ GeV, and $L = 3 \times 10^{-17}\,\mathrm{GeV}^{-1}$. Here the unit of the longitudinal axis is GeV^4. The same convention is used in Figs. 3–5.

Table 2. Numerical values of $\mathrm{Li}_n(-1)$ for some values of n.

n	2	3	4	5	6
$\mathrm{Li}_n(-1)$	−0.822	−0.90	−0.947	−0.96	−0.986

For sufficiently small values of ϕ, the mass-independent terms are dominant and almost determine the behavior of the potential near $\phi = 0$. Here, we consider two cases with $\theta = 0$ and $\theta = \pi$.

(i) Case with $\theta = 0$

In this case, the polylogarithm functions are just the ζ functions, i.e., $\mathrm{Li}_n(e^0) = \mathrm{Li}_n(1) = \zeta(n)$, and positive constants for $n > 1$. There are no differences between the charged fermions and neutral ones in point of contribution to the potential. The potential is approximated as

$$V(\phi, \theta = 0) \simeq \frac{3}{\pi^2}\frac{1}{\phi^2 L^4}\left[(c_1 + c_2 - 2)\,\zeta(5) - \frac{2}{3}L^2\phi^{2/3}\left(c_1 m^2 + c_2\mu^2\right)\zeta(3) - \frac{1}{3}L^3\phi\left(c_1 m^3 + c_2\mu^3\right)\zeta(2)\right]. \tag{20}$$

We see that the first term is dominant for $\phi \ll 1$, and the potential is repulsive near the origin if $c_1 + c_2 > 2$, i.e.,

$$c_1 + c_2 \geq 3, \tag{21}$$

where c_1 and c_2 take zero or positive integers. As ϕ becomes larger, i.e., $L^3 m^3 \phi \simeq 1$, the second and third terms begin to exceed the first term; then the potential becomes negative and attractive. Taking into account the behavior of the potential for a large value of ϕ, we conclude that $V(\phi, \theta = 0)$ has a minimum. A typical shape of $V(\phi, \theta = 0)$ is shown in Fig. 2. This result agrees with that given in previous work [13,15,16].

(ii) Case with $\theta = \pi$

In this case, the polylogarithm functions can be evaluated as follows:

$$\mathrm{Li}_n\left(e^{i\pi}\right) = \mathrm{Li}_n(-1) = -1 + \frac{1}{2^n} - \frac{1}{3^n} + \frac{1}{4^n} - \cdots = \left(2^{1-n} - 1\right)\zeta(n). \tag{22}$$

Some sample values of $\mathrm{Li}_n(-1)$ are given in Table 2. $\mathrm{Li}_n(-1)$ take negative values, and then flip the signs of the terms proportional to c_1 in (19).

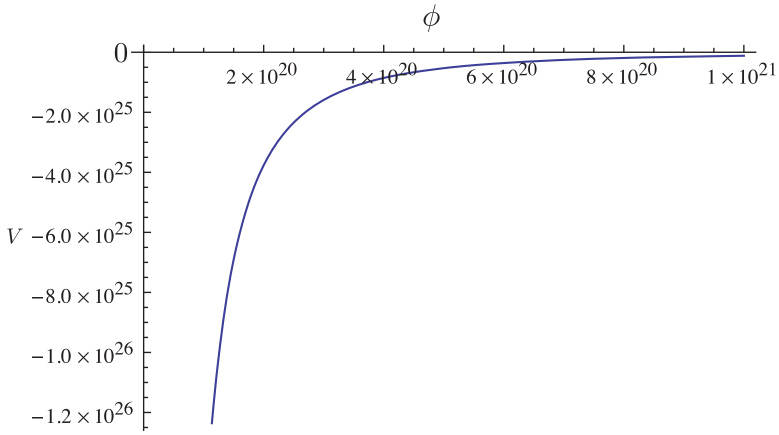

Fig. 3. $V(\phi, \theta)$ for $\theta = \pi$ with $c_1 = 3$, $c_2 = 0$, $m = 1 \times 10^{10}$ GeV, and $L = 3 \times 10^{-17}$ GeV^{-1}.

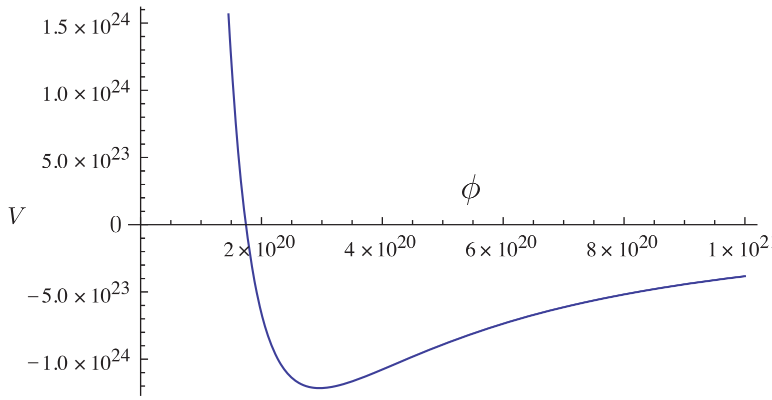

Fig. 4. $V(\phi, \theta)$ for $\theta = \pi$ with $c_1 = 1$, $c_2 = 4$, $m = 1 \times 10^{10}$ GeV, $\mu = 1 \times 10^{10}$ GeV, and $L = 3 \times 10^{-17}$ GeV^{-1}.

The potential is given by

$$V(\phi, \theta = \pi) \simeq \frac{3}{\pi^2}\frac{1}{\phi^2 L^4}\Bigg[\Big\{c_2 - 2 - c_1\big(1 - 2^{-4}\big)\Big\}\zeta(5) - \frac{2c_2}{3}L^2\mu^2\phi^{2/3}\zeta(3) - \frac{c_2}{3}L^3\mu^3\phi\zeta(2)$$
$$+ \frac{2c_1}{3}L^2 m^2\phi^{2/3}\big(1 - 2^{-2}\big)\zeta(3) + \frac{c_1}{3}L^3 m^3\phi\big(1 - 2^{-1}\big)\,\zeta(2)\Bigg]. \tag{23}$$

In the absence of neutral fermions, i.e., $c_2 = 0$, $V(\phi, \theta = \pi)$ is attractive near the origin and approaches negative infinity for any c_1. A typical shape of $V(\phi, \theta = \pi)$ is shown in Fig. 3.

In the introduction of neutral fermions, the potential can have a finite minimum, if the following condition is fulfilled:

$$c_2 > 2 + c_1. \tag{24}$$

This comes from the notion that the coefficient of the first term in $V(\phi, \theta = \pi)$ should be positive. Note that the condition (21) automatically holds if (24) is satisfied, because $c_1 + c_2 > 2 + 2c_1 \geq 2$.

Figure 4 shows a typical shape of the potential with a finite minimum (24) being satisfied.

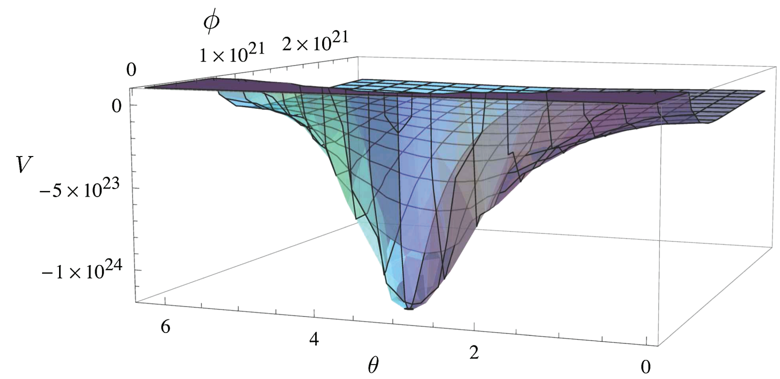

Fig. 5. The potential $V(\phi, \theta)$ with $c_1 = 1$, $c_2 = 4$, $m = 1 \times 10^{10}$ GeV, $\mu = 1 \times 10^{10}$ GeV and $L = 3 \times 10^{-17}\,\text{GeV}^{-1}$.

3.2. Stability

First, we study the θ dependence of $V(\phi, \theta)$. The θ-dependent term in $V(\phi, \theta)$ has the form

$$\text{ReLi}_n\left(e^{-Lm\phi^{1/3}} e^{i\theta}\right) = \sum_{k=1}^{\infty} \frac{\cos k\theta}{k^n} e^{-kLm\phi^{1/3}}. \tag{25}$$

Its maximum and minimum with respect to θ for fixed values of ϕ can be found by computing its derivatives in θ. By noting

$$\frac{d}{d\theta}\text{ReLi}_n\left(e^{-Lm\phi^{1/3}} e^{i\theta}\right) = -\sum_{k=1}^{\infty} \frac{\sin k\theta}{k^{n-1}} e^{-kLm\phi^{1/3}} = 0 \tag{26}$$

and the second derivative of (25)

$$\frac{d^2}{d\theta^2}\text{ReLi}_n\left(e^{-Lm\phi^{1/3}} e^{i\theta}\right) = -\sum_{k=1}^{\infty} \frac{\cos k\theta}{k^{n-2}} e^{-kLm\phi^{1/3}}, \tag{27}$$

we find that the maximum and minimum occur at $\theta = 0$ and $\theta = \pi$, respectively.

Hence, we have only to examine the behavior of $V(\phi, \theta)$ around $\theta = \pi$ for the stabilization of the radius. As seen in the previous subsection, the potential has a finite minimum in the presence of neutral fermions whose numbers c_2 are bigger than $c_1 + 2$, and then the radion is stabilized at a certain finite value of ϕ.

A typical shape of $V(\phi, \theta)$ is depicted in Fig. 5. From this figure, we see that the true minimum of the potential is located on the line of $\theta = \pi$. The values of ϕ and the potential at the minimum depend on parameters such as m, μ, and L. Their values do not modify the shape of the potential drastically, except for the cases such that $m \to \infty$, $\mu \to \infty$, $L = 0$, and $L = \infty$.

Up to now, we have considered the case with massive fermions. Here, we give a comment on the contributions from massless fermions. Their contributions have the same forms as those of the graviton and gauge boson, i.e., $1/\phi^2 L^4$ up to their coefficients, and change the long-distance (large ϕ) behavior of the potential. There are three cases such that i) $m = \mu = 0$, ii) $m \neq 0$, $\mu = 0$, and iii) $m = 0$, $\mu \neq 0$. For cases i) and ii), $V(\phi, \theta)$ does not have a stable minimum for any choice of the other parameters. In contrast, the condition (24) is satisfied and the potential has a stable minimum for case iii).

4. Conclusion and discussion

We have studied the stabilization of an extra-dimensional radius of S^1 in the presence of a Wilson line phase of an extra $U(1)$ gauge symmetry on a 5D space-time with a flat background metric and without branes, using the effective potential relating the radion and the Wilson line phase at the one-loop level. The Wilson line phases are incorporated in the introduction of charged fermions. We have investigated the behavior of the potential for both large and small values of the radion, and found that the potential does not have a finite minimum in the case with only charged fermions as matter fields. The stabilization of the radion is realized in the presence of neutral fermions whose number is bigger than the number of charged ones by two.

Our result is different from those in a specific gauge-Higgs unification model [11,12], and this is mainly caused by the difference in the setup. In the model with the Randall–Sundrum metric, the Casimir energy from the graviton is exponentially suppressed because the graviton is localized around the UV brane. The bulk gauge bosons spread over the bulk S^1/Z_2 and make sizable contributions to the potential; their gauge brane-localized kinetic terms are then crucial in stabilizing the radion. In contrast, we have considered a model with a flat background metric in which every bulk field stretches out on S^1 and can make sizable contributions to the potential; the existence of bulk fermions is then essential to the stabilization of the radion and the Wilson line phase.

It is straightforward to extend our analysis to 6D models. The radion and the Wilson line phases in 6D models are expected to have the same properties as those in 5D ones, by imposing periodic boundary conditions on fields.

On remaining subject is the application of our potential $V(\phi, \theta)$ to an inflation model. By identifying the extra-dimensional component of the 5D gauge field and/or the scalar component of the 5D metric as the inflaton, we examine whether the potential reproduces realistic inflation parameters or not. In other words, we have to study whether there is an allowed region of parameters in our potential to satisfy all the constraints, in order to realize the slow-roll inflation scenario [17]. Now the radion and the Wilson line phase may compete in giving rise to inflation. Because the property of the radion differs from that of the Wilson line phase, there is an interesting possibility that hybrid inflation [18] occurs besides extranatural inflation [10] and radion inflation [7]. We are interested in identifying which scalar dominates the energy density of the universe, which can then play the role of the inflaton.

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