THEORETICAL STUDIES ON THE TRAVERSE AND NON-TRAVERSE MOTION IN WINDING

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INTRODUCTION

When a thread is wound up on a bobbin in factories, a mechanism of traverse motion is usually used in order to wind up regularly, to interrupt adhering the threads each other, to unwind smoothly etc. In the case of traverse winding, as we treated in the previous paper[2)], the configuration of the thread wound up is expressed by a function of winding velocity, traverse velocity, distance between the bobbin and the traverse bar etc. But in the winding mechanism of the lifting machines such as crane or hoist it is possible to wind up a wire regularly without traverse, and also in the case of rereeling of a thread in home sewing machine, the traverse motion is not usually used. When the thread is wound on the bobbin, the thread shows a tendency to take the most stable place and this property is used efficiently in these machines.

The condition of the "Non-traverse motion winding" and the most suitable type of bobbin for it were investigated.

FUNDAMENTAL EQUATION OF THE TRAVERSE MOTION

A cylindrical bobbin is used for simple analysis. In Fig. 1, take coordinate x in the direction of the traverse bar 1 and coordinate ξ in parallel to the axis (5) of the bobbin including point p, where the thread passed through a thread guide (2) on the traverse bar comes in contact with the bobbin 3. And the relation between the traverse motion $x=x(t)$ and the configulation of the thread wound up on the cylindrical bobbin $\xi=\xi(t)$ is

$$\frac{d\xi}{dt}=(x-\xi)\frac{v}{l} \qquad \cdots\cdots(1)$$

where l : distance between the bobbin and the traverse bar and

v : winding velocity $(=r\omega_0)$.

For convenience, let $l/v=l/r=c$ be a constant which has a dimension of time. Then equation (1) becomes

$$c\frac{d\xi}{dt}+\xi=x. \qquad \cdots\cdots(2)$$

It is called "Fundamental equation of traverse motion".

When the motion of the traverse bar $x=x(t)$ is given analytically, this equation can be solved. For example, in the case of simple harmonic motion;

$$x=a\sin\omega t \qquad \cdots\cdots(3)$$

we get in steady state;

$$\xi=b\sin(\omega t+\varphi) \qquad \cdots\cdots(3')$$

where $2a$: traverse (double amplitude)
b : amplitude of responsed motion.

From equation (3) and (3′), it becomes

$$\xi=\frac{a}{\sqrt{1+(c\omega)^2}}\sin\{\omega t-\tan^{-1}(c\omega)\} \qquad \cdots\cdots(4)$$

Therefore, the frequency responses are;

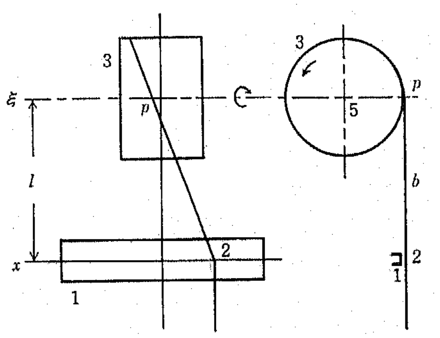

Fig. 1 Schematic diagram of traverse motion winding
1 : traverse bar 2 : thread guide on the traverse bar 3 : bobbin

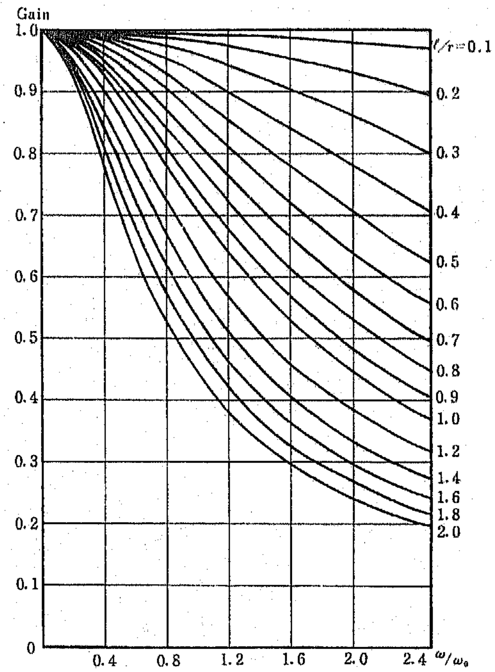

Fig. 2 Relation between the gain and ω/ω_0 with l/r as parameter

$$\left.\begin{array}{ll}\text{amplitude ratio (gain)} & b/a = 1/\sqrt{1+(c\omega)^2} \\ \text{phase lag} & \varphi = -\tan^{-1}(c\omega)\end{array}\right\}. \quad \cdots\cdots\cdots(5)$$

From equation (5) it is easily to see that both amplitude ratio and phase lag are functions of the winding constant l/r and angular velocity ratio ω/ω_0, and that amplitude of thread wound up on the bobbin is always smaller than that of traverse motion. The relation between the gain and ω/ω_0 is shown in Fig. 2 with l/r as parameter. We carried out this experiment, using the Scotch Yoke mechanism as the traverse, and found that the observed results almost coincide with the theoretical analysis.

MECHANISM OF NON-TRAVERSE WINDING

Mechanism of non-traverse winding is shown in Fig. 3. If the thread is assumed to have not any friction, flexual and torsional rigidity, the position and the configulation of thread wound up on the flanged bobbin without traverse, depend upon the diameters of bobbin and thread and the position of thread guide. Consider that the thread of d in diameter is wound through the fixed thread guide (G), which is at the distance of l from the bobbin and f from a production of flange of the bobbin as shown in Fig. 3, and that the flanged bobbin has a diameter of D and a width of w.

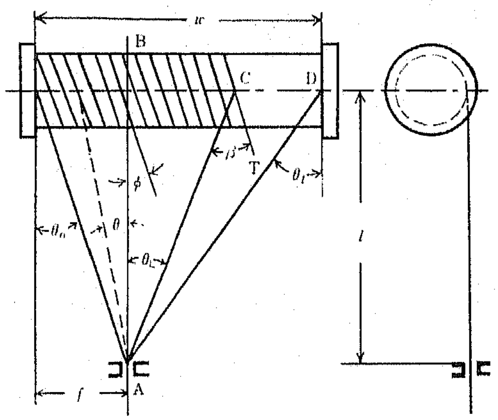

Fig. 3 Illustration of non-traverse winding

To wind up the thread on the bobbin regularly as closed coil, the angle θ_0 between flange and thread of the initial state of winding must not be larger than a helical angle of the closed coil ϕ determined by the diameters of the thread and the bobbin. When the angle θ_0 is larger than this helical angle ϕ, it is impossible to wind up the thread on the bobbin as closed coil, and the thread does not contact with each other. Therefore, the critical condition to wind up just as closed coil is

$$\tan\theta_0 = \frac{d}{\pi D} = \frac{f}{l} \quad \cdots\cdots\cdots(6)$$

where d : diameter of the thread
D : diameter of the bobbin
f : distance between a production of the flange and the thread guide
l : tangential length from the thread guide to the bobbin

As the thread is wound, winding angle θ in Fig. 3 decreases and becomes negative. But the helical angle on the bobbin is always kept at constant angle ϕ if the compressive deformation of thread is neglected. The absolute value of θ increases gradually until the thread cannot lie on the bobbin and climbs on the thread previously wound. And the subsequent thread is wound up on the first layer to the inverse direction as in Fig. 4. We call this phenomenon "Crossing over" and denote this critical winding angle by θ_L.

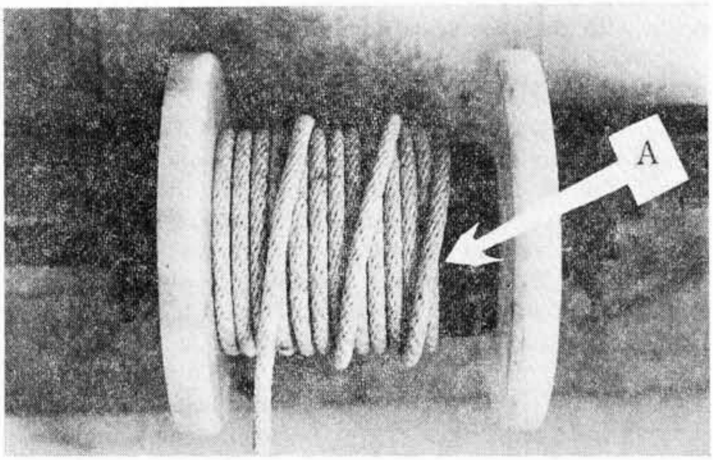

Fig. 4 Phenomenon of "crossing over" A : crossing over

In order to wind up the thread to the other side of the bobbin, the angle θ_t, which is the absolute value of the winding angle at the other side of the bobbin as Fig. 3, must not be larger than θ_L. In making the second layer, though the diameter of the coil increases and the winding angle θ becomes larger than ϕ, it is possible to wind up the thread regularly as closed coil in a certain range of θ_t. Because it is effected by the under layer with regular furrows produced by closely contact threads. The relation between θ_0 and θ_t is usually ; $\theta_0 < \theta_t$

SUITABLE SHAPE OF THE BOBBIN FOR NON-TRAVERSE WINDING

The curve of the thread wound up on the cylindrical bobbin is the helix which has the helical angle ϕ and is independent of winding angle θ. In the case of the conical bobbin, the helical angle at any point is not constant to wind up closely since the distance of centers of adjacent threads is constant at any point. The projection of this curve on the plane perpendicular to the bobbin axis is a spiral of ARCHIMEDES as;

$$\Lambda = k\Theta$$

where Λ : distance from the origin
Θ : polar angle
k : proportional constant

The value of θ on each point is not always coincided with the helical angle ϕ. To wind up closely, θ must not be larger than the helical angle ϕ. As the thread wound previously is pushed by the subsequent threads, the thread assembly mounted on the bobbin is apt to fall down. This pushing force or "Side pressure" of the neighbouring threads causes crossing over and the pressure on the flange. To wind without the side pressure, the winding angle θ should be always equal to the helical angle ϕ and the geodesic curvature of the thread on the bobbin should be zero. But generally it is impossible that both the two conditions are satisfied at the same time. Take coordinates as in Fig. 5, where x : bobbin axis, r : radius of the bobbin, $\tan \varepsilon$: gradient of the bobbin curve at x and d : diameter of the thread. The shape of bobbin, where the winding angle θ is always equal to the helical angle ϕ, is expressed as a solution of differential equation.

$$(f-x)\sqrt{1+\left(\frac{dr}{dx}\right)^2}=\frac{dl}{2\pi r} \qquad \cdots\cdots(7)$$

Integrating this differential equation under the condition; $r=r_0$ at $x=0$, we can obtain the shape of the bobbin $r(x)$. It was calculated numerically by means of the Runge–Kutta method (details in appendix 1) and the results were shown in Fig. 6. This curve is hyperbolic and has the asymptote $x=f$

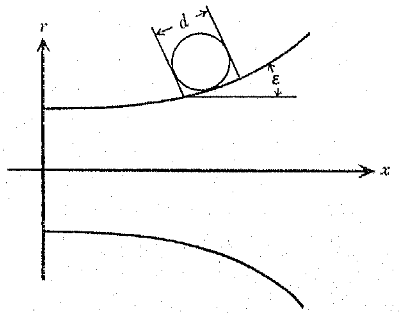

Fig. 5 Coordinates system of the bobbin and the thread

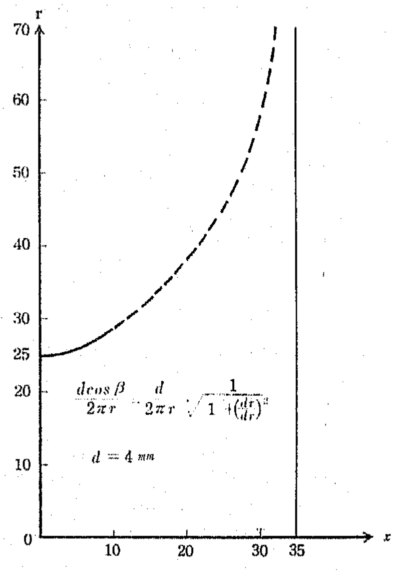

Fig. 6 One of the theoretical bobbin shape

at the middle of bobbin. The available region of this ideal shape is limited in the region of small x.

MECHANISM OF CROSSING OVER OF THE THREAD

As the thread is wound until the winding angle θ becomes θ_L, the phenomenon "Crossing over" of thread will occur, and the thread forms the second layer of open coil before arriving at the other flange. Since the helical angle of thread is very small in practical winding, it is assumed, in order to simplify the theoretical analysis, that a wound thread on the bobbin forms a torus. Equation of the torus is given in Cartesian coordinates as follows,

$$x = (R_1 + R_2 \sin \varphi) \cos \theta$$
$$y = (R_1 + R_2 \sin \varphi) \sin \theta$$
$$z = R_2 \cos \varphi$$

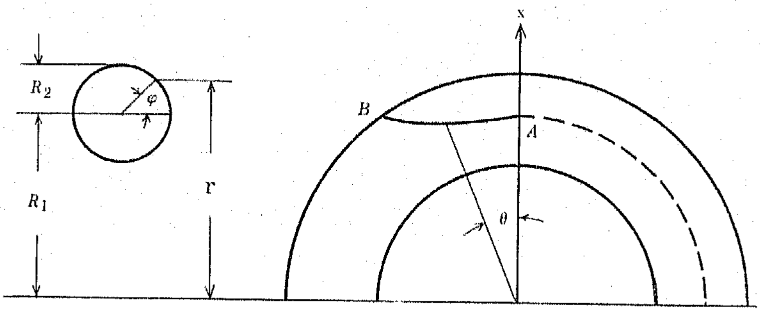

Fig. 7 Geodesic line (AB) on the torus of the thread

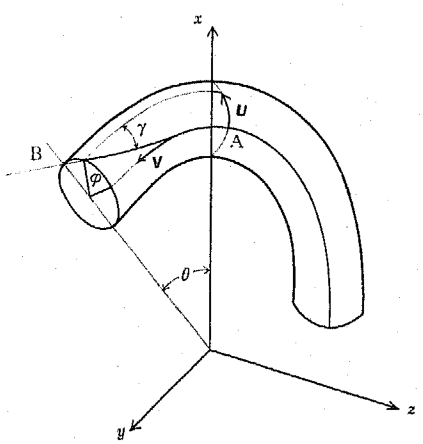

Fig. 8 Geodesic line (AB) on the torus of the thread

where the origin (0) of the coordinates is at the center of the torus, an axis z is in the direction of the torus axis (bobbin axis) through the origin, an axis x is perpendicular to the axis z, θ is rotating angle around the axis z and φ is the rotating angle around the center line of the torus ring (Fig. 7 and 8).

When the subsequent thread is wound up to the surface of the torus of the previously wound thread, the contact point of the winding thread on the torus lies on the curve determined by the statical condition. If there is no inter-thread-friction, the curve of the contact point will

be a geodesic line, because in which state, the tensioned thread on the torus surface is in the most stable state. When we take the curvilinear coordinates $\boldsymbol{u}$, $\boldsymbol{v}$ as $u = R_2\varphi$ $v = \theta$, the first kind of fundamental quantities of the differential geometry $\boldsymbol{E}$, $\boldsymbol{F}$ and $\boldsymbol{G}$ are; $\boldsymbol{E}=1$, $\boldsymbol{F}=0$, $\boldsymbol{G}=(R_1+R_2\sin\varphi)^2$,
where R_1: radius of the center line of the torus ring,
R_2: radius of a cross section of the torus ring.
The equation of geodesic line on a torus surface is generally given as:

$$\frac{\partial\gamma}{\partial v}=\frac{\partial\sqrt{G}}{\partial\sqrt{u}} \qquad \cdots\cdots(9)$$

where γ is the angle between geodesic line and the curvilinear coordinate u. By substituting equation (8) to equation (9) and assuming $\gamma \fallingdotseq \tan\gamma$, we have

$$\frac{d^2\varphi}{d\theta^2}=\pm\frac{R_1}{R_2}\cos\varphi\,(1+\frac{R_2}{R_1}\sin\varphi) \qquad \cdots\cdots(10)$$

Integrating equation (10) with the initial condition;

$$\varphi = 0 \text{ and } \frac{d\varphi}{d\theta}= 0 \text{ at } \theta =0$$

$$\theta = \frac{1}{\sqrt{\frac{2R_2}{R_1}}}\int_0^{\varphi}\frac{d\varphi}{\sqrt{\sin\varphi(1+\frac{R_2}{2R_1}\sin\varphi)}} \qquad \cdots\cdots(11)$$

This equation is solved with the graphical integration and the result is plotted against φ with the full line for the case of $\frac{R_2}{R_1}= 0.074$ in Fig. 9 and for the case of $\frac{R_2}{R_1}=0.5$ in Fig. 10 respectively.

Since $\frac{R_2}{R_1}\sin\varphi$ is negligibly small compared with unity in practical applications, equation (11) is simplified by neglecting of $\frac{R_2}{R_1}\sin\varphi$ and by putting $\sin\varphi=\cos^2\phi$ (details in appendix 2);

$$\theta = \frac{1}{\sqrt{\frac{R_1}{R_2}}}\int_0^{\phi}\frac{d\phi}{\sqrt{1-\frac{1}{2}\sin^2\phi}} \qquad \cdots\cdots(11')$$

The solution of this equation is plotted against φ with the dotted line for the case of $\frac{R_2}{R_1}=0.074$ in Fig. 8 and for the case of $\frac{R_2}{R_1} = 0.5$ in Fig. 9. It is seen that the solution of the equation (11′) agrees very good with the solution of the equation (11) in the case of $\frac{R_2}{R_1}=0.074$, but in the case of $\frac{R_2}{R_1}=0.5$ the differnce between the two solutions is comparatively large. Since in practical application, the ratio of $\frac{R_2}{R_1}$ is usually less than the value of 0.074, the term $\frac{R_2}{R_1}\sin\varphi$ may be neglected.

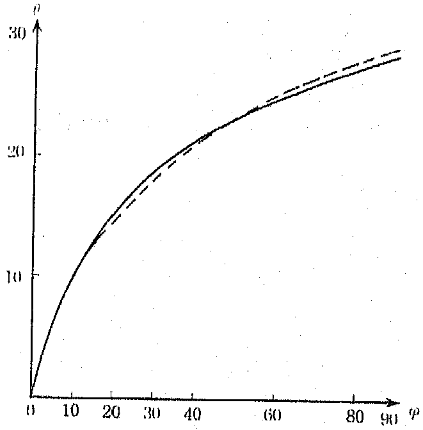

Fig. 9

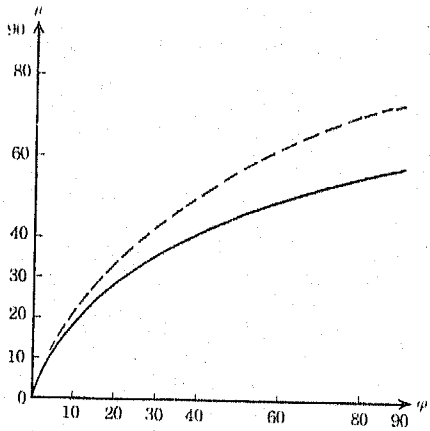

Fig. 10

Relation between the rotation angle of the torus (θ) and the rotating angle of the torus ring (φ) on the geodesic line.

The angle β between the thread AB and the plane perpendicular to the torus axis through a contact point on the torus in Fig. 11, is described against φ using the geometrical relations as follows :

$$\sin\beta=\sin\gamma\cdot\sin\varphi$$

$$\tan\gamma=\frac{R_2}{R_1+R_2\sin\varphi}\frac{d\varphi}{d\theta}.$$

The result is shown in Fig. 12. As the helical angle which is produced by the wound thread on the bobbin is kept at θ_0, the relation between β and θ is given as $\beta=\theta+\phi$. Since in the case of no inter-thread friction, "Crossing over" occurs at the point $\varphi=90°$ on the geodesic line on the torus, we can calculate the crossing over angle β_L by using the above result. For example, β_L is 20° in the case of $\frac{R_2}{R_1}=0.074$ (Fig. 12).

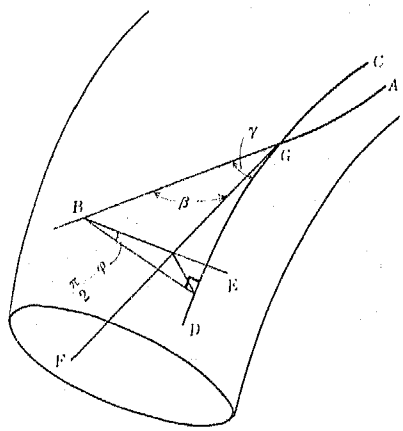

Fig. 11 Geometrical relation between the crossing over angle β and the geodesic line on the torus
AB : geodesic line, CD : line on a constant
BE : direction of parallel to the torus' axis
FG : direction of perpendicular to the torus' axis

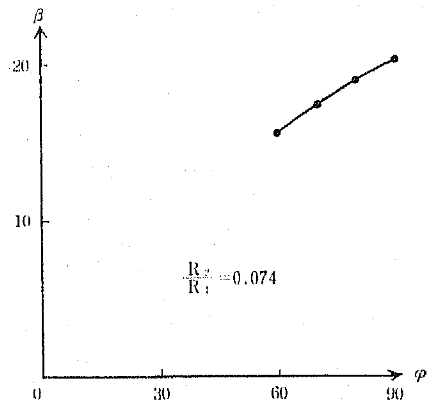

Fig. 12 Relation between φ and β in the case of no friction (calculated)

But practically the frictional force acts on the contact region, so the curve of contact on the torus is given by the equilibrium condition (Fig. 13);

$$T_n = 2T \cdot \sin \frac{ds}{2R}$$

as $\sin \frac{ds}{2R} \doteqdot \frac{ds}{2R}$

$$T_n = T \frac{ds}{R}$$

$$P = T_n \cos \delta = T \frac{ds}{R} \cos \delta$$

$$F = T_n \sin \delta = T \frac{ds}{R} \sin \delta$$

substituting P, F to the equation $\mu P = F$

$$T \frac{ds}{R} \cos \delta \cdot \mu = T \frac{ds}{R} \sin \delta$$

$$\frac{\cos \delta}{R} \equiv 1/\rho_n \text{ and } \frac{\sin \delta}{R} \equiv 1/\rho_b$$

so, we have $1/\rho_n \cdot \mu = 1/\rho_b$

where T_n : normal force to osculating plane
T : tention of the thread
s : length along the thread
$\frac{1}{R}$: curvature of the thread
$\frac{1}{\rho_b}$: geodesic curvature of the thread
$\frac{1}{\rho_n}$: normal curvature of the thread
P: normal force to tangent plane
F: frictional force acting perpendicular to the thread in tangent plane
μ : frictional coefficient
δ : angle between T and P

So, the curve of thread on the torus does not completely coincide with the geodesic line. It is considerably difficult to analyze this curve, but as $\frac{1}{\rho_n}$ is nearly zero in the case of small region of φ, the effect of friction is negligible, and it is expected that the crossing over angle β_L must decrease as frictional coefficient μ increases.

EXPERIMENTS AND RESULTS

Four kinds of bobbins were used in this experiment (Fig. 14). Bobbin **A** was made of polystyrene being covered with a thin sheet of emery. Bobbin $\mathbf{B}_1$, $\mathbf{B}_2$ and **C** were made of mild steel and their shapes and sizes are seen in Fig. 14. The threads used for this experiment were nylon braids of 4mm

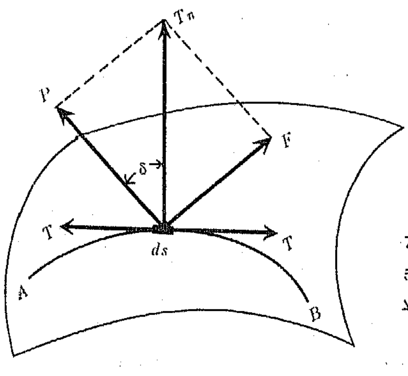

Fig. 13 Mechanical equilibrium of the thread on the torus with effect of friction AB: thread

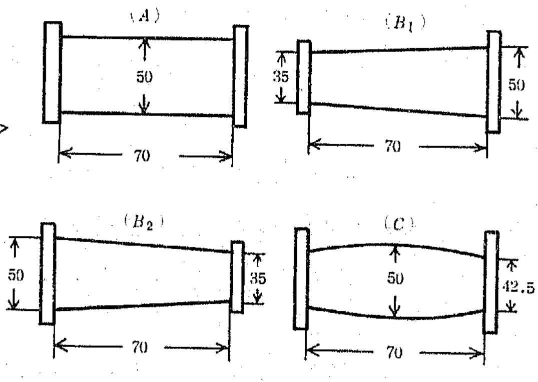

Fig. 14 Bobbins used in this studies

and 4.5mm and cotton strings of 4 mm and 4.5 mm in diameter. Winding velocities were kept about 10cm/sec in every case and two kinds of winding ways i.e. **Z** and **S** were used.

a) Initial angles of the first and the second layers (θ_0 and θ_t)

During this experiment, load of 150g in tention was applied to the thread constantly. The experimental results, obtained by using nylon braid of 4.5 mm in diameter, show that the variations of observed values of θ_0 and θ_t to wind the thread closely were very small and these values were sufficiently reproducible. They are shown in Table (1). There is some discrepancy between **Z** winding and **S** winding, although the other conditions are quite

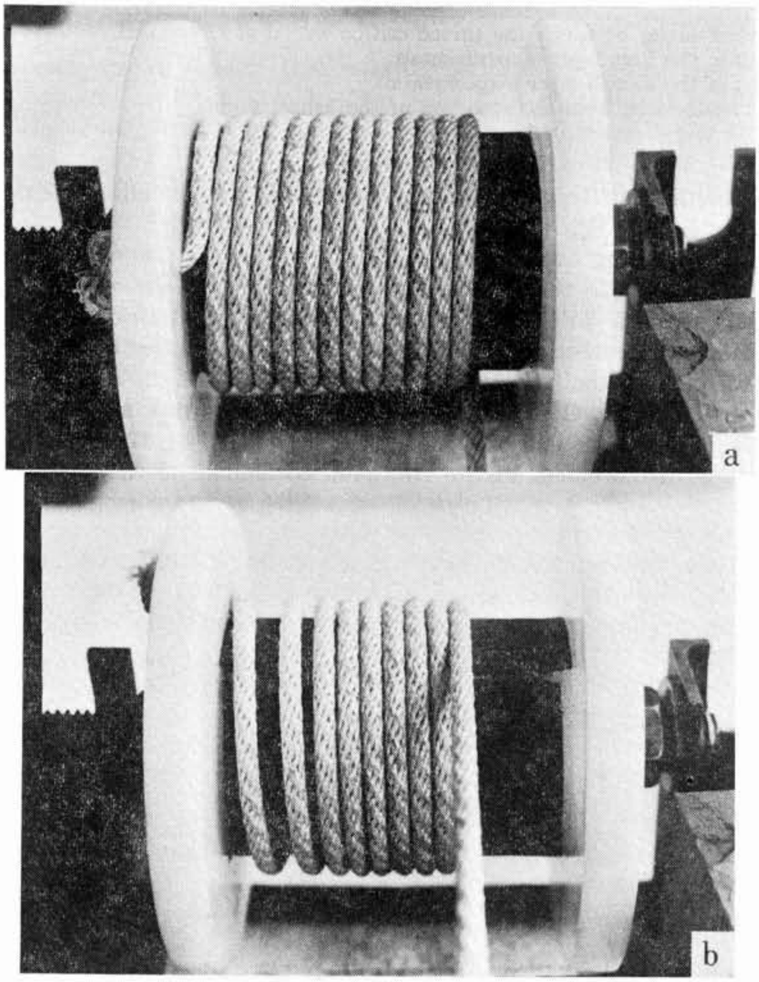

Fig. 15 Effect of the liveliness
(a) : **S** winding (b) : **Z** winding

Table 1 Diffrence in θ_0 and θ_t between S winding and Z winding

Bobbins	θ_c	Windings	θ_0	θ_t
A	1°38′	Z	1°15′	2°50′
		S	2°30′	2°14′
B_1	2°20′	Z	1°18′	3°20′
		S	4°30′	1°42′
B_2	1°37′	Z	−0°20′	5°00′
		S	2°00′	4°05′
C	1°55′	Z	1°06′	5°48′
		S	2°55′	5°00′

Critical angle, of which the thread can be wound as closed coil,
θ_0 : in the first layer (experimental)
θ_t : in the second layer (expeumental)
θ_c : calculated from the condition of the helical angle

equal. It seems that this phenomenon is caused by the liveliness (Z or S twist) of the thread (Fig. 15).

b) Angle of crossing over ; β

We examined the effect of frictional coefficient of threads for the angle of crossing over; β. In this experiment, we used a cylindrical bobbin, 50mm in diameter, and three kinds of threads, one of which was coated with MoS_2 powder to modify the frictional coefficient of threads. These details are denoted in Table (2). Observed values of β were within about ±5 % from average value of them and they are in Fig. 16 and Fig. 17. It is found that β decreses with increment of the frictional coefficient of the thread as theo-

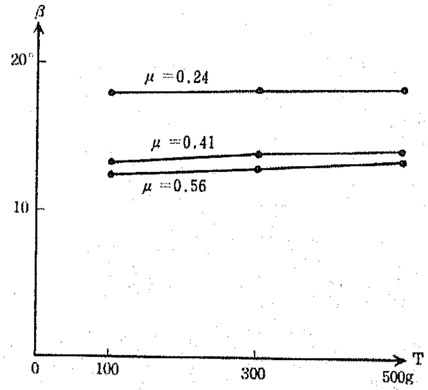

Fig. 16 Experimental result between the angle of crossing ovre β and tension

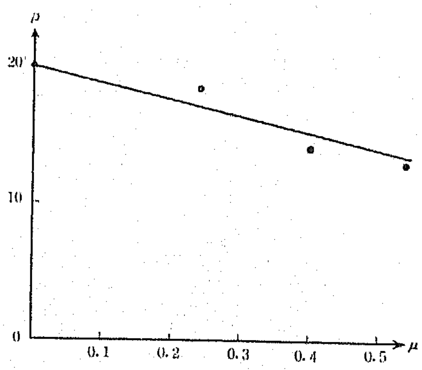

Fig. 17 Experimental result between the angle of crossing over β and frictional coefficient μ

Table 2 Diameter and frictional coefficient (μ) of used thread

threads	diameter	frictional coefficient (μ)
cotton	4.0 mm	0.56
cotton coated with MoS_2 powder	4.0 mm	0.41
nylon	4.0 mm	0.24

retical result expected. It seems that β is independent of tension applied. Regarding the unwinding of the thread, the condition of the crossing over is also useful.

CONCLUSIONS

a) In non-traverse winding, the relative position of thread guide to the bobbin is determined by θ_0 and θ_t. The angle θ_0 is determined by the helical angle of closed coil with some correction due to liveliness but the muximum angle of θ_t is larger than the helical angle by the effect of the under layer of the threads. The angle θ_t must be determined by experiments since it is so much influenced by flexual rigidity, torsional rigidity* and friction as in the case of β. Generally $\theta_0 < \theta_t \leqslant \beta$.

It is desirable that the thread guide is far from the bobbin to make θ_0 and θ_t small and that the coefficient of friction of threads is small.

b) The suitable shape of the bobbin for non-traverse winding is determined by the conditions of

(1) the helical angle should be equal to θ and

(2) the geodesic curvature of the thread should be zero, but both the conditions are not satisfied at the same time. From the condition (1), it is found that the shape of the bobbin must be hyperbolic revolution.

c) In the case of no friction the angle of crossing over is determined by the equation (11), (11′) and (12) which are obtained by using differential geometry on the torus surface. If friction acts between threads, geodesic curvature is not zero and the equation of the thread is more complicated, but the experimental results almost agrees with theoretical conclusion, that is, the angle of crossing over (β) decreases with increasing of frictional coefficient (μ), and theoretical angle in the case of no friction almost coincides with the angle extraporated from observed value under in friction.

SUMMARY

The solution of the fundamental equation between the traverse motion and the configuration of the thread on the bobbin $(\frac{r\omega_0}{l}\cdot\frac{d\xi}{dt}+\xi=x)$ was already reported. In this report, the conditions of the non-traverse winding

* When the thread falls down in the furrow of the thread layer, it is observed that the thread slips down or rolls down with torsions.

(a suitable type of the bobbin, position of the thread guide, crossing over and so on) were studied comparing the theoretical analysis with the experimental results and some fundamental conclusions were obtained.

In order to wind up the thread on the cylindrical bobbin as closed coil without traverse motion, the initial winding angle must not be larger than helical angle, but in the second layer, the thread can be wound by the effect of the first layer even when the initial winding angle is somewhat larger than it.

This critical angle is effected by friction, rigidity of the thread etc., and the experimental results show a tendency expected from the theoretical analysis.

The conditions to avoid the side pressure between nighbonring threads were considered and the suitable type of the bobbin was analyzed in a region of small geodesic curvature.

The conditions of crossing over of the thread were derived under the cases in which friction is neglected and is taken into consideration. The condition for the thread without friction was solved. The analytical results almost coincided with the experimental results.

APPENDIX

1) $$(f-x)\sqrt{1+\left(\frac{dr}{dx}\right)^2}=\frac{dl}{2\pi r} \qquad \cdots\cdots\cdots(A)$$

put $r=r_0$ at $x=0$, $f-x=t$, $\frac{t}{l}=\mathrm{X}$ and $\frac{r}{r_0}=\mathrm{Y}$

then
$$\mathrm{X}=\frac{d}{2\pi r_0\,\mathrm{Y}\sqrt{1+(-r_0)^2\left(\frac{d\mathrm{Y}}{dt}\right)^2}}$$
$$=\frac{d}{2\pi r_0\,\mathrm{Y}\sqrt{1+\left(\frac{r_0}{l}\right)^2\left(\frac{d\mathrm{Y}^2}{d\mathrm{X}^2}\right)}}$$

it becomes $\left(\frac{d\mathrm{Y}}{d\mathrm{X}}\right)^2=\mathrm{H}^2\frac{1}{\mathrm{K}^2\cdot\mathrm{Y}\cdot\mathrm{X}^2}-\frac{1}{\mathrm{K}^2}$

where $\frac{r_0}{l}=\mathrm{K}$ $\qquad \frac{d}{2\pi r_0}=\mathrm{H}.$

2) $$\frac{d^2\varphi}{d\theta^2}=\pm\frac{R_1}{R_2}\cos\varphi\left(1+\frac{R_2}{R_1}\sin\varphi\right) \qquad \cdots\cdots\cdots(a)$$

$$\frac{d\theta}{d\varphi}=\frac{1}{\sqrt{\frac{2R_1}{R_2}}\cdot\sqrt{\sin\varphi\left(1+\frac{R_2}{2R_1}\sin\varphi\right)}}$$

$$\theta=\frac{1}{\sqrt{\frac{2R_1}{R_2}}}\int_0^{\varphi}\frac{d\varphi}{\sqrt{\sin\varphi\left(1+\frac{R_2}{2R_1}\sin\varphi\right)}} \qquad \cdots\cdots\cdots(b)$$

If we neglect $\frac{R_2}{R_1}\sin\varphi$, the equation (a) becomes

$$\frac{d^2\varphi}{d\theta^2}=\pm\frac{R_1}{R_2}\cos\varphi. \qquad \cdots\cdots(a')$$

It is solved as follows

$$\left(\frac{d\varphi}{d\theta}\right)^2=2\frac{R_1}{R_2}\sin\varphi$$

$$d\theta=\frac{d\varphi}{\sqrt{\frac{2R_1}{R_2}\sin\varphi}}$$

$$\theta=\frac{1}{\sqrt{2\frac{R_1}{R_2}}}\int_0^{\varphi}\frac{d\varphi}{\sqrt{\sin\varphi}}$$

putting $\sin\varphi=\cos^2\phi$,

$$\theta=-\sqrt{\frac{R_2}{R_1}}\int\frac{d\phi}{\sqrt{1-\frac{1}{2}\sin^2\phi}} \qquad \cdots\cdots(c)$$

The equation (c) is the first kind of complete elliptic integral and it can be solved by using the table of elliptic integral.

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