ON COEFFICIENTS OF PERMEABILITY OF SAND-LAYERS

By

Hachiro SASAKI*

(Civil Eng. Dept., Faculty of Engineering.)

Synopsis: In this paper, the relationship between the form of grain and the grading of sand and the coefficient of permeability is stated, and the writer mentions an application of the coefficient of effective size and he also tries to show the coefficient of permeability by a graphical method.

I. Coefficient of Form of Grain

Generally, the form of a body is a factor in the resistance of the body in the flow of fluids. In the flow of percolation, the form of grain forming a porous media also has an influence on permeability and this is treated as a so-called coefficient of form. The writer considers that this value is not constant, but it is changeable to some extent, in the events of any variation in grading and porosity. Therefore, it is considered that the coefficient of form of grain should be at least compared with each other at the same grading.

(1) PERCOLATION-FORMULA OF SPECIFIC-AREA-TYPE AND COEFFICIENT OF FORM OF GRAIN

$$v=k\cdot h/l \qquad \text{.........Darcy's law**.........} \quad (1)$$

The coefficient of permeability k in (1) is shown as follows:

$k=f$(viscosity of fluid)f(form of grain)f(porosity)f(diameter of grain)

or

$$k=f(\eta)f(f)f(p)f(d) \qquad \text{..........................} \quad (2)$$

* Assistant Professor of Shinshu University, Nagano, Japan.

$f(f)$, $f(p)$, $f(d)$**** in (2) are generally shown by many authorities.(1) Supposing fluid to be water and temperature to be 10°C

$$k=\beta f(p)f(d) \qquad \cdots\cdots (3)$$

or

$$\beta=k/f(p)f(d) \qquad \cdots\cdots (4)$$

where

$$\beta=f(\eta)f(f)=c/\eta_0$$

That is to say, the writer means that β is "coefficient of form" including "water; temperature 10°C".

If β' represents the coefficient of form of grain of sands in which are mixed several sorts of sands of uniform sized groups, β' will be, in specific-area-type formula,(1) considered proportional to the surface area of the grains of the sand and be calculated as the follows:

$$\beta'=\frac{\left(\beta_1\frac{g_1}{\gamma_1}\frac{1}{d_1}+\beta_2\frac{g_2}{\gamma_2}\frac{1}{d_2}+\cdots\cdots\right)\Big/\left(\frac{g_1}{\gamma_1}+\frac{g_2}{\gamma_2}+\cdots\cdots\right)}{\left(\frac{g_1}{\gamma_1}\frac{1}{d_1}+\frac{g_2}{\gamma_2}\frac{1}{d_2}+\cdots\cdots\right)\Big/\left(\frac{g_1}{\gamma_1}+\frac{g_2}{\gamma_2}+\cdots\cdots\right)}$$

or

$$\beta'=\frac{\Sigma\beta_N\cdot\frac{g_N}{\gamma_N d_N}}{\Sigma\frac{g_N}{\gamma_N d_N}} \qquad \cdots\cdots (5)^{(2)}$$

If specific gravities $\gamma_1=\gamma_2=\cdots\cdots=\gamma_N$

$$\beta'=\frac{\Sigma\beta_N\frac{g_N}{d_N}}{\Sigma\frac{g_N}{d_N}} \qquad \cdots\cdots (6)$$

If β measured in the equation (4) nearly coincides with β' calculated in the equation (6), β may be considered the coefficient of form of the grain of the sand, but if not, β will be regarded rather as the coefficient of form of capillary tube in the porous media formed of various groups of sand grains. The writer now adopts the latter's point of view and then wants to make an investigation in the following. Here, the writer will show a comparison of

** If the coefficient of permeability k_0 of the porous media only is considered

$$k_0=v\eta l/h$$

*** The values of c, C represented by $f(f)=c$ and $f(f)f(p)=C$ are shown in the reference (1).

**** $f(f)$ in Kozeny's or Zunker's type formula is shown as the coefficient of form of a grain of sand.

Table 1. Coefficients of form of several mixed-sands.

Original sand groups	d $cm\times10^{-2}$	** β_0	
a	2 ～ 4	76.60	Very rugged and brittle quartz sand.
b	4 ～ 5	60.09	
c	5 ～ 7.5	79.79	$\gamma=2.559$
d	7.5～10	65.46	
e	10 ～15	72.13	

* Forms of gradu-ation	Mixed percentages g					Zunker's effe. dia. d_w $cm\times10^{-2}$	** Porosity p %	** k Measured permeability at 10°C. $cm.s^{-1}\times10^{-3}$	Coeff. of form	
	a	b	c	d	e				β Mea-sured	β' Cal-culated by (6)
I	76	8	10	3	3	3.18	44.41	25.42	88.7	79.4
II	48	2	2	8	40	4.50	38.46	26.86	88.3	75.4
III	35	11	20	14	20	4.64	42.36	37.10	75.3	75.0
IV	13	17	58	8	4	5.20	44.41	60.45	78.9	74.5
V	6	5	14	15	60	8.17	42.45	133.40	86.6	72.0
V	6	5	14	15	60	8.17	44.41	161.50	85.4	72.0

* To refer (5).

** Measured by Mr. Donat; refer (4), s. 229.

those equations (4) and (6) in Tabel-1, applying $f(p)$ of Mr. Kozeny[3] and the measured data of Mr. Donat which are reliable.

As you see in the table, the relation of β and β' seems to have each different effect, in I-II-V groups and III-IV groups respectively, owing to the difference of combined ratio even if the grains come from the same place of source.

(2) COEFFICIENT OF FORM AND COEFFICIENT OF EFFECTIVE SIZE IN THE PERCOLATION-FORMULAE OF EFFECTIVE-SIZE-TYPE.

In the formulae of effective-size d_e type,[1] so-called $f(f)$ differs from the case (1) and strictly speaking, there is element of $f(d)$ to some degree included in $f(f)$. Therefore, it seems better to make β include in the $f(f)$

such a coefficient as the coefficient of effective size α mentioned by the writer.[5]

(a) For Hazen's formula.

As Mr. Hazen's formula has been experimented for the sand at special uniformity, a great error will be committed if it be applied to other sand having different graduation. But it may be made universal through the following method. For an example, $C=f(f)f(p)=116$ shown by Hazen[1] for a loose packed sand is the value in the case of $u=1.5\sim2.5$, but in order to get the value of C in the case of uniform grains, α is to be applied and

$$f(f_{u=n})\approx f(f_{u=m})\left(\frac{\alpha_{u=n}}{\alpha_{u=m}}\right)^2$$

......for Hazen's effective type......SASAKI......(7)

therefore

$$C_{u=1}\approx C_{u=2}\times\left(\frac{\alpha_{u=1}}{\alpha_{u=2}}\right)^2\approx 116\times\left(\frac{1}{1.6}\right)^2=45^*$$

That is to say, it is almost equal to the values of C shown by Mr. Seelheim, etc.[1]

(b) For Terzaghi's formula.

(i) Dr. Terzaghi carefully gives each different value of β as 646 ($u=2.04$), 696 ($u=2.50$) and 460 ($u=1.40$) to three different kinds of sand which have different u respectively and come from same place of source. In this method it is rather difficult to apply β to the sand having other graduation, but if the equation (7) be taken the application may be possible.

For instance, supposing β in the case of $u=2$

For sand No. 3 ($u=2.04$) ; $\beta_{u=2}=646\times(1.6/1.62)^2=630$

,, No. 4 ($u=2.50$) ; $\beta_{u=2}=696\times(1.6/1.80)^2=550$

,, No. 5 ($u=1.40$) ; $\beta_{u=2}=460\times(1.6/1.29)^2=707$

and thus the values of $\beta_{u=2}$ come closer to each other than they stand at first.

(ii) Other examples of the application of the coefficient of effective size to the coefficient of form.

A: The case where the difference of u is comparatively large.

Table-2 A shows Mr. Donat's measured values. The sands have small permeability and the grains are rugged and brittle. The mean value of the coefficient of form for each mixed sand of I~V calculated according to Terzaghi's formula is $\beta=447$ and the amount of deviations of each value is $\sum|\Delta\beta|=85.1$. These deviations $\Delta\beta$ seem to have a connection with u or α.

* α is generally equal $1+\lambda\log u$, but $\alpha=1+2\log u$ approximately may be used.[5]

Table 2. Correction done by Writer for β accordance with u.

Case A: The grading is non-uniform and the form of grain of sand is very rugged and brittle.

Sand	** Measured permeability at 10°C k $cm.s^{-1}$ $\times 10^{-3}$	** Porosity p %	Hazen's-effective-size d_e $cm \times 10^{-2}$	Uniformity coefficient u	Coefficients of form: β			
					Calculated *		Corrected by (7)	
					β_T	$\Delta\beta$ %	$\beta_{u=2}$	$\Delta\beta$ %
I	25.42	44.41	2.20	1.59	359	−19.7	440	+12.0
II	26.86	38.46	2.32	4.31	561	+25.5	382	− 2.8
III	37.10	42.36	2.44	2.74	498	+11.4	352	−10.4
IV	60.45	44.41	3.46	1.84	343	−23.2	389	− 1.0
V	133.40	42.45	4.47	2.45	465	+ 4.0	405	+ 3.0
V	161.50	44.41	4.47	2.45	453	+ 1.3	395	+ 2.0
					Mean **447**	$\sum\lvert\,\rvert=85.1$	Mean **393**	$\sum\lvert\,\rvert=31.2$

Case B: The grading is pretty uniform and the form of grain of sand is some horny and weny.

No. of sand	Combined ratio %		Hazen's-effective-size d_e $cm \times 10^{-2}$	** Porosity p %	** Measured permeability at 10°C k $cm.s^{-1}$ $\times 10^{-3}$	Uniformity coefficient u	Coefficients of form: β			
	No. 1	No. 8					Calculated *		Corrected by (7)	
							β_T	$\Delta\beta$ %	$\beta_{u=2}$	$\Delta\beta$ %
1	100	0	3.10	39.16	35.00	1.16	384	− 6.1	768	− 2.4
2	75	25	3.13	38.25	35.82	1.22	418	+ 2.2	712	− 5.1
3	50	50	3.20	39.71	44.00	1.33	431	+ 5.4	709	− 5.5
4	25	75	3.40	37.92	45.15	1.30	460	+12.5	782	+ 4.3
5	15	85	3.67	37.85	44.35	1 24	390	− 4.7	709	− 5.5
6	15	85	3.67	39.01	56.10	1.24	441	+ 8.6	805	+ 7.3
7	10	90	4.00	38.48	51.75	1.15	362	−11.5	735	− 2.0
8	0	100	4.10	38.52	57.70	1.12	381	− 6.9	778	+ 3.7
							Mean **409**	$\sum\lvert\,\rvert$ $=57.9$	Mean **750**	$\sum\lvert\,\rvert$ $=35.8$

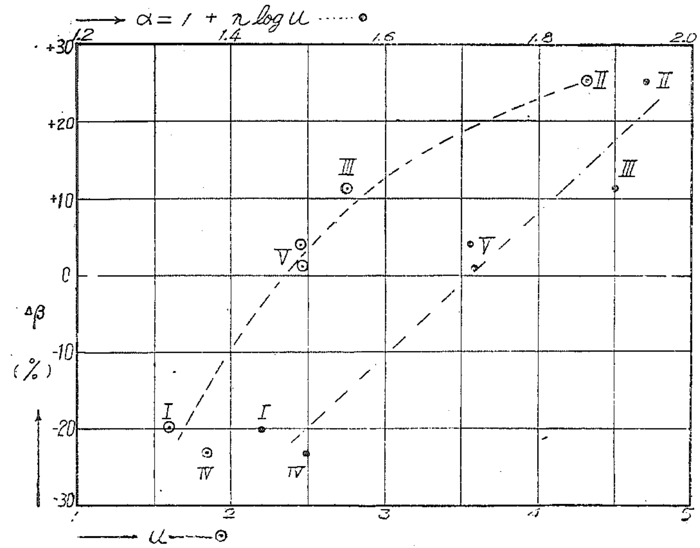
* Calculated by Terzaghi's formula; refer (6), s. 119, 1925 or (1).

** Measured by Mr. Donat; refer (4), s. 229.

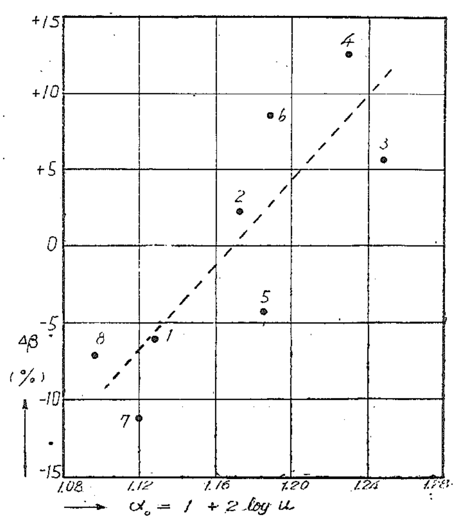
Refer Fig.-1 (a). Therefore, if they are converted into $\beta_{u=2}$ as well as in (a), the mean value of $\beta_{u=2}=393$ and $\sum|\Delta\beta|=31.2$ are obtained. That is to say, the amount of deviations will almost be reduced to a third.

B: The case of pretty uniform-grains.

Table-2 B shows eight sorts of the mixed sand got by combining two kinds of different sand ($d_e=0.031$ and $0.041\,cm$), according to Mr. Donat's data.(4) The grains of the sand are rather smoother than sand-A and the



(a) Case of u is not small.



(b) Case of u is very small.

Fig. 1. Deviations of coefficient of form $\Delta\beta$ concerned with grading.

permeability is greater, too. $\beta=409$ and $\sum|\Delta\beta|=57.9$ is got by these measured values. $\Delta\beta$ in Fig.-1(b) seems to be influenced by u. Then, $\beta_{u=2}=750$ and $\sum|\Delta\beta|=35.8$ will be got if β be converted by the equation (7); that is, the amount of deviations will approximately be reduced to a half. When we compare these coefficients of form with each other, a caution must be paid, otherwise it will be misunderstood that sand-A may be smoother and has a greater permeability (447>409) than sand-B. But such a mistake will not be committed (393<750) if the coefficients of form be compared with each other in the same graduation by the writer's method.

II. An Approximate Graphical Method to get Coefficient of Permeability

(1) HOW TO GET d_w, SPECIFIC-AREA-TYPE OR ZUNKER-TYPE EFFECTIVE DIAMETER, FROM A GRADING CURVE.

(a) Graphical method (Refer Fig.-2).

AB in the semi-log paper is a grading curve of a certain mixed-sand. Get the intersecting point b of AB and 90% line of the abscissa. Next, get the point c of intersection of the perpendicular line from b and 10% line of the abscissa. Connect c with a which is the intersection of the line of 60% and the perpendicular line drawn from A, the point of smallest size on the grading curve. Letting d to be the intersecting point of ca and AB, the reading of the abscissa corresponding to the point d will nearly give d_w.

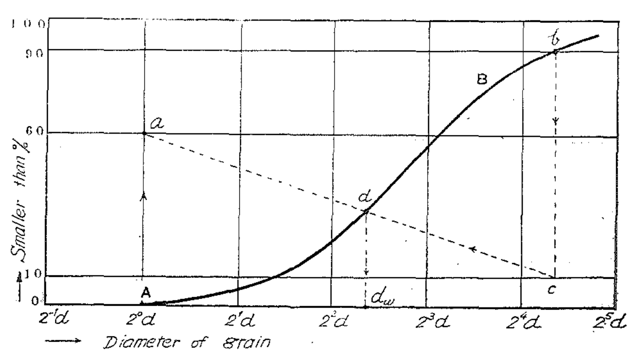


Fig. 2. Approximate determination of d_w, Zunker's effective diameter, from a certain grading curve.

(b) Accuracy.

The values of d_w of about seventy grading curves having various shapes have been obtained by the graphical method and they are compared with the one which has been obtained by the calculation. Namely, (i) this graphical method gives pretty satisfactory value to the grading curves belonging to the type of normal statistical distribution; (ii) in grading curves belonging to other types, the mean value of the graphical method will be smaller by about 0~5% than by the calculation if it is $u<5$, and inversely it will be bigger if it is $u>5$.

(2) HOW TO GET COEFFICIENT OF PERMEABILITY FROM GRADING CURVE OF A CERTAIN MIXED-SAND.

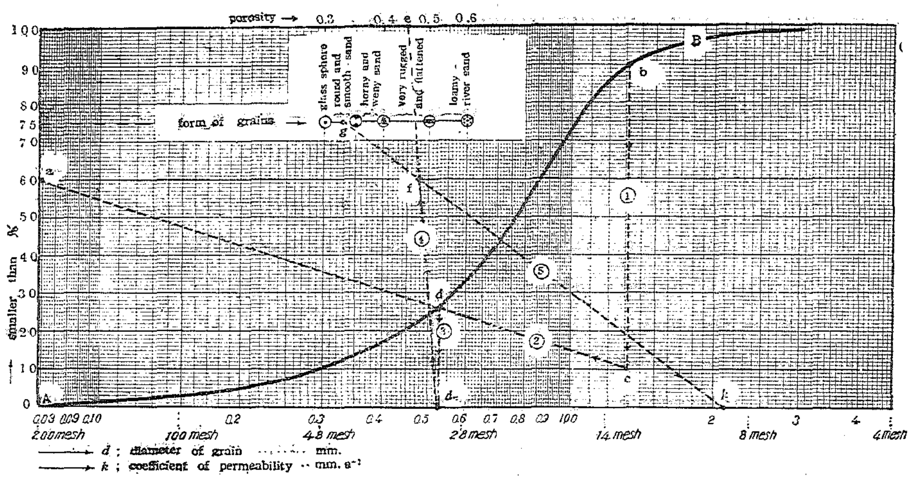


Fig. 3. Graphical method to get k (at 10°C) from the grading curve of a certain mixed-sand.

An example will be shown in Fig.-3.

(a) Given data;

the grading curve $=AB$;

the form of grain = smooth and round sand;

the porosity $p=0.45$,

the temperature of water $=10°C$.

(b) Graphical method;

(i) $d_w \approx 0.525\,mm$: Graphically by the approximate method shown in (1).

(ii) $k_{10°C}$: Take d_w at the bottom line of the abscissa and take e (corresponding to $p=0.45$) at the top line of the abscissa, then draw ed_w—line. And get the intersecting point f of ed_w—line and 60% line. Next, connect f with g at the line of 75% of the abscissa (the reading of g corresponds to smooth and round sand). Read the intersecting point of the extension of gf and the bottom line, then

$$k_{10°C}=2.1\,mm.\,s^{-1}=0.21\,cm.\,s^{-1}$$

(c) Comparison of the graphical method with several permeability formulae (at 10°C).

1. By Zunker's.[1]

$$d_w = 1\Big/\sum\frac{g_N}{d_N} = 1\Big/\left(\frac{0.025}{0.01}+\frac{0.07}{0.02}+\frac{0.22}{0.04}+\frac{0.58}{0.08}+\frac{0.125}{0.16}+\frac{0.02}{0.32}\right)$$
$$= 0.0524\ cm$$

$$k = \frac{c_z}{\eta}\left(\frac{p}{1-p}\right)^2 d_w^2 = \frac{1.5}{0.0131}\times\left(\frac{0.45}{1-0.45}\right)^2\times 0.0524^2 = 0.21\ cm.s^{-1}$$

2. By Kozeny's.[1]

$$k = \frac{c_k}{\eta}\frac{p^3}{(1-p)^2}d_w^2 = \frac{3.6}{0.013}\times\frac{0.45^3}{(1-0.45)^2}\times 0.0524^2 = 0.23\ cm.s^{-1}$$

3. By Hazen's.[1]

$$k = (60\sim150)\,d_e^2 = (60\sim150)\times 0.03^2 = \underset{\text{(dense packing)}}{0.054} \sim \underset{\text{(loose packing)}}{0.135}\ cm.s^{-1}$$

4. By writer's.[1][5]

$$k = (0.7+0.03t)\,\beta u\left(\frac{p}{1-p}\right)^2 d_e^2 = (0.7+0.03\times10)\times120\times2.9\times\left(\frac{0.45}{1-0.45}\right)^2$$
$$\times 0.03^2 = 0.21\ cm.s^{-1}$$

Summary: In this paper, the writer observed the coefficient of form in the percolation formulae of effective size-type from the point of view of the grading and made its negotiability greater by his α. Then, he got an approximate effective diameter of the specific-area-type (so-called Zunker's type) from the grading curve and showed graphically a method to get the coefficient of permeability in Darcy's law. This method includes porosity, coefficient of form of grain and form of grading curve, and, therefore, it has more reliability than Hazen's formula.

Acknowledgement: The writer wishes to express his cordial gratitude to Dr. Kazuyuki TSURUMI and to Dr. Tomoyasu YŪKI, Dean of Faculty of Engineering of Shinshū University, for their very kind guidance.

References:

(1) H. Sasaki, Tech. Rept. of Shinshū Univ., Vol. 2, 1953.
(2) F. Yamazaki, Nogyō-Doboku-Kenkyū, Vol. 14, No. 2, 1942.
(3) J. Kozeny, Wasserkraft u. Wasserwirtschaft, 22, 1927.
(4) J. Donat, Wasserkraft u. Wasserwirtschaft, 25, 1929.
(5) H. Sasaki, Journal of Shinshū Univ. Vol. II, 1952.
(6) K. Terzaghi, "Erdbaumechanik", s. 113, Tabelle 27, 1925.