Preparation of Standing Tree Volume Tables for Japanese Cypress in Shinshu University Forest in Tera

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"The principal variables ordinarily associated with standing tree volumes are dbh, stem length in terms of merchantable or total height, tree form or taper, species, and locality.[1]" In undertaking to prepare a volume table, usually, dbh only or dbh and tree height are used as the independent variables, since their values are taken just as observed, independent of any of the condition of the problem, and volume which is being estimated is set to the dependent variable, since its estimated value depends upon those of the other variable or variables. "Tree volume tables that are based on the single variable of dbh are commonly refered to as *local* volume tables, those that require the user to also obtain estimates of tree height and possibly form or taper are referred to as *standard* volume tables.[1]"

In this report, we tried to prepare the standing tree volume table based on two independent variables of dbh and tree height and the volume table based on the single variable of dbh for Japanese cypress (*Chamaecyparis obtusa*) in Shinshu University Forest in Tera. That is, we tried to prepare the *standard* and *local* volume tables for Japanese cypress.

Although a number of mathematical and graphical method "such as the least squares, alinement chart, and harmonized curve method[7]" have been used for preparating volume tables in the past, "the preferred approach today is by *multiple regression analysis*.[1]" "Regression equations involving several independent variables and hundreds of sample observations can be quickly and efficiently solved by use of electronic computers.[1]" So we have been able to prepare in a short period.

To Mr. SHIMAZAKI, Mr. HAYASHI and all the students of our seminary, we have been greatly indebted for gathering the nece necessary materials. And we are indebted to Miss HAYASHI for her assistance. To all of them, our most greateful acknowledgement must paid.

I Field Sample

The basic data of this report have been gathered in Shinshu University Forest in Tera, Ina City, Nagano Prefecture, in October 1968.

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For the preparation of the volume tables for Japanese cypress, sample trees were taken at random from clear fellings in the 4th Compartment. Sample trees were measured in two meters interval sections, and at the same time the dbh and the stem length of them were measured. For this report 142 sample trees have been used. These sample trees distribute as in Table 1.

Table 1. Distribution of Sample Trees

Height(m) \ DBH(cm)	10	12	14	16	18	20	22	24	26	28	30	Total
7	1											1
8												
9	1											1
10	1											1
11		1		1								2
12			1									1
13				1								1
14			1	2	1			1				5
15		1	1	3	7	2	4	4	3	1		26
16			1	4	5	7	1	1	1	1		21
17				3	14	15	4	1	2	1		40
18					3	8	9	6	4	1	1	32
19						1	4	1	2			8
20							1		1			2
21								1				1
Total	3	2	4	14	30	33	23	15	13	4	1	142

II Analysis of Data

The exact volumes of all sample trees have been obtained by computing the volume of short sections whose lengths are uniformly two meters, by the *Huber's formula*, and then adding the results together. The volume of the stump and top must, of course, be computed separately and added in. The dbh, the tree height and the volume on all sample trees are tabulated in Table 2.

§ 1 Rejecting the abnormal data

As all gathered materials are available as a rule, they should not be rejected. But we have frequently the abnormal data arising from errors in calculations and faults in measurement and others, so it is necessary to test whether such abnormal data should be rejected or not.

When the volume on ordinate is discribed on log-log section paper as dbh or tree height on abscissa, it appears in a straight line at every point with a very narrow width (Fig. 1 and 2).

Table 2. List of Sample Tree

Sample Number	DBH d(cm)	Tree Height h(m)	Volume v(m³)	X_1 logd	X_2 logh	Y logv+3
1	20.6	16.5	0.2864	1.31386	1.21748	2.45697
2	19.3	16.6	0.2795	1.28555	1.22010	2.44638
3	16.1	15.5	0.1816	1.20682	1.19033	2.25911
4	17.9	16.9	0.2139	1.25285	1.22788	2.33021
5	24.6	18.3	0.3821	1.39093	1.26245	2.58217
6	20.8	17.3	0.2735	1.31806	1.23806	2.43695
7	20.5	16.5	0.2816	1.31175	1.21748	2.44963
8	21.5	17.3	0.3096	1.33243	1.23806	2.49080
9	23.2	18.9	0.3742	1.36548	1.27646	2.57310
10	19.3	17.5	0.2414	1.28555	1.24303	2.38273
11	18.4	15.6	0.2317	1.26481	1.19312	2.36492
12	20.1	17.5	0.2826	1.30319	1.24303	2.45117
13	20.1	16.8	0.2774	1.30319	1.22530	2.44310
14	19.9	15.1	0.1894	1.29885	1.17897	2.27738
15	27.3	15.6	0.4153	1.43616	1.19312	2.61836
16	24.8	17.9	0.4014	1.39445	1.25285	2.60357
17	23.2	18.0	0.3483	1.36548	1.25527	2.54195
18	21.6	17.5	0.3371	1.33445	1.24303	2.52775
19	19.3	17.5	0.2556	1.28555	1.24303	2.40756
20	22.2	19.3	0.3978	1.34635	1.28555	2.59966
21	24.7	17.5	0.4043	1.39267	1.24303	2.60670
22	25.3	18.1	0.4579	1.40312	1.25767	2.66077
23	17.3	16.7	0.2160	1.23804	1.22271	2.33445
24	23.6	16.9	0.3410	1.37291	1.22788	2.53275
25	17.6	16.6	0.1989	1.24551	1.22010	2.29863
26	19.9	17.2	0.2605	1.29885	1.23552	2.41580
27	10.9	10.3	0.0240	1.03742	1.01283	1.38021
28	29.5	18.3	0.5378	1.46982	1.26363	2.73062
29	24.6	15.3	0.3186	1.39093	1.18469	2.50324
30	17.9	15.4	0.1495	1.25285	1.18752	2.17464
31	18.9	14.8	0.1900	1.27646	1.17026	2.27875
32	26.0	15.7	0.3723	1.41497	1.19589	2.57100
33	17.6	16.7	0.2146	1.24551	1.22271	2.33162
34	19.5	16.7	0.2381	1.29003	1.22271	2.37675
35	16.5	15.7	0.1733	1.21748	1.19589	2.23879
36	20.5	17.4	0.2836	1.31175	1.24054	2.45270
37	23.3	20.5	0.4515	1.36735	1.31175	2.64565
38	16.7	14.5	0.1233	1.22271	1.16136	2.09096
39	19.0	16.7	0.2535	1.27875	1.22271	2.40397
40	17.3	16.4	0.2462	1.23804	1.21484	2.39128
41	21.1	17.9	0.2631	1.32428	1.25285	2.42012
42	16.1	13.7	0.1272	1.20682	1.13672	2.10448

Sample Number	DBH d(cm)	Tree Height h(m)	Volume v(m³)	X_1 logd	X_2 logh	Y logv+3
43	19.3	17.6	0.2531	1.28555	1.24551	2.40329
44	22.8	17.5	0.3491	1.35793	1.24303	2.54294
45	17.2	14.5	0.1753	1.23552	1.16113	2.24378
46	21.9	18.2	0.3086	1.34044	1.26007	2.48939
47	15.8	17.2	0.1764	1.19865	1.23552	2.24649
48	17.7	15.3	0.2103	1.24797	1.18469	2.32283
49	25.3	19.5	0.5128	1.40312	1.29003	2.70994
50	16.9	16.2	0.1893	1.22788	1.20951	2.27715
51	23.8	17.5	0.3714	1.37657	1.24303	2.56984
52	13.9	15.2	0.1076	1.14301	1.18184	2.03181
53	20.5	16.9	0.2498	1.31175	1.22788	2.39759
54	9.2	9.0	0.0303	0.96378	0.95424	1.48144
55	19.7	16.0	0.2244	1.29446	1.20412	2.35102
56	25.2	18.9	0.4173	1.40140	1.27646	2.62044
57	20.0	17.2	0.2596	1.30103	1.23552	2.41430
58	19.4	16.8	0.2389	1.28780	1.22530	2.37821
59	19.0	17.2	0.2303	1.27875	1.23552	2.36229
60	20.1	16.6	0.2416	1.30319	1.22141	2.38309
61	21.6	17.8	0.2633	1.33445	1.25042	2.42045
62	20.8	17.3	0.2699	1.31806	1.23804	2.43120
63	22.0	17.9	0.3400	1.34242	1.25285	2.53147
64	18.0	16.8	0.1974	1.25527	1.22530	2.29534
65	17.6	18.4	0.2369	1.24551	1.26481	2.37456
66	22.0	18.4	0.2950	1.34242	1.26481	2.46982
67	16.2	17.3	0.1873	1.20951	1.23804	2.27253
68	17.7	17.6	0.2180	1.24797	1.24551	2.33845
69	21.0	16.7	0.2866	1.32221	1.22271	2.45727
70	16.4	12.7	0.1466	1.21484	1.10380	2.16613
71	18.8	15.1	0.1811	1.27415	1.17897	2.25791
72	12.7	14.9	0.0914	1.10380	1.17318	1.96070
73	15.7	15.2	0.1442	1.19589	1.18184	2.15896
74	17.8	17.2	0.2015	1.25042	1.23552	2.30427
75	19.0	15.7	0.2352	1.27875	1.19589	2.37143
76	22.1	18.6	0.3429	1.34439	1.26951	2.53516
77	15.6	16.2	0.1524	1.19312	1.20951	2.18298
78	19.4	17.5	0.2673	1.28780	1.24303	2.42699
79	22.2	17.3	0.3017	1.34635	1.23804	2.47957
80	17.4	16.5	0.1673	1.24054	1.21748	2.22349
81	18.1	16.7	0.2134	1.25767	1.22271	2.32919
82	14.0	14.4	0.1114	1.14612	1.15836	2.04688
83	19.0	17.1	0.2548	1.27875	1.23299	2.40619
84	21.3	17.4	0.3118	1.32837	1.24054	2.49387
85	17.0	16.8	0.1773	1.23044	1.22530	2.24870
86	17.3	14.9	0.1555	1.23804	1.17318	2.19173
87	24.3	17.9	0.4229	1.38560	1.25285	2.62623
88	17.7	16.4	0.2016	1.24797	1.21484	2.30449
89	22.5	18.1	0.3579	1.35218	1.25767	2.55376

Sample Number	DBH d(cm)	Tree Height h(m)	Volume v(m³)	X_1 logd	X_2 logh	Y logv+3
90	18.4	17.6	0.2509	1.26481	1.24551	2.39950
91	19.2	16.0	0.2473	1.28330	1.20412	2.39322
92	20.6	17.5	0.2920	1.31386	1.24303	2.46538
93	11.5	11.0	0.0584	1.06069	1.04139	1.76641
94	17.4	14.9	0.1742	1.24054	1.17318	2.24104
95	16.0	13.9	0.1414	1.20412	1.14301	2.15044
96	20.6	14.5	0.2214	1.31386	1.16136	2.34517
97	21.2	18.0	0.3309	1.32633	1.25527	2.51969
98	14.8	15.6	0.1558	1.17026	1.19312	2.19256
99	22.0	19.1	0.3598	1.34242	1.28103	2.55606
100	25.9	17.9	0.4209	1.41329	1.25285	2.62417
101	22.2	18.5	0.3386	1.34635	1.26717	2.52968
102	10.7	7.3	0.0305	1.02938	0.86332	1.48429
103	20.7	17.7	0.2775	1.31597	1.24797	2.44326
104	19.7	17.9	0.2614	1.29446	1.25285	2.41730
105	18.0	17.0	0.2103	1.25527	1.23044	2.32283
106	22.8	15.9	0.3345	1.35793	1.20139	2.52439
107	23.1	14.9	0.2796	1.36361	1.17318	2.44653
108	19.2	16.0	0.2551	1.28330	1.20412	2.40671
109	22.1	16.6	0.3133	1.34439	1.22010	2.49596
110	25.1	18.0	0.3976	1.39967	1.25527	2.59944
111	27.8	18.3	0.4364	1.44404	1.26245	2.63688
112	15.6	10.6	0.0955	1.19312	1.02530	1.98023
113	28.6	15.0	0.4780	1.45636	1.17609	2.67942
114	23.0	15.3	0.2919	1.36172	1.18469	2.46523
115	28.1	16.5	0.4338	1.44870	1.21748	2.63728
116	20.8	18.7	0.3411	1.31806	1.27184	2.53288
117	22.6	18.5	0.4010	1.35410	1.26717	2.60314
118	17.3	16.0	0.2338	1.23804	1.20412	2.36884
119	18.9	17.3	0.2386	1.27646	1.23804	2.37767
120	25.8	19.1	0.4271	1.41161	1.28103	2.63052
121	26.7	17.0	0.4063	1.42651	1.23044	2.60884
122	18.5	14.1	0.1853	1.26717	1.14921	2.26787
123	18.4	17.1	0.2475	1.26481	1.23299	2.39357
124	21.0	17.3	0.2898	1.32221	1.23804	2.46209
125	14.7	12.3	0.0938	1.16731	1.08990	1.97220
126	22.7	15.4	0.2830	1.35602	1.18752	2.45178
127	22.5	15.4	0.2775	1.35218	1.18752	2.44326
128	26.8	15.3	0.4225	1.42813	1.18469	2.62582
129	25.0	15.0	0.3123	1.39794	1.17609	2.49457
130	26.2	14.9	0.3478	1.41830	1.17318	2.54132
131	19.8	14.7	0.2230	1.29666	1.16731	2.34830
132	23.8	16.0	0.3500	1.37657	1.20412	2.54406
133	24.7	14.4	0.2511	1.39269	1.15836	2.39984
134	24.0	15.3	0.3168	1.38021	1.18469	2.50078
135	21.5	15.3	0.2838	1.33243	1.18469	2.45301
136	26.0	17.2	0.3980	1.41497	1.23552	2.59988

Sample Number	DBH d(cm)	Tree Height h(m)	Volume v(m³)	X_1 logd	X_2 logh	Y logv+3
137	21.0	16.0	0.2468	1.32221	1.20412	2.39234
138	19.1	15.5	0.2160	1.28103	1.19033	2.33445
139	15.8	15.0	0.1495	1.19865	1.17609	2.17464
140	25.4	15.2	0.3496	1.40483	1.18184	2.54357
141	25.4	18.0	0.3720	1.40483	1.25527	2.57054
142	19.8	16.0	0.2317	1.29666	1.20412	2.36492

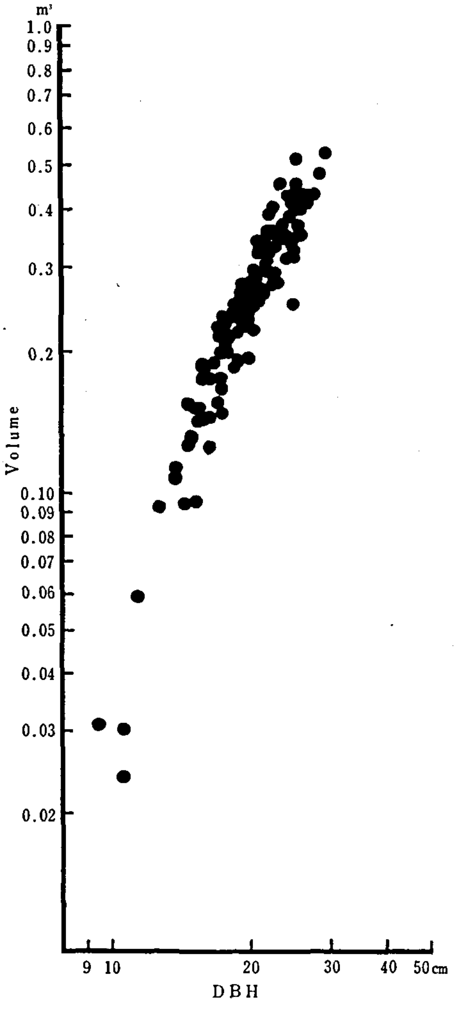

Fig. 1. The Relation between Volume and DBH

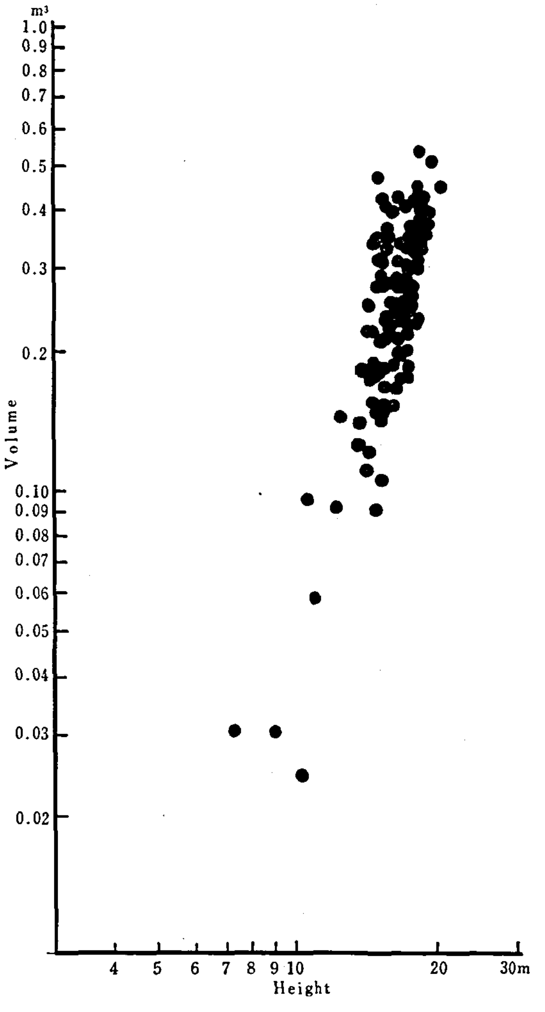

Fig. 2. The Relation between Volume and Tree Height

Accordingly, the *Yamamoto's formula* seems to be suitable and is applied here.

$$v = 10^a \ d^{b_1} \ h^{b_2} \qquad (1)$$

where v is tree volume in m, d is dbh in cm, h is tree height in m, a, b_1 and b_2 are constants.

By logarithmic transforming, formula (1) may be written

$$log\ v = a + b_1\ log\ d + b_2\ log\ h \qquad (2)$$

The formula (2) is transformed as follows:

$$Y = a + b_1\ X_1 + b_2\ X_2 \qquad (3)$$

In this paper, the region of rejection about abnormal values is determined by the variance of the observation from the value estimated by the equation (3).

In given materials, as volumes have been multipled by 1,000 for convenience, estimation equation is shown as follows:

$$\hat{Y} = -1.531921 + 1.701806\ X_1 + 1.409693\ X_2 \qquad (4)$$

Variance from regression and standard deviation of estimate:

$$\begin{aligned} S^2_{y \cdot x_1 x_2} &= \Sigma(y_i - \hat{y})^2/n - 3 \\ &= \Sigma(y_i - y)^2 - b_1\Sigma(x_{1i} - \bar{x}_1)\ (y_i - \bar{y}) \\ &\qquad - b_2\Sigma(x_{2i} - \bar{x}_2)\ (y_i - \bar{y})/n - 3 \qquad (5) \\ &= 6.542731 - 1.701806 \times 2.413863 \\ &\quad - 1.409693 \times 1.462261/139 \\ &= 0.005729 \\ S_{y \cdot x_1 x_2} &= 0.075692 \end{aligned}$$

And those which exceed as much as standard deviation of the value estimated from equation (4) are taken out of all. Therefore, the width of rejection region in this examination is taken narrower. As the result of rejection, four trees, No. 27, 30, 40 and 118, are taken out.

§ 2 Testing applicability of the *standard* volume table now in use

The standard volume table now in use for Japanese cypress in Tera University Forest is that prepared by Management Section of Forest Bureau. Here, we try to test the applicability of the *standard* volume table with the above mentioned materials on Japanese cypress in Tera forest.

When x represents actual tree volume and y value from volume table, the relation is represented as $y = b\ x$. If the actual volume agrees with that from the table, y is equal to x and every point appears on a straight line at 45 degrees to both axes. When the hypothesis $b = 1$ is set, unless this hypothesis is rejected, this table shall be considered to be applicable.

Now, in given materials

$$b = \Sigma\ x_i y_i / \Sigma x_i \qquad (6)$$

$= 12.019359/11.472656$
$= 1.047653$

Regression equation:

$$y = 1.047653\ x \tag{7}$$

Variance from regression and standard deviation of estimate:

$$S^2_{yx} = \Sigma (y_i - b\ x_i)^2/n - 1$$
$$= \Sigma y_i^2 - b\ \Sigma x_i y_i/n - 1 \tag{8}$$
$$= 12.670755 - 1.047653 \times 12.019359/137$$
$$= 0.000574$$
$$S_{yx} = 0.023958$$

For the test of applicability of the *standard* volume table, the value of t_0 is as follows:

$$t_0 = (b - 1)\sqrt{\Sigma (x_i - \bar{x})^2}/S_{yx}$$
$$= (b - 1)\sqrt{\Sigma x_i^2 - (\Sigma x_i)^2/n}/S_{yx} \tag{9}$$
$$= 2.2935$$

On looking up Table of t, we observe that for $d.f. = 120$, the value of t for 1% level is 2.617 and our t_0 is smaller than this. But the value of t for 5% level for $d.f. = 120$ is 1.980 and our t_0 is greater than this. Therefore, we know that the values of this *standard* volume table have the significant difference with 5% level to the actual volumes in Tera forest.

III Volume Table

§ 3 Preparing the *local* volume table

In Fig. 1 in Chapter II, it was shown that the relation between logarithm of volume (v m³) and logarithm of dbh (d cm) appears in a straight line. So the following formula seems to be suitable and is applied here.

$$v = 10^a\ d^b \tag{10}$$

where a and b are constants.

By logarithmic transforming, formula (10) may be written

$$\log v = a + b \log d \tag{11}$$

The formula (11) is transformed as follows :

$$Y = a + b\ X \tag{12}$$

"The *coefficient* of *regression*"[3] and "*constant* of *regression*"[3] for the *local* volume table for Japanese cypress are calculated as follows:

$$b = n\ \Sigma x_i y_i - \Sigma x_i\ \Sigma y_i/n\ \Sigma x_i^2 - (\Sigma x_i)^2 \tag{13}$$
$$= \frac{138 \times 433.041352 - 179.825840 \times 330.681710}{138 \times 235.293041 - (179.825840)^2}$$
$$= 2.213185$$

$$a = \Sigma\, y_i/n - b\,\Sigma\, x_i/n \tag{14}$$
$$= 330.681710/138 - 2.213185 \times 179.825840/138$$
$$= -0.487727$$

For convenience, y_i are added 3 to logarithm of volume in calculating process. The regression line, $Y = a + b\,X$, therefore is for this case

$$Y = -3.487727 + 2.213185\,X \tag{15}$$

Variance from regression and standard deviation of estimate:

$$S^2_{yx} = \Sigma\,(y_i - \hat{y}_i)^2/n - 2$$
$$= \Sigma\, y_i^2 - a\,\Sigma\, y_i - b\,\Sigma\, x_i y_i \tag{16}$$
$$= 797.866055 + 0.487727 \times 330.681710 - 2.213185 \times 433.041352$$
$$= 0.005499$$
$$S_{yx} = 0.074153$$

Fiducial limits of parameter β of b:

$$b \pm t_{0.05}\, S_{yx}/\sqrt{\Sigma\,(x_i - \bar{x})^2} \tag{17}$$
$$= 2.213185 \pm 1.960 \times 0.074153/\sqrt{0.964544}$$
$$= 2.213185 \pm 0.147988$$

Fiducial limits of the regression :

$$\hat{y}_0 \pm t_{0.05}\, S_{yx} \sqrt{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}} \tag{18}$$

Correction coefficient :

$$f = 10^{1.1513\sigma^2} \tag{19}$$
$$= 10^{1.1513 \times 0.005499}$$
$$= 1.0146$$

The volume and the fiducial limits for each dbh were calculated by use of the equations (15) and (18), and then the values were multipled by the correction coefficient *1.0146*. The prepared *local* volume table for standing tree of Japanese cypress is shown in Table 3.

Table 3. Local Volume Table for Japanese Cypress

DBH (cm)	Volume (m^3)	DBH (cm)	Volume (m^3)	DBH (cm)	Volume (m^3)	DBH (cm)	Volume (m^3)
4	0.0071±0.0019	12	0.0808±0.0069	20	0.2500±0.0073	28	0.5265±0.0305
6	0.0175±0.0035	14	0.1135±0.0072	22	0.3087±0.0099	30	0.6133±0.0417
8	0.0329±0.0049	16	0.1526±0.0069	24	0.3743±0.0149	32	0.7075±0.0552
10	0.0539±0.0063	18	0.1980±0.0067	26	0.4468±0.0218	34	0.8091±0.0704

§ 4 Preparing the *standard* volume table

In equation (3) in Chapter II, it was shown that an equation could be arrived at to express the average relation between volume (Y), dbh (X_1) and tree height (X_2):

$$Y = a + b_1 X_1 + b_2 X_2 \tag{3}$$

"If the criterion of "rightness" is taken as that which will make the standard deviation of the residuals, when volume is estimated from the other two variables, as small as possible, the value of a, b_1 and b_2 which will give this result can be determined by a direct mathematical process, known as the method of *linear multiple regression*".[3]

The best values for a, b_1 and b_2 in the multiple regression equation (3) can be worked out by an extension of the same process used in working out the values for the estimating equation when only one independent variable was considered.

$$b_1 = \frac{\Sigma(x_{1i}-\bar{x}_1)(y_i-\bar{y})\cdot\Sigma(x_{2i}-\bar{x}_2)^2 - \Sigma(x_{1i}-\bar{x}_1)(x_{2i}-\bar{x}_2)\cdot\Sigma(x_{2i}-\bar{x}_2)(y_i-\bar{y})}{\Sigma(x_{1i}-\bar{x}_1)^2\cdot\Sigma(x_{2i}-\bar{x}_2)^2 - [\Sigma(x_{1i}-\bar{x}_1)(x_{2i}-\bar{x}_2)]^2} \tag{20}$$

$$= 1.676339$$

$$b_2 = \frac{\Sigma(x_{1i}-\bar{x}_1)^2\cdot\Sigma(x_{2i}-\bar{x}_2)(y_i-\bar{y}) - \Sigma(x_{1i}-\bar{x}_1)(x_{2i}-\bar{x}_2)\cdot\Sigma(x_{1i}-\bar{x}_1)(y_i-\bar{y})}{\Sigma(x_{1i}-\bar{x}_1)^2\cdot\Sigma(x_{2i}-\bar{x}_2)^2 - [\Sigma(x_{1i}-\bar{x}_1)(x_{2i}-\bar{x}_2)]^2} \tag{21}$$

$$= 1.303679$$

$$a = \bar{y} - b_1 \bar{x}_1 - b_2 \bar{x}_2$$
$$= -1.368886$$

The regression line, $Y = a + b_1 X_1 + b_2 X_2$, therefore is for this case

$$Y = -4.368886 + 1.676339\ X_1 + 1.303679\ X_2 \tag{23}$$

Variance from regression and standard deviation of estimate is calculated from equation (6).

$$S^2_{y\cdot x_1x_2} = 0.001899$$
$$S_{y\cdot x_1x_2} = 0.043572$$

Fiducial limits of parameter β_1 of b_1 and β_2 of b_2:

$$b_1 \pm t_{0.05} S_{y\cdot x_1x_2}\sqrt{\frac{\Sigma(x_{2i}-\bar{x}_2)^2}{\Sigma(x_{1i}-\bar{x}_1)^2\cdot\Sigma(x_{2i}-\bar{x}_2) - [\Sigma(x_{1i}-\bar{x}_1)(x_{2i}-\bar{x}_2)]^2}} \tag{24}$$

$$= 1.676339 \pm 0.108820$$

$$b_2 \pm t_{0.05} S_{y\cdot x_1x_2}\sqrt{\frac{\Sigma(x_{1i}-\bar{x}_1)^2}{\Sigma(x_{1i}-\bar{x}_1)^2\cdot\Sigma(x_{2i}-\bar{x}_2) - [\Sigma(x_{1i}-\bar{x}_1)(x_{2i}-\bar{x}_2)]^2}} \tag{25}$$

$$= 1.303679 \pm 0.158879$$

Fiducial limits of the regression:

Table 4 Standard Volume Table for Japanese Cypress

Height(m) \ DBH(cm)	4	6	8	10	12	14	16	18
3	0.0018±0.0004	0.0036±0.0008	0.0058±0.0014	0.0085±0.0021	0.0116±0.0030	0.0150±0.0040		
4	0.0027±0.0005	0.0053±0.0010	0.0085±0.0016	0.0125±0.0025	0.0169±0.0035	0.0218±0.0047	0.0273±0.0061	
5	0.0036±0.0006	0.0070±0.0011	0.0115±0.0018	0.0166±0.0027	0.0226±0.0038	0.0292±0.0052	0.0366±0.0067	
6	0.0045±0.0007	0.0089±0.0012	0.0145±0.0019	0.0211±0.0028	0.0286±0.0040	0.0371±0.0054	0.0463±0.0071	0.0565±0.0090
7	0.0055±0.0008	0.0110±0.0014	0.0178±0.0020	0.0212±0.0024	0.0350±0.0041	0.0453±0.0054	0.0567±0.0071	0.0690±0.0092
8	0.0066±0.0010	0.0131±0.0015	0.0211±0.0021	0.0307±0.0029	0.0417±0.0039	0.0540±0.0053	0.0674±0.0070	0.0822±0.0092
9		0.0152±0.0017	0.0246±0.0022	0.0358±0.0029	0.0485±0.0038	0.0629±0.0051	0.0787±0.0066	0.0959±0.0087
10		0.0174±0.0019	0.0288±0.0024	0.0410±0.0030	0.0557±0.0037	0.0722±0.0048	0.0902±0.0062	0.1099±0.0082
11			0.0320±0.0027	0.0464±0.0030	0.0631±0.0036	0.0817±0.0043	0.1021±0.0056	0.1244±0.0073
12			0.0358±0.0030	0.0521±0.0034	0.0707±0.0036	0.0916±0.0040	0.1145±0.0049	0.1394±0.0066
13			0.0398±0.0034	0.0578±0.0037	0.0785±0.0039	0.1015±0.0038	0.1270±0.0042	0.1548±0.0055
14				0.0636±0.0044	0.0864±0.0042	0.1119±0.0040	0.1399±0.0038	0.1704±0.0043
15					0.0946±0.0050	0.1224±0.0046	0.1531±0.0039	0.1865±0.0037
16						0.1332±0.0055	0.1665±0.0049	0.2029±0.0040
17							0.1802±0.0060	0.2196±0.0052
18							0.1942±0.0080	0.2366±0.0070
19							0.2083±0.0098	0.2539±0.0095
20								0.2715±0.0122
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								

Table 4

Height(m) \ DBH(cm)	20	22	24	26	28	30	32	34
3								
4								
5								
6								
7								
8	0.0981±0.0115							
9	0.1144±0.0112	0.1342±0.0139	0.1553±0.0174					
10	0.1312±0.0105	0.1540±0.0136	0.1781±0.0168					
11	0.1485±0.0096	0.1743±0.0126	0.2017±0.0158	0.2306±0.0199				
12	0.1664±0.0088	0.1952±0.0115	0.2259±0.0151	0.2583±0.0187	0.2925±0.0235			
13	0.1847±0.0072	0.2167±0.0102	0.2507±0.0133	0.2867±0.0174	0.3247±0.0223	0.3645±0.0272		
14	0.2034±0.0060	0.2387±0.0084	0.2762±0.0119	0.3158±0.0161	0.3576±0.0203	0.4014±0.0260	0.4473±0.0316	
15	0.2226±0.0040	0.2611±0.0072	0.3022±0.0101	0.3455±0.0142	0.3912±0.0192	0.4392±0.0250	0.4894±0.0307	0.5417±0.0372
16	0.2421±0.0043	0.2841±0.0056	0.3282±0.0090	0.3759±0.0133	0.4256±0.0175	0.4778±0.0234	0.5323±0.0303	0.5892±0.0370
17	0.2620±0.0046	0.3074±0.0054	0.3557±0.0084	0.4068±0.0120	0.4606±0.0172	0.5171±0.0223	0.5762±0.0294	0.6377±0.0363
18	0.2823±0.0066	0.3312±0.0071	0.3832±0.0090	0.4383±0.0120	0.4963±0.0165	0.5571±0.0229	0.6207±0.0292	0.6870±0.0364
19	0.3029±0.0089	0.3554±0.0091	0.4112±0.0105	0.4702±0.0138	0.5324±0.0177	0.5978±0.0234	0.6660±0.0300	0.7372±0.0376
20	0.3239±0.0121	0.3800±0.0127	0.4397±0.0138	0.5028±0.0158	0.5693±0.0201	0.6391±0.0251	0.7121±0.0321	0.7882±0.0386
21	0.3451±0.0156	0.4049±0.0159	0.4685±0.0175	0.5358±0.0200	0.6067±0.0238	0.6811±0.0280	0.7589±0.0342	0.8400±0.0412
22	0.3667±0.0187	0.4302±0.0202	0.4978±0.0215	0.5693±0.0234	0.6446±0.0253	0.7236±0.0326	0.8063±0.0379	0.8924±0.0455
23		0.4559±0.0241	0.5275±0.0259	0.6033±0.0284	0.6831±0.0321	0.7668±0.0376	0.8545±0.0436	0.9457±0.0501
24			0.5576±0.0306	0.6377±0.0338	0.7220±0.0382	0.8105±0.0429	0.9032±0.0496	0.9997±0.0568
25				0.6725±0.0395	0.7615±0.0433	0.8549±0.0486	0.9525±0.0560	1.1104±0.0675
26					0.8015±0.0503	0.8997±0.0564	1.0025±0.0629	1.1160±0.0722
27						0.9451±0.0630	1.1103±0.0740	1.1216±0.0769
28						0.9910±0.0719	1.1154±0.0809	1.1272±0.0817
29							1.1206±0.0857	1.1329±0.0866
30							1.1258±0.0905	1.1388±0.0915

$$\hat{y} \pm t_{0.05}\, S_{y \cdot x_1 x_2} \times \sqrt{\frac{1}{n} + \frac{\Sigma(x_{2i}-\bar{x}_2)^2(X_1-\bar{x}_1)^2 + \Sigma(x_{1i}-\bar{x}_1)^2(X_2-\bar{x}_2)^2 - 2\Sigma(x_{1i}-\bar{x}_1)(x_{2i}-\bar{x}_2)(X_1-\bar{x}_1)(X_2-\bar{x}_2)}{\Sigma(x_{1i}-\bar{x}_1)^2 \cdot \Sigma(x_{2i}-\bar{x}_2)^2 - [\Sigma(x_{1i}-\bar{x})(x_{2i}-\bar{x}_2)]^2}} \quad (26)$$

Correction coefficient :

$$f = 10^{1.1513 \times 0.005499}$$
$$= 1.0050$$

The volume and the fiducial limits for each dbh and tree height were calculated by use of the equation (23) and (26), and then the values were multipled by the correction coefficient *1.0050.* The prepared standard volume table for standing tree of Japanese cypress is shown in Table 4.

IV Stochastical Consideration

For the prepared volume table, we tried to test the applicability. Upon the test for the two tables, it is admitted that both the tables have not any significant difference (Table 5). However, with the elements containing no errors, the error of volume estimated by means of the *standard* volume table is smaller than that by means of the *local* volume table (Table 6).

Table 5 Test of Applicability of Volume Table

Volume Table	Regression Equation	Test of b		
		b—1	S_{yx}	to
local	$y=1.007748x$	0.007748	0.039562	0.2258
standard	$y=0.995902x$	−0.004098	0.020968	0.2254
now in use	$y=1.047653x$	0.047653	0.023958	2.2935※

Table 6 Test of Uniformity of Variance from Regression

Volume Table	Variance	F_0
local	$V_1=0.001565$	$F_0=V_1/V_2=3.5568$※※
standard	$V_2=0.000440$	$F_0=V_3/V_2=1.3045$※
now in use	$V_3=0.000574$	$F_0=V_1/V_3=2.7265$※※

Even in the case of trees with the equal dbh and the equal tree height, its volumes represents various values, and the values shown in the table represent only the likelihood estimate. Usually, therefore, there is a large error in the estimation of the volume of a single tree by a volume table. Taking account of the result represented in Table 6, the *standard* table is more available than the *local* table for the estimation of the volume of a single tree by a volume table.

But the error percent of the total volume tends to fail lower with the increase of the total volume, so the *local* table will be more available than the *standard* table for the estimation of the volume of the larger forest stand, because of its easi ness in the field work.

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手良演習林のヒノキ立木材積表の調製

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要　　約

手良演習林のヒノキ立木材積表として現在使用されている林野庁計画課編の「長野地方ヒノキ・サワラ・ヒバ・コウヤマキ立木材積表」は，その適合度がかならずしも高くないので，手良演習林のヒノキに対する立木材積表の調製を試みた。

本報告では一変数材積表と二変数材積表とを調製したが，その材積式は次のようである。

$$\log v = -3.487727 + 2.213185 \log d \qquad (16)$$

$$\log v = -4.368886 + 1.676339 \log d + 1.303679 \log h \qquad (23)$$

v：材積(m^3)，　d：胸高直径(cm)，　h：樹高(m)

材積表は表3ならびに表4に示してある。これら両表の推計学的検討の結果は，次のように要約できる。

1　両表とも手良演習林のヒノキに対しては適合度が高い

2　推定値の持つ誤差は，一変数材積表の方が二変数材積表よりも大きいから，単木材積の推定に際しては，二変数材積表の方が好ましい

3　しかし林分材積の場合のように，多数の林木材積の和を求めるような時には，外業上の有利さから，むしろ一変数材積表がすすめられる

なお本報告での立木材積表は，142本の胸高直径10cm～30cm，樹高7m～21mという割合狭い範囲の資料を用いて調製されたものであるから，なお今後に修正することが必要であろう。