

Misleading Coupling Unification and Lifshitz Type Gauge Theory

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We propose a new grand unification scenario for ensuring proton stability. Our scenario is based on the idea that the proton decay problem is an artificial one, which is caused by the identification of the gauge-coupling unification scale with the grand unification scale or the grand unified symmetry breaking scale. We propose a Lifshitz type gauge theory as a candidate for realizing our scenario.

§1. Introduction

Grand unification is an attractive idea and enables the unification of forces and the (partial) unification of quarks and leptons in each family.¹⁾ The introduction of supersymmetry (SUSY) increases the likelihood of realizing grand unification. In the minimal supersymmetric standard model (MSSM), the gauge couplings meet at $M_X = 2.1 \times 10^{16}$ GeV if the superpartners and Higgs particles exist below or at approximately $O(1)$ TeV.^{2),**)} It is natural to surmise that the grand unification occurs at M_X and that the physics above M_X can be described by a supersymmetric grand unified theory (SUSY GUT). This scenario is very attractive, but, in general, it suffers from problems related to Higgs multiplets. A typical problem is the proton decay problem⁴⁾ in the minimal SUSY GUT.⁵⁾

The proton decay problem originates from the significant contribution from the operators of dimensionality five. Stronger constraints on the colored Higgs mass M_{H_C} and the sfermion mass $m_{\tilde{f}}$ have been obtained (e.g., $M_{H_C} > 6.5 \times 10^{16}$ GeV for $m_{\tilde{f}} < 1$ TeV) from analysis including a Higgsino dressing diagram with right-handed matter fields in the minimal SUSY $SU(5)$ GUT.⁶⁾ Extra particles such as X and Y gauge bosons, colored Higgs bosons and their superpartners are expected to acquire heavy masses of $O(M_U)$ after the breakdown of grand unified symmetry. Here M_U is the grand unification scale or the grand unified symmetry breaking scale. The constraints contradict the relation $M_{H_C} = O(M_U)$ if M_U is identified with the gauge-coupling unification scale M_X . From this observation, we conjecture that *the proton stability can be made compatible with the grand unification if M_X is not directly related to M_U and the order of M_U is larger than that of M_X* . This might lead to the idea that gauge couplings agree accidentally at M_X .

In this paper, we propose a new grand unification scenario for ensuring proton stability on the basis of the above conjecture and the standpoint that *the scale M_X has a physical meaning beyond the fact that the agreement of gauge couplings*

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^{**)} The gauge-coupling unification based on SUSY was predicted in Ref. 3).

occurs accidentally there. We discuss a Lifshitz type gauge theory as a candidate for realizing our scenario.

This paper is organized as follows. In the next section, we elaborate our scenario. In §3, we discuss a candidate for realizing our scenario. In §4, we present conclusions and a discussion. In Appendix A, we study radiative corrections using a Lifshitz type Abelian gauge theory.

§2. Our scenario

The proton stability can deteriorate through contributions from the operators of dimensionality five including the colored Higgsinos in the minimal SUSY $SU(5)$ GUT. Stronger constraints on the colored Higgs mass M_{HC} and the sfermion mass $m_{\tilde{f}}$ have been obtained (e.g., $M_{HC} > 6.5 \times 10^{16}$ GeV for $m_{\tilde{f}} < 1$ TeV) from analysis including a Higgsino dressing diagram with right-handed matter fields.^(6),*) Extra particles such as X and Y gauge bosons, colored Higgs bosons and their superpartners usually acquire heavy masses of $O(M_U)$ after the breakdown of grand unified symmetry at M_U . The root of the proton decay problem stems from the relation $M_{HC} = O(M_U)$ with the identification between the grand unified symmetry breaking scale M_U and the gauge-coupling unification scale at $M_X = 2.1 \times 10^{16}$ GeV. Conventionally, one tackles the problem by extending the model leaving the identification untouched.

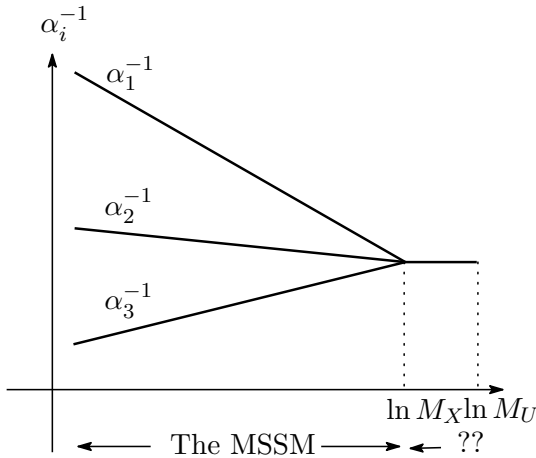


Fig. 1. Evolution of the three gauge couplings.

the proton decay processes are sufficiently suppressed. After the breakdown of grand unified symmetry, the running of standard model (SM) gauge couplings g_i , in general, differs. In this case, it is necessary to explain why the g_i meet again at M_X . It might occur owing to a decoupling of extra particles. In this paper, we explore a more exotic scenario that the g_i run (or stand) together from M_U to M_X resulting

We explore a new possibility of constructing a problem-free model. The underlying idea of our scenario is that *the gauge-coupling unification at M_X produces misleading information that the gauge symmetries are also unified at M_X . The proton stability can be made compatible with the grand unification if the grand unified symmetry breaking scale is not M_X but larger than M_X . The scale M_X has a physical meaning beyond the fact that the agreement of gauge couplings occurs accidentally there.* The outline of our scenario is as follows. A GUT describes physics above M_U , which is larger than M_X , in which

*) In Ref. 7), an intriguing possibility is pointed out that there are some uncertainties in estimating the operators of dimensionality five and that even the minimal SUSY $SU(5)$ GUT survives.

from a specific dynamics.^{*)} Then the structure of the theory changes around M_X as a result of a phase transition irrelevant to the grand unification and the different running of g_i starts there. A typical running of gauge couplings is depicted in Fig. 1. The α_i ($i = 1, 2, 3$) are structure constants constructed from g_i . Through two types of phase transitions, the MSSM will be derived as follows:

$$\int dt d^D \mathbf{x} \mathcal{L}_U \xrightarrow{M_U} \int dt d^{\tilde{D}} \mathbf{x} \mathcal{L}_I \xrightarrow{M_X} \int d^A x \mathcal{L}_{\text{MSSM}}, \quad (2.1)$$

where \mathcal{L}_U , \mathcal{L}_I and $\mathcal{L}_{\text{MSSM}}$ are the Lagrangian density of the GUT, an intermediate theory and the MSSM, and we have the possibility that the structure of space-time is also changed.

For our scenario, it is natural to ask the following questions:

1. What type of GUT describes the physics above M_U ?
2. How is the grand unified symmetry broken down to a smaller one at M_U ?
3. What type of theory describes the physics at the region between M_U and M_X ?
4. What type of dynamics makes g_i run (or stand) together from M_U to M_X ?
5. What type of phase transition occurs to derive the MSSM around M_X ?

Unfortunately, we presently have no definite answers to these questions, although we provide some conjectures. Let us look for possible answers by taking the fourth question as a clue. The simplest possibility is that no gauge couplings (almost) run between M_U and M_X , i.e., the beta functions of g_i (almost) vanish. We expect that this can stem from specific super-renormalizable interactions. Then a phase transition occurs and a theory with super-renormalizable interactions changes into one with renormalizable interactions. We consider this possibility, which is inspired by models with a Lifshitz type fixed point.^{9),10)} For the first and third questions, we expect that the theory has a Lifshitz type fixed point with $z > 1$ above M_X , a grand unified symmetry above M_U and the SM symmetry below M_U , and it changes into a renormalizable theory with $z = 1$ similarly to the MSSM around M_X . In this case, the Poincaré invariance and SUSY are regarded as accidental symmetries that emerge in the lower-energy scale less than M_X or in the infrared (IR) region.^{**)}

§3. A candidate

We expect that a specific gauge theory that possesses a fixed point with anisotropic scaling characterized by a dynamical critical exponent $z > 1$ will realize our scenario, but more careful consideration is needed to construct a complete theory. Here, we present a candidate gauge theory and discuss its features.

^{*)} The idea that the proton decay rate is suppressed by an increase of the unification scale has been proposed with interesting examples on the basis of the existence of extra vectorlike split multiples.⁸⁾ In particular, the pattern of running gauge couplings in “fake unification” is the same as that in our scenario, but the causes are different, i.e., the running is closely related to the existence of extra particles in the scenario in Ref. 8) and to a specific dynamics in our scenario.

^{**)} In Ref. 11), properties and renormalizability for quantum field theories with Lorentz symmetry breaking terms have been studied intensively on the basis of “weighted power counting”. Furthermore, extensions of the SM have been proposed for this framework.¹²⁾

3.1. Preparation

Space-time is assumed to be factorized into a product of D -dimensional Euclidean space \mathbf{R}^D and time \mathbf{R} , whose coordinates are denoted by x^i ($i = 1, \dots, D$) (or \mathbf{x}) and t , respectively. In this paper, we mostly study the case with $D = 3$. The dimensions of x^i and t are defined as

$$[x^i] = -1, \quad [t] = -z, \quad (3.1)$$

where z is the dynamical critical exponent, which characterizes anisotropic scaling $x^i \rightarrow bx^i$ and $t \rightarrow b^z t$ at the fixed point. The system does not possess the relativistic invariance for $z \neq 1$ but it possesses spatial rotational invariance and translational invariance. The Lorentz invariance is expected to emerge after the transition from $z \neq 1$ to $z = 1$.

Let us first consider a simple model on $\mathbf{R}^3 \times \mathbf{R}$ with a complex scalar field ϕ and two types of spinor fields φ and η described by the action

$$S_0 = \int dt d^3 \mathbf{x} \left[\left| \frac{\partial \phi}{\partial t} \right|^2 - \frac{1}{\kappa^2} \left| \frac{\partial^2 \phi}{\partial x^i \partial x^i} \right|^2 + \varphi^\dagger i \frac{\partial}{\partial t} \varphi + \eta^\dagger i \frac{\partial}{\partial t} \eta + \frac{1}{\xi^2} \varphi^\dagger \frac{\partial^2}{\partial x^i \partial x^i} \eta + \frac{1}{\xi^2} \eta^\dagger \frac{\partial^2}{\partial x^i \partial x^i} \varphi \right]. \quad (3.2)$$

Both φ and η are 2-component spinors defined on \mathbf{R}^3 and they transform as

$$\varphi(x) \rightarrow \varphi' \rightarrow \varphi'(x') = S(O)\varphi(x), \quad \eta(x) \rightarrow \eta' \rightarrow \eta'(x') = S(O)\eta(x), \quad (3.3)$$

$$S(O) \equiv e^{-\frac{i}{4}\omega_{ij}\sigma^{ij}}, \quad \sigma^{ij} \equiv \frac{i}{2}(\sigma^i\sigma^j - \sigma^j\sigma^i), \quad (3.4)$$

under the 3-dimensional rotation $x^i \rightarrow x'^i = O_j^i x^j$. Here, the σ^i are Pauli matrices, the ω_{ij} are parameters related to the rotation angles θ^i with $\omega_{ij} = -\varepsilon_{ijk}\theta^k$ and O_j^i is the 3×3 orthogonal matrix given by $O_j^i = (e^\omega)_j^i$. $S(O)$ satisfies the following relations:

$$S^\dagger(O)\sigma^i S(O) = O_j^i \sigma^j, \quad S^\dagger(O)S(O) = I, \quad (3.5)$$

where I is the 2×2 unit matrix. The terms such as $\varphi^\dagger \partial_i \partial_i \varphi$ and $\eta^\dagger \partial_i \partial_i \eta$ are also invariant under the spatial rotation, but we assume that they are absent. Then (3.2) is rewritten as

$$S_0 = \int dt d^3 \mathbf{x} \left[\left| \frac{\partial \phi}{\partial t} \right|^2 - \frac{1}{\kappa^2} \left| \frac{\partial^2 \phi}{\partial x_i \partial x_i} \right|^2 + \bar{\psi} i \gamma^0 \frac{\partial}{\partial t} \psi + \frac{1}{\xi^2} \bar{\psi} \frac{\partial^2}{\partial x_i \partial x_i} \psi \right], \quad (3.6)$$

where we define a 4-component spinor field as $\psi \equiv (\varphi, \eta)^t$, $\bar{\psi} \equiv \psi^\dagger \gamma^0$. Here, γ^0 corresponds to the time component of the gamma matrices and is given by

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}. \quad (3.7)$$

The engineering dimensions of fields (ϕ, ψ) and couplings (κ^2, ξ^2) are given by

$$[\phi] = \frac{3-z}{2}, \quad [\psi] = \frac{3}{2}, \quad [\kappa^2] = 4-2z, \quad [\xi^2] = 2-z. \quad (3.8)$$

The system has a free-field fixed point with $z = 2$.

Next let us adopt the gauge principle. The gauge fields (A_t, A_i) are introduced and the derivatives $(\partial_t \equiv \partial/\partial t, \partial_i \equiv \partial/\partial x^i)$ are replaced with the covariant derivatives (D_t, D_i) using

$$\partial_t \Rightarrow D_t \equiv \partial_t + igA_t, \quad \partial_i \Rightarrow D_i \equiv \partial_i + igA_i, \quad (3.9)$$

where g is a gauge coupling. The engineering dimensions of A_t and A_i are

$$[A_t] = z - [g], \quad [A_i] = 1 - [g]. \quad (3.10)$$

Gauge field strengths (F_{ti}, F_{ij}) are constructed from the commutators of covariant derivatives as

$$[D_t, D_i] = igF_{ti}, \quad [D_i, D_j] = igF_{ij}. \quad (3.11)$$

If the gauge fields are dynamical, the kinetic term of the gauge fields should be added. In this way, we obtain the following gauge-invariant action:

$$S = \int dt d^3 \mathbf{x} \left[\text{tr}(F_{ti})^2 - \frac{1}{2\lambda^2} \text{tr}(D_k F_{ij})^2 + |D_t \phi|^2 - \frac{1}{\kappa^2} |D_i D_i \phi|^2 + \bar{\psi} i \gamma^0 D_t \psi + \frac{1}{\xi^2} \bar{\psi} D_i D_i \psi \right], \quad (3.12)$$

where tr represents the trace of the gauge generators. If the differential operator $\partial_i \partial_i$ for φ and η in (3.2) is regarded as $(\sigma^i \partial_i)^2$, the last term in (3.12) is replaced by $-\xi^{-2} \bar{\psi} (\gamma^i D_i)^2 \psi$. Here, the γ^i correspond to the space components of gamma matrices, are given by

$$\gamma^i = \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix} \quad (3.13)$$

and satisfy the relation $\gamma^i \gamma^j + \gamma^j \gamma^i = -2\delta^{ij}$. We add $\text{tr}(D_k F_{ij})^2$ as the ‘potential term’ for A_i to the action S and this form can be generalized in the case with a higher spatial derivative. The ‘potential term’ comprises terms other than the kinetic term including time derivatives. If we use the detailed balance condition, a term proportional to $\text{tr}(D_i F_{ik} D_j F_{jk})$ is introduced. Here, the detailed balance condition means that the action (S_{D+1}) in $D+1$ dimensions is constructed from the action (S_D) in D dimensions through a particular procedure using functional derivatives. For example, in a model with a real scalar field Φ , the action is constructed as

$$S_{D+1} = \int dt d^D \mathbf{x} \left[\frac{1}{2} \left(\frac{\partial \Phi}{\partial t} + \frac{1}{2\kappa} \frac{\delta S_D}{\delta \Phi} \right)^2 \right], \quad (3.14)$$

up to the total derivative term. The dimension of $[\text{tr}(F_{ti})^2]$ is given by $3 + z$. Then the dimensions of (A_t, A_i) and (g, λ^2) are

$$[A_t] = \frac{1+z}{2}, \quad [A_i] = \frac{3-z}{2}, \quad [g] = \frac{z-1}{2}, \quad [\lambda^2] = 4 - 2z. \quad (3.15)$$

The effective field theory, in general, contains operators whose dimensions are equal to or less than those in S , which are not forbidden by symmetry. We add the following relevant terms directly to S :

$$\Delta S = \int dt d^3 \mathbf{x} \left[-\frac{\tilde{c}_A^{2z-2}}{2} \text{tr}(F_{ij})^2 - \tilde{c}_\phi^{2z-2} |D_i \phi|^2 + \tilde{c}_\psi^{z-1} \bar{\psi} i \gamma^i D_i \psi + \dots \right], \quad (3.16)$$

where \tilde{c}_A , \tilde{c}_ϕ and \tilde{c}_ψ are parameters whose mass dimension is one and the ellipsis stands for terms related to other operators such as the Yukawa interactions and the self-interactions of scalar fields. The third term in (3.16) is written in terms of 2-component spinors φ and η as

$$\tilde{c}_\psi^{z-1} \bar{\psi} i \gamma^i D_i \psi = \tilde{c}_\psi^{z-1} \left(\varphi^\dagger i \sigma^i D_i \varphi - \eta^\dagger i \sigma^i D_i \eta \right). \quad (3.17)$$

It is straightforward to construct a model with a complex scalar field ϕ , a $2^{[(D+1)/2]}$ -component spinor field ψ and gauge fields (A_t, A_i) on $\mathbf{R}^D \times \mathbf{R}$. The superficial degree of divergence (D_s) is defined by

$$D_s \equiv D + z - \frac{D-z}{2} B - \left(\frac{D+z}{2} - 1 \right) B_t - \frac{D}{2} F - \sum_k [g^k] n_k - \dots, \quad (3.18)$$

where B , B_t , F and n_k are the numbers of external lines for ϕ and A_i , external lines for A_t , external lines for ψ and vertices including the k th power of g , respectively. The ellipsis represents contributions from other couplings.

In the case with $z = 2$ and $D = 3$, the dimensions of fields and couplings become

$$[\phi] = \frac{1}{2}, \quad [\psi] = \frac{3}{2}, \quad [A_t] = \frac{3}{2}, \quad [A_i] = \frac{1}{2}, \quad (3.19)$$

$$[g] = \frac{1}{2}, \quad [\kappa^2] = [\xi^2] = [\lambda^2] = 0, \quad (3.20)$$

and D_s is given by

$$D_s = 5 - \frac{1}{2} B - \frac{3}{2} B_t - \frac{3}{2} F - \frac{1}{2} \sum_k k n_k. \quad (3.21)$$

The theory is super-renormalizable by power counting and infinities appear for a finite number of diagrams if they exist. In Appendix A, we study radiative corrections at the one-loop level using an Abelian gauge theory and find that radiative corrections for gauge coupling do not contain any infinities owing to the gauge invariance. We expect that the same property holds for the Lifshitz type extension of the MSSM and SUSY GUT.

3.2. A chiral model

On the construction of a GUT or an intermediate theory, a simple group or the SM gauge group should be chosen as a gauge group with suitable particle contents (including chiral fermions). Here, we do not specify the gauge group or particle contents for simplicity. We discuss a model with $z = 3$ on the space-time $\mathbf{R}^3 \times \mathbf{R}$, including chiral fermions with a higher spatial derivative term. A candidate is given by the action

$$\begin{aligned}
 S_{z=3} &= S + \Delta S , \\
 S &= \int dt d^3 \mathbf{x} \left[\text{tr}(F_{ti})^2 - \frac{1}{2\lambda^2} \text{tr}(D_k D_l F_{ij})^2 + \sum_f \left(\varphi_f^\dagger i D_t \varphi_f - \frac{1}{\xi^2} \varphi_f^\dagger \mathbf{D}^2 i \sigma^i D_i \varphi_f \right) \right. \\
 &\quad \left. + \sum_h \left(|D_t \phi_h|^2 - \frac{1}{\kappa^2} |\mathbf{D}^2 D_i \phi_h|^2 \right) \right] , \\
 \Delta S &= \int dt d^3 \mathbf{x} \left[-\frac{\tilde{c}_A^4}{2} \text{tr}(F_{ij})^2 + \sum_f \tilde{c}_f^2 \varphi_f^\dagger i \sigma^i D_i \varphi_f - \sum_h \tilde{c}_h^4 |D_i \phi_h|^2 + \dots \right] , \quad (3.22)
 \end{aligned}$$

where φ_f are 2-component spinor fields, ϕ_h are scalar fields, $\mathbf{D}^2 = D_i D_i$ and the ellipsis stands for terms related to other operators such as the Yukawa interactions and the self-interactions of scalar fields. We can, in general, add various types of gauge-invariant operators whose dimension is less than or equal to 6. Here, we take simple operators in $S_{z=3}$. For example, we assume that operators such as $\varphi_f^\dagger \varphi_f$ and $\varphi_f^\dagger \mathbf{D}^2 \varphi_f$ are absent and do not appear through radiative corrections. If $\varphi_f^\dagger \varphi_f$ exists, the Lorentz invariance does not appear in the IR region. We expect that the above type of action without specific operators is derived from a more fundamental theory.

The engineering dimensions of the fields and parameters are given by

$$[A_t] = 2 , \quad [A_i] = 0 , \quad [\varphi_f] = \frac{3}{2} , \quad [\phi_h] = 0 , \quad (3.23)$$

$$[g] = 1 , \quad [\lambda^2] = [\xi^2] = [\kappa^2] = 0 , \quad [\tilde{c}_A] = [\tilde{c}_f] = [\tilde{c}_h] = 1 \quad (3.24)$$

and D_s is given by

$$D_s = 6 - 2B_t - \frac{3}{2}F - \sum_k k n_k . \quad (3.25)$$

This theory is also super-renormalizable by power counting. It is important to study the ultraviolet (UV) behavior of our model more carefully.

From a naive dimensional analysis, ΔS dominates ‘potential terms’ in S at the energy scale below \tilde{c}_A , \tilde{c}_f and \tilde{c}_h , and we expect that relativistic invariance appears there and that the transition occurs from the theory with $z = 3$ to that with $z = 1$. To become relativistic below M_X , fine tuning among parameters is required such that $\tilde{c}_A = \tilde{c}_f = \tilde{c}_h \equiv M_\ell$ to a high accuracy and M_ℓ is one or two orders larger than M_X . We will discuss constraints on these parameters obtained from experiments in

the next subsection. The relation $\tilde{c}_A = \tilde{c}_f = \tilde{c}_h \equiv M_\ell$ can be realized in terms of renormalization group (RG) invariants.^{*)} Then $M_\ell^2 t$ is regarded as an ordinary time variable x_0 in the relativity. Hence, we expect that each term in ΔS has a common origin that \tilde{c}_A , \tilde{c}_f and \tilde{c}_h take a common value at the beginning and that they do not receive radiative corrections. If the fine-tuning relation is maintained for all particles and the theory has suitable particle contents (gauge bosons, chiral fermions, Higgs bosons and their superpartners) with suitable gauge quantum numbers, the MSSM can be derived as the theory below M_X and the running of gauge couplings in the IR region is verified.

3.3. Constraints from experiments

In the Lifshitz type extension of the MSSM with $z > 1$, the dispersion relation for free fields is given by

$$E^2 = \mathbf{p}^2 c_k^2 + m_k^2 c_k^4 + \frac{\zeta_k}{M_\ell^{2z-2}} \mathbf{p}^{2z} c_k^{4-2z}, \quad (3.26)$$

where $\mathbf{p}^{2z} = (\mathbf{p}^2)^z$, $c_k = c(\tilde{c}_k/M_\ell)^{z-1}$ (c is the speed of light) and ζ_k are dimensionless parameters and every particle (except gauge bosons related to unbroken gauge symmetries) acquires mass m_k after the SUSY and/or electroweak symmetry breaking. Here, c_k is an ‘own velocity’ (the maximal attainable velocity if $\zeta_k > 0$) for each particle labeled by k .

Let us discuss constraints on parameters obtained from experimental data. Lorentz violating dispersion relations such as (3.26) are tested by the photon emission from a high-energy particle.¹⁵⁾ A process such as $C \rightarrow C + \gamma$ is forbidden (at tree level) by the energy-momentum conservation in the case of Lorentz invariant dispersion relations such as $E^2 = \mathbf{p}^2 c^2 + m_C^2 c^4$ for a massive charged particle C and $E^2 = \mathbf{p}^2 c^2$ for a photon γ . It, however, can occur in the presence of Lorentz symmetry breaking terms. We consider the case with the following dispersion relations:

$$E^2 = \mathbf{p}^2 c_C^2 + m_C^2 c_C^4 + \frac{\zeta_C}{M_\ell^{n-2}} \mathbf{p}^n c_C^{4-n}, \quad (3.27)$$

$$E^2 = \mathbf{p}^2 c_\gamma^2 + \frac{\zeta_\gamma}{M_\ell^{n-2}} \mathbf{p}^n c_\gamma^{4-n}, \quad (3.28)$$

where $n = 2z$. Using the kinematics, we derive the following relation:

$$\begin{aligned} \frac{\zeta_\gamma}{M_\ell^{n-2}} \mathbf{p}_\gamma^n c_\gamma^{4-n} &= (\mathbf{p}^2 + \mathbf{p}'^2) (c_C^2 - c_\gamma^2) \\ &+ 2 (m_C^2 c_C^4 + \mathbf{p} \cdot \mathbf{p}' c_\gamma^2 - EE') + \frac{\zeta_C}{M_\ell^{n-2}} (\mathbf{p}^n + \mathbf{p}'^n) c_C^{4-n}, \end{aligned} \quad (3.29)$$

^{*)} There has been a proposal that the Lorentz invariance appears at an attractive IR fixed point.¹³⁾ In Ref. 14), the RG evolution of \tilde{c}_h has been studied for the Lifshitz type scalar field theory with $z = 2$ and $D = 4$ ($D = 10$), and it has been pointed out that severe fine-tuning in the UV region seems to be inevitable. This result can give a strong constraint on theories with extra dimensions.

where \mathbf{p}_γ is the momentum of a photon, and \mathbf{p} (\mathbf{p}') and E (E') are the momentum and energy of an incoming (outgoing) particle C . E and E' satisfy (3·27) and the corresponding relation when \mathbf{p} is replaced by \mathbf{p}' , respectively. The stability of C at a high momentum \mathbf{p}_h means that (3·29) does not hold at and below \mathbf{p}_h , leading to the following constraints:

$$\frac{|c_C^2 - c_\gamma^2|}{c_C^2} \lesssim \frac{m_C^2 c_C^2}{\mathbf{p}_h^2}, \quad \zeta_k \lesssim \frac{m_C^2 M_\ell^{n-2}}{\mathbf{p}_h^n} c_k^n, \quad (3\cdot30)$$

where k is C or γ . We use the relation $c_C^2 = c_\gamma^2$ up to the very small value expected from the first constraint to derive the second constraint. The most stringent constraints originate from the case that C is a proton (p^+ *) and are given by**)

$$\frac{|c_p^2 - c_\gamma^2|}{c_p^2} \lesssim \frac{m_p^2 c_p^2}{\mathbf{p}_h^2} = O(10^{-22}), \quad \zeta_k \lesssim \frac{m_p^2 M_\ell^{n-2}}{\mathbf{p}_h^n} c_k^n = O(10^{6n-34}), \quad (3\cdot31)$$

where we use $\mathbf{p}_h = 10^{20}$ eV from Akeno Giant Air Shower Array (AGASA) data¹⁷⁾ and $m_p = 938$ MeV. We take $M_\ell = 10^{17}$ GeV. The first constraint requires that $\tilde{c}_p = \tilde{c}_\gamma$ to a high accuracy of order $O(10^{-22} M_\ell)$. From the second constraint, we find that the magnitude of ζ_k becomes $O(1)$ if $n \geq 6$. In the case that C is an electron (e^-), the constraints are given by

$$\frac{|c_e^2 - c_\gamma^2|}{c_e^2} \lesssim \frac{m_e^2 c_e^2}{\mathbf{p}_h^2} = O(10^{-16}), \quad \zeta_k \lesssim \frac{m_e^2 M_\ell^{n-2}}{\mathbf{p}_h^n} c_k^n = O(10^{12n-40}/2^{2-n}), \quad (3\cdot32)$$

where we use $\mathbf{p}_h = 50$ TeV from observations of high-momentum gamma rays originating from the Crab nebula¹⁸⁾ and $m_e = 0.51$ MeV. We take $M_\ell = 10^{17}$ GeV. The first constraint requires that $\tilde{c}_e = \tilde{c}_\gamma$ to a high accuracy of order $O(10^{-16} M_\ell)$. From the second constraint, we find that the magnitude of ζ_k becomes $O(1)$ if $n \geq 4$.

If the magnitude of the Lorentz symmetry breaking term is within the range of those observed in future for a photon with an ultrahigh momentum, we can conclude that *light led us to the concept of special relativity, but (ironically) it also might teach us to violate the relativistic invariance, leading to a new physics beyond the SM or the MSSM.*

§4. Conclusions and discussion

We have proposed a new grand unification scenario for ensuring proton stability on the basis of the conjecture that *the proton stability can be made compatible with the grand unification if M_X is not related to M_U* and the standpoint that *the scale M_X has a physical meaning beyond the fact that the agreement of gauge couplings occurs accidentally there.* In our scenario, M_U is assumed to be larger than M_X to suppress the proton decay sufficiently so that gauge couplings agree from M_U to

*) Strictly speaking, a proton does not belong to the SM particles as an elementary particle, and hence ζ_p would be zero in our Lifshitz type model.

**) Various constraints on maximal attainable velocities for particles were derived in Ref. 16).

M_X resulting from a specific dynamics. The candidate for realizing our scenario is a Lifshitz type gauge theory with $z > 1$, which possesses specific super-renormalizable interactions. The structure of the theory changes around M_X after a phase transition irrelevant to the grand unification, and the different running of g_i starts there. The transition occurs from the theory with $z > 1$ to that with $z = 1$, and the ordinary renormalizable MSSM is expected to be maintained with the emergence of Poincaré invariance and SUSY below M_X .

There are open questions regarding our scenario and/or model. The most serious problem is a fine-tuning problem related to the speed of light. The relativistic invariance requires fine tuning among parameters such as $\tilde{c}_A = \tilde{c}_f = \tilde{c}_h \equiv M_\ell$ to a high accuracy for all particles that survive in our low-energy world. This originates from the standpoint that the Lorentz invariance emerges as an accidental symmetry. The relation $\tilde{c}_A = \tilde{c}_f = \tilde{c}_h \equiv M_\ell$ can be realized in terms of RG invariants. Then the RG invariance (or no running of parameters) would become one of conditions in selecting a realistic model. We need to study radiative corrections or RG behavior in the space of all the couplings in our model, bearing the fine-tuning problem in mind. Furthermore, M_ℓ might be a fundamental constant in a fundamental theory, e.g., the string scale in string theory. In fact, it is known that the string scale is about 20 times larger than M_X in the heterotic string theory. Alternatively, we might need to reconsider the relation between the kinematics of each particle and the structure of space-time. Another notorious problem is the triplet-doublet Higgs mass splitting problem. There is a possibility that mass splitting can be realized upon the orbifold breaking¹⁹⁾ if a higher-dimensional gauge theory is retained above M_U . It will be interesting to study this problem in our framework. It is also important to explore the phenomenological implications of the violation of Lorentz invariance at a high-energy scale, considering the phenomenological aspects of Hořava-Lifshitz gravity.*)

Even if our present model does not work, the study of elementary particle physics using Lifshitz type quantum field theories is attractive, and it is worth exploring high-energy theories on the basis of our proposal. In any event, we would like to believe that nature uses the idea of grand unification to effectively organize physical laws.**), ***)

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*) Recently, various aspects of Hořava-Lifshitz gravity have been intensively investigated from its renormalizability to its cosmological implications.²⁰⁾

**) Recently, the unconventional idea that grand unification is realized in an unphysical world has been proposed.²¹⁾

***) On the other hand, there is an interesting proposal that the merging of gauge couplings can originate from not the grand unification but the property of an underlying fermionic vacuum and that there is no possibility of proton decay.²²⁾

Appendix A

— Radiative Corrections in Lifshitz Type Abelian Gauge Theory —

We study radiative corrections at the one-loop level using a Lifshitz type Abelian gauge theory with $z = 2$ on the space-time $\mathbf{R}^3 \times \mathbf{R}$. The theory has a $U(1)$ gauge field (A_t, A_i) and a fermion ψ , which is a 4-component spinor (defined on \mathbf{R}^3) and has a unit $U(1)$ charge e . The action is given by

$$S = \int dt d^3 \mathbf{x} \left[\frac{1}{2} (F_{ti})^2 - \frac{1}{4} (\partial_k F_{ij})^2 + \bar{\psi} i \gamma^0 D_t \psi + \bar{\psi} D_i D_i \psi - \frac{\tilde{c}_A^2}{4} (F_{ij})^2 + \tilde{c}_\psi \bar{\psi} i \gamma^i D_i \psi - \tilde{m}^2 \bar{\psi} \psi \right], \quad (\text{A}\cdot 1)$$

where $F_{ti} = \partial_t A_i - \partial_i A_t$, $F_{ij} = \partial_i A_j - \partial_j A_i$, $D_t \equiv \partial_t + ieA_t$ and $D_i \equiv \partial_i + ieA_i$. The engineering dimensions of the fields and parameters are given by

$$[A_t] = \frac{3}{2}, \quad [A_i] = \frac{1}{2}, \quad [\psi] = \frac{3}{2}, \quad [e] = \frac{1}{2}, \quad [\tilde{c}_A] = [\tilde{c}_\psi] = [\tilde{m}] = 1. \quad (\text{A}\cdot 2)$$

The Feynman rules are given by

$$\frac{i}{\gamma^0 E_p - \mathbf{p}^2 - \tilde{c}_\psi \boldsymbol{\gamma} \cdot \mathbf{p} - \tilde{m}^2 + i\varepsilon} \quad \text{for the propagator of } \psi, \quad (\text{A}\cdot 3)$$

$$\frac{-i(\mathbf{p}^2 + \tilde{c}_A^2)}{E_p^2 - \mathbf{p}^4 - \tilde{c}_A^2 \mathbf{p}^2 + i\varepsilon} \quad \text{for the propagator of } A^t, \quad (\text{A}\cdot 4)$$

$$\frac{i}{E_p^2 - \mathbf{p}^4 - \tilde{c}_A^2 \mathbf{p}^2 + i\varepsilon} \quad \text{for the propagator of } A^i, \quad (\text{A}\cdot 5)$$

$$-ie\gamma^0 \quad \text{for the vertex among } \bar{\psi}, \psi \text{ and } A^t, \quad (\text{A}\cdot 6)$$

$$-ie(p_i + p'_i + \tilde{c}_\psi \gamma_i) \quad \text{for the vertex among } \bar{\psi}, \psi \text{ and } A^i, \quad (\text{A}\cdot 7)$$

$$-2ie^2 \delta_{ij} \quad \text{for the vertex among } \bar{\psi}, \psi, A^i \text{ and } A^j, \quad (\text{A}\cdot 8)$$

where $\mathbf{p}^4 \equiv (\mathbf{p}^2)^2$. We take $\partial_t A_t + (\nabla^2 - \tilde{c}_A^2) \partial_i A_i = 0$ as the gauge-fixing condition to derive the propagators of A_t and A_i .

Now let us study radiative corrections at the one-loop level. Here, we do not consider the terms $\frac{\tilde{c}_A^2}{4} (F_{ij})^2$ and $\tilde{c}_\psi \bar{\psi} i \gamma^i D_i \psi$ for simplicity.*) The vacuum polarizations at the one-loop level are given by

$$i\Pi_{tt}(q) = -e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3 \mathbf{p}}{(2\pi)^4} \text{Tr} \left(\gamma^0 \frac{1}{\gamma^0 E_p - \mathbf{p}^2 - \tilde{m}^2} \cdot \gamma^0 \frac{1}{\gamma^0 (E_p - E_q) - (\mathbf{p} - \mathbf{q})^2 - \tilde{m}^2} \right), \quad (\text{A}\cdot 9)$$

$$i\Pi_{it}(q) = -e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3 \mathbf{p}}{(2\pi)^4} \text{Tr} \left(\frac{2p_i - q_i}{\gamma^0 E_p - \mathbf{p}^2 - \tilde{m}^2} \right)$$

*) The qualitative features remain almost the same upon the introduction of these terms. We will report the results of analysis including these terms elsewhere.

$$\begin{aligned}
i\Pi_{ij}(q) = & -e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\mathbf{p}}{(2\pi)^4} \left[\text{Tr} \left(\frac{2p_i - q_i}{\gamma^0 E_p - \mathbf{p}^2 - \tilde{m}^2} \right. \right. \\
& \cdot \gamma^0 \frac{1}{\gamma^0(E_p - E_q) - (\mathbf{p} - \mathbf{q})^2 - \tilde{m}^2} \Big) , \quad (\text{A}\cdot 10) \\
& \cdot \frac{2p_j - q_j}{\gamma^0(E_p - E_q) - (\mathbf{p} - \mathbf{q})^2 - \tilde{m}^2} \\
& \left. + \text{Tr} \left(\frac{2\delta_{ij}}{\gamma^0 E_p - \mathbf{p}^2 - \tilde{m}^2} \right) \right] , \quad (\text{A}\cdot 11)
\end{aligned}$$

where $q = (E_q, \mathbf{q})$ (E_q and \mathbf{q} are the energy and momentum of the external gauge boson, respectively) and Tr represents the trace for the spinor indices. Linear, quadratic and tertiary divergent parts in $\Pi_{tt}(q)$, $\Pi_{it}(q)$ and $\Pi_{ij}(q)$ can be removed from the condition of gauge invariance. The physics must not be changed under the gauge transformations $A_t \rightarrow A_t + \partial_t \Lambda$ and $A_i \rightarrow A_i + \partial_i \Lambda$ ($\varepsilon_t \rightarrow \varepsilon_t + \lambda E_q$ and $\varepsilon_i \rightarrow \varepsilon_i + \lambda q_i$ in terms of polarization vectors ε_t and ε_i). This requirement leads to the relations $E_q \Pi_{tt} - \sum_i q_i \Pi_{it} = 0$ and $E_q \Pi_{it} - \sum_j q_j \Pi_{ij} = 0$. Using expressions (A·9) – (A·11), we derive the following relations:

$$\begin{aligned}
E_q \Pi_{tt} - \sum_i q_i \Pi_{it} = & ie^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\mathbf{p}}{(2\pi)^4} \text{Tr} \left(\gamma^0 \frac{1}{\gamma^0(E_p - E_q) - (\mathbf{p} - \mathbf{q})^2 - \tilde{m}^2} \right. \\
& \left. - \gamma^0 \frac{1}{\gamma^0 E_p - \mathbf{p}^2 - \tilde{m}^2} \right) , \quad (\text{A}\cdot 12)
\end{aligned}$$

$$\begin{aligned}
E_q \Pi_{it} - \sum_j q_j \Pi_{ij} = & ie^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\mathbf{p}}{(2\pi)^4} \text{Tr} \left(\frac{2p_i - q_i}{\gamma^0(E_p - E_q) - (\mathbf{p} - \mathbf{q})^2 - \tilde{m}^2} \right. \\
& \left. - \frac{2p_i + q_i}{\gamma^0 E_p - \mathbf{p}^2 - \tilde{m}^2} \right) . \quad (\text{A}\cdot 13)
\end{aligned}$$

If the integrals were finite, we could make them zero after changing the variables of integration in the first terms. Actually, we can regularize $\Pi_{tt}(q)$, $\Pi_{it}(q)$ and $\Pi_{ij}(q)$ to the following forms while maintaining the gauge invariance

$$\begin{aligned}
\Pi_{tt}(q) &= \mathbf{q}^2 \Pi(q) , \quad \Pi_{it}(q) = E_q q_i \Pi(q) , \\
\Pi_{ij}(q) &= E_q^2 \delta_{ij} \Pi(q) + (\mathbf{q}^2 \delta_{ij} - q_i q_j) \tilde{\Pi}(q) , \quad (\text{A}\cdot 14)
\end{aligned}$$

where $\Pi(q)$ is a finite quantity. For example, $i\Pi(0)$ is calculated as

$$\begin{aligned}
i\Pi(0) &= -e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\mathbf{p}}{(2\pi)^4} \text{Tr} \left(\frac{1}{(\gamma^0 E_p - (\mathbf{p}^2 + \tilde{m}^2))^3} \right) \\
&= -e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\mathbf{p}}{(2\pi)^4} \text{Tr} \left(\frac{(\gamma^0 E_p + (\mathbf{p}^2 + \tilde{m}^2))^3}{(E_p^2 - (\mathbf{p}^2 + \tilde{m}^2)^2)^3} \right) \\
&= -4ie^2 \int_{-\infty}^{\infty} \frac{dE d^3\mathbf{p}}{(2\pi)^4} \frac{-3E^2(\mathbf{p}^2 + \tilde{m}^2) + (\mathbf{p}^2 + \tilde{m}^2)^3}{(E^2 + (\mathbf{p}^2 + \tilde{m}^2)^2)^3} \\
&= -4ie^2 \int_{-\infty}^{\infty} \frac{d^3\mathbf{p}}{(2\pi)^4} \left(\frac{-3}{8} \frac{\pi}{\mathbf{p}^2 + \tilde{m}^2} + \frac{3}{8} \frac{\pi}{\mathbf{p}^2 + \tilde{m}^2} \right) = 0 , \quad (\text{A}\cdot 15)
\end{aligned}$$

in which the Wick rotation is performed ($E_p \rightarrow iE$).

In the same way, we can calculate the fermion self-energy and the vertex functions. The expression for the fermion self-energy is given at the one-loop level,

$$\begin{aligned}
 -i\Sigma(q) = & e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\mathbf{p}}{(2\pi)^4} \frac{(2\mathbf{q} - \mathbf{p})^2}{E_p^2 - \mathbf{p}^4} \frac{1}{\gamma^0(E_q - E_p) - (\mathbf{q} - \mathbf{p})^2 - \tilde{m}^2} \\
 & + 6e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\mathbf{p}}{(2\pi)^4} \frac{1}{E_p^2 - \mathbf{p}^4} \\
 & + e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\mathbf{p}}{(2\pi)^4} \frac{-\mathbf{p}^2}{E_p^2 - \mathbf{p}^4} \frac{1}{\gamma^0(E_q - E_p) - (\mathbf{q} - \mathbf{p})^2 - \tilde{m}^2}, \quad (\text{A}\cdot 16)
 \end{aligned}$$

where E_q and q_i are the energy and momentum of the external fermion, respectively. The expressions for the vertex functions among $\bar{\psi}$, ψ and A^t (A^i) are given at the one-loop level,

$$\begin{aligned}
 -i\Lambda_t(q, q') = & e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\mathbf{p}}{(2\pi)^4} \gamma^0 \frac{1}{E_p^2 - \mathbf{p}^4} \frac{(2\mathbf{q} - \mathbf{p}) \cdot (2\mathbf{q}' - \mathbf{p})}{\gamma^0(E_q - E_p) - (\mathbf{q} - \mathbf{p})^2 - \tilde{m}^2} \\
 & \cdot \frac{1}{\gamma^0(E_{q'} - E_p) - (\mathbf{q}' - \mathbf{p})^2 - \tilde{m}^2} \\
 & + e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\mathbf{p}}{(2\pi)^4} \gamma^0 \frac{-\mathbf{p}^2}{E_p^2 - \mathbf{p}^4} \frac{1}{\gamma^0(E_q - E_p) - (\mathbf{q} - \mathbf{p})^2 - \tilde{m}^2} \\
 & \cdot \frac{1}{\gamma^0(E_{q'} - E_p) - (\mathbf{q} - \mathbf{p})^2 - \tilde{m}^2}, \quad (\text{A}\cdot 17)
 \end{aligned}$$

$$\begin{aligned}
 -i\Lambda_i(q, q') = & e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\mathbf{p}}{(2\pi)^4} \frac{1}{E_p^2 - \mathbf{p}^4} \frac{(2\mathbf{q} - \mathbf{p}) \cdot (2\mathbf{q}' - \mathbf{p})}{\gamma^0(E_q - E_p) - (\mathbf{q} - \mathbf{p})^2 - \tilde{m}^2} \\
 & \cdot \frac{(q + q' - 2p)_i}{\gamma^0(E_{q'} - E_p) - (\mathbf{q}' - \mathbf{p})^2 - \tilde{m}^2} \\
 & + 2e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\mathbf{p}}{(2\pi)^4} \frac{(2q - p)_i}{E_p^2 - \mathbf{p}^4} \frac{1}{\gamma^0(E_q - E_p) - (\mathbf{q} - \mathbf{p})^2 - \tilde{m}^2} \\
 & + 2e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\mathbf{p}}{(2\pi)^4} \frac{(2q' - p)_i}{E_p^2 - \mathbf{p}^4} \frac{1}{\gamma^0(E_{q'} - E_p) - (\mathbf{q}' - \mathbf{p})^2 - \tilde{m}^2} \\
 & + e^2 \int_{-\infty}^{\infty} \frac{dE_p d^3\mathbf{p}}{(2\pi)^4} \frac{-\mathbf{p}^2}{E_p^2 - \mathbf{p}^4} \frac{(q + q' - 2p)_i}{\gamma^0(E_q - E_p) - (\mathbf{q} - \mathbf{p})^2 - \tilde{m}^2} \\
 & \cdot \frac{1}{\gamma^0(E_{q'} - E_p) - (\mathbf{q}' - \mathbf{p})^2 - \tilde{m}^2}, \quad (\text{A}\cdot 18)
 \end{aligned}$$

where E_q and q_i (E'_q and q'_i) are the energy and momentum of the incoming (outgoing) fermion, respectively. We find that the above vertex functions are UV finite. Furthermore, the following identities hold:

$$\Lambda_t(q, q) = -\frac{\partial}{\partial E_q} \Sigma(q), \quad \Lambda_i(q, q) = \frac{\partial}{\partial q_i} \Sigma(q). \quad (\text{A}\cdot 19)$$

These are the counterparts of the Ward identity in QED, which guarantee that the fermion self-energy and the three-point vertex functions do not contribute the renormalization of gauge coupling by their cancellation. We find that $D_s < 0$ for diagrams including loops related to the vertex among $\bar{\psi}$, ψ , A^i and A^j and that the proper vertex functions among them are also UV finite.

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