Source Functions in Bose-Einstein Correlations and Analyses of Data in S + Pb Reaction at 200 GeV/c

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Abstract

We consider several source functions in the Bose-Einstein correlations (BEC) and show their interrelations between 4-dimensional and 3-dimensional configuration space. We use two frameworks, i.e., a conventional one and that of the laser optical (LO) approach. The data of the BEC by NA44 Collaboration are analysed by our formulae. It is found that the BEC expressed by the Gaussian distribution, an exponential function and power functions of Lorentzian show almost the same χ^2-values in both frameworks. From estimated values of the root mean square in the conventional formula, $R_{rms}=\sqrt{\langle r^2\rangle}$, the following physical picture is obtained: $1.2A_T^{1/3} \lesssim R_{rms}(\mathrm{S+Pb\ at\ 200\ GeV}/c) \lesssim 1.2(A_T^{1/3}+A_P^{1/3})$, where A_T and A_P are target and projectile masses.

Introduction: The study of the Bose-Einstein correlations (BEC), i.e., the identical particle exchange effect [1], is an interesting subject currently investigated in high energy physics [2, 3, 4, 5] and in heavy-ion reactions [6, 7]. In the analyses, a Gaussian distribution or an exponential function have been used. It is reported that the data favour the latter [7].

On the other hand, to investigate a problem relating to source functions, a proper theoretical study of them in the 4-dimensional configuration space has been give in ref. [8]. However, authors of ref. [8] did not extract pure spatial behaviour of source functions in the configuration space.

In this paper we examine in some detail several source functions in 4-dimensional and 3-dimensional configuration space; like the Gaussian distribution, the exponential form, and others, in identical particle exchange effect. In the following we concentrate on the second order BEC function. We introduce the notation E_{2B} expressing the identical particle exchange effect in two boson production. The ratio of the number of identical boson pair ($N^{(2\pm)}$) to the number of uncorrelated boson pair (N^{BG}), as a function of the relative boson momentum $Q^2=-(p_1-p_2)^2$, is given by

$$N^{(2\pm)}/N^{BG}=1+\lambda E_{2B}^2. \qquad (1)$$

Equation (1) is named the conventional formula, because the degree of coherence λ is introduced in a phenomenological point of view [9]. This physical quantity is reflecting the coherence property in production, rescattering effect and resonance effect, etc. [10]. As many Collaborations use eq. (1), first we utilize it in the present paper, to compare some results.

In the later part we use the laser optical (LO) approach as the second framework [8] : The ratio $N^{(2\pm)}/N^{BG}$ is expressed as

$$N^{(2\pm)}/N^{BG}=1+2p(1-p)E_{2B}+p^2E_{2B}^2. \qquad (2)$$

Here p denotes the chaoticity parameter, i.e., the degree of chaoticity,

$$p=\langle n_{ch}\rangle/\langle n\rangle, \qquad \langle n\rangle=\langle n_{ch}\rangle+\langle n_c\rangle, \qquad (3)$$

where $\langle n_{ch}\rangle$ and $\langle n_c\rangle$ means the average number of chaotically and coherently produced boson, respectively.

4-dimensional source functions : In order to calculate E_{2B} in the Lorentz invariant form, we have to write an amplitude corresponding to a diagram in Fig. 1,

$$A_{12}=\frac{1}{\sqrt{2}}\left[e^{ip_1(x_A-x_1)+ip_2(x_B-x_2)}+e^{ip_1(x_A-x_2)+ip_2(x_B-x_1)}\right]. \qquad (4)$$

where p_1 and p_2 are momenta of identical particles.

Assuming that the source function for chaotically produced boson is $\rho(x)$ and taking into account the Wick rotation, E_{2B}^2 can be calculated as

$$E_{2B}^2=\frac{1}{2}\int\left[e^{i(p_1-p_2)(x_1-x_2)}+\text{c. c.}\right]\rho(x_1)\rho(x_2)d^4x_1d^4x_2. \qquad (5)$$

Typical source functions in 4-dimensional configuration space are shown in Table I. Combining them with eq. (5), we can obtain explicit expressions for E_{2B}^2. To explain our present study, a flow chart of our procedure is given in Fig. 2.

The source function in 3-dimensional configuration space is obtained as

$$\int_{-\infty}^{\infty}\rho(\xi)d\tau\equiv\rho(r), \qquad (6)$$

Table I : 4-dimensional, extracted 3-dimentional source functions and identical particle exchange functions, E_{2B}^2's.

Source function		R_{rms}	E_{2B}^2
4-dim. $\rho(\xi)$ $\xi^2=r^2+\tau^2(\tau=ct)$	3-dim. $\rho(r)$ $r^2=x^2+y^2+z^2$	$\sqrt{\frac{3}{4}\langle\xi^2\rangle}=\sqrt{\langle r^2\rangle}$	
$\frac{1}{(2\pi)^2R^4}\exp\left(-\frac{\xi^2}{2R^2}\right)$	$\frac{1}{(2\pi)^{3/2}R^3}\exp\left(-\frac{r^2}{2R^2}\right)$	$\sqrt{3}R$	$\exp(-R^2Q^2)$
$\frac{3}{4\pi^2R^4}\frac{1}{(1+\xi^2/R^2)^{5/2}}$	$\frac{1}{\pi^2R^3}\frac{1}{(1+r^2/R^2)^2}$	—	$\exp(-2RQ)$
$\frac{1}{12\pi^2R^4}\exp(-\xi/R)$	$\frac{1}{6\pi^2R^3}(r/R)K_1(r/R)$	$\sqrt{15}R$	$\frac{1}{(1+R^2Q^2)^5}$
$\frac{1}{4\pi^2R^4}\frac{\exp(-\xi/R)}{\xi/R}$	$\frac{1}{2\pi^2R^3}K_0(r/R)$	$3R$	$\frac{1}{(1+R^2Q^2)^3}$

where $\xi^2 = r^2 + \tau^2$, $r^2 = x^2 + y^2 + z^2$, and $\tau^2 = (ct)^2$. The root mean squares are calculated as

$$\langle \xi^2 \rangle = 2\pi^2 \int_0^\infty \xi^5 \rho(\xi)\, d\xi, \tag{7}$$

$$\langle r^2 \rangle = 4\pi \int_0^\infty r^4 \rho(r)\, dr, \tag{8}$$

$$R_{rms} = \sqrt{\frac{3}{4}\langle \xi^2 \rangle} = \sqrt{\langle r^2 \rangle}. \tag{9}$$

Here it should be minded that in the second column in Table I, the finite R_{rms} is not obtained. Because of this fact, we consider several source functions in addition to the Gaussian distribution and the exponential function.

Spatial 3-dimensional source function times temporal source function : The following source function is often used in studies of the BEC in heavy-ion reactions [6, 11] :

$$\rho(r,\, t) = \frac{1}{(2\pi)^2 R^3 R_t} \exp\left(-\frac{r^2}{2R^2} - \frac{t^2}{2R_t^2}\right), \tag{10}$$

where R (spatial interaction size) and R_t (time interaction region) can take different values. In this paragraph, an extended scheme of eq. (10) is studied as

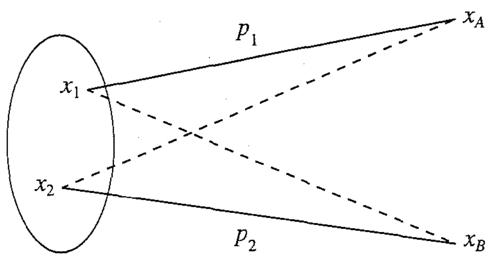

Fig. 1 Identical boson exchange diagram.

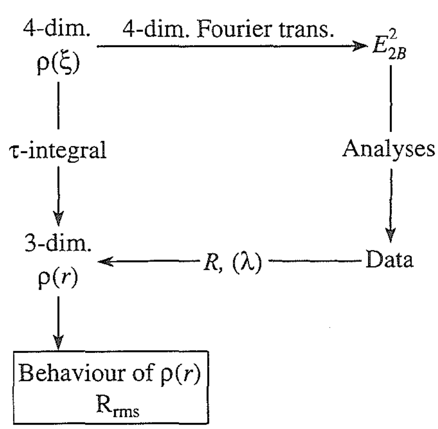

Fig. 2 Flow chart for study of interrelation between source functions $\rho(\xi)$ and $\rho(r)$

$$\rho(r,\ t)=f(r)\times\frac{1}{(2\pi)^{1/2}R_t}\exp\left(-\frac{t^2}{2R_t^2}\right), \tag{11}$$

where $f(r)$ is a 3-dimensional spatial source function. In eq. (11), the temporal source function is assumed by the Gaussian distribution. The identical particle exchange effect is calculated as follows

$$E_{2B}^2=E_{2B}^2(q,\ R)\times\exp(-R_t^2q_0^2), \tag{12a}$$

$$E_{2B}^2(q,\ R)=\frac{1}{2}\int e^{i(\mathbf{p}_1-\mathbf{p}_2)\cdot(\mathbf{x}_1-\mathbf{x}_2)}f(x_1)f(x_2)\,d^3x_1d^3x_2. \tag{12b}$$

where $q^2=(\mathbf{p}_1-\mathbf{p}_2)^2$, and $q_0^2=(p_{10}-p_{20})^2$. In Table II, we show several source functions, $f(r)$'s, relating to nuclear potential-like source functions in 3-dimensional configuration space. From comparisons between Tables I and II, we see that the Gaussian distribution and the power function of Lorentzian have respectively the same forms, except for the arguments, R^2Q^2 and Rq. The flow chart of the second procedure is given in Fig. 3.

For the Woods-Saxon potential-like source function, we use the following approximation

$$\int\frac{\sin(Qr)\,r}{1+\exp[(r-R)/a]}dr\cong\int_0^R\sin(Qr)\,rdr+\frac{\pi a}{6}\frac{d}{dr}[\sin(Qr)\,r]\Big|_{r=R}, \tag{13}$$

with an assumption of $(R\gg a)$,

Formulae for the data in the rest frame of pair : It should be noted that the functions, $E_{2B}^2(q,\ R)$'s, are useful to analyse the data obtained in the rest frame of a pair. In this frame we have $Q^2=-(p_1-p_2)^2=(\mathbf{p}_1-\mathbf{p}_2)^2=q^2$, because $p_{10}=p_{20}$, and there is no correlation effect in the variable q_0.

Table II : 3-dimensional spatial source functions and identical particle exchange functions, $E_{2B}^2(q,R)$'s. E_{2B}^2 and temporal correlation function should be combined, if necessary.

Source function : 3-dim. $f(r)$	$R_{rms}=\sqrt{\langle r^2\rangle}$	$E_{2B}^2(q,R)$	$E_{2B}^2(q_0,Rt)$
		[Note that $q^2=(\mathbf{p}_1-\mathbf{p}_2)^2$, and $x=Rq$]	
$\frac{1}{(2\pi)^{3/2}R^3}\exp\left(-\frac{r^2}{2R^2}\right)$	$\sqrt{3}R$	$\exp(-x^2)$	
$\frac{1}{\pi^2R^3}\frac{1}{(1+r^2/R^2)^2}$	—	$\exp(-2x)$	
$\frac{1}{8\pi R^3}\exp(-r/R)$	$\sqrt{12}R$	$\frac{1}{(1+x^2)^4}$	$\exp(-R_t^2q_0^2)$
$\frac{1}{4\pi R^3}\frac{\exp(-r/R)}{r/R}$	$\sqrt{6}R$	$\frac{1}{(1+x^2)^2}$	
$\frac{3}{4\pi R^3[1+(\pi a/R)^2]}\frac{1}{1+\exp[(r-R)/a]}$ (Woods-Saxon potential-like)	$\sqrt{\frac{3}{5}\frac{1+\frac{10}{3}(\pi a/R)^2}{1+(\pi a/R)^2}}R$	$\left[\frac{1}{2(1+\xi^2)}\left(\frac{\sin x}{x}+\cos x\right)+\frac{3}{1+1/\xi^2}\frac{1}{x^2}\left(\frac{\sin x}{x}-\cos x\right)\right]^2$ $[\xi=R/(\pi a)]$	

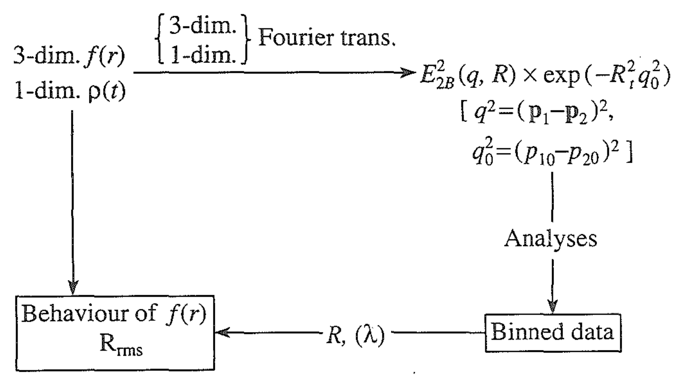

Fig. 3 Flow chart for study of 3-dimensional source function $f(r)$ times 1-dimensional temporal source function $\rho(t)$.

Table III : Various parameters estimated by means of eq. (1).

E_{2B}^2	c	R[fm]	λ	γ	χ^2/NDF
$\exp(-R^2Q^2)$	0.800 ± 0.003	4.506 ± 0.327	0.461 ± 0.042	—	17.5/16
	0.824 ± 0.010	5.015 ± 0.421	0.450 ± 0.044	-0.178 ± 0.065	10.9/15
$\exp(-2RQ)$	0.794 ± 0.004	3.550 ± 0.333	0.774 ± 0.081	—	12.0/16
	0.804 ± 0.014	3.776 ± 0.470	0.779 ± 0.084	-0.065 ± 0.087	11.4/15
$\frac{1}{(1+R^2Q^2)^5}$	0.798 ± 0.003	2.219 ± 0.167	0.490 ± 0.045	—	13.4/16
	0.819 ± 0.010	2.443 ± 0.218	0.482 ± 0.047	-0.146 ± 0.070	9.4/15
$\frac{1}{(1+R^2Q^2)^3}$	0.796 ± 0.003	3.068 ± 0.240	0.512 ± 0.047	—	11.4/16
	0.814 ± 0.011	3.333 ± 0.316	0.506 ± 0.050	-0.119 ± 0.073	9.0/15

Analyses of the data of NA44 Collaboration by means of eq. (1) : The NA44 Collaboration has analysed the data of the second order BEC in S+Pb reaction at 200 GeV/c per nucleon [7]. Authors of ref. [7] have used their own conventional forms :

$$N^{(2+)}/N^{BG}=c(1+\lambda e^{-R^2Q^2}), \qquad (14a)$$

$$N^{(2+)}/N^{BG}=c(1+\lambda e^{-2RQ}). \qquad (14b)$$

Their results are contained in Table III. In our analyses, we use two formulae with/without a correction factor $(1+\gamma Q)$ in eq. (1):

$$N^{(2\pm)}/N^{BG}=c(1+\lambda E_{2B}^2)\times\begin{Bmatrix}1\\(1+\gamma Q)\end{Bmatrix}. \qquad (15)$$

The correction factor is necessary to account the long-range correlation in Q. As you see in Table III, χ^2-values estimated by eq. (1) with $(1+\gamma Q)$ are smaller than those without $(1+\gamma Q)$. From comparisons of χ^2-values, it is difficult to distinguish a preferred source function among them. Indeed, these facts are also seen in Fig. 4.

The root mean square, $R_{rms}=\langle r^2\rangle$, is also shown in Table IV. From it, we see that estimated R_{rms}'s are almost the same order.

Behaviours of source functions $\rho(r)$'s : By making use of parameters in Table III and $\rho(r)$ in Table I, we can calculate behaviours of $\rho(r)$ and $4\pi r^2\rho(r)$. As you see in Fig. 5(b), the source functions of $E_{2B}^2=1/(1+R^2Q^2)^K$ (K=3, 5) show almost the same behaviours. This fact corresponds to two similar curves in Fig. 4(b).

Table Ⅳ : The root mean squared values, R_{rms}, in S + Pb reaction.

E_{2B}^2	R_{rms} [fm]
$\exp(-R^2Q^2)$	8.69 ± 0.73
$\exp(-2RQ)$	—
$\dfrac{1}{(1+R^2Q^2)^5}$	9.46 ± 0.85
$\dfrac{1}{(1+R^2Q^2)^3}$	10.00 ± 0.95

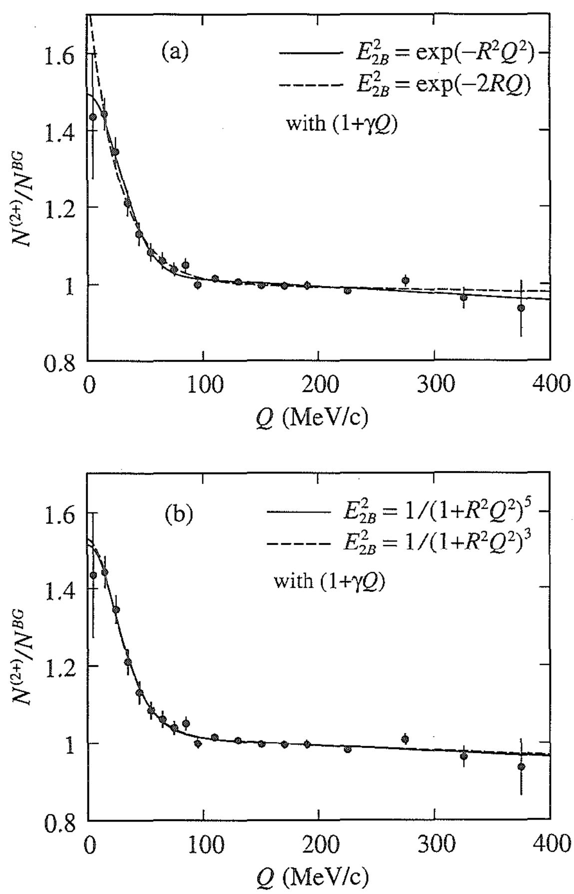

Fig. 4 Analyses of the data of NA44 Collaboration : (a) Gaussian distribution and exponential function are used. (b) Power functions of Lorentzian are used.

Analyses of the same data by means of eq. (2) : Combining several expressions E_{2B} and eq. (2) with the correction factor $(1+\gamma Q)$, we can analyse the data of NA44 Collaboration. In this case the parameter p is free. Our results are shown in Table V. There are no significant differences between Tables III and V. However, R's are larger than those of Table III.

In this case, moreover, we can predict the third order BEC by

$$N^{(3-)}/N^{BG}=1+6p(1-p)E_{3B}+3p^2(3-2p)E_{3B}^2+2p^3E_{3B}^3, \tag{16}$$

where E_{3B} is a function of the $Q_{3B}^2=Q_{12}^2+Q_{23}^2+Q_{31}^2$. The predictions with parameters in Table V are depicted in Fig. 6.

Concluding remarks : (i) We show interrelations of source functions in BEC is 4-dimensional and 3-dimensional configuration space. (See Table I and Fig. 2.) By this procedure, we can infer interaction region through analyses of the BEC.

(ii) To analyse the binned data, and the data in the rest frame of a pair, formulae

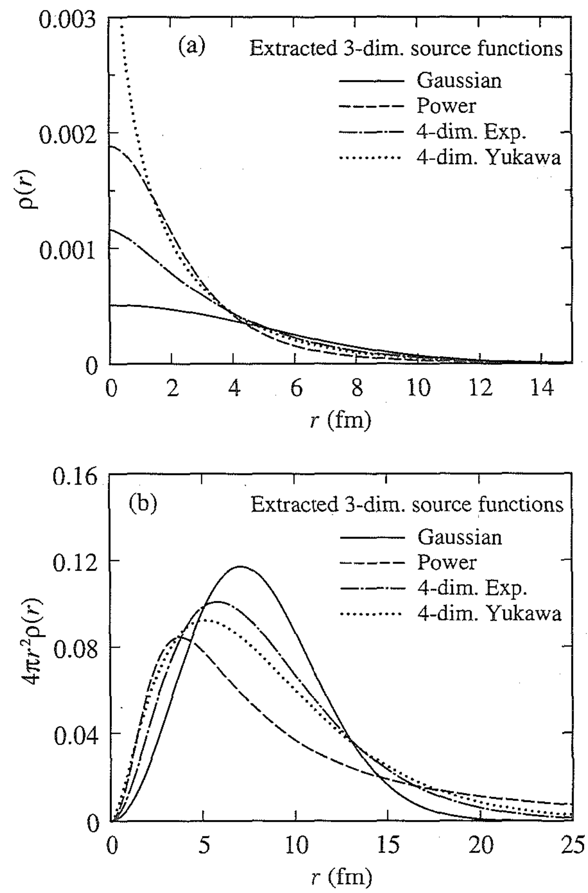

Fig. 5 Extracted 3-dimensional source functions. We use parameters estimated by the present analyses with $(1+\gamma Q)$: (a) $\rho(r)$. (b) $4\pi r^2\rho(r)$.

Table V : Various parameters estimated by means of eq. (2).

E_{2B}	c	R[fm]	p	γ	χ^2/NDF
$\exp(-R^2Q^2/2)$	0.823 ± 0.010	6.816 ± 0.557	0.264 ± 0.031	-0.173 ± 0.066	10.5/15
$\exp(-RQ)$	0.796 ± 0.015	5.964 ± 0.639	0.596 ± 0.127	-0.025 ± 0.096	12.5/15
$\frac{1}{(1+R^2Q^2)^{5/2}}$	0.810 ± 0.012	3.506 ± 0.329	0.310 ± 0.038	-0.098 ± 0.076	8.9/15
$\frac{1}{(1+R^2Q^2)^{3/2}}$	0.796 ± 0.013	4.856 ± 0.512	0.347 ± 0.045	-0.033 ± 0.084	9.2/15

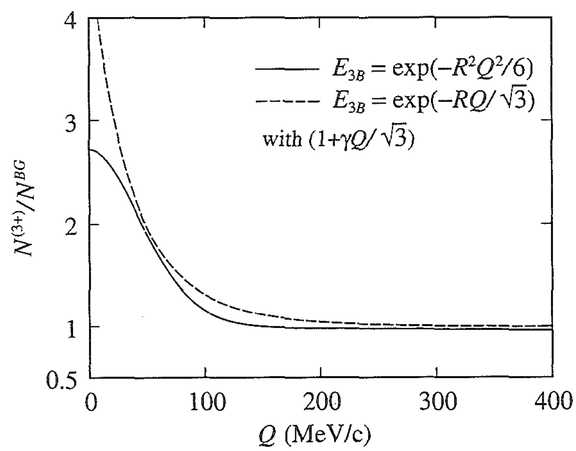

Fig. 6 Predictions of the third order BEC. $Q=\sqrt{Q_{3B}^2}=\sqrt{3}Q_{12}$.

(with/without the factor $\exp(-R_t^2q_0^2)$) in Table II are available.

(iii) From analyses of the data of NA44 Collaboration, the importance of the correction factor $(1+\gamma Q)$ is found. (See Table III.)

(iv) It is found that the Gaussian distribution, the exponential function and power functions of Lorentzian for E_{2B}^2 show almost the same χ^2-values in analyses of the data of NA44 Collaboration in both frameworks.

(v) Moreover, the R_{rms}'s are almost the same order in S+Pb reaction. As you see in Table IV, we obtain that $R_{rms}(\text{S+Pb}) \approx 9 \sim 10$ fm. To explain large value of R_{rms} in S+Pb reaction, we would accept the following inequality, provided that a simple geometrical interpretation is allowed,

$$1.2A_T^{1/3} \lesssim R_{rms}(\text{S+Pb}) \lesssim 1.2(A_T^{1/3}+A_T^{1/3}) \approx 3.8 \text{ fm} + 7.1 \text{ fm} = 10.9 \text{ fm}. \qquad (17)$$

This value is somewhat larger than those of ref. [6] : R_{rms}(Si+Au at 14.6A GeV/c) $\approx 7.48 \pm 0.73$ fm and $R_{rms}(\text{Si+Al}) \approx 6.47 \pm 0.69$ fm.

(vi) The higher order BEC may be useful [12], to confirm parameters R and λ. In the LO approach, the third order BEC is predicted. (See Fig. 6.) In a future, comparisons with experimental analyses are possible.

(vii) To get more information from the BEC of NA44 Collaboration, the binned data are needed. In this case formulae in Table II are available, and various comparisons with analyses by E802 Collaboration [6] are possible.

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