Tunnel Structure Analysis by the Operational Method
First Report

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1. Introduction

Tunnel structures resting on an elastic foundation have found their frequent applications in practice, but it would seem that they have never been rigorously solved, mainly because of the difficulty in treating members resting on elastic foundation. At times, such structural systems have been treated by replacing the foundation members by ordinary flexural members subjected to given distributed loads. Since the distribution of the reaction from foundation is unknown, it is sometimes assumed to be some given applied force. In order to be compatible with oblique or horizontal loads, as well as vertical loads, it is absolutely necessary to extend the usual Winkler assumption for the lateral deflection and the corresponding normal reaction to that of tangential or longitudinal displacement. Under these two kinds of Winkler assumption, the foundation beams or the combined structures involving such beam or beams can be treated without difficulty by means of the operational method avoiding simultaneous equations. It is added also that the method may be converted into the recursive displacement method in which unknowns are the column assemblage of all nodal displacements and the entire stiffness matrix results in the form of a tridiagonal matrix.

The structure will consist of one horizontal straight member, (1), which rests on the elastic foundation, two vertical straight members, (2) and (4), and one circular member, (3), as shown in Fig. 1. The load conditions are entirely arbitrary.

2. Complete State Vectors

The complete state vector of a member is a sixth-order column matrix,

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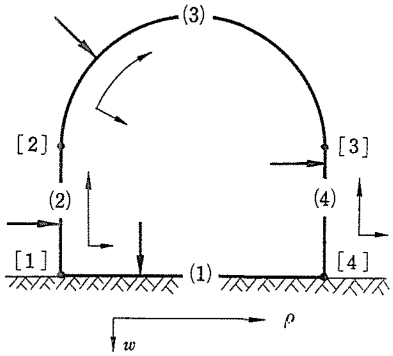

Fig. 1. Tunnel Structure.

consisting of the longitudinal displacement, the axial force, the lateral deflection, the flexural slope, the flexural moment, and the shearing force. Then the state vectors at any point ρ are given by the equations[1)]

$$\mathbf{W}_i(\rho) = \mathbf{R}_i(\rho)\mathbf{X}_i, \tag{1}$$

and

$$\mathbf{W}'_i(\rho) = \mathbf{R}_i(\rho)[\mathbf{X} + \mathbf{K}]_i, \tag{2}$$

for which $i = 1, 2, 3, 4$ (Fig. 1). Here $\mathbf{W}_i(\rho)$ and $\mathbf{W}'_i(\rho)$ hold for normal and conjugate domains respectively, $\mathbf{R}_i(\rho)$ is the complete abscissa matrix of size 6-by-6, $\mathbf{X}_i$ is the sixth-order eigenmatrix,[2)] and $\mathbf{K}_i$ is the load-matrix which is compatible with any external loads.[3)] Eqs. 1 and 2 are the approach equations for respective members properly.

It should be noticed herein that the right sides of Eqs. 1 and 2 exhibit the complete classification of data, and then attention can be focused at attacking the eigenmatrix $\mathbf{X}$ only.

3. Connection Conditions at Corners and Shift Formulas

Take the first corner [1] in Fig. 1, and then the connection condition at this corner is given by the equation

$$\mathbf{W}_2(0) = \mathbf{P}_1\mathbf{W}_1(0), \tag{3}$$

in which $\mathbf{P}_1$ denotes the projector. Eq. 3 then will yield the desired shift formula

$$\mathbf{X}_2 = \mathbf{L}_1\mathbf{X}_1, \tag{4}$$

providing

$$\mathbf{L}_1 = [\mathbf{R}_2(0)]^{-1}\mathbf{P}_1\mathbf{R}_1(0). \tag{5}$$

Similar considerations at corner [2], [3], and [4] will yield the desired shift formulas

$$\mathbf{X}_3 = \mathbf{L}_2(\mathbf{X}_2 + \mathbf{K}_2), \tag{6}$$

$$\mathbf{X}_4 + \mathbf{K}_4 = \mathbf{L}_3(\mathbf{X}_3 + \mathbf{K}_3), \tag{7}$$

and

$$\mathbf{X}_1 + \mathbf{K}_1 = \mathbf{L}_4\mathbf{X}_4, \tag{8}$$

providing

$$\mathbf{L}_2 = [\mathbf{R}_3(0)]^{-1}\mathbf{R}_2(1), \tag{9}$$

$$\mathbf{L}_3 = [\mathbf{R}_4(1)]^{-1}\mathbf{P}_3\mathbf{R}_3(1), \tag{10}$$

$$\mathbf{L}_4 = [\mathbf{R}_1(1)]^{-1}\mathbf{P}_4\mathbf{R}_4(0). \tag{11}$$

It is concluded that four equations of type of Eq. 3 express all the necessary connection conditions at the four corners of the present tunnel structure (Fig. 1).

4. Final Equation

Eqs. 4, 6, 7, and 8 will yield the following equation for determining the first eigenmatrix $\mathbf{X}_1$:

$$\mathbf{X}_1 = (\mathbf{L}_4\mathbf{L}_3\mathbf{L}_2\mathbf{L}_1 - \mathbf{E})^{-1}\lfloor \mathbf{E}, \quad -\mathbf{L}_4\mathbf{L}_3\mathbf{L}_2, \quad -\mathbf{L}_4\mathbf{L}_3, \quad \mathbf{L}_4 \rfloor \begin{bmatrix} \mathbf{K}_1 \\ \mathbf{K}_2 \\ \mathbf{K}_3 \\ \mathbf{K}_4 \end{bmatrix}, \tag{12}$$

or

$$\mathbf{X}_1 = \lfloor \mathbf{G} \rfloor \{ \mathbf{K} \}. \tag{13}$$

Eq. 12 or 13 is the desired final equation, and hence the present tunnel structure has been solved. Here $\lfloor \mathbf{G} \rfloor$ is a 6-by-24 rectangular matrix depending on only the geometry and material properties of the tunnel structure and hence it is called the geometry matrix, while $\{\mathbf{K}\}$ is the assembled load-matrix of size 24-by-1 which is compatible with any kind and number of external loads to be applied on the structure.

5. Numerical Example

The tunnel structure will consist of four members; the first being a beam resting on elastic member, the second and third being ordinary beams without foundation, and the fourth being a semi-circular arch. Its geometry and material properties are taken to be as follows:

Table 1.

E (t/m^2)	I (m^4)	A (m^2)	j (t/m^2)	k (t/m^2)
2 100 000. 0	0. 010 8	0. 36	351. 53	1 054. 6

in which E is Young's modulus, I is the moment of inertia of the cross section, A is the cross-sectional area, j is the shear modulus of foundation, and k is the usual normal modulus of foundation; all of these constants being measured with the ton-meter unit. Table 2 shows numerical results for the cited tunnel structure, which give the evaluation of complete state vectors of component members at several intermittent points. Fig. 2 gives the flexural moment and axial force for each member and the normal and tangential reactions, R and V say, from the foundation, which are extracted from Table 2.

6. Conclusions

In virtue of the perfectly classified data of configurations and external loading conditions assumed for each member of the tunnel structure, the analysis can be carried out systematically, and the necessary operators are obtained readily by the compatibility and equilibrium conditions at connection points. It permits recursion avoiding large-size simultaneous equations.

It should be added that the extension of the prevailing Winkler assumption for the lateral deflection to the longitudinal displacement has made it possible to analize the structural systems subjected to any loading conditions, including oblique or horizontal loads as well as vertical ones.

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Table 2. Evaluation of State Vectors.

		$u \times 10^3$ (m)	$w \times 10^3$ (m)	$\theta \times 10^3$	F (t)	S (t)	M (t−m)	R t/m	V t/m
(1)	0	−0.056	9.107	0.112	−14.179	23.561	22.106	9.604	−0.030
	1	−0.045	8.899	0.523	−14.162	18.452	9.508	9.385	−0.024
	2	−0.034	8.537	0.641	−14.150	13.531	−0.075	9.004	−0.018
	3	−0.022	8.173	0.545	−14.141	8.848	−6.778	8.619	−0.012
	4	−0.011	7.912	0.307	−14.136	4.367	−10.733	8.344	−0.006
	5	0.0	7.818	0.0	−14.134	0.0	−12.041	8.245	0.0
	6	0.011	7.912	−0.307	−14.136	−4.367	−10.733	8.344	0.006
	7	0.022	8.173	−0.545	−14.141	−8.848	−6.778	8.619	0.012
	8	0.034	8.537	−0.641	−14.150	−13.531	−0.075	9.004	0.018
	9	0.045	8.899	−0.523	−14.162	−18.452	9.508	9.385	0.024
	10	0.056	9.107	−0.112	−14.179	−23.561	22.106	9.604	0.030
(2)	11	−9.107	0.056	−0.112	−23.561	14.179	−22.106	—	—
	12	−9.125	0.144	0.373	−23.561	9.559	−15.002	—	—
	13	−9.144	0.473	0.706	−23.561	5.299	−10.563	—	—
	14	−9.163	0.974	0.953	−23.561	1.399	−8.572	—	—
	15	−9.181	1.613	1.179	−23.561	−2.141	−8.812	—	—
	16	−9.200	2.395	1.438	−23.561	−5.321	−11.068	—	—
(3)	17	−9.200	2.395	1.438	−23.561	−5.321	−11.068	—	—
	18	−8.251	3.852	1.641	−19.571	0.764	−13.002	—	—
	19	−6.815	5.430	1.662	−14.564	4.005	−10.527	—	—
	20	−4.889	6.885	1.372	−9.844	4.497	−6.322	—	—
	21	−2.559	7.918	0.777	−6.517	2.837	−2.731	—	—
	22	0.0	8.292	0.0	−5.321	0.0	−1.346	—	—
	23	2.559	7.918	−0.777	−6.517	−2.837	−2.731	—	—
	24	4.889	6.885	−1.372	−9.844	−4.497	−6.322	—	—
	25	6.815	5.430	−1.662	−14.564	−4.005	−10.527	—	—
	26	8.251	3.852	−1.641	−19.571	−0.764	−13.002	—	—
	27	9.200	2.395	−1.438	−23.561	5.321	−11.068	—	—
(4)	28	−9.200	−2.395	−1.438	−23.561	5.321	11.068	—	—
	29	−9.181	−1.613	−1.179	−23.561	2.141	8.812	—	—
	30	−9.163	−0.794	−0.953	−23.561	−1.399	8.572	—	—
	31	−9.144	−0.473	−0.706	−23.561	−5.299	10.563	—	—
	32	−9.125	−0.144	−0.373	−23.561	−9.559	15.002	—	—
	33	−9.107	−0.056	0.112	−23.561	−14.179	22.106	—	—

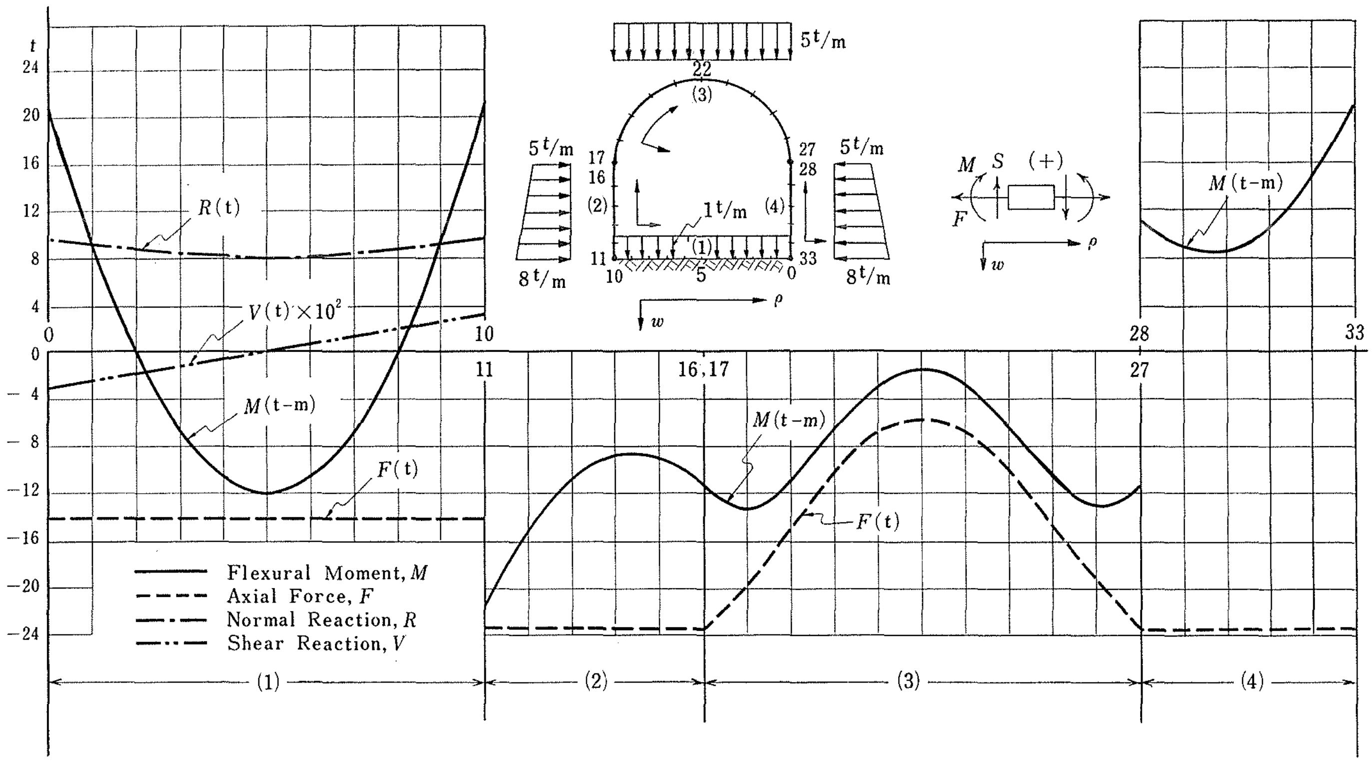

Fig. 2. Flexural Moment and Axial Force in Respective Members, together with Reactions from Foundation.

References

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Appendix. —Notation

The following symbols are used in this paper:

A = cross-sectional area;
EI = flexural rigidity;
F = axial force;
$\lfloor \mathbf{G} \rfloor$ = geometry matrix; Eq. 13;
j = shear modulus of foundation;
k = normal modulus of foundation;
$\mathbf{K}$ = load-matrix; Eq. 2;
$\{\mathbf{K}\}$ = load-matrix assemblage; Eq. 13;
$\mathbf{L}$ = shiftor; Eqs. 4, 6, 7, and 8;
M = bending moment;
$\mathbf{P}$ = projector; Eqs. 3, 10, and 11;
R = normal reaction;
$\mathbf{R}(\rho)$ = abscissa matrix; Eq. 1;
S = shearing force;
u = axial displacement;
V = tangential reaction;
$\mathbf{W}(\rho)$, $\mathbf{W}'(\rho)$ = state vector for normal and conjugate domain respectively; Eqs. 1 and 2;
w = lateral deflection;
$\mathbf{X}$ = 6-by-1 eigenmatrix; Eq. 1;
θ = flexural slope;
ρ = x/L, dimensionless current abscissa; Eq. 1;
$\lfloor\ \rfloor$ = row vector; and
$\{\ \}$ = column vector.