A STUDY ON GRADING CURVE AND EFFECTIVE DIAMETER OF SOIL

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I. Introduction

In a case that in a certain granular material, every grain is different in size and in shape, supposing a spherical uniform grain which gives the same capillary attraction or permeability such as the granular material, we call its diameter "Effective diameter".

Hazen[(1)] empirically advocated so-called "Hazen's effective size d_e" and Krüger, Zunker and Kozeny theoretically[(2)] did "Zunker's effective diameter d_w". d_e is obtained easily from grading curve and is used in a wide range, but is not applicable for very non-uniform soil. As one of methods to specify the physical pecuriality of soil, Hazen took uniformity coefficient u with it, but he does not treat it calculatively.

On the other hand, limitting granular materials so far as the range of fine sand or silt in the smaller part of grains, it is observed[(2)] that d_w is very suitable as effective diameter. As one of methods to connect the relation between d_e and d_w, the author took the way connecting u to d_e calculatively. By the way, he tried to enlarge the application of this convenient Hazen's effective size, even in the case of excessively non-uniformity of soil.

II. Relation between two effective diameters, d_e and d_w, in various types of grading curve

(1) III-type of grading curve.

In such a case taking at the abscissa on a semi-log paper the number of log 2 cycles by Tyler's or A.S. sieve and at the ordinate the passage of grains, the grading curve which is represented as a straight line is in convenience sake called III-type grading curve. Refer Fig. 1.

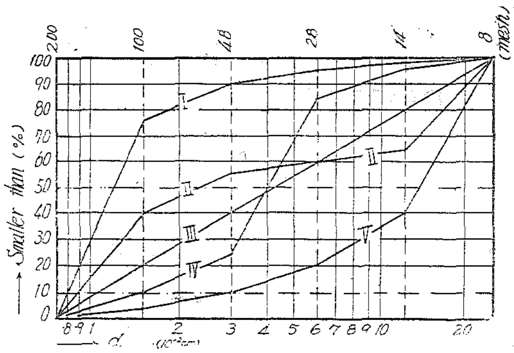

Fig. 1. Five types of grading curve.

In this curve, the following equations are formed. Refer Fig. 2.

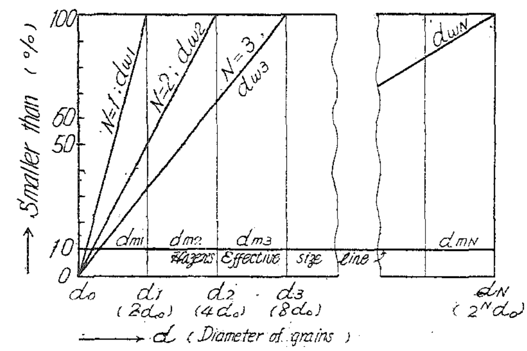

Fig. 2. Grading curves of III-type.

(a) For Hazen's effective size

$$d_e = d_0 10^{\frac{N}{10}\log 2} \qquad \text{(1)}$$

$$d_{60} = d_0 10^{\frac{6N}{10}\log 2} \qquad \text{(2)}$$

$$u = 10^{\frac{N}{2}\log 2} \qquad \text{(3)}$$

where

N: the number of log 2 cycles, the grading curve strides,

d_0: the minimum size of grain, the grading curve ends.

(b) For Zunker's effective diameter

Generally

$$\frac{1}{d_{wN}} = U = \sum_{1}^{N} \frac{g_N}{d_{mN}} \qquad \text{(4)}^{(2)}$$

therefore

$$d_{wN} = \frac{2^{N-1}N}{2^{N-}+2^{N-2}+\cdots+2^{N-N}} d_{w1} \qquad \text{(5)}$$

where

$$1/d_{m_1}=0.4343\ (1/d_0-1/d_1)/(\log d_1-\log d_0) \qquad \cdots\cdots (6)$$

and

d_{m1}: the mean diameter (from the point of view of surface area) of a group of grains, exsists between d_0 and d_1 of the screen sizes,

g_1 : the percentage of the weight of the group,

d_{w1}: Zunker's effective diameter, owned by the grading curve in such a case as $N=1$,

d_{wN}: Zunker's effective diameter, owned by the grading curve in such a case as it strides on N log 2 cycles.

(c) Coefficient of effective size.

Letting

$$d_w = \alpha\, d_e \qquad \cdots\cdots (7)$$

then, by (1) ~ (7)

$$\alpha = \frac{\dfrac{2^{N-1}N}{2^{N-1}+2^{N-2}+\cdots+2^{N-N}}d_{w1}}{d_0\, 10^{\frac{N}{10}\log 2}} \qquad \cdots\cdots (8)$$

where

α: coefficient of effective size (a tentative name) ; the number, ordinarily smaller than numerically 1 affected by the shape of grading curve.

In III-type curve, in the range of $u=1\sim16$, equation (8) may be represented as the following form with a sufficient accuracy.

$$\alpha = 1+2\log u \qquad \cdots\cdots\text{III-type}\cdots\cdots (9)$$

Therefore, from (7) and (9)

$$d_w \approx (1+2\log u)\, d_e \qquad \cdots\cdots\text{Author}\cdots\cdots (10)$$

On the other hand, Dr. Yoshida indicates formula (11) experimentally.

$$d_y = (0.75+0.25\, u)\, d_e \qquad \cdots\cdots\text{Yoshida}^{(3)}\cdots\cdots (11)$$

(2) I, II, IV and V-types of grading curve and coefficient of grading form.

Generally formula (9) may be modified such as the following, except III-type curve.

$$\alpha = 1 + \lambda \log u \qquad \cdots\text{I, II, IV, V-types}\cdots (12)$$

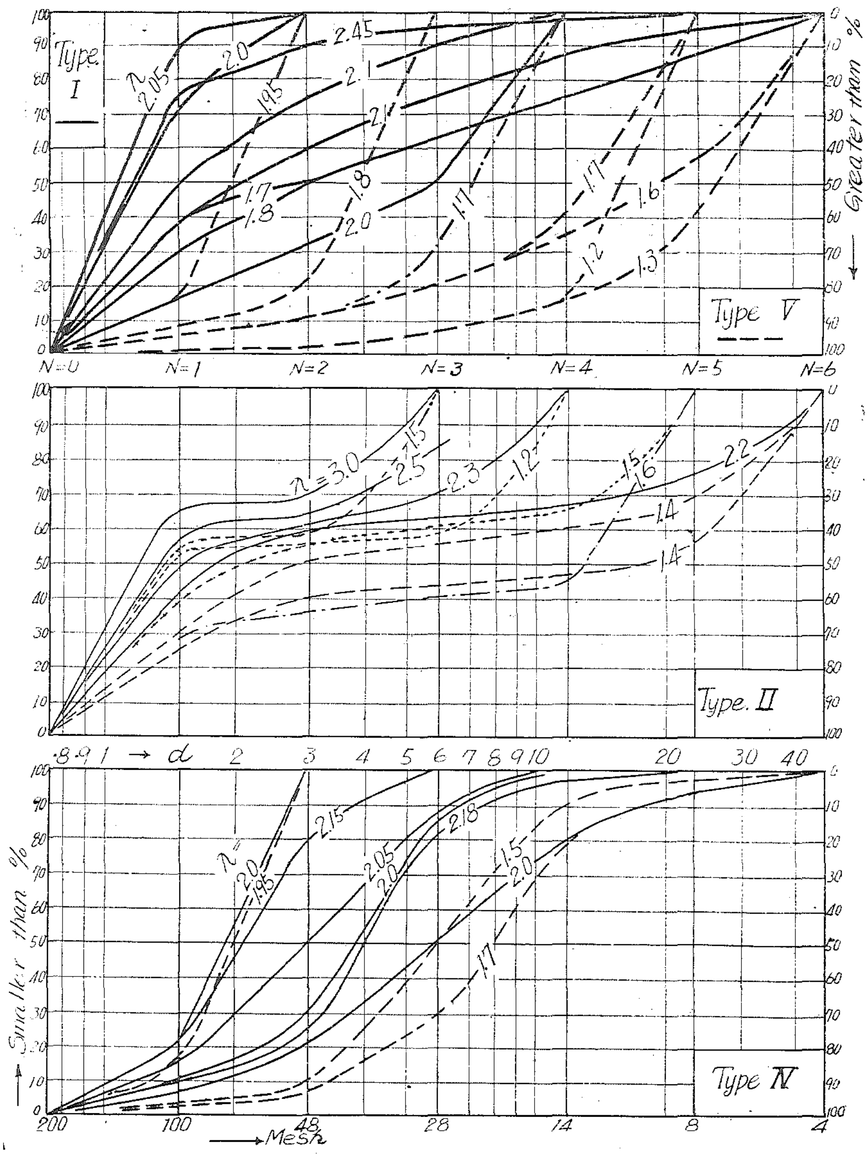

Fig. 3. Values of λ of various grading curves.

where

λ: coefficient of grading form (a tentative name); the number varying with the form of grading curve.

Fig. 3 shows the value of λ to the various grading curves.

III. Examples of application of coefficient of effective size and coefficient of grading form by Author

(1) Enlargement of range of application of d_e for percolating formula.

(a) Coefficient of effective size and measured examples.

Generally, in order to get the true effective diameter d_t of a combined sand, the measured value of the permeability of the sand shall be substituted in the Slichter's theoretical formula, which is consisted on the layer of uniform spheres, then the diameter d_t is supposed to be calculated, conversely.

That is,

$$k = \frac{c_s}{\eta}\frac{1}{x}f(d_t) \quad \cdots\cdots \text{Slichter}^{(4)} \cdots\cdots (13)$$

Now, letting

$$\alpha_0 = d_t/d_e \quad \cdots\cdots (14)$$

α_0 may be got from (13) and (14), based on the measured values of permeability k which have been experienced by the authorities, and is compared with α which is shown by (12), then Fig. 4 is obtained. According to the above, it may be understood that the author's formula (12) well coincides to the measured values by the authorities, concerning the wide range of u. That is, the most part seems to be adapted to the case which $\lambda=2$. In the case of unadaptation from the above, it becomes $\lambda=1$, $\lambda=0$ or $\lambda<0$, influenced by the each characteristic of soil.

Observing in each part of Fig. 4, No. 1 & 2 are $d_e=0.09\sim0.83$ mm, $u=2\sim9$, $p=0.25\sim0.48$, that is, the range of ordinary "sand", so the formula can show the most satisfactory result. No. 3 is the case of $d_e=3$ & 5 mm, $u=1.4\sim2$, which shows that the formula also coinsides to the pebble layer.[5] No. 4[6] is $d_e=0.3\sim0.4$ mm, $u=1.1\sim1.3$, $p\approx0.38$, that is, the uniform medium sand, so it coinsides to Yoshida's formula. No. 5[6] is $d_e=0.2\sim1$ mm, $u=1.1\sim4.3$, $p=0.38\sim0.5$, very rugged sand, and in the case of $p\approx0.5$, it becomes $\lambda<0$ (namely $d_t<d_e$) exceptionally. As one of these reasons it may be considered that the application of (13) becomes unreasonable, the porosity of the sand being beyond the limit of Slichter's theoretical formula. No. 6[7] is the result for sandy loam ($d_e=0.0027$ mm, $u=40$), silty loam ($d_e=0.00267$ mm, $u=27.2$) and silty sand ($d_e=0.0060$ mm, $u=46.2$) of $p\approx0.50\sim0.68$, but (13) formula is not suitable as the same reason as No. 5. Namely, in this case the shape of void is supposed to

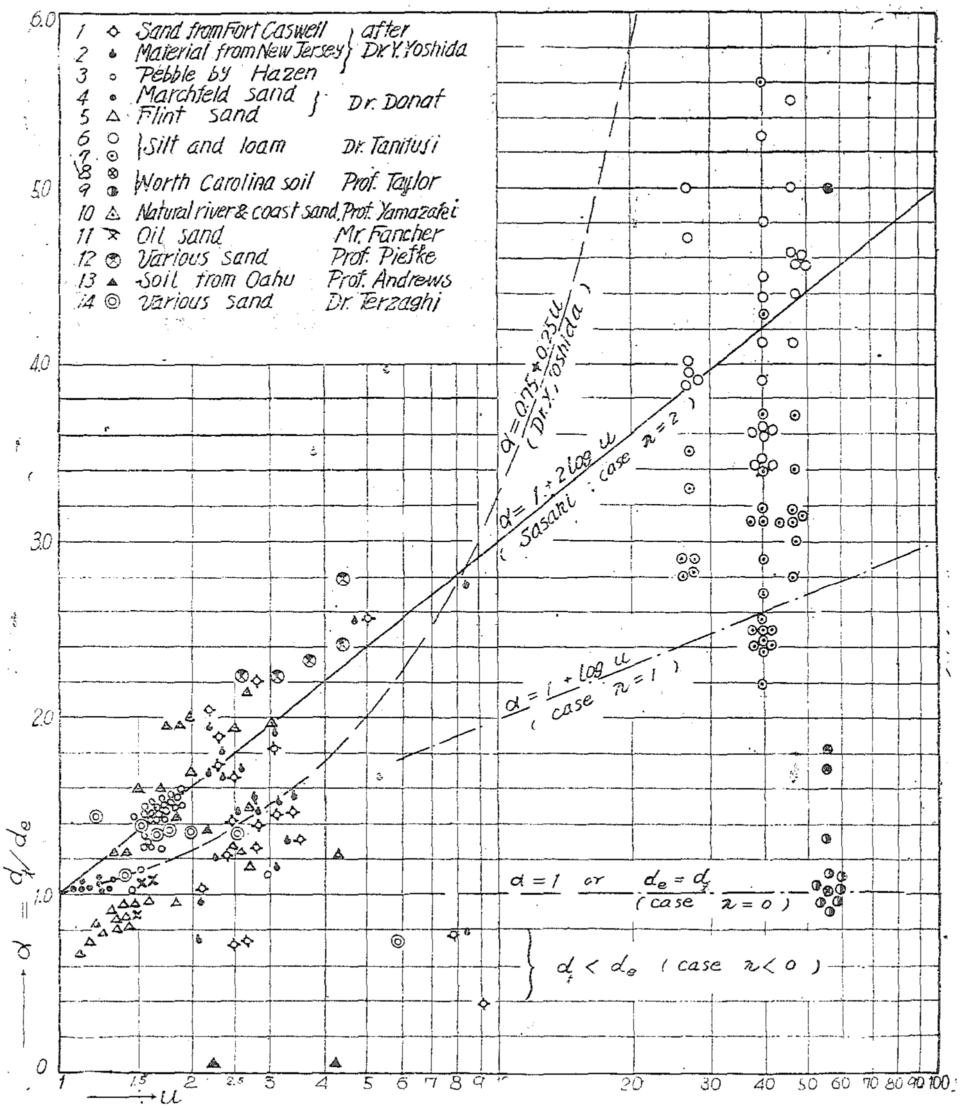

Fig. 4. Comparison of the values of d_t/d_e with the coefficient of effective size α.
d_t: True effective diameter from Slichter's formula,
d_e: Hazen's effective size, λ: Coefficient of grading form,
u: Uniformity coefficient.

be rather suitable in the case that in the formulae of Zunker, Kozeny or Terzaghi. Now, therefore, if show the coefficient of effective size calculated conversely by Terzaghi's formula,[8] instead of Slichter's, it becomes to No. 7. According to this, it seems that the range from $\alpha=1+2\log u$ to $\alpha=1+\log u$ represents the coefficient of effective size well to the soil which have very large u as same as silty loam, etc. No. 8 & 9[9] are calculated for the materials of earth dam (d_e=0.0018 mm, u=55.5, p=0.36~0.47) as same as No. 7. No. 8 is the result got from the non-consolidated condition and No. 9 is the result got from the consolidated condition at 1~5.5 ton/ft^2. In case of the latter, it becomes $\alpha_0=1$ namely $d_t \approx d_e$ accidentally. No. 10[10] is the result got from the natural river and coast sand in Japan (d_e=0.06~0.7 mm, u=1.4~3.1, p=0.36~0.46), so it coinsides to the author's formula. No. 11[11] is oil sand of d_e=0.09~0.13 mm, u=1.5, $p \approx 0.27$ and seems to be $\lambda=o$ ($d_t \approx d_e$). No. 12[12] is the various combined sand of d_e=0.07~0.37 mm, u=2.6~4.4, and it coinsides well, too. No. 13[13] is the soil (d_e=0.0046 mm, u=2.2, p=0.45; d_e=0.0078 mm, u=4.2, p=0.31) having loss on ignition≈20% and is an extreme example in the case of $\lambda<0$. No. 14[8] is the result to d_e=0.12~0.64 mm, u=1.2~5.8, p=0.35~0.495.

In the above view point, if it is the ordinary sand (includes pebble), its effective diameter may be almost represented by the case which $\lambda=2$ of the formula. In the case of very rugged and flattened sand or such as core materials, especially at consolidated condition, for example, in which the permeability is very small, it has the tendency of $\lambda=1$, $\lambda=0$ or $\lambda<o$.

(b) Percolation formula by Author.

Author proposes the following as a practical formula of percolation to the non-uniform sand, taking the convenience of the Hazen's type and the reliability of the Zunker's type.[2]

$$k=\frac{c}{\eta}\left(\frac{p_0}{1-p}\alpha d_e\right)^2 \quad \cdots\cdots\text{Author}\cdots\cdots (15)$$

or practically

$$k=\beta\left(\frac{p_0}{1-p}\alpha d_e\right)^2(0.7+0.03\,t) \quad \cdots\cdots (15)'$$

or approximately

$$k \approx \beta u\left(\frac{p}{1-p}\right)^2 d_e^2(0.7+0.03\,t) \quad \cdots\cdots (15)''$$

where,

k; coef. of permeability in soil
(Darcy's transmission constant) ··················cm s^{-1}

d_e; Hazen's effective size ··················cm

p; porosity (p_0=effective porosity)

c; an empirical coefficient depending upon the shapes of grains and voids in porous media ....$cm^{-2}\ g\ s^{-2}$

η; coefficient of viscosity of fluid ....$cm^{-1}\ g\ s^{-1}$

t; temperature in degrees centigrade

β; coefficient of form of grains (Table 2) ....$cm^{-1}\ s^{-1}$

α; coef. of effective size (Table 1) : $\alpha = 1 + \lambda \log u \approx 1 + 2 \log u$

Table 1. Values of coefficient of effective size. (case $\lambda=2$)

u=1	2	3	4	5	6	7	8	9	10	15	20
α=1.0	1.6	2.0	2.2	2.4	2.6	2.7	2.8	2.9	3.0	3.4	3.6

Table 2. General values of coefficient of form of grains.

grains	β
1. glass sphere	140~180
2. round and smooth dune sand	140
3. round and smooth river sand	100~140
4. horny and weny sand	70~ 80
5. edged glass powder	60
6. very rugged or flattened irregular river sand	30~ 50
7. very rugged, brittle and a little loamy river sand	10~ 30

(2) A numerical designation of a grading curve.

(a) Classification and designation.

The grading curve has the two factors, independent on the each as follows;

1. The range of grains; 2. The form of curve.

Therefore even if the ranges of grains of soil are same, according to the variation of mixing rate of the grains, the form of grading curve changes as well as the characteristic of the soil does. (To refer Table 3) The ways to express the range of grains are the one to add the both limits of fine and coarse and one to represent it by the various converted diameters and etc. To the form of curve, one way is to express by the sign or number. These are tried by many researchers, having merits and defects, it has become a difficult problem for numerically representing the grading curve of soil.

Table 3. Types of grading curve and several characteristics.

<table>
<tr><th rowspan="2">Degree of skew of grading</th><th rowspan="2">Type</th><th colspan="3">Quantity of each groups</th><th rowspan="2">* Degree of mature</th><th rowspan="2">** Density etc.</th><th rowspan="2">Probable existence</th></tr>
<tr><th>fine</th><th>medium</th><th>coarse</th></tr>
<tr><td>Steep</td><td>I</td><td>almost</td><td>a little</td><td>a little</td><td>maturity</td><td>Uniform materials, least dense, compacted very little.</td><td>*** Semi-hydraulic fill dam.</td></tr>
<tr><td rowspan="4">Number of log 2 cycles N is not so large</td><td>II</td><td>much</td><td>a little</td><td>much</td><td></td><td>Pack well and very dense:</td><td>Boulder-clay of glacialed regions: Bitting and Supporting type of subgrade.</td></tr>
<tr><td>III</td><td colspan="3">equal</td><td>inter-mediate</td><td>Smaller density than IV—type</td><td>**** Near to good un-graded materials of impervious section of rolled-fill dam.</td></tr>
<tr><td>IV</td><td>a little</td><td>much</td><td>a little</td><td></td><td>Normal statistical distribution; nearly symmetrical—S shaped curve (very dense)</td><td>Most existence in natural soil.</td></tr>
<tr><td>V</td><td>a little</td><td>a little</td><td>almost</td><td>immaturity</td><td>Near to maximum density grading curve</td><td>**** Good graded materials of impervious section: Uniform class 2 of Kendorco classification *****</td></tr>
<tr><td>Low, N; large</td><td>V</td><td colspan="3">vary as exponential</td><td colspan="3">Minimum void ideal curve. Type of best stability subgrade by Talbot and U.S. Bureau of P.R.</td></tr>
</table>

* Campbell, Trans. A.S.C.E., No. 104, 1939.
** Burmister, Do. Disscus.
*** Plummer and Dore, "Soil Mechanics and Foundations", Fig. 213, 1940.
**** Lee, Proc. A.S.C.E., Selection of Materials for Rolled Fill Earth Dams, 1936.
***** Quabin Reservoir project of Mass., Trans. A.S.C.E., Vol. 103, 1938.

(b) Author's designation; $d_e-u-\lambda$ method (Designated by effective size —uniformity coefficient—coefficient of grading form)

Author proposes the trial which add the λ, mentioned in the above, to d_e and u, and express the grading curve by it.

Now, he picks up 5 types (to refer Fig. 1) which are considered to be the typical type of curve, and shows the example as Table 4, comparing with the other designations concretely.

Table 4. Comparison of designations of grading curves of five types shown in Fig. 1.

Types of grading curve	Effective diameters $cm \times 10^{-2}$			Method of Fineness modulus	Campbell*		Weinig ϕ	Burmister**		Author $d_e - u - \lambda$ $cm \times 10^{-2}$
	d_e	d_w	d_p $\phi\phi$		Diameter-grade designation D (*mm*)	Grade line deviation (*mm*)		Mean size percentage	Range of particle (*mm*)	
I	.82	1.21	1.85	0.42	D 0.12-G 1.7	+0.013	C	0.69		.82-1.57-2.45
II	.89	1.98	7.03	1.80	D 0.24-G 5.7	+17	S	0.57		.89-6.61-1.49
III	1.05	2.63	6.32	2.00	D 0.42-G 5.0	0	L	0.50	S 0.074–	1.05-5.65-2.00
IV	1.47	3.00	4.49	1.85	D 0.39-G 3.3	−0.03	S	0.49	G 2.36	1.47-3.01-2.13
V	2.95	6.48	11.59	3.26	D 1.33-G 5.2	−0.28	K	0.38		2.95-5.08-1.68

*, **, Refer Table 3.

ϕ Weinig, Trans. A.S.C.E., Discuss. No. 104, 1939.

$\phi\phi$ Puri, Soil Science, 1939.

The characteristics of this designation:

(a) It can put into a practical use d_e & u which have been taken in the general use.

(b) It can connect d_e to the capillary diameter by λ and u. (λ is proportional inversely to the ratio of surface area of soil.)

(c) It can represent the physical properties of a mixture, some degree, for instance:

(i) Comparing two soil, these d_e and u are equal respectively and these λ are different, usually is the more rich in the group of minimum grain the less of λ.

(ii) It is a dense type (impervious) generally which λ is small, and a loose type (pervious) which λ is large.

IV. Conclusion

Mr. Hazen's effective size d_e, has been preferably used wide in order to represent the physical properties of soil, is fairly uncertain as the means to express the capillary and the permeability, but its certainty is developed by adding the author's coefficient of effective size α. The percolation formula (15) is indicated as the applied example. Moreover, he shows the application of α tentatively concerning the numerical designation of the grading curve of soil.

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