International Journal of Industrial Engineering, 26(6), 931-958, 2022

PRICING AND INVESTMENT STRATEGY FOR DIGITAL TECHNOLOGY IN A SUPPLY CHAIN

Nguyen Thi My Hanh[1] and Jen-Ming Chen[1,*]

[1]Institute of Industrial Management
National Central University
Taoyuan City, Taiwan, 32001, R.O.C
[*]Corresponding author's e-mail: jmchen@mgt.ncu.edu.tw

This study addresses the problem of the pricing and investment strategy for smart technology under a supply chain with one manufacturer and one retailer. The models are constructed to investigate the strategic choices of supply chain members for investing in digital/smart technology under three scenarios: the M–system, wherein only the manufacturer fully pays for the investment cost; the S–system, wherein the manufacturer and retailer share the investment cost; and the R–system, wherein the retailer fully pays for the investment cost. We formulate analytical models to determine the optimal wholesale price, retail price and investment strategy in a Stackelberg game setting. Our findings show that the S–system is the most appropriate choice for both the manufacturer and retailer. We also suggest the appropriate investment sharing ratio to achieve Pareto improvement under such an arrangement.

Keywords: Investment Sharing, Pricing, Channel Coordination, Pareto Improvement, Wholesale-Price-Only Contract.

(Received on August 18, 2022; Accepted on October 7, 2022)

1. INTRODUCTION

Every industry wants to generate more profits with lower production costs. Cost reduction is the holy grail for all manufacturing firms (Matalycki *et al*., 2010). Reducing production costs substantially improves firm performance and competitiveness, and thus it is common for firms to invest in reducing production costs (Porter, 1998). Many manufacturing firms, such as those manufacturing home appliances, automobiles, electronics and food, have implemented lean production systems to enhance process efficiency and reduce marginal costs (Netland and Ferdows, 2014). Indeed, planning to reduce production costs may influence and be influenced by the manufacturer's decisions. Cost reduction also helps the manufacturer reduce wholesale prices, which decreases the double marginalization effects in the supply chain. Consequently, it may benefit the downstream counterparts in the supply chain.

The literature has investigated many ways of reducing variable production costs. Pillai (2015) examined four factors that may help in reducing solar panel production costs: reduction in raw material cost, increasing the number of solar panels manufactured in China, technological innovation and increasing industry-level investment. In this study, we consider the aggregate effect of investment on production cost reduction. Kovács and Kot (2017) proposed an adequate facility layout to minimize production costs. Many firms have implemented continuous process improvements, such as lean operations, to reduce manufacturing costs. Buer *et al*. (2021) stated that lean manufacturing considerably improves operational performance. Tiwari *et al*. (2020) constructed a sustainable lean production model for manufacturing firms, which saved $120,142 annually because of higher production efficiency. The Tiwari *et al*. (2020) study belongs to the stream of research on lean production improvements, while ours is a study on digital lean improvement.

The next industrial revolution, Industry 4.0, also known as the new digital industrial transformation, is changing the way firms operate. Smart factories based on Industry 4.0 combine physical and cyber technologies, resulting in more complex and precise technologies. Industry 4.0 has transformed manufacturing processes, resulting in intelligent manufacturing that promises self-sufficient production via the use of machines and devices that interact with each other through digital connections (Xu *et al*., 2018; Mahmud *et al*., 2019; Castelo-Branco *et al*., 2019; Manresa *et al*., 2021). Smart technology benefits firms by reducing cycle times and costs, increasing productivity and producing higher-quality goods. Chen *et al*. (2020) proposed that smart packaging systems can help reduce food loss and wastage and reduce production costs. Ardolino *et al*. (2018) discussed the importance of adopting digital technologies for service transformation in industrial firms. They concluded that the Internet of Things (IoT) is fundamental to any service transformation. Li (2020) proposed that applying smart technologies to the production process improves operational efficiency. They found that one tobacco firm in China saved approximately $7 million through labor cost reduction and inventory utilization after upgrading its enterprise resource planning system with digital technology. In other words, the system can be collectively considered a digital lean paradigm shift. Manresa *et al*. (2021) concluded that new technology and organizational practice have a positive impact on the operational performance of manufacturing firms.

Despite the appeal of reducing production costs and adopting smart technology in manufacturing processes, the literature has not explored investment policies for smart technology in manufacturing. Therefore, this study develops a two-echelon supply chain model, with one manufacturer and one retailer, to investigate the pricing problem under different scenarios of investment in smart technology. In doing so, we try to achieve digital leanness through investing in technology.

In addition, with increasing globalization, supply chain coordination is crucial for firms to achieve their goals (Singh *et al*., 2018). The literature has intensively considered channel coordination between sellers and buyers in a supply chain. Cachon (2003) reviewed studies on coordinating a decentralized supply chain using contracting schemes, such as profit-sharing (Fu *et al*., 2018; Fu and Ma, 2019; Ventura *et al*., 2021), revenue-sharing (Zhong *et al*., 2019; Avinadav, 2020; Bart *et al*., 2021), risk-sharing (Adhikari *et al*., 2020; Andersson *et al*., 2020) and investment-sharing contracts (Mohammadi *et al*., 2019; Liu *et al*., 2021; Xing *et al*., 2022). Channel efficiency can be improved by removing the effects of double marginalization and/or distorted information using properly designed contracts.

Investment sharing is a common practice. For example, the construction firm Caterpillar Inc. and its dealers jointly invest in improving operations and service quality and building trust through rewards to maintain relationships (Fites, 1996). The firm also invests in advanced technology to innovate and restructure its manufacturing process to make the firm leaner and more responsive to customers' needs. In the automobile industry, Toyota and Honda have consistently increased investment in their suppliers and rely on goodwill and trust in collaborating with many vendors (Aoki and Lennerfors, 2013). Many original equipment manufacturers invest in their contracted manufacturer's mastery of the production process (Arruñada and Vázquez, 2006). To the best of our knowledge, this study is the first to use an investment-sharing mechanism to solve the coordination problem in a two-echelon supply chain, wherein the supply chain profits increase, and all participants benefit (or at least no one is worse off). The knowledge management (KM) approach is very useful to the adoption of digital technologies (Alvarenga *et al*., 2020). However, this study focuses on model formulation and analysis, and the KM approach is beyond its scope. The KM approach causes difficulties in model formulation and analysis of long-term equilibrium because of the supply-demand dynamic created by an investment-sharing mechanism. Therefore, this study proposes and analyzes three models under static settings to avoid distractions.

Based on our industry observations, we construct our model under three scenarios to investigate the investment policies: the M–system, wherein the manufacturer fully pays for the investment cost ($\delta = 1$); the S–system, wherein the manufacturer and retailer share the investment cost ($0 < \delta < 1$); and the R–system, wherein the retailer fully pays for the investment cost ($\delta = 0$). The main purpose behind constructing the models is to determine the optimal wholesale and retail prices and investment strategy under a Stackelberg leader-follower game setting. In addition, we investigate how the wholesale price, retail price and channel profits are affected by the choice of investment policy. Our research findings propose that the S–system is the most appropriate choice for both the manufacturer and retailer. We also suggest the appropriate investment-sharing ratio so that Pareto improvement can be achieved under such an arrangement, that is, where one party is better off, and the other is not worse off. As Brandenburger and Nalebuff (2021) proposed, negotiation focuses on the investment-sharing agreement between the stakeholders, and the value created by such an agreement should be fairly divided.

Our study answers the following questions.

1) Should digital/smart technology be adopted to reduce manufacturing costs of the manufacturer? If yes, who should be responsible for the investment cost?
2) What are the optimal wholesale and retail prices under different investment scenarios?
3) What is the best investment strategy for firms under different conditions?

The rest of this paper is organized as follows. Section 2 presents the related literature. Section 3 provides the problem context, assumptions, notations and benchmark case. Section 4 develops the model under the three investment scenarios and presents the managerial implications of each. Section 5 presents a numerical study and explains the strategic choice. Section 6 concludes the paper by summarizing our main contributions and future research directions.

2. LITERATURE REVIEW

This study is related to two streams of research: coordination in the supply chain and digital/smart technology adoption in the manufacturing process. Next, we examine both literature streams and analyze their differences and similarities.

Many scholars have investigated supply chain coordination (Zhong *et al*., 2019; Shafiq and Luong, 2021; Bart *et al*., 2021; Fu *et al*., 2018; Min *et al*., 2019; Ventura *et al*., 2021; Adhikari *et al*., 2020; Andersson *et al*., 2020; Hanh *et al*., 2022). Zhong *et al*. (2019) investigated supply chain coordination in the e-commerce industry using the Stackelberg game theory under a revenue-sharing contract. Shafiq and Luong (2021) determined the optimal order quantity under a revenue-sharing contract. Bart *et al*. (2021) reviewed 148 studies and discussed the various aspects of revenue contracts. Fu *et al*. (2018) designed a profit-sharing coordination framework using a distributionally resilient Stackelberg game model under uncertain demand and price distribution scenarios. They found that a profit-sharing contract generates more earnings for the supply chain participants than wholesale pricing. Min *et al*. (2019) applied the revenue-sharing contract to a multiple-echelon supply chain and concluded that a revenue-sharing contract helps the supply chain members achieve a win-win outcome. Ventura *et al*. (2021) created a profit-sharing model for allocating order quantity to the chosen providers in a two-tier supply chain with one buyer and several suppliers. Adhikari *et al*. (2020) investigated the effect of buyback contracts and options contracts on supply chain performance and the improvement in supply chain performance with the use of a risk-sharing contract in a textile firm. Andersson *et al*. (2020) studied a risk-sharing mechanism as a managed entry agreement in a healthcare system. Hanh *et al*. (2022) proposed the best coordination mechanism for a three-echelon supply chain under different scenarios.

Other studies have also considered an investment-sharing mechanism in supply chain coordination (Ribeiro, 2016; Mohammadi *et al*., 2019; Xing *et al*., 2022; Liu *et al*., 2021). Ribeiro (2016) investigated the competition between three facility-based firms in the presence of investment-sharing agreements. Mohammadi *et al*. (2019) considered revenue and preservation technology investment sharing (RPTIS) in a fresh-product supply chain. They concluded that RPTIS increases the profits of both the individual firm and the supply chain and reduces product waste. Xing *et al*. (2022) examined a two-echelon supply chain with one start-up supplier and one manufacturer wherein the manufacturer supports quality innovation by the supplier via an investment-sharing contract. The results showed that an investment-sharing contract helped both entities achieve a win-win outcome. Liu *et al*. (2021) investigated the impact of investment in blockchain technology on a port supply chain and identified the most suitable investment strategy.

To the best of our knowledge, this study is the first to apply an investment-sharing mechanism to coordinate between channel members and determine the optimal profit-sharing ratio for all entities in a supply chain such that they can achieve Pareto improvement, that is, the supply chain profit increases and all participants are better off or at least no one is worse off.

This study is also closely related to the application of digital/smart technologies to the manufacturing process, such as automation, cyber-physical systems, cloud computing, artificial intelligence (AI), IoT, big data, robotics and semi-autonomous industrial techniques (Kamble *et al*., 2018). This study focuses on investment in AI technologies that enable smart scheduling, preventive maintenance and production optimization, assembling, and testing processes. Many studies have studied the importance of these technologies on operations and supply chain management. For instance, Leng *et al*. (2020) reconfigured the manufacturing system in a catering business by automating process planning, which significantly reduced manufacturing costs. Lu *et al*. (2020) proposed the future of mass customization via responsive autonomous manufacturing operations at competitive costs using a smart manufacturing process and system automation. Ma *et al*. (2021) investigated the impact of investing in carbon emissions-reducing technology to meet the preferences of green customers. According to Lim *et al*. (2021), Industry 4.0 aims to develop efficient and low-cost production with flexible workflows for producing high-quality personalized products. Avilés-Sacoto *et al*. (2019) listed smart technologies and identified their impact on supply chain and manufacturing operations. Big data and AI analytics play an important role in helping businesses increase supply chain efficiency (Giannakis and Louis, 2016), satisfy consumers, add more economic value (Zhan and Tan, 2020) and reduce manufacturing costs (Choi *et al*., 2018). Other technologies, such as RFID (Biswal *et al*., 2021), blockchain (Dutta *et al*., 2020; Shi *et al*., 2021) and 3D printing (Beltagui *et al*., 2020), have been implemented in supply chain management. All of these technologies are considered digital/smart manufacturing technologies, as they make the manufacturing line "smarter," which means lower costs, higher efficiency and more customer value. This study emphasizes one function of digital/smart technology—reducing the manufacturer's production cost—and investigates an investment-sharing strategy for the supply chain.

The literature has revealed that reducing production costs is critical to all firms. Jha *et al*. (2020) stated that production cost is an important factor in measuring the feasibility of a sintering process. Therefore, they considered biomass for the sintering process instead of the conventional method to reduce production costs to help firms stay competitive. Blakey-Milner *et al*. (2021) concluded that the reduction of production costs and lead times are two of the fundamental objectives of metal additive manufacturing in aerospace applications. Kaur and Singh (2021) proposed that additive manufacturing could reduce manufacturing costs to help firms become cost-competitive. Kovács and Kot (2017) concluded that to stay in a competitive market, manufacturing firms must produce cost-effective products, that is, minimize production costs and increase effectiveness. The benefit generated by the firms is determined based on the following factors: revenue (Pistolesi *et al*., 2017), manufacturing costs (Fan *et al*., 2019), energy costs (Gadaleta *et al*., 2019; Cheng *et al*., 2021), maintenance expenses (Vogl *et al*., 2019), waste treatment fees (Santos *et al*., 2019) and inventory costs (Chen and Kuo, 2019; Castellano *et al*., 2019). Most of the aforementioned studies have concluded that the performance of firms that implement smart manufacturing systems is much better than that of firms using traditional manufacturing due to the higher cost-savings (Alqahtani *et al*., 2019; Gadaleta *et al*., 2019). In 2018, Tesla built a state-of-the-art smart manufacturing system in Shanghai to reduce its production costs by improving capacity utilization and localizing parts (Cheng *et al*., 2022).

Despite the appeal of production cost reduction and the importance of adopting smart technology in manufacturing processes, the literature has not explored investment policies for improving manufacturing processes through smart technology. Therefore, this study develops a two-echelon supply chain model with one manufacturer and one retailer to investigate the pricing problem for investing in smart technology to reduce manufacturing costs under different scenarios. In addition, this study proposes the best investment strategy and optimal investment-sharing ratio for the entire supply chain. The reviewed literature is summarized in Table 1.

Table 1. Summary of the related literature

Authors	Year	Smart technology investment	Supply chain scenario/ structure	Investment sharing percentage	Pricing strategy	Coordination scenarios	Production cost reduction	Solution method used
Wang and Li	2012	X	X	X	✓	X	X	Closed-form optimization model
Giri and Barhan	2016	X	✓	X	✓	✓	X	Closed-form game-theoretical model
Li	2020	✓	✓	X	✓	✓	X	Closed-form game-theoretical model
Liu *et al.*	2021	✓	X	X	X	✓	✓	Optimization model
Hanh *et al.*	2022	X	✓	X	✓	✓	X	Closed-form game-theoretical model and heuristics approach
Xing *et al.*	2022	✓	✓	X	X	✓	X	Closed-form optimization model
Our study		✓	✓	✓	✓	✓	✓	Closed-form game-theoretical model

Note: ✓: Yes, X: No

3. THE PROBLEM CONTEXT AND THE BENCHMARK CASE

3.1 Assumptions and Notations

This study considers a manufacturer who sells products to the end consumers through a retailer. In this supply chain, it is allowed to invest in a specific smart technology to reduce the manufacturer's unit production cost. For instance, investing in automating the manufacturing process reduces manpower costs and improves work productivity. $K(c_m)$ denotes the amount of investment required to implement smart technology to lower the cost from c_0 to $c_m \epsilon$ [0, c_0]. We assume that $K(c_m) = A - Bln(c_m)$, where B and A are given positive constants. This cost formulation has been used in other studies, such as Porteus (1985, 1986), Billington (1987) and Li (2020). B denotes whether it is economical to invest in smart technology; a large B signifies that making the production line "smarter" is costly or that it is expensive to decrease production costs to a target value. The demand function is expressed as linear, $\alpha - \beta p$, where p denotes the retailer's market price, α represents the potential market size, and β denotes the sensitivity coefficient of demand.

The supply chain entities interact with each other under a leader-follower Stackelberg game—the manufacturer is the leader who offers the wholesale price w, and the retailer is the follower who responds by ordering quantity $q(p)$, where p is the retail price determined by the retailer. Moreover, the manufacturer initiates the investment in digital technology. However, the investment cost can be fully paid by the manufacturer or the retailer or shared by both. This assumption aligns with industry observations and has also been used in the supply chain framework of other studies (e.g., Li *et al*., 2016; McGuire and Staelin, 1983).

Hence, we examine our model under the following three scenarios of investment policies.

1) M–system: The manufacturer fully pays for the investment cost ($\delta = 1$).
2) S–system: The manufacturer and retailer share the investment cost ($0 < \delta < 1$).
3) R–system: The retailer fully pays for the investment cost ($\delta = 0$).

We first study the case with no investment as the benchmark case.

Hereafter, we let subscript $i \in (m, r)$ represent the manufacturer ($i = m$) and the retailer ($i = r$), and subscript $j \in (1, 2, 3, 4)$ denote the investment scenarios. Demand is assumed to be price-dependent. The demand function in scenario j is

$$q_j = \alpha - \beta p_j. \quad (1)$$

The notations used in the study are summarized as follows.

q_j	Retailer's demand in scenario j
p_j	Retailer's unit selling price in scenario j
w_j	Manufacturer's unit wholesale price in scenario j
α	Potential market size
β	Sensitivity coefficient of demand
c_0	Manufacturer's variable manufacturing cost per unit without investment
$c_{m,j}$	Manufacturer's variable manufacturing cost per unit in scenario j ($j = 2, 3, 4$)
c_r	Retailer's unit variable cost
K_j	Investment cost in scenario j to reduce c_0 to $c_{m,j}$, $K_j(c_{m,j}) = A - Bln(c_{m,j})$
δ	Manufacturer's investment cost-sharing percentage, $0 \leq \delta \leq 1$
θ	The fractional per unit time opportunity cost of capital
$\pi_{i,j}$	Profit of manufacturer ($i = m$), retailer ($i = r$), or supply chain ($i = SC$), generated in scenario j, $\pi_{SC,j} = \pi_{m,j} + \pi_{r,j}$

We formulate the decision problems of a two-echelon supply chain under no-investment and with-investment settings. For comparison, we begin with the no-investment scenario as a benchmark case.

3.2 No-Investment Scenario: The Benchmark Case

In this case, the manufacturer first determines the wholesale price w. Next, the retailer determines the retail price p. We first manage the retailer's problem using the standard procedure for solving a Stackelberg game. The objective function of the retailer is as follows:

$$\underset{p_1}{\text{Max}}\ \pi_{r,1} = (p_1 - w_1 - c_r)q_1. \tag{2}$$

Lemma 1 shows the result of taking the first derivative of Equation (2) with respect to p_1, then setting it equal to 0 and solving the first-order necessary condition. Appendix A provides detailed proof for Lemma 1.

Lemma 1. *Under the no-investment scenario, the equilibrium selling price of the retailer is as follows:*

$$p_1(w_1) = \frac{\alpha + w_1\beta + \beta c_r}{2\beta}. \tag{3}$$

Substituting Equation (3) into Equations (1) and (2) yields the equilibrium demand and retailer's profit as follows:

$$q_1(w_1) = \tfrac{1}{2}(\alpha - w_1\beta - \beta c_r), \tag{4}$$
$$\pi_{r,1}(w_1) = -\tfrac{1}{2}(w - c_0)(-\alpha + w_1\beta + \beta c_r). \tag{5}$$

Given the retailer's response, the manufacturer aims to find the optimal wholesale price w_1 to maximize the profit in Equation (6):

$$\underset{w_1}{\text{Max}}\,\pi_{m,1} = (w_1 - c_0)(\alpha - \beta p_1) = (w_1 - c_0)q_1(w_1). \tag{6}$$

Note that the retail price presented in Lemma 1 is the function of w_1. Substituting Equation (3) into Equation (6) yields

$$\pi_{m,1}(w_1) = -\tfrac{1}{2}(w_1 - c_0)(-\alpha + w_1\beta + \beta c_r). \tag{7}$$

We find the equilibrium wholesale price by obtaining the first derivative of Equation (7) with respect to w_1 and then setting it equal to 0. We obtain the equilibrium retail price by substituting the equilibrium wholesale price in Equation (3). The results are shown in Theorem 1, and the complete proof is presented in Appendix A.

Theorem 1. *The unique optimal solutions for the manufacturer's wholesale price and retailer's selling price in the benchmark case are given as follows.*

$$w_1^* = \frac{\alpha+\beta c_0-\beta c_r}{2\beta}, \tag{8}$$
$$p_1^* = \frac{3\alpha+\beta c_0+\beta c_r}{4\beta}. \tag{9}$$

Substituting w_1^* and p_1^* into Equation (4) yields the optimal selling quantity,

$$q_1^* = \frac{1}{4}\left(\alpha - \beta(c_0 + c_r)\right). \tag{10}$$

We obtain the profits of the manufacturer and retailer by substituting the equilibrium solutions into Equations (2) and (6) as follows.

$$\pi_{m,1}^* = \frac{(\alpha-\beta(c_0+c_r))^2}{8\beta}, \tag{11}$$
$$\pi_{r,1}^* = \frac{(\alpha-\beta(c_0+c_r))^2}{16\beta}. \tag{12}$$

The channel-wide profit is $\pi_{SC,1}^* = \pi_{m,1}^* + \pi_{r,1}^*$. The mathematical attributes of the equilibrium solutions are presented in Proposition 1. Appendix A provides the complete proof for Proposition 1.

Proposition 1. *The relationships between the unique optimal solutions obtained by the benchmark case and the parameters are shown as follows.*

(i) p_1^* *increases with an increase in* c_0, c_r, *and* α
(ii) w_1^* *increases with an increase in* c_0 *and* α *but decreases with a decrease in* c_r
(iii) q_1^* *increases with an increase in* α *but decreases with a decrease in* c_0 *and* c_r.

Proposition 1 shows that higher costs lead to higher prices, which result in lower demand.

4. INVESTMENT SCENARIOS

Next, we investigate the scenarios where it is allowed to invest in a specific smart technology to reduce the manufacturer's unit production cost. Let δ be the manufacturer's share of the investment cost. This study investigates three scenarios (M, S, and R). In the M–system, the manufacturer can fully invest in its own technology without any involvement from the retailer ($\delta = 1$). In the S–system, the manufacturer shares the investment cost with the retailer ($0 < \delta < 1$). In the R–system, the retailer fully pays for the investment cost ($\delta = 0$).

4.1 M–system ($\delta = 1$)

Under the M–system, the manufacturer decides to invest in the new technology to reduce the variable manufacturing cost. By doing so, the firm can attract more customers by offering a lower wholesale price while increasing its profit. For example, Tesla's smart manufacturing system in Shanghai invested in production equipment to optimize its manufacturing lines in 2018 (Cheng *et al*., 2022). Then in 2020, due to the production cost reduction, Tesla reduced the car prices by 18%, which helps to attract more consumers.

The decision sequence of this scenario is as follows: i) the manufacturer (the leader) first determines the investment cost $K_2(c_{m,2})$ and the wholesale price w_2, and ii) the retailer (the follower) sets the retail price p_2. We first manage the retailer's problem using the standard procedure for solving a Stackelberg game. The objective function of the retailer is given by:

$$\underset{p_2}{\text{Max}}\, \pi_{r,2} = (p_2 - w_2 - c_r)q_2. \tag{13}$$

The manufacturer's objective function is given as

$$\underset{w_2,\, K_2(c_{m,2})}{\text{Max}}\, \pi_{m,2} = (w_2 - c_{m,2})q_2 - \theta K_2(c_{m,2}). \tag{14a}$$

Substituting $K_2(c_{m,2}) = A - Bln(c_{m,2})$ into Equation (14a) yields the equivalent Equation:

$$\max_{w_2,\, c_{m,2}} \pi_{m,2} = (w_2 - c_{m,2})q_2 - \theta(A - Bln(c_{m,2})). \tag{14b}$$

Lemma 2 shows the results after generating the first derivative of Equation (13) with respect to p_2, setting it equal to 0 and solving the first-order necessary condition. Appendix B gives the proof for Lemma 2 in detail.

Lemma 2. *When the manufacturer fully pays for the investment cost, the retailer's equilibrium selling price is as follows:*

$$p_2(w_2) = \frac{\alpha + w_2\beta + \beta c_r}{2\beta} \tag{15}$$

Given the retailer's response, the manufacturer finds the optimal wholesale price w_2 and production cost $c_{m,2}$ to maximize the profit in Equation (14b). For a given $c_{m,2}$, we obtain the equilibrium wholesale price as follows,

$$w_2(c_{m,2}) = \frac{\alpha + c_{m,2}\beta - c_r\beta}{2\beta}. \tag{16}$$

Substituting Equation (16) into the manufacturer's profit function yields

$$\pi_{m,2}(c_{m,2}) = -A\theta + \frac{(\alpha - (c_{m,2} + c_r)\beta)^2}{8\beta} + B\theta\, ln(c_{m,2}). \tag{17}$$

We obtain the optimal solution by computing the first derivative of Equation (17) with respect to $c_{m,2}$, setting it equal to 0 and solving the Equation. Defining $c_2 = \frac{\alpha - c_r\beta - \sqrt{-16B\theta\beta + (-\alpha + c_r\beta)^2}}{2\beta}$ yields the necessary and sufficient conditions to determine the manufacturer's optimal variable cost in Lemma 3. Appendix B provides the proof for Lemma 3.

Lemma 3.

(1) When $B < \frac{c_0(\alpha - c_r\beta - \beta c_0)}{4\theta}$, the manufacturer's optimal variable cost is $c^*_{m,2} = c_2 < c_0$.

(2) When $B > \frac{(c_r\beta - \alpha)^2}{16\theta\beta}$, the manufacturer's optimal variable cost is $c^*_{m,2} = c_0$.

(3) When $\frac{c_0(\alpha - c_r\beta - \beta c_0)}{4\theta} \le B \le \frac{(c_r\beta - \alpha)^2}{16\theta\beta}$, the manufacturer's optimal variable cost is given by:

$$c^*_{m,2} = \begin{cases} c_2 \; if \; c_0 > \frac{\alpha}{2\beta} - \frac{c_r}{2} \; and \; B < \frac{(c_0 - c_2)(\beta(c_2 + c_0 + 2c_r) - 2\alpha)}{8\,\theta\, ln\left(\frac{c_0}{c_2}\right)}, \\ c_0, otherwise. \end{cases} \tag{18}$$

Lemma 3 provides the optimal investment decision for the manufacturer on whether they should invest in smart technology to reduce production costs from c_0 to the target cost $c_{m,2}$ or do nothing and maintain the original production cost at c_0. The decision is determined by three factors: the cost factor of digital technology investment B for making the manufacturing line smarter, the original production cost c_0 and the opportunity cost of investing in digital technology θ.

Note that the retail price presented in Lemma 2 is the function of w_2. Substituting the given $c^*_{m,2}$ determined in Lemma 3 and Equation (15) into Equation (14b) yields

$$\pi_{m,2}(w_2) = -A\theta + \frac{1}{2}(c^*_{m,2} - w_2)(-\alpha + (c_r + w_2)\beta) + B\theta ln(c^*_{m,2}). \tag{19}$$

We find the equilibrium wholesale price by generating the first derivative of Equation (19) with respect to w_2 and setting it equal to 0. The equilibrium retail price is obtained by substituting the equilibrium wholesale price in Equation (15). We show the results in Theorem 2 and the exhaustive proof in Appendix B.

Theorem 2. *The unique equilibrium solutions for the manufacturer's wholesale price and the retailer's selling price in the M-system are given as follows:*

$$w_2^* = \frac{\alpha + c_{m,2}^*\beta - c_r\beta}{2\beta}, \tag{20}$$

$$p_2^* = \frac{3\alpha + (c_{m,2}^* + c_r)\beta}{4\beta}. \tag{21}$$

Substituting w_2^* and p_2^* into Equation (1) yields the equilibrium selling quantity as

$$q_2^* = \frac{\alpha}{4} + \frac{1}{4}\left(-c_{m,2}^* - c_r\right)\beta. \tag{22}$$

Substituting the equilibrium solutions into Equations (13) and (14b) yields the profits of the manufacturer and retailer as follows:

$$\pi_{m,2}^* = -A\Theta + \frac{\left(\alpha - (c_{m,2}^* + c_r)\beta\right)^2}{8\beta} + B\Theta ln(c_{m,2}^*), \tag{23}$$

$$\pi_{r,2}^* = \frac{(\alpha - (c_{m,2}^* + c_r)\beta)^2}{16\beta}. \tag{24}$$

The channel-wide profit is $\pi_{SC,2}^* = \pi_{m,2}^* + \pi_{r,2}^*$. Proposition 2 presents the mathematical properties of the equilibrium solutions. The complete proof is provided in Appendix B.

Proposition 2. *The relationships between the equilibrium solutions obtained by the M–system and the parameters are shown as follows:*

(i) p_2^* *increases with an increase in* $c_{m,2}^*$, c_r, *and* α,
(ii) w_2^* *increases with an increase in* $c_{m,2}^*$ *and* α *but decreases with a decrease in* c_r, *and*
(iii) q_2^* *increases with an increase in* α *but decreases with a decrease in* $c_{m,2}^*$, *and* c_r.

Proposition 2 states that higher costs lead to higher prices, which result in lower demand. Moreover, when the potential market is large, the manufacturer and retailer can increase their selling price and demand.
Corollary 1 provides managerial insights for decision-makers on whether they should invest in smart technology.

Corollary 1.
(1) *When* B *is less than the threshold* $\frac{c_0(\alpha - c_r\beta - \beta c_0)}{4\Theta}$, *which depends on* c_0 *and* Θ, *production costs should be reduced.*
(2) *When* B *is greater than the threshold* $\frac{(c_r\beta - \alpha)^2}{16\Theta\beta}$, *the firm should not invest.*
(3) *When* B *lies between the two thresholds, the scenario can be categorized into two subcases:*
(i) *when* c_0 *is small* ($c_0 < \frac{\alpha}{2\beta} - \frac{c_r}{2}$), *no investment should be undertaken;*
(ii) *otherwise, the decision should be made based on the relationship between* c_0 *and* B *presented in Equation (18). The target production cost* c_2 *increases with an increase in* B, *which means that investment will decrease when digital technology becomes more expensive.*

4.2 S–system ($0 < \delta < 1$)

In this scenario, the manufacturer and retailer share the investment cost with ratios of δ and $1 - \delta$, respectively. According to Chakraborty *et al*. (2019), Walmart is a quality-oriented giant retailer; therefore, it works closely and shares the investment cost with its suppliers to ensure that the quality of products meets or exceeds Walmart's standards. According to the Annual Progress Report of Apple Inc. (2021), Apple invests in its suppliers by providing them with platforms, tools and resources. The platforms provide online tools for improving supplier operations and refining new processes. Based on mutually beneficial investments and collaborations, the original equipment manufacturer (Apple) and its suppliers customize the capability-building program. The manufacturer first determines the investment cost $K_3(c_{m,3})$ and wholesale price w_3. Next, the retailer defines the market price p_3. We first manage the retailer's problem using the standard procedure of solving a Stackelberg game. The objective function of the retailer is as follows:

$$\underset{p_3}{\text{Max}}\, \pi_{r,3} = (p_3 - w_3 - c_r)q_3 - (1 - \delta)\theta K_3(c_{m,3}). \tag{25}$$

The manufacturer's objective function is given by

$$\underset{w_3,\, K_3(c_{m,3})}{\text{Max}} \pi_{m,3} = \left(w_3 - c_{m,3}\right)q_3 - \delta\theta K_3(c_{m,3}). \tag{26a}$$

Substituting $K_3(c_{m,3}) = A - Bln(c_{m,3})$ into Equation (26a) yields the equivalent Equation

$$\max_{w_3,\, c_{m,3}} \pi_{m,3} = (w_3 - c_{m,3})q_3 - \delta\theta(A - Bln(c_{m,3})). \tag{26b}$$

Lemma 4 shows the result after generating the first derivative of Equation (25) with respect to p_3, then setting it equal to 0 and solving the first-order necessary condition. Appendix C provides the proof for Lemma 4 in detail.

Lemma 4. *When the manufacturer and retailer share the investment cost, the equilibrium price is as follows.*

$$p_3(w_3) = \frac{\alpha + w_3\beta + \beta c_r}{2\beta}. \tag{27}$$

Given the retailer's response, the manufacturer finds the optimal wholesale price w_3 and production cost $c_{m,3}$ to maximize the profit shown in Equation (26b). For a given $c_{m,3}$, we obtain the following equilibrium wholesale price:

$$w_3(c_{m,3}) = \frac{\alpha + c_{m,3}\beta - c_r\beta}{2\beta}. \tag{28}$$

Substituting Equation (28) into the manufacturer's profit function yields

$$\pi_{m,3}(c_{m,3}) = -A\theta\delta + \frac{(\alpha - (c_{m,2} + c_r)\beta)^2}{8\beta} + B\theta\delta\, ln(c_{m,3}). \tag{29}$$

We obtain the optimal solution after generating the first derivative of Equation (29) with respect to $c_{m,3}$, then setting it equal to 0 and solving the Equation. Defining $c_3 = \frac{\alpha - c_r\beta - \sqrt{-16B\theta\beta\delta + (-\alpha + c_r\beta)^2}}{2\beta}$ yields the necessary and sufficient conditions to determine the manufacturer's optimal variable cost in Lemma 5. Appendix C gives detailed proof for Lemma 5.

Lemma 5.

(1) When $B < \frac{c_0(\alpha - c_r\beta - \beta c_0)}{4\theta\delta}$, the manufacturer's optimal variable cost is $c^*_{m,3} = c_3 < c_0$.

(2) When $B > \frac{(c_r\beta - \alpha)^2}{16\theta\beta\delta}$, the manufacturer's optimal variable cost is $c^*_{m,3} = c_0$.

(3) When $\frac{c_0(\alpha - c_r\beta - \beta c_0)}{4\theta\delta} \le B \le \frac{(c_r\beta - \alpha)^2}{16\theta\beta\delta}$, the manufacturer's optimal variable cost is given by

$$c^*_{m,3} = \begin{cases} c_3 \ if \ c_0 > \frac{\alpha}{2\beta} - \frac{c_r}{2} \ and \ B < \frac{(c_0 - c_3)(\beta(c_3 + c_0 + 2c_r) - 2\alpha)}{8\,\theta\,\delta\, ln\left(\frac{c_0}{c_3}\right)}, \\ \qquad\qquad otherwise, c_0. \end{cases} \tag{30}$$

Lemma 5 provides the optimal investment decision for the manufacturer on whether to invest in smart technology to reduce the production cost from c_0 to the target cost $c_{m,3}$ or do nothing and maintain the original production cost at c_0. The decision is determined by three factors: the cost factor for digital technology investment B for making the manufacturing line smarter, the initial production cost c_0 and the opportunity cost of investing in digital technology θ.

Note that the retail price presented in Lemma 4 is the function of w_3. Substituting $c^*_{m,3}$ determined in Lemma 5 and Equation (27) into Equation (26b) yields

$$\pi_{m,3}(w_3) = -A\theta\delta + \frac{1}{2}(c^*_{m,3} - w_3)(-\alpha + (c_r + w_3)\beta) + B\theta\delta ln(c^*_{m,3}). \tag{31}$$

We obtain the equilibrium wholesale price in the S-system by obtaining the first derivative of Equation (31) with respect to w_2 and setting it equal to 0. Substituting the equilibrium wholesale price in Equation (27) yields the equilibrium retail price. We show the results in Theorem 3 and the exhaustive proof in Appendix C.

Theorem 3. *The unique equilibrium solutions for the manufacturer's wholesale price and retailer's selling price in the S-system are given by*

$$w^*_3 = \frac{\alpha + c^*_{m,3}\beta - c_r\beta}{2\beta}, \tag{32}$$

$$p^*_3 = \frac{3\alpha + (c^*_{m,3} + c_r)\beta}{4\beta}. \tag{33}$$

Substituting w_3^* and p_3^* into Equation (1) yields the equilibrium selling quantity

$$q_3^* = \frac{\alpha}{4} + \frac{1}{4}\left(-c_{m,3}^* - c_r\right)\beta. \tag{34}$$

The profits of the manufacturer and retailer are generated by replacing the equilibrium solutions shown in Theorem 3 into Equations (25) and (26b), respectively, as

$$\pi_{m,3}^* = -A\theta\delta + \frac{\left(\alpha-(c_{m,3}^*+c_r)\beta\right)^2}{8\beta} + \delta B\theta ln(c_{m,3}^*), \tag{35}$$

$$\pi_{r,3}^* = \frac{(\alpha-(c_{m,3}^*+c_r)\beta)^2}{16\beta} + A\theta(-1+\delta) - B\theta(-1+\delta)ln(c_{m,3}^*). \tag{36}$$

The channel-wide profit is $\pi_{SC,3}^* = \pi_{m,3}^* + \pi_{r,3}^*$. We show the mathematical attributes of the equilibrium solutions in Proposition 3 and the detailed proof in Appendix C.

Proposition 3. *The relationships between the equilibrium solutions obtained by the S-system and the parameters are shown as follows:*

(i) p_3^* *increases with an increase in* $c_{m,3}^*$, c_r, *and* α,
(ii) w_3^* *increases with an increase in* $c_{m,3}^*$ *and* α, *but decreases with a decrease in* c_r,
(iii) q_3^* *increases with an increase in* α, *but decreases with a decrease in* $c_{m,3}^*$, *and* c_r, *and*
(iv) $\pi_{m,3}^*$ *decreases with a decrease in* δ *while* $\pi_{r,3}^*$ *increases with an increase in* δ.

Proposition 3 states that both the manufacturer and retailer prefer to undertake the smaller share of the investment cost because the lower their cost, the higher will be their profits. Corollary 2 presents managerial insights for the decision-makers for determining when to invest.

Corollary 2.
(1) When B *is lower than the threshold* $\frac{c_0(\alpha-c_r\beta-\beta c_0)}{4\theta\delta}$, *which depends on* c_0 *and* θ, *production costs should be reduced.*
(2) When B *is higher than the threshold* $\frac{(c_r\beta-\alpha)^2}{16\theta\beta\delta}$, *the firm should not invest.*
(3) When B *lies between the two thresholds, the situation is categorized into two subcases:*
  (i) when c_0 *is small* ($c_0 < \frac{\alpha}{2\beta} - \frac{c_r}{2}$), *no investment should be undertaken;*
  (ii) otherwise, the decision should be made depending on the relationship between c_0 *and* B *as shown in Equation (30). Note that the target production cost* c_3 *increases with an increase in* B, *which means that when digital technology is expensive, the investment amount should decrease.*

4.3 R–system ($\delta = \mathbf{0}$)

In this scenario, the retailer agrees to pay the full amount of the investment cost. This can be demonstrated by the case of Foxconn and the telecom operator Orange (formerly France Telecom) (Hu *et al*., 2019). Foxconn manufactures phones based on the requirements of Orange. The telecom operator is responsible for investing in product research and development, while Foxconn only focuses on production.

The decision-making sequence in the R–system is as follows: i) the retailer decides the investment cost $K_4(c_{m,4})$, ii) the manufacturer decides the wholesale price w_4 and iii) the retailer determines the retail price p_4. We first manage the retailer's problem using the standard procedure for solving a Stackelberg game. The objective function of the retailer is

$$\underset{p_4,\, K_4(c_{m,4})}{\text{Max}}\ \pi_{r,4} = (p_4 - w_4 - c_r)q_4 - \theta K_4(c_{m,4}). \tag{37a}$$

Substituting $K_4\left(c_{m,4}\right) = A - Bln(c_{m,4})$ into Equation (37a) yields the equivalent Equation

$$\underset{p_4,\, c_{m,4}}{\text{Max}}\ \pi_{r,4} = (p_4 - w_4 - c_r)q_4 - \theta(A - Bln(c_{m,4})). \tag{37b}$$

The manufacturer's objective function is given as follows:

$$\underset{w_4}{\text{Max}}\, \pi_{m,4} = (w_4 - c_{m,4})q_4. \tag{38}$$

Lemma 6 gives the results after obtaining the first derivative of Equation (37b) with respect to p_4, then setting it equal to 0 and solving the first-order necessary condition. Appendix D provides the proof for Lemma 6 in detail.

Lemma 6. *When the retailer fully pays for the investment cost, the equilibrium price is as follows.*

$$p_4(w_4) = \frac{\alpha + w_4\beta + \beta c_r}{2\beta}. \tag{39}$$

Given the retailer's response, the manufacturer aims to find the optimal wholesale price w_4 to maximize profit, as shown in Equation (38). For a given $c_{m,4}$, we have the equilibrium wholesale price

$$w_4(c_{m,4}) = \frac{\alpha + c_{m,4}\beta - c_r\beta}{2\beta}. \tag{40}$$

Substituting Equation (40) into the retailer's profit function yields

$$\pi_{r,4}(c_{m,4}) = -A\theta + \frac{(\alpha - (c_{m,4} + c_r)\beta)^2}{16\beta} + B\theta\, ln(c_{m,4}). \tag{41}$$

We obtain the optimal solution by generating the first derivative of Equation (41) with respect to $c_{m,4}$, then setting it equal to 0 and solving the Equation. Defining $c_4 = \frac{\alpha - c_r\beta - \sqrt{-32B\theta\beta + (-\alpha + c_r\beta)^2}}{2\beta}$ yields the necessary and sufficient conditions to determine the manufacturer's optimal variable cost in Lemma 7. Appendix D gives the proof for Lemma 7 in detail.

Lemma 7.

(1) When $B < \frac{c_0(\alpha - c_r\beta - \beta c_0)}{8\theta}$, the manufacturer's optimal variable cost is $c_{m,4}^* = c_4 < c_0$.

(2) When $B > \frac{(c_r\beta - \alpha)^2}{32\theta\beta}$, the manufacturer's optimal variable cost is $c_{m,4}^* = c_0$.

(3) *When $\frac{c_0(\alpha - c_r\beta - \beta c_0)}{8\theta} \le B \le \frac{(c_r\beta - \alpha)^2}{32\theta\beta}$, the manufacturer's optimal variable cost is given by*

$$c_{m,4}^* = \begin{cases} c_4 \text{ } if \text{ } c_0 > \frac{\alpha}{2\beta} - \frac{c_r}{2} \text{ } and \text{ } B < \frac{(c_0 - c_4)(\beta(c_4 + c_0 + 2c_r) - 2\alpha)}{16\theta\, ln\left(\frac{c_0}{c_4}\right)}, \\ \qquad otherwise, c_0\,. \end{cases} \tag{42}$$

Lemma 7 provides the optimal investment decision for the retailer on whether to invest in smart technology to reduce the production cost from c_0 to the target cost $c_{m,4}$ or do nothing and maintain the original production cost at c_0. The decision is determined by three factors: the cost factor for digital technology investment B of making the manufacturing line smarter, the initial production cost c_0 and the opportunity cost of investing in digital technology θ.

Note that the retail price presented in Lemma 6 is the function of w_4. Substituting $c_{m,4}^*$ determined in Lemma 7 and Equation (39) into Equation (38) yields

$$\pi_{m,4}(w_4) = \frac{1}{2}(c_{m,4}^* - w_4)(-\alpha + (c_r + w_4)\beta). \tag{43}$$

We obtain the equilibrium wholesale price by generating the first derivative of Equation (43) with respect to w_4 and setting it equal to 0. Substituting the equilibrium wholesale price in Equation (39) yields the equilibrium retail price. We show the results in Theorem 4 and the exhaustive proof in Appendix D.

Theorem 4. *The unique equilibrium solutions for the manufacturer's wholesale price and retailer's selling price in the R–system are given as follows:*

$$w_4^* = \frac{\alpha + c_{m,4}^*\beta - c_r\beta}{2\beta}, \tag{44}$$

$$p_4^* = \frac{3\alpha + (c_{m,4}^* + c_r)\beta}{4\beta}. \tag{45}$$

Substituting w_4^* and p_4^* into Equation (1) yields the equilibrium selling quantity

$$q_4^* = \frac{\alpha}{4} + \frac{1}{4}(-c_{m,4}^* - c_r)\beta, \tag{46}$$

Substituting the equilibrium solutions into Equations (37b) and (38) yields the profits of the manufacturer and retailer, respectively, as follows:

$$\pi_{m,4}^* = \frac{(\alpha-(c_{m,4}^*+c_r)\beta)^2}{8\beta}, \tag{47}$$
$$\pi_{r,4}^* = -A\theta + \frac{(\alpha-(c_{m,4}^*+c_r)\beta)^2}{16\beta} + B\theta Ln(c_{m,4}^*). \tag{48}$$

The channel-wide profit is $\pi_{SC,4}^* = \pi_{m,4}^* + \pi_{r,4}^*$. We show the mathematical attributes of the equilibrium solutions in Proposition 4. Appendix D provides the complete proof.

Proposition 4. *The relationship between the equilibrium solutions obtained by the R–system and the parameters are given as follows:*

(i) p_4^* *increases with an increase in* $c_{m,4}^*$, c_r, *and* α,
(ii) w_4^* *increases with an increase in* $c_{m,4}^*$ *and* α, *but decreases with a decrease in* c_r, *and*
(iii) q_4^* *increases with an increase in* α, *but decreases with a decrease in* $c_{m,4}^*$, *and* c_r.

Proposition 4 states that both retail and wholesale prices increase with an increase in the unit manufacturing cost because higher costs lead to higher prices.

Corollary 3 presents managerial insights for the decision-makers in determining when to invest under the R–system.

Corollary 3.

(1) *When* B *is lower than the threshold* $\frac{c_0(\alpha-c_r\beta-\beta c_0)}{8\theta}$, *which depends on* c_0 *and* θ, *production costs should be reduced.*
(2) *When* B *is higher than the threshold* $\frac{(c_r\beta-\alpha)^2}{32\theta\beta}$, *the firm should not invest.*
(3) *When* B *lies between the two thresholds, the situation is categorized into two subcases:*
 (i) *when* c_0 *is small* ($c_0 < \frac{\alpha}{2\beta} - \frac{c_r}{2}$), *no investment should be undertaken;*
 (ii) *otherwise, the result should be made considering the relationship between* c_0 *and* B *as shown in Equation (42). The target production cost* c_4 *increases with an increase in* B, *which means when digital technology is expensive, the investment amount should decrease.*

4.4 Analysis and Managerial Implications

This section characterizes different qualitative properties of the solutions obtained by the models under the with-investment and no-investment settings.

Proposition 5. The manufacturer's wholesale price and retailer's selling price generated by the models under the with-investment and no-investment settings have the following properties:

(i) $w_3^* \leq w_2^* \leq w_4^* \leq w_1^*$, and
(ii) $p_3^* \leq p_2^* \leq p_4^* \leq p_1^*$.

Appendix E provides the proof for Proposition 5 in detail.

Proposition 5 implies that investment in smart technology leads to a lower wholesale price, which helps reduce the retailer's selling price to the end users. This is because an investment in smart technology reduces the production cost of the manufacturer, which results in a lower wholesale price.

Proposition 6. The manufacturer's profit generated by the models under the with-investment and no-investment settings has the following properties.

(i) $\pi_{m,4}^* \geq \pi_{m,1}^*$,
(ii) $\pi_{m,3}^* \geq \pi_{m,1}^*$, provided $B \leq \frac{(c_0-c_3)(-2\alpha+(c_0+c_3+2c_r)\beta)}{8\delta\theta\, ln(c_3)} + \frac{A}{ln(c_3)}$, and vice versa,
(iii) $\pi_{m,2}^* \geq \pi_{m,1}^*$, provided $B \leq \frac{(c_0-c_2)(-2\alpha+(c_0+c_2+2c_r)\beta)}{8\theta\, ln(c_2)} + \frac{A}{ln(c_2)}$, and vice versa,
(iv) $\pi_{m,3}^* \geq \pi_{m,4}^*$, provided $B \leq \frac{(c_4-c_3)(-2\alpha+(c_3+c_4+2c_r)\beta)}{8\theta\delta\, ln(c_3)} + \frac{A}{ln(c_3)}$, and vice versa,
(v) $\pi_{m,2}^* \geq \pi_{m,4}^*$, provided $B \leq \frac{(c_4-c_2)(-2\alpha+(c_4+c_2+2c_r)\beta)}{8\theta\, ln(c_2)} + \frac{A}{ln(c_2)}$, and vice versa, and

(vi) $\pi^*_{m,3} \geq \pi^*_{m,2}$, provided $c_2 < c_3^{\delta}$ and $B \geq \frac{(c_2-c_3)(-2\alpha+(c_2+c_3+2c_r)\beta)}{8\delta\theta\, ln(c_3)} + \frac{A(\delta-1)}{\delta ln(c_3)-ln\,(c_2)}$ or $c_2 > c_3^{\delta}$ and $B \leq \frac{(c_2-c_3)(-2\alpha+(c_2+c_3+2c_r)\beta)}{8\delta\theta\, ln(c_3)} + \frac{A(\delta-1)}{\delta ln(c_3)-ln\,(c_2)}$, and vice versa.

Appendix E gives the proof for Proposition 6 in detail.

Proposition 6 shows that the manufacturer's profit is always higher in the R–system than in the no-investment setting. Proposition 6 also provides the strategic choice for the manufacturer's investment decision under different conditions. For instance, property (v) states that when B is less than or equal to the threshold $\frac{(c_4-c_2)(-2\alpha+(c_4+c_2+2c_r)\beta)}{8\theta\, ln(c_2)} + \frac{A}{ln(c_2)}$, the manufacturer generates more profits in the M–system than in the R–system.

Proposition 7. The retailer's profit generated by the models under the with-investment and no-investment settings has the following properties:

(i) $\pi^*_{r,2} \geq \pi^*_{r,1}$,

(ii) $\pi^*_{r,3} \geq \pi^*_{r,1}$, provided $B \leq \frac{(c_0-c_3)(-2\alpha+(c_0+c_3+2c_r)\beta)}{16\theta(1-\delta)\, ln(c_3)} + \frac{A}{ln(c_3)}$, and vice versa,

(iii) $\pi^*_{r,4} \geq \pi^*_{r,1}$, provided $B \leq \frac{(c_0-c_4)(-2\alpha+(c_0+c_4+2c_r)\beta)}{16\theta\, ln(c_4)} + \frac{A}{ln(c_4)}$, and vice versa,

(iv) $\pi^*_{r,4} \geq \pi^*_{r,2}$, provided $B \leq \frac{(c_2-c_4)(-2\alpha+(c_2+c_4+2c_r)\beta)}{16\theta\, ln(c_4)} + \frac{A}{ln(c_4)}$, and vice versa,

(v) $\pi^*_{r,3} \geq \pi^*_{r,4}$, provided $c_4 < c_3^{(1-\delta)}$ and $B \geq \frac{(c_3-c_4)(-2\alpha+(c_3+c_4+2c_r)\beta)}{16\theta((\delta-1)\, ln(c_3)+ln(c_4))} + \frac{\delta A}{(\delta-1)ln(c_3)+ln\,(c_4)}$ or $c_4 > c_3^{(1-\delta)}$ and $B \leq \frac{(c_3-c_4)(-2\alpha+(c_3+c_4+2c_r)\beta)}{16\theta((\delta-1)\, ln(c_3)+ln(c_4))} + \frac{\delta A}{(\delta-1)ln(c_3)+ln\,(c_4)}$, and vice versa, and

(vi) $\pi^*_{r,3} \geq \pi^*_{r,2}$, provided $B \leq \frac{(c_2-c_3)(-2\alpha+(c_2+c_3+2c_r)\beta)}{16\theta(1-\delta)\, ln(c_3)} + \frac{A}{ln(c_3)}$, and vice versa.

Appendix E gives the proof for Proposition 7 in detail.

Proposition 7 states that the retailer's profit in the M–system is always higher than in the no-investment scenario. Proposition 7 also provides the strategic choice for the retailer's investment decision under different conditions.

Proposition 8. The channel-wide profits generated by the models under the with-investment and no-investment settings have the following properties:

(i) $\pi^*_{SC,2} \geq \pi^*_{SC,1}$, provided $B \leq \frac{3(c_0-c_2)(-2\alpha+(c_0+c_2+2c_r)\beta)}{16\theta\, ln(c_2)} + \frac{A}{ln(c_2)}$, and vice versa,

(ii) $\pi^*_{SC,3} \geq \pi^*_{SC,1}$, provided $B \leq \frac{3(c_0-c_3)(-2\alpha+(c_0+c_3+2c_r)\beta)}{16\theta\, ln(c_3)} + \frac{A}{ln(c_3)}$, and vice versa,

(iii) $\pi^*_{SC,4} \geq \pi^*_{SC,1}$, provided $B \leq \frac{3(c_0-c_4)(-2\alpha+(c_0+c_4+2c_r)\beta)}{16\theta\, ln(c_4)} + \frac{A}{ln(c_4)}$, and vice versa,

(iv) $\pi^*_{SC,3} \geq \pi^*_{SC,2}$, provided $c_3 < c_2$ and $B \geq \frac{3(c_2-c_3)(-2\alpha+(c_2+c_3+2c_r)\beta)}{16\theta\, ln(c_2/c_3)}$ or $c_3 > c_2$ and $B \leq \frac{3(c_2-c_3)(-2\alpha+(c_2+c_3+2c_r)\beta)}{16\theta\, ln(c_2/c_3)}$, and vice versa,

(v) $\pi^*_{SC,4} \geq \pi^*_{SC,2}$, provided $c_4 < c_2$ and $B \geq \frac{3(c_2-c_4)(-2\alpha+(c_2+c_4+2c_r)\beta)}{16\theta\, ln(c_2/c_4)}$ or $c_4 > c_2$ and $B \leq \frac{3(c_2-c_4)(-2\alpha+(c_2+c_4+2c_r)\beta)}{16\theta\, ln(c_2/c_4)}$, and vice versa, and

(vi) $\pi^*_{SC,4} \geq \pi^*_{SC,3}$, provided $c_4 < c_3$ and $B \geq \frac{3(c_3-c_4)(-2\alpha+(c_3+c_4+2c_r)\beta)}{16\theta\, ln(c_3/c_4)}$ or $c_4 > c_3$ and $B \leq \frac{3(c_3-c_4)(-2\alpha+(c_3+c_4+2c_r)\beta)}{16\theta\, ln(c_3/c_4)}$, and vice versa.

Appendix E provides the proof for Proposition 8 in detail.

Proposition 8 states the strategic choice for the channel-wide investment decision under different conditions. For example, property (i) shows that when B is less than or equal to the threshold $\frac{3(c_0-c_2)(-2\alpha+(c_0+c_2+2c_r)\beta)}{16\theta\, ln(c_2)} + \frac{A}{ln(c_2)}$, the channel-wide profits are higher in the M–system than in the no-investment scenario.

Corollary 4. The investment decisions generated by the models under the with-investment and no-investment settings are shown in Table 2, which summarizes Corollaries 1, 2 and 3.

Tables 2a, 2b and 2c demonstrate that under the S–system, when the manufacturer and retailer share the investment cost, the probability of undertaking investment is greater than in the M– and R–systems, while the probability of not investing is the highest in the R–system.

Table 2. Strategic investment decision choices under the three scenarios

Table 2a. Under the M–system

B	-∞	$\frac{c_0(\alpha - c_r\beta - \beta c_0)}{4\theta}$		$\frac{(c_r\beta - \alpha)^2}{16\theta\beta}$	+∞
$\delta = 1$	Invest	$c_0 < \frac{\alpha}{2\beta} - \frac{c_r}{2}$: no investment; otherwise, the decision is made based on the relationship between c_0 and B as shown in Equation (18)			Not invest

Table 2b. Under the S–system

B	-∞	$\frac{c_0(\alpha - c_r\beta - \beta c_0)}{4\theta\delta}$		$\frac{(c_r\beta - \alpha)^2}{16\theta\beta\delta}$	+∞
$0 < \delta < 1$	Invest	$c_0 < \frac{\alpha}{2\beta} - \frac{c_r}{2}$: no investment; otherwise, the decision is made based on the relationship between c_0 and B as shown in Equation (30)			Not invest

Table 2c. Under the R–system

B	-∞	$\frac{c_0(\alpha - c_r\beta - \beta c_0)}{8\theta}$		$\frac{(c_r\beta - \alpha)^2}{32\theta\beta}$	+∞
$\delta = 0$	Invest	$c_0 < \frac{\alpha}{2\beta} - \frac{c_r}{2}$: no investment; otherwise, the decision is made based on the relationship between c_0 and B as shown in Equation (42)			Not invest

5. NUMERICAL STUDY

We conducted a case study in a paper manufacturing firm based in Thanh Hoa province, Vietnam. The firm was established in 2000 and manufactured paper and paper-based products. It is a small and medium-sized enterprise (SME) that produces less than 50,000 tons of paper annually. The firm has considered investing in smart technology to reduce its production cost. However, they have found it difficult to determine whether such investment is worthwhile. As the investment cost would be substantial for a small firm such as theirs, they had been considering whether sharing the investment cost with the buyer would be more beneficial. Moreover, the firm was finding it challenging to price its products. We use the numerical example of this case to demonstrate the pricing strategy and smart technology investment problem in this study. Our above-mentioned models seem incapable of thoroughly analyzing the pricing decision and profit functions. Hence, we conducted this exhaustive numerical study to supplement our models. We present the numerical results to quantify these attributes and provide managerial implications for the pricing decision and strategic choice associated with the no-investment and with-investment scenarios. We focus on three aspects: the decision properties and tendencies in the four scenarios, the comparison of supply chain efficiency in the with-investment and no-investment scenarios and the strategic choices of the manufacturer, retailer and supply chain under the different scenarios. The numerical case study is organized as follows. We first present the numerical results of the base case under the four scenarios, then compare the performance when there is an investment in smart technology under the 2^k factorial design, provide the strategic choice for each entity and the entire supply chain and present the Pareto improvement.

5.1 The Base Case

After a series of numerical tests, we verified that the following set of parameters are significant to serve as our base setting: $\alpha = 0.7$, $\beta = 0.6$, $c_0 = 0.32$, $\theta = 0.1$, $A = 0.05$, $B = 0.1$, $\delta = 0.5$ and $c_r = 0.2$. This setting allows us to keep the results as clear as possible while showing the effects of the four scenarios.

Table 3 shows the numerical results. The results obtained under the four scenarios are as follows:

(i) $p_1^* > p_4^* > p_2^* > p_3^*$, which leads to $q_1^* < q_4^* < q_2^* < q_3^*$. This is quite straightforward; a higher price leads to lower demand.

(ii) $w_3^* < w_2^* < w_4^* < w_1^*$. When investment is undertaken to reduce the manufacturer's production cost, the manufacturer can reduce its selling price to the retailer while maintaining its profits.

(iii) The pricing behaviors in (i) and (ii) are consistent with Proposition 5.

(iv) $\pi^*_{m,2} < \pi^*_{m,1} < \pi^*_{m,3} < \pi^*_{m,4}$. This is because of the effect of the payment responsibility of the investment cost, which leads to the manufacturer sharing the investment cost with the retailer or convincing the retailer to undertake the full investment. If the manufacturer has to fully pay for the investment cost, it would prefer not to invest.

(v) $\pi^*_{r,4} < \pi^*_{r,3} < \pi^*_{r,1} < \pi^*_{r,2}$. This is because of the effect of the payment responsibility of the investment cost, which leads to the retailer sharing the investment cost with the manufacturer or allowing the manufacturer to undertake the full investment. If the retailer has to fully pay for the investment cost, it would prefer not to invest.

(vi) $\pi^*_{SC,1} < \pi^*_{SC,4} < \pi^*_{SC,2} < \pi^*_{SC,3}$.

From (iii), (iv) and (v), we find that if the manufacturer and retailer act individually, each has a preference about investing, as both entities may not always benefit from an investment in smart technology. However, from the supply chain's perspective, it is always better to invest in smart technology regardless of who undertakes the investment. In such a case, profit or cost-sharing (i.e., sharing the risk associated with the business) is a useful tool to distribute the extra profit of the gainer with the other members. This profit or cost-sharing encourages less profitable participants to join the firm, thereby benefitting the entire supply chain and potentially increasing the overall earnings of the firm.

Table 3 also shows that the supply chain profit increases by 25.81% when the manufacturer and retailer share the investment cost, whereas the supply chain profit increases by 24.48% when the manufacturer fully pays the investment cost. Supply chain efficiency increases by only 4.33% when the retailer fully pays for the investment cost.

Table 3. Pricing, demand and profits generated under the four scenarios

Scenario / **d.v.**	Not invest	Invest		
	$j = 1$	M–system ($j = 2$)	S–system ($j = 3$)	R–system ($j = 4$)
		$\delta = 1$	$0 < \delta < 1$	$\delta = 0$
w^*_j	0.6833	0.5207	0.5012	0.5667
p^*_j	1.0050	0.9437	0.9340	0.9667
$\pi^*_{m,j}$	0.0314	0.0287	0.0458	0.0480
$\pi^*_{r,j}$	0.0157	0.0298	0.0133	0.0011
$\pi^*_{SC,j}$	0.0471	0.0585	0.0591	0.0491
Improvement (%)		**24.48**	**25.81**	**4.33**

5.2 The Factorial Design

Next, we focus on the performance of the supply chain under a 2^k factorial design with k = 3, where the low, medium and high levels of two factors are $B = 0.05, 0.075$ and 0.1, $\delta = 0.3, 0.5$ and 0.7, respectively. The other parameters that are selected from the base setting are: $\alpha = 0.7$, $\beta = 0.6$, $c_0 = 0.32$, $\Theta = 0.1$, $A = 0.05$ and $c_r = 0.2$.

Based on the above set of parameters, Table 4 shows the numerical results for the selling prices and profits when the manufacturer and retailer share the investment cost. The model generates lower prices and higher profits for the manufacturer but lower profits for the retailer when the manufacturer has a lower share of the investment cost. The model generates lower prices for the manufacturer but higher profits for both the manufacturer and retailer when it is more economical to invest in smart technology (cheaper to make the production smarter).

Table 4. Pricing and profits generated when the retailer and manufacturer share investment cost

B	δ	c_m	w	p	q	π_m	π_r	π_{SC}
0.05	0.3	0.01046	0.48856	0.92761	0.14343	0.06023	0.01483	0.07506
	0.5	0.01756	0.49211	0.92939	0.14237	0.05495	0.02117	0.07613
	0.7	0.02477	0.49572	0.93119	0.14128	0.05009	0.02622	0.07632
0.075	0.3	0.01577	0.49122	0.92894	0.14263	0.05698	0.00862	0.06560
	0.5	0.02659	0.49663	0.93165	0.14101	0.05018	0.01704	0.06722
	0.7	0.03768	0.50217	0.93442	0.13935	0.04401	0.02349	0.06750
0.1	0.3	0.02115	0.49391	0.93029	0.14183	0.05398	0.00303	0.05701
	0.5	0.03581	0.50124	0.93395	0.13963	0.04584	0.01335	0.05919
	0.7	0.05096	0.50881	0.93774	0.13736	0.03855	0.02101	0.05957

Figure 1 shows that under different economy levels of investment cost-sharing, the supply chain always obtains the highest profit when the manufacturer shares 70% of the investment cost. Furthermore, it is cheaper to make the production line smarter, as the supply chain will benefit (i.e., increase profits), which also matches reality.

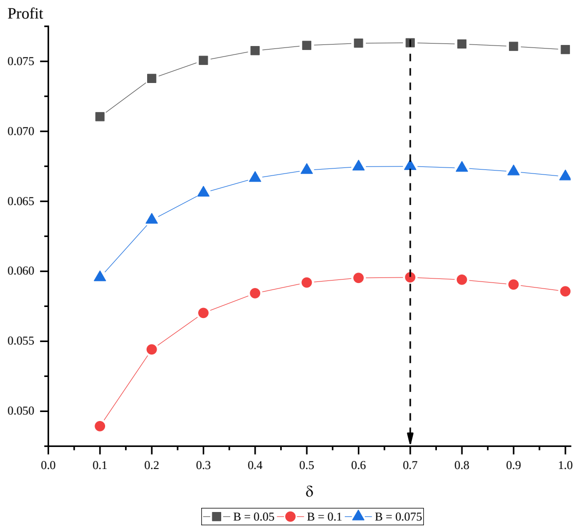


Figure 1. Supply chain profits under three levels of B

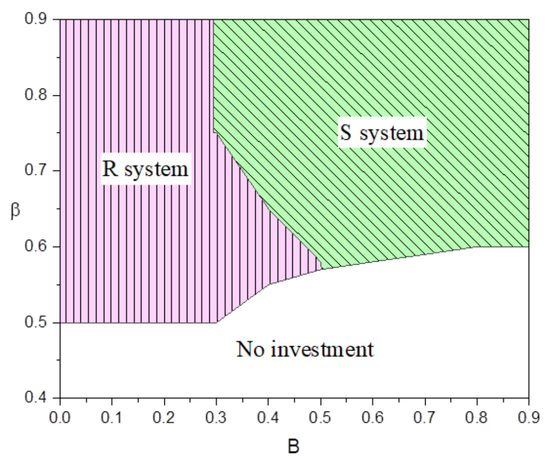
5.3 Strategic Choice and Managerial Implications

Next, we focus on comparative performance and strategic choice in the four scenarios. We investigate the effect of the factorial combination of the parameter pairs by changing the values of each factor from 0.1 to 0.9. We calculate 81 cases to compare the profits of the manufacturer, retailer and supply chain, which is graphically illustrated in Figures 2–4. The results show that the strategic decision can be determined by several factors in the scenarios, which means that the values of the factors influence the investment decision.

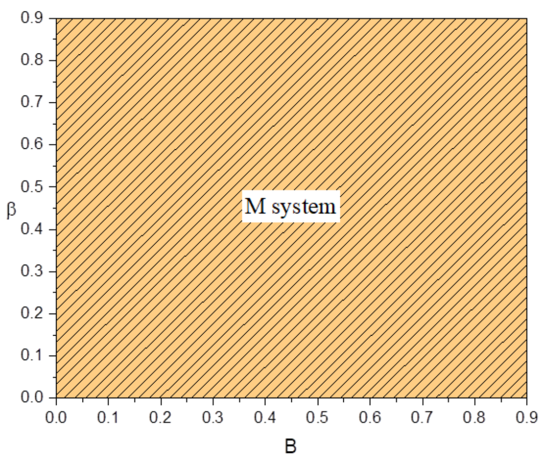
As Figures 2 and 3 illustrate, the manufacturer tends not to invest because of a low sensitivity coefficient of demand β. Figure 2 shows that the manufacturer prefers the R–system for a lower economy of investment in smart technology but prefers the S–system because of the high sensitivity coefficient of demand and economy of investment in smart technology; in other words, it is more expensive to make the production line smarter.

When the manufacturer has a higher investment share and sensitivity coefficient of demand, the S–system generates more profit for the supply chain, whereas when the manufacturer has a lower investment share and higher sensitivity coefficient of demand, the M–system generates more supply chain profits (shown in Figure 3).

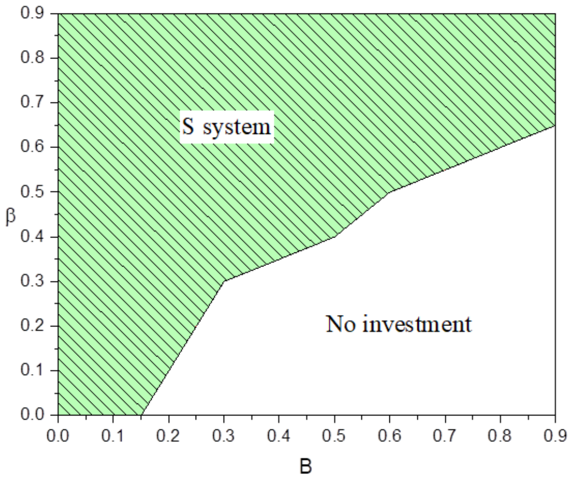
Figures 2–4 show that the retailer always prefers the M– or S–system, wherein the manufacturer fully pays for the investment cost or both parties share the investment cost, respectively, whereas the manufacturer prefers the no investment scenario, R–system, or S–system, wherein there is no investment, the retailer fully pays for the investment cost, or they both share the investment cost, respectively. Therefore, sharing the investment cost is the most appropriate strategy for both the manufacturer and retailer.



(a) Manufacturer’s choice

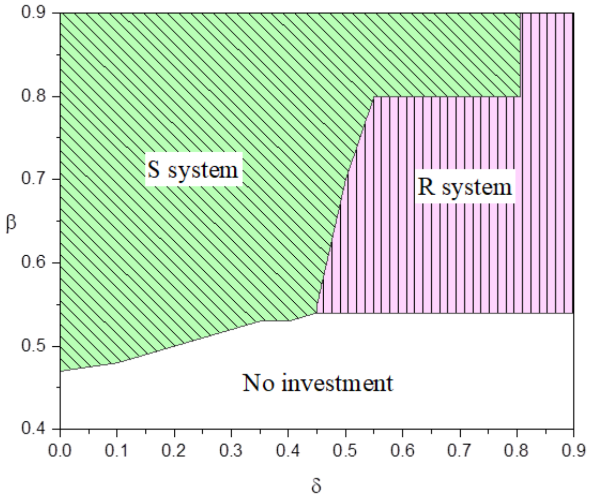


(b) Retailer’s choice

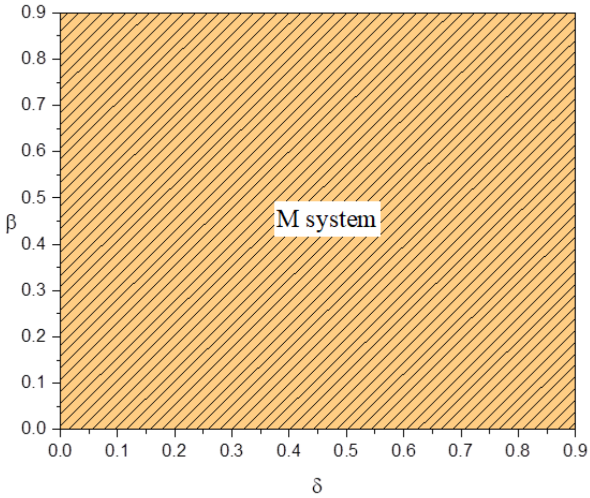


(c) Supply chain choice

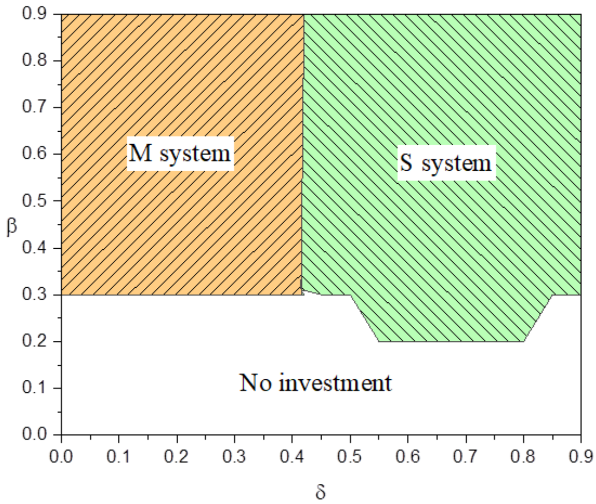
Figure 2. Effect of the economy of investing in smart technology and demand sensitivity on the firm's choice



(a) Manufacturer's choice

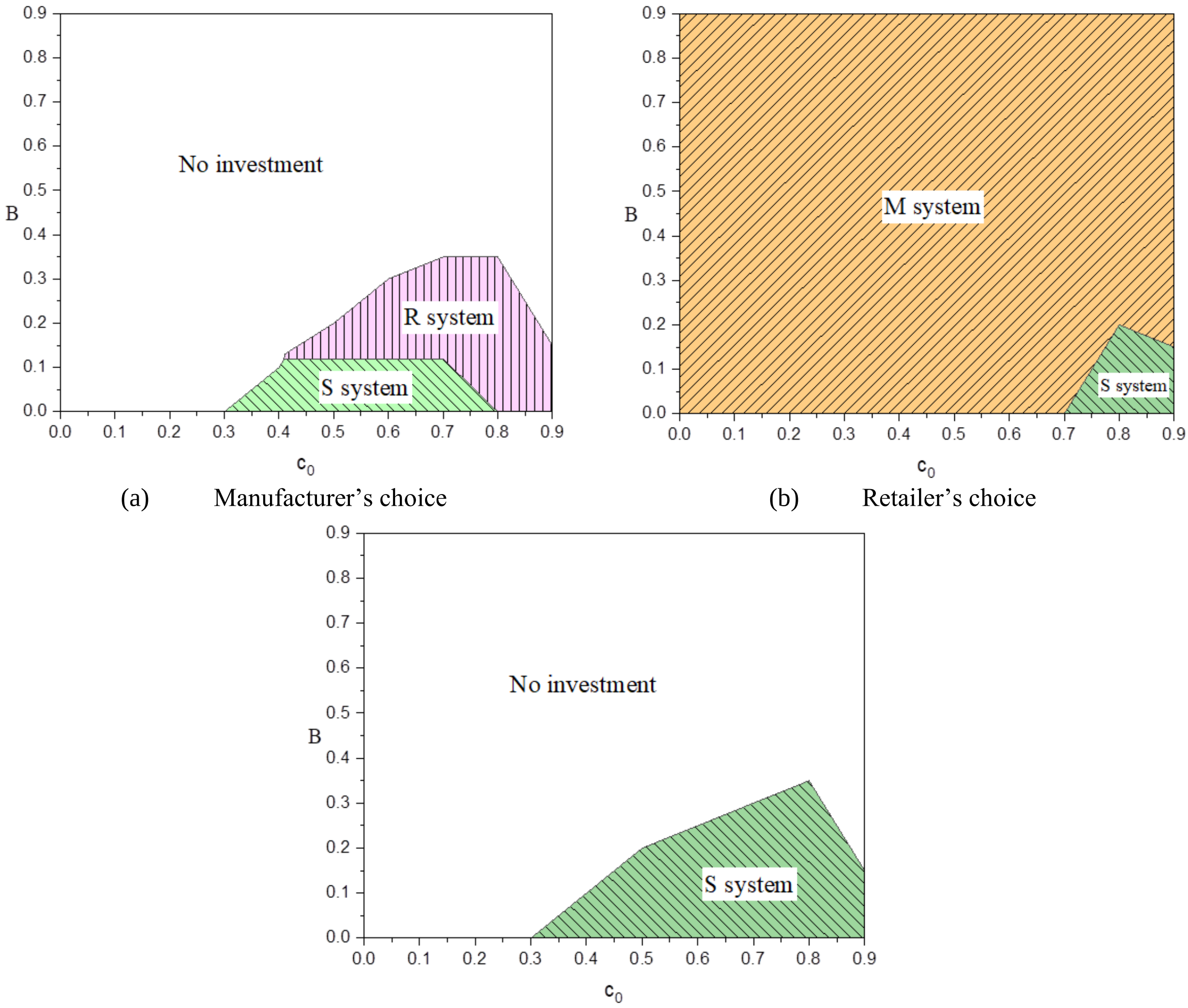


(b) Retailer's choice



(c) Supply chain choice

Figure 3. Effect of the cost-sharing ratio and demand sensitivity on the firm's choice



(a) Manufacturer's choice (b) Retailer's choice

(c) Supply chain choice

Figure 4. Effect of the original production cost and the economy of investing in smart technology on the firm's choice

We derive the following managerial insights from the results obtained in this study.

(i) The investment decision depends on many factors, such as the sensitivity coefficient of demand, original manufacturing cost without investment, opportunity cost and the economy of investing in smart technology. Collaboration between the manufacturer and the retailer in sharing the investment cost benefits the entire supply chain (Chakraborty *et al*., 2019). Therefore, the manufacturer should invite the retailer to collaborate by explaining the benefits rather than bearing the investment cost alone. However, it is challenging to fairly split the benefits across the supply chain members. In this regard, Brandenburger and Nalebuff (2021) proposed an equal split of the additional value.

(ii) Our results also suggest that investment in smart technology should be undertaken when the sensitivity coefficient of demand is medium or high. It is important for the manufacturer to consider the original manufacturing cost without investment and the economy of investing in smart technology. When the original manufacturing cost is high and investment in smart technology is low, it is always better to invest in smart technology to reduce production costs.

5.4 Pareto Improvement

Part 5.3 shows that the S–system is the most appropriate choice for both the manufacturer and retailer. Therefore, we further analyze investment cost sharing. The profits of both the manufacturer and retailer are higher under the S–scenario when the share is between 56% and 92% than under the no-investment scenario (shown in Figure 5).

Figure 6 shows that under the S–system, the supply chain always earns more profits than under the no-investment scenario.

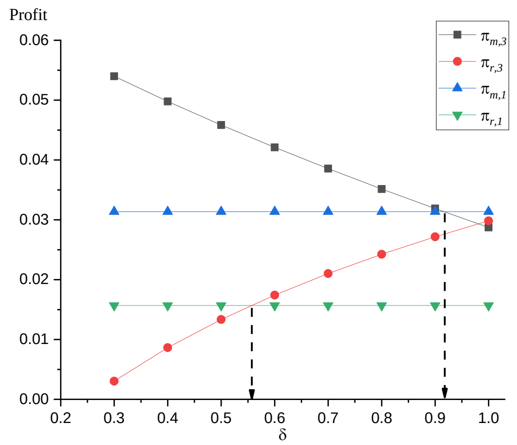


Figure 5. Effect of the cost-sharing percentage on the profits of the manufacturer and retailer under the no-investment and investment-sharing scenarios.

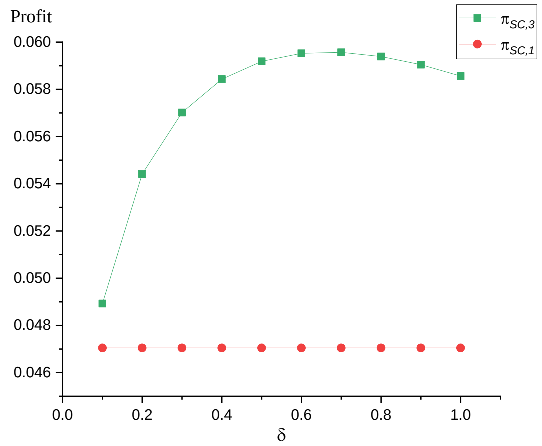


Figure 6. Effect of the cost-sharing percentage on the profits of the supply chain under the no-investment and investment-sharing scenarios.

6. CONCLUSION

This study proposes a model wherein the production cost is considered a decision variable instead of a parameter, and investing in digital technology is incorporated in a clear formula. We first study a case wherein no investment is undertaken as the benchmark and then investigate three investment scenarios (i.e., M–, S– and R–systems). Furthermore, we apply an investment-sharing mechanism to coordinate between the supply chain members such that they achieve Pareto improvement and determine the optimal sharing ratio for each entity in the supply chain. We also propose the appropriate cost-sharing range under the S–system, wherein both the manufacturer and retailer achieve Pareto improvement.

This study makes three important contributions to the literature. First, to the best of our knowledge, this study is the first to investigate the pricing problem under different scenarios of investing in smart technology. Second, this study considers an investment-sharing mechanism for supply chain coordination. Third, we suggest an optimal sharing ratio to help the supply chain members achieve Pareto improvement. Future studies could examine different coordination contracts, such as trade-ins, revenue sharing, rebates and cooperative advertisement; different types of demand functions with uncertainty; and a variety of game theoretical model settings. Another promising research direction might involve conducting an empirical survey on investment sharing in a variety of industries.

REFERENCES

Adhikari, A., Bisi, A., and Avittathur, B. (2020). Coordination Mechanism, Risk Sharing, and Risk Aversion in A Five-Level Textile Supply Chain under Demand and Supply Uncertainty. *European Journal of Operational Research*, 282(1): 93-107. DOI: Https://Doi.Org/10.1016/J.Ejor.2019.08.051

Alqahtani, A. Y., Gupta, S. M., and Nakashima, K. (2019). Warranty and Maintenance Analysis of Sensor Embedded Products Using Internet of Things in Industry 4.0. *International Journal of Production Economics*, 208: 483-499. DOI: Https://Doi.Org/10.1016/J.Ijpe.2018.12.022.

Alvarenga, A., Matos, F., Godina, R., and CO Matias, J. (2020). Digital Transformation and Knowledge Management in The Public Sector. Sustainability, 12(14): 5824.

Andersson, E., Svensson, J., Persson, U., and Lindgren, P. (2020). Risk Sharing in Managed Entry Agreements - A Review of The Swedish Experience. *Health Policy*, 124(4): 404-410. DOI: Https://Doi.Org/10.1016/J.Healthpol.2020.02.002.

Aoki, K. and Lennerfors, T. T. (2013). Global Business The New, Improved Keiretsu. *Harvard Business Review,* 91(9): 109.

Apple Inc. (2021). People and Environment in Our Supply Chain, 2021 Annual Progress Report. DOI: Https://Www.Apple.Com/Supplier-Responsibility/Pdf/Apple_SR_2021_Progress_Report.Pdf.

Ardolino, M., Rapaccini, M., Saccani, N., Gaiardelli, P., Crespi, G., and Ruggeri, C. (2018). The Role of Digital Technologies for The Service Transformation of Industrial Companies. *International Journal of Production Research,* 56(6): 2116-2132. DOI: Https://Doi.Org/10.1080/00207543.2017.1324224.

Arruñada, B. and Vázquez, X. H. (2006). When Your Contract Manufacturer Becomes Your Competitor. *Harvard Business Review,* 84(9): 135.

Avilés-Sacoto, S. V., Avilés-González, J. F., Garcia-Reyes, H., Bermeo-Samaniego, M. C., Cañizares-Jaramillo, A. K., and Izquierdo-Flores, S. N. (2019). A Glance of I4.0 at Supply Chain and Inventory Management. *International Journal of Industrial Engineering: Theory, Applications and Practice*, *26*(4). DOI: Https://Doi.Org/10.23055/Ijietap. 2019.26.4.5175.

Avinadav, T. (2020). The Effect of Decision Rights Allocation on A Supply Chain of Perishable Products under A Revenue-Sharing Contract. *International Journal of Production Economics*, 225: 107587. DOI: Https://Doi.Org/10.1016/ J.Ijpe.2019.107587

Bart, N., Chernonog, T., and Avinadav, T. (2021). Revenue-Sharing Contracts in Supply Chains: A Comprehensive Literature Review. *International Journal of Production Research,* 59(21): 6633-6658. DOI: Https://Doi.Org/10.1016/ J.Ifacol.2019.11.425

Beltagui, A., Kunz, N., and Gold, S. (2020). The Role of 3D Printing and Open Design on Adoption of Socially Sustainable Supply Chain Innovation. *International Journal of Production Economics*, 221: 107462. DOI: Https://Doi.Org/10.1016/J.Ijpe.2019.07.035

Billington, P. J. (1987). The Classic Economic Production Quantity Model with Setup Cost as A Function of Capital Expenditure. *Decision Sciences*, 18(1): 25-42. DOI: Https://Doi.Org/10.1111/J.1540-5915.1987.Tb01501.X.

Biswal, A. K., Jenamani, M., and Kumar, S. K. (2021). Product Subsidy and Expected Consumption with Inventory Inaccuracy: Implications of RFID Adoption in Indian Public Distribution System. *Computers and Industrial Engineering,* 159: 107527. DOI: Https://Doi.Org/10.1016/J.Cie.2021.107527

Blakey-Milner, B., Gradl, P., Snedden, G., Brooks, M., Pitot, J., Lopez, E., Leary, M., Berto, F., and Du Plessis, A. (2021). Metal Additive Manufacturing in Aerospace: A Review. *Materials and Design*, 209: 110008. DOI: Https://Doi.Org/ 10.1016/J.Matdes.2021.110008

Brandenburger, A. and Nalebuff, B. (2021). The Rules of Co-Opetition. *Harvard Business Review*, 99(1): 48-57.

Buer, S.V., Semini, M., Strandhagen, J. O., and Sgarbossa, F. (2021). The Complementary Effect of Lean Manufacturing and Digitalisation on Operational Performance. *International Journal of Production Research,* 59(7): 1976-1992. DOI: Https://Doi.Org/10.1080/00207543.2020.1790684

Cachon, G. P. (2003). Supply Chain Coordination with Contracts. *Handbooks in Operations Research and Management Science*, 11: 227-339. DOI: Https://Doi.Org/10.1016/S0927-0507(03)11006-7

Castellano, D., Gallo, M., Grassi, A., and Santillo, L. C. (2019). Batching Decisions in Multi-Item Production Systems with Learning Effect. *Computers and Industrial Engineering*, 131: 578-591. DOI: Https://Doi.Org/10.1016/ J.Cie.2018.12.068

Castelo-Branco, I., Cruz-Jesus, F., and Oliveira, T. (2019). Assessing Industry 4.0 Readiness in Manufacturing: Evidence for The European Union. *Computers in Industry*, 107: 22-32. DOI: Https://Doi.Org/10.1016/J.Compind.2019.01.007.

Chakraborty, T., Chauhan, S. S., and Ouhimmou, M. (2019). Cost-Sharing Mechanism for Product Quality Improvement in A Supply Chain under Competition. International Journal of Production Economics, 208: 566-587. DOI: Https://Doi.Org/10.1016/J.Ijpe.2018.12.015.

Chen, S., Brahma, S., Mackay, J., Cao, C., and Aliakbarian, B. (2020). The Role of Smart Packaging System in Food Supply Chain. *Journal of Food Science*, 85(3): 517-525. DOI: Https://Doi.Org/10.1111/1750-3841.15046.

Chen, Z.Y. and Kuo, R. (2019). Combining SOM and Evolutionary Computation Algorithms for RBF Neural Network Training. *Journal of Intelligent Manufacturing*, 30(3): 1137-1154. DOI: Https://Doi.Org/10.1007/S10845-017-1313-7.

Cheng, F., Chen, T., and Chen, Q. (2022). Cost-Reducing Strategy Or Emission-Reducing Strategy? The Choice of Low-

Carbon Decisions under Price Threshold Subsidy. *Transportation Research Part E: Logistics and Transportation Review,* 157: 102560. DOI: Https://Doi.Org/10.1016/J.Tre.2021.102560

Cheng, X., Yan, C.B., and Gao, F. (2021). Energy Cost Optimisation in Two-Machine Bernoulli Serial Lines under Time-Of-Use Pricing. *International Journal of Production Research*, 1-17. DOI: Https://Doi.Org/10.1080/00207543.2021.1936265

Choi, T. M., Wallace, S. W., and Wang, Y. (2018). Big Data Analytics in Operations Management. *Production and Operations Management*, 27(10): 1868-1883. DOI: Https://Doi.Org/10.1111/Poms.12838

Dutta, P., Choi, T. M., Somani, S., and Butala, R. (2020). Blockchain Technology in Supply Chain Operations: Applications, Challenges and Research Opportunities. *Transportation Research Part E: Logistics and Transportation Review*, 142: 102067. DOI: Https://Doi.Org/10.1016/J.Tre.2020.102067

Fan, B., Yang, W., and Zhang, Z. (2019). Solving The Two-Stage Hybrid Flow Shop Scheduling Problem Based on Mutant Firefly Algorithm. *Journal of Ambient Intelligence and Humanized Computing*, 10(3): 979-990. DOI: Https://Doi.Org/10.1007/S12652-018-0903-3

Fites, D. V. (1996). Make Your Dealers Your Partners. *Harvard Business Review,* 74(2): 84.

Fu, H. and Ma, Y. (2019). Optimization and Coordination of Decentralized Supply Chains with Vertical Cross-Shareholding. *Computers and Industrial Engineering*, 132: 23-35. DOI: Https://Doi.Org/10.1016/J.Cie.2019.04.009

Fu, Q., Sim, C. K., and Teo, C. P. (2018). Profit Sharing Agreements in Decentralized Supply Chains: A Distributionally Robust Approach. *Operations Research*, *66*(2), 500-513. DOI: Https://Doi.Org/10.1287/Opre.2017.1677

Gadaleta, M., Pellicciari, M., and Berselli, G. (2019). Optimization of The Energy Consumption of Industrial Robots for Automatic Code Generation. *Robotics and Computer-Integrated Manufacturing*, 57: 452-464. DOI: Https://Doi.Org/10.1016/J.Rcim.2018.12.020

Giannakis, M. and Louis, M. (2016). A Multi-Agent Based System with Big Data Processing for Enhanced Supply Chain Agility. *Journal of Enterprise Information Management*, 29(5).

Giri, B. C. and Bardhan, S. (2016). Coordinating A Two-Echelon Supply Chain with Environmentally Aware Consumers. *International Journal of Management Science and Engineering Management*, 11(3): 178–185. DOI: Https://Doi.Org/10.1080/17509653.2015.1004203

Hanh, N. T. M., Chen, J. M., and Van Hop, N. (2022). Pricing Strategy and Order Quantity Allocation with Price-Sensitive Demand in Three-Echelon Supply Chain. *Expert Systems with Applications*, 206: 117873. DOI: Https://Doi.Org/10.1016/J.Eswa.2022.117873.

Hu, J., Hu, Q., and Xia, Y. (2019). Who Should Invest in Cost Reduction in Supply Chains? *International Journal of Production Economics*, 207: 1-18. DOI: Https://Doi.Org/10.1016/J.Ijpe.2018.10.002.

Jha, G., Soren, S., and Mehta, K. D. (2020). Life Cycle Assessment of Sintering Process for Carbon Footprint and Cost Reduction: A Comparative Study for Coke and Biomass-Derived Sintering Process. *Journal of Cleaner Production*, 259: 120889. DOI: Https://Doi.Org/10.1016/J.Jclepro.2020.120889

Kamble, S. S., Gunasekaran, A., and Gawankar, S. A. (2018). Sustainable Industry 4.0 Framework: A Systematic Literature Review Identifying The Current Trends and Future Perspectives. *Process Safety and Environmental Protection*, 117: 408-425. DOI: Https://Doi.Org/10.1016/J.Psep.2018.05.009.

Kaur, I. and Singh, P. (2021). State-Of-The-Art in Heat Exchanger Additive Manufacturing. *International Journal of Heat and Mass Transfer*, 178: 121600. DOI: Https://Doi.Org/10.1016/J.Ijheatmasstransfer.2021.121600.

Kovács, G. and Kot, S. (2017). Facility Layout Redesign for Efficiency Improvement and Cost Reduction. *Journal of Applied Mathematics and Computational Mechanics*, 16(1): 63-74. 10.17512/Jamcm.2017.1.06.

Leng, J., Liu, Q., Ye, S., Jing, J., Wang, Y., Zhang, C., Zhang, D., and Chen, X. (2020). Digital Twin-Driven Rapid Reconfiguration of The Automated Manufacturing System Via An Open Architecture Model. *Robotics and Computer-*

Integrated Manufacturing, 63: 101895. DOI: Https://Doi.Org/10.1016/J.Rcim.2019.101895.

Li, X. (2020). Reducing Channel Costs by Investing in Smart Supply Chain Technologies. *Transportation Research Part E: Logistics and Transportation Review*, 137: 101927. DOI: Https://Doi.Org/10.1016/J.Tre.2020.101927.

Li, X., Li, Y., Cai, X., and Shan, J. (2016). Service Channel Choice for Supply Chain: Who Is Better Off by Undertaking The Service? *Production and Operations Management*, *25*(3), 516-534. DOI: Https://Doi.Org/10.1111/Poms.12392

Lim, K. Y. H., Le, N. T., Agarwal, N., and Huynh, B. H. (2021). Digital Twin Architecture and Development Trends on Manufacturing Topologies. *Implementing Industry 4.0*, 259.

Liu, J., Chen, X., Xiang, Y., Huo, D., and Liu, J. (2021). Optimal Planning and Investment Benefit Analysis of Shared Energy Storage for Electricity Retailers. *International Journal of Electrical Power and Energy Systems*, 126: 106561. DOI: Https://Doi.Org/10.1016/J.Ijepes.2020.106561

Lu, Y., Xu, X., and Wang, L. (2020). Smart Manufacturing Process and System Automation–A Critical Review of The Standards and Envisioned Scenarios. *Journal of Manufacturing Systems*, 56: 312-325. DOI: Https://Doi.Org/10.1016/J.Jmsy.2020.06.010

Ma, S., He, Y., Gu, R., and Li, S. (2021). Sustainable Supply Chain Management Considering Technology Investments and Government Intervention. *Transportation Research Part E: Logistics and Transportation Review,* 149: 102290. DOI: Https://Doi.Org/10.1016/J.Tre.2021.102290

Mahmud, R., Toosi, A. N., Ramamohanarao, K., and Buyya, R. (2019). Context-Aware Placement of Industry 4.0 Applications in Fog Computing Environments. *IEEE Transactions on Industrial Informatics*, 16(11): 7004-7013.

Manresa, A., Bikfalvi, A., and Simon, A. Investigating The Impact of New Technologies and Organizational Practices on Operational Performance: Evidence from Spanish Manufacturing Companies. *Cent Eur J Oper Res,* 29: 1317–1327 (2021). DOI: Https://Doi.Org/10.1007/S10100-020-00692-8

Matalycki, M., Kiturko, O., and Basiewicz, N. (2010). Application of HM-Networks in Problems of Transport Logistics. *Scientific Research of The Institute of Mathematics and Computer Science*, 9(2): 153-172.

Mcguire, T. W. and Staelin, R. (1983). An Industry Equilibrium Analysis of Downstream Vertical Integration. *Marketing Science*, 2(2): 161-191. DOI: Https://Doi.Org/10.1287/Mksc.2.2.161

Min, Z., Luong, H. T., and Shafiq, M. (2019). Coordination of A Multi-Echelon Supply Chain Using Spanning Revenue Sharing Contract. *International Journal of Industrial Engineering: Theory, Applications and Practice*, 26(6). DOI: Https://Doi.Org/10.23055/Ijietap.2019.26.6.3901

Mohammadi, H., Ghazanfari, M., Pishvaee, M. S., and Teimoury, E. (2019). Fresh-Product Supply Chain Coordination and Waste Reduction Using A Revenue-And-Preservation-Technology-Investment-Sharing Contract: A Real-Life Case Study. *Journal of Cleaner Production*, 213: 262-282. DOI: Https://Doi.Org/10.1016/J.Jclepro.2018.12.120

Netland, T. and Ferdows, K. (2014). What to Expect from Corporate Lean Programs. *MIT Sloan Manage. Rev.* 55(4): 83-89.

Pillai, U. (2015). Drivers of Cost Reduction in Solar Photovoltaics. *Energy Economics*, 50: 286-293. DOI: Https://Doi.Org/10.1016/J.Eneco.2015.05.015

Pistolesi, F., Lazzerini, B., Dalle Mura, M., and Dini, G. (2017). EMOGA: A Hybrid Genetic Algorithm with Extremal Optimization Core for Multiobjective Disassembly Line Balancing. *IEEE Transactions on Industrial Informatics*, 14(3): 1089-1098.

Porter, M. E. (1998). Clusters and The New Economics of Competition (Vol. 76). *Harvard Business Review Boston.*

Porteus, E. L. (1985). Investing in Reduced Setups in The EOQ Model. *Management Science*, 31(8): 998-1010. DOI: Https://Doi.Org/10.1287/Mnsc.31.8.998.

Porteus, E. L. (1986). Optimal Lot Sizing, Process Quality Improvement and Setup Cost Reduction. *Operations Research*, 34(1): 137-144. DOI: Https://Doi.Org/10.1287/Opre.34.1.137.

Ribeiro, V. M. (2016). Exclusion and Regulatory Intervention in Investment Sharing Agreements. *Telecommunications Policy,* 40(10-11): 1032-1051. DOI: Https://Doi.Org/10.1016/J.Telpol.2016.07.002.

Santos, J., Muñoz-Villamizar, A., Ormazábal, M., and Viles, E. (2019). Using Problem-Oriented Monitoring to Simultaneously Improve Productivity and Environmental Performance in Manufacturing Companies. *International Journal of Computer Integrated Manufacturing*, 32(2): 183-193. DOI: Https://Doi.Org/10.1080/0951192X. 2018.1552796

Shafiq, M. and Luong, H. T. (2021). Optimal Quantity Allocation Decision in Revenue Sharing Contract. *International Journal of Industrial Engineering: Theory, Applications and Practice*, 28(6). DOI: Https://Doi.Org/10.23055/ Ijietap.2021.28.6.6847

Shi, X., Yao, S., and Luo, S. (2021). Innovative Platform Operations with The Use of Technologies in The Blockchain Era. *International Journal of Production Research*, 1-19. DOI: Https://Doi.Org/10.1080/00207543.2021.1953182.

Singh, H., Garg, R., and Sachdeva, A. (2018). Supply Chain Collaboration: A State-Of-The-Art Literature Review. *Uncertain Supply Chain Management*, 6(2): 149-180. DOI: Https://Doi.Org/10.5267/J.Uscm.2017.8.002.

Tiwari, P., Sadeghi, J. K., and Eseonu, C. (2020). A Sustainable Lean Production Framework with A Case Implementation: Practice-Based View Theory. *Journal of Cleaner Production*, 277: 123078. DOI: Https://Doi.Org/10.1016/ J.Jclepro.2020.123078.

Wang, X. and Li, D. (2012). A Dynamic Product Quality Evaluation Based Pricing Model for Perishable Food Supply Chains. *Omega*, 40(6): 906–917. DOI: Https://Doi.Org/10.1016/J.Omega.2012.02.001.

Ventura, J. A., Bunn, K. A., Venegas, B. B., and Duan, L. (2021). A Coordination Mechanism for Supplier Selection and Order Quantity Allocation with Price-Sensitive Demand and Finite Production Rates. *International Journal of Production Economics*, 233: 108007. DOI: Https://Doi.Org/10.1016/J.Ijpe.2020.108007.

Vogl, G. W., Weiss, B. A., and Helu, M. (2019). A Review of Diagnostic and Prognostic Capabilities and Best Practices for Manufacturing. *Journal of Intelligent Manufacturing*, 30(1): 79-95. DOI: Https://Doi.Org/10.1007/S10845-016-1228-8.

Xing, G., Xia, B., and Guo, J. (2022). Contract Choice for Upstream Innovation in A Finance - Constrained Supply Chain. *International Transactions in Operational Research*, 29(3): 1897-1914. DOI: Https://Doi.Org/10.1111/Itor.13033.

Zhan, Y. and Tan, K. H. (2020). An Analytic Infrastructure for Harvesting Big Data to Enhance Supply Chain Performance. *European Journal of Operational Research*, 281(3): 559-574. DOI: Https://Doi.Org/10.1016/J.Ejor.2018.09.018

Zhong, Y., Guo, F., Wang, Z., and Tang, H. (2019). Coordination Analysis of Revenue Sharing in E-Commerce Logistics Service Supply Chain with Cooperative Distribution. *Sage Open*, *9*(3): 2158244019870536. DOI: Https://Doi.Org/ 10.1177/2158244019870536

APPENDIX A

Proof of Lemma 1 & Theorem 1:

It is easy to see that $\pi_{r,1} = (p_1 - w_1 - c_r)(\alpha - \beta p_1)$ is concave in p_1. We have $\frac{\partial^2 \pi_{r,1}}{\partial^2 p_1} = -2\beta < 0$. Taking the first derivative subject to p_1, then setting it be equal to 0, we have:

$$\frac{\partial \pi_{r,1}(p_1)}{\partial p_1} = \alpha - p_1\beta - \beta(p_1 - w_1 - c_r) = 0. \tag{49}$$

Solving Equation (49), we have the equilibrium selling price of the retailer $p_1(w_1)$ shown in Lemma 1.
Substituting $p_1(w_1)$ from Lemma 1 into Equation (6), we have $\pi_{m,1}(w_1)$. Generating the first derivative of $\pi_{m,1}(w_1)$ subject to w_1 and setting the result equal to zero, we have:

$$\frac{\partial \pi_{m,1}(w_1)}{\partial w_1} = -\frac{1}{2}\beta(w_1 - c_0) + \frac{1}{2}(\alpha - w_1\beta - \beta c_r) = 0. \quad (50)$$

The unique equilibrium wholesale price of the manufacturer in Equation (8) in Theorem 1 is obtained by solving Equation (50). The unique optimal selling price of the retailer is obtained by substituting Equation (8) into Equation (3).
Checking the values of the second derivatives of Equations (2) and (6) subject to p_1 and w_1, respectively, we can prove the uniqueness of the equilibrium prices:

$$\frac{\partial^2 \pi_{r,1}}{\partial^2 p_1} = -2\beta < 0,$$
$$\frac{\partial^2 \pi_{m,1}}{\partial^2 w_1} = -\beta < 0,$$

which is absolutely negative for $\beta > 0$, and

$$\left(\frac{\partial^2 \pi_{r,1}}{\partial^2 p_1}\right)\left(\frac{\partial^2 \pi_{m,1}}{\partial^2 w_1}\right) - \left(\frac{\partial^2 \pi_{r,1}}{\partial p_1 \partial w_1}\right)\left(\frac{\partial^2 \pi_{m,1}}{\partial w_1 \partial p_1}\right) = 3\beta^2 > 0.$$

The profit function is jointly concave in w_1 and p_1, respectively. Therefore, we can conclude that the solution of the first-order necessary condition is the unique optimal solution. □

Proof of Proposition 1:

The proof can be completed by proving the values of the first derivative of the decision variables p_1^*, w_1^*, and q_1^* subject to the parameter are absolutely negative or positive:

$$\frac{\partial p_1^*}{\partial c_0} = \frac{1}{4} > 0, \frac{\partial p_1^*}{\partial c_r} = \frac{1}{4} > 0, \frac{\partial p_1^*}{\partial \alpha} = \frac{1}{4\beta} > 0,$$
$$\frac{\partial w_1^*}{\partial c_0} = \frac{1}{2} > 0, \frac{\partial w_1^*}{\partial \alpha} = \frac{1}{2\beta} > 0, \frac{\partial w_1^*}{\partial c_r} = -\frac{1}{2} < 0, \text{ and}$$
$$\frac{\partial q_1^*}{\partial \alpha} = \frac{3}{4} > 0, \frac{\partial q_1^*}{\partial c_0} = -\frac{\beta}{4} < 0, \frac{\partial q_1^*}{\partial c_r} = -\frac{\beta}{4} < 0. \ □$$

APPENDIX B

Proof of Lemma 2:

It is easy to see that $\pi_{r,2}(p_2)$ is concave in p_2. We have $\frac{\partial^2 \pi_{r,2}}{\partial^2 p_2} = -2\beta < 0$. Generating the first derivative of Equation (13) subject to p_2 and letting the result equal to zero, we have:

$$\frac{\partial \pi_{r,2}(p_2)}{\partial p_2} = \alpha - p_2\beta + (c_r - p_2 + w_2)\beta = 0. \quad (51)$$

Solving Equation (51), we can find $p_2(w_2)$ in the Lemma 2. □

Proof of Lemma 3:

Obtaining the first-order derivative of Equation (17), we have $\frac{\partial \pi_{m,2}}{\partial c_{m,2}} = \frac{B\theta}{c_{m,2}} + \frac{1}{4}\left(-\alpha + \left(c_{m,2} + c_r\right)\beta\right)$, leading to the following two cases:

(I) When $B \geq \frac{(c_r\beta - \alpha)^2}{16\theta\beta}$, we have $\frac{\partial \pi_{m,2}}{\partial c_{m,2}} \geq 0$, and thus $c_{m,2}^* = c_0$.

(II) When $B < \frac{(c_r\beta - \alpha)^2}{16\theta\beta}$, we have two roots, $c_2 = \frac{\alpha - c_r\beta - \sqrt{-16B\theta\beta + (-\alpha + c_r\beta)^2}}{2\beta}$, $c_2' = \frac{\alpha - c_r\beta + \sqrt{-16B\theta\beta + (-\alpha + c_r\beta)^2}}{2\beta}$, which leads to the two following subcases:

(i) When $c_0 \leq \frac{\alpha}{2\beta} - \frac{c_r}{2}$, we have $c_0 \leq c_2'$, and thus $\pi_{m,2}$ is increasing in $c_{m,2} \epsilon [0, c_2]$ and decreasing in $c_{m,2} \epsilon [c_2, c_2']$. It then follows that $c_{m,2}^* = \min(c_0, c_2)$. By comparing the value of c_0 and $c_2 = \frac{\alpha - c_r\beta - \sqrt{-16B\theta\beta + (-\alpha + c_r\beta)^2}}{2\beta}$, we have $c_{m,2}^* = \begin{cases} c_2 \ if \ B < \frac{c_0(\alpha - c_r\beta - \beta c_0)}{4\theta}, \\ c_0, \quad otherwise. \end{cases}$

(ii) When $c_0 > \frac{\alpha}{2\beta} - \frac{c_r}{2}$, with $B < \frac{c_0(\alpha - c_r\beta - \beta c_0)}{4\theta}$, we have $c_2 \le \frac{1}{2} < c_0 \le c'_2$ and thus $\pi_{m,2}$ is increasing in $c_{m,2} \epsilon [0, c_2]$ and decreasing in $c_{m,2} \epsilon [c_2, c_0]$, which yields $c^*_{m,2} = c_2$; with $\frac{c_0(\alpha - c_r\beta - \beta c_0)}{4\theta} \le B \le \frac{(c_r\beta - \alpha)^2}{16\theta\beta}$, we have $c_2 \le \frac{1}{2} \le c'_2 \le c_0$, which yields $c^*_{m,2} = \begin{cases} c_2 \; if \; B < \frac{(c_0 - c_2)(\beta(c_2 + c_0 + 2c_r) - 2\alpha)}{8\,\theta\, ln(\frac{c_0}{c_2})}, \\ c_0, \quad otherwise, \end{cases}$ after comparing the values of $\pi_{m,2}(c_0)$ and $\pi_{m,2}(c_2)$. Combining the above cases of (I) and (II), we have the result shown in Lemma 3. □

Proof of Theorem 2:

Substituting $c^*_{m,2}$ from Lemma 3 and $p_2(w_2)$ from Lemma 2 into Equation (14b), we have $\pi_{m,2}(w_2)$. Generating the first derivative of $\pi_{m,2}(w_2)$ subject to w_2 and setting the result equal to zero, we have:

$$\frac{\partial \pi_{m,2}(w_2)}{\partial w_2} = \frac{1}{2}(c^*_{m,2} - w_2)\beta + \frac{1}{2}(\alpha - (c_r + w_2)\beta) = 0. \quad (52)$$

The unique equilibrium wholesale price of the manufacturer in Equation (20) is obtained by solving Equation (52). The unique equilibrium selling price of the retailer shown in Equation (21) is obtained by substituting Equation (20) into Equation (15).
Checking the values of the second derivatives of Equations (13) and (14) subject to p_2 and w_2, respectively, we can prove the uniqueness of the equilibrium prices:

$$\frac{\partial^2 \pi_{r,2}}{\partial^2 p_2} = -2\beta < 0,$$
$$\frac{\partial^2 \pi_{m,2}}{\partial^2 w_2} = -\beta < 0,$$

which is absolutely negative for $\beta > 0$, and

$$\left(\frac{\partial^2 \pi_{r,2}}{\partial^2 p_2}\right)\left(\frac{\partial^2 \pi_{m,2}}{\partial^2 w_2}\right) - \left(\frac{\partial^2 \pi_{r,2}}{\partial p_2 \partial w_2}\right)\left(\frac{\partial^2 \pi_{m,2}}{\partial w_2 \partial p_2}\right) = 2\beta^2 > 0.$$

The profit function is jointly concave in w_2 and p_2, respectively. Hence, we can conclude that the solution of the first-order necessary condition is the unique optimal solution. □

Proof of Proposition 2:

To complete the proof, we present that the values of the first derivative of the decision variables p^*_2, w^*_2, and q^*_2 subject to the parameter are absolutely negative or positive.

$$\frac{\partial p^*_2}{\partial c^*_{m,2}} = \frac{1}{4} > 0, \frac{\partial p^*_2}{\partial c_r} = \frac{1}{4} > 0, \frac{\partial p^*_2}{\partial \alpha} = \frac{3}{4\beta} > 0,$$
$$\frac{\partial w^*_2}{\partial c^*_{m,2}} = \frac{1}{2} > 0, \frac{\partial w^*_2}{\partial \alpha} = \frac{1}{2\beta} > 0, \frac{\partial w^*_2}{\partial c_r} = -\frac{1}{2} < 0, \text{ and}$$
$$\frac{\partial q^*_2}{\partial \alpha} = \frac{1}{4} > 0, \frac{\partial q^*_2}{\partial c^*_{m,2}} = -\frac{\beta}{4} < 0, \frac{\partial q^*_2}{\partial c_r} = -\frac{\beta}{4} < 0. \; □$$

APPENDIX C

Proof of Lemma 4:

It is easy to see that $\pi_{r,3}(p_3)$ is concave in p_3. We have $\frac{\partial^2 \pi_{r,3}}{\partial^2 p_3} = -2\beta < 0$. Generating the first derivative of Equation (25) subject to p_3 and setting the result equal to zero, we have:

$$\frac{\partial \pi_{r,3}(p_3)}{\partial p_3} = \alpha - p_3\beta + (c_r - p_3 + w_3)\beta = 0. \quad (53)$$

Solving Equation (53), we can find $p_3(w_3)$ in Lemma 4. □

Proof of Lemma 5:

Taking the first-order derivative of Equation (29), we have $\frac{\partial \pi_{m,3}}{\partial c_{m,3}} = \frac{1}{4}(-\alpha + (c_{m,3} + c_r)\beta) + \frac{B\theta\delta}{c_{m,3}}$, leading to the following two cases:

(I) When $B \geq \frac{(c_r\beta-\alpha)^2}{16\theta\beta\delta}$, we have $\frac{\partial \pi_{m,3}}{\partial c_{m,3}} \geq 0$, and thus $c^*_{m,3} = c_0$.

(II) When $B < \frac{(c_r\beta-\alpha)^2}{16\theta\beta\delta}$, we have two roots, $c_3 = \frac{\alpha - c_r\beta - \sqrt{-16B\theta\beta\delta+(-\alpha+c_r\beta)^2}}{2\beta}$, $c'_3 = \frac{\alpha - c_r\beta + \sqrt{-16B\theta\beta\delta+(-\alpha+c_r\beta)^2}}{2\beta}$, which leads to the following two subcases.

(i) When $c_0 \leq \frac{\alpha}{2\beta} - \frac{c_r}{2}$, we have $c_0 \leq c'_3$, and thus $\pi_{m,3}$ is increasing in $c_{m,3} \epsilon [0, c_3]$ and decreasing in $c_{m,3} \epsilon [c_3, c'_3]$. It then follows that $c^*_{m,3} = \min(c_0, c_3)$. By comparing the value of c_0 and $c_3 = \frac{\alpha - c_r\beta - \sqrt{-16B\theta\beta\delta+(-\alpha+c_r\beta)^2}}{2\beta}$, we have $c^*_{m,3} = \begin{cases} c_3 \ if \ B < \frac{c_0(\alpha - c_r\beta - \beta c_0)}{4\theta\delta}, \\ c_{0,} \quad otherwise. \end{cases}$

(ii) When $c_0 > \frac{\alpha}{2\beta} - \frac{c_r}{2}$, with $B < \frac{c_0(\alpha - c_r\beta - \beta c_0)}{4\theta\delta}$, we have $c_3 \leq \frac{1}{2} < c_0 \leq c'_3$ and thus π_m is increasing in $c_{m,3} \epsilon [0, c_3]$ and decreasing in $c_{m,3} \epsilon [c_3, c_0]$, which yields $c^*_{m,3} = c_1$; with $\frac{c_0(\alpha - c_r\beta - \beta c_0)}{4\theta\delta} \leq B \leq \frac{(c_r\beta-\alpha)^2}{16\theta\beta\delta}$, we have $c_3 \leq \frac{1}{2} \leq c'_3 \leq c_0$, which yields $c^*_{m,3} = \begin{cases} c_3 \ if \ B < \frac{(c_0-c_3)(\beta(c_3+c_0+2c_r)-2\alpha)}{8\,\theta\,\delta\, ln(\frac{c_0}{c_3})}, \\ c_0, \quad otherwise \end{cases}$ after comparing the values of $\pi_{m,3}(c_0)$ and $\pi_{m,3}(c_3)$. We have the result given in Lemma 5 by combining the above cases of (I) and (II). □

Proof of Theorem 3:

Substituting $c^*_{m,3}$ from Lemma 5 and $p_3(w_3)$ from Lemma 4 into Equation (26b), we have $\pi_{m,3}(w_3)$. Generating the first derivative of $\pi_{m,3}(w_3)$ subject to w_3 and setting the result equal to zero, we have:

$$\frac{\partial \pi_{m,3}(w_3)}{\partial w_3} = \frac{1}{2}(c_{m,3} - w_3)\beta + \frac{1}{2}(\alpha - (c_r + w_3)\beta) = 0. \tag{54}$$

The unique optimal wholesale price of the manufacturer in Equation (32) is obtained by solving Equation (54). The unique optimal selling price of the retailer shown in Equation (33) is obtained by substituting Equation (32) into Equation (27).

To prove the uniqueness of the equilibrium prices, we take the second derivatives of (25) and (26b) subject to p_3 and w_3:

$$\frac{\partial^2 \pi_{r,3}}{\partial^2 p_3} = -2\beta < 0,$$
$$\frac{\partial^2 \pi_{m,3}}{\partial^2 w_3} = -\beta < 0,$$

which is absolutely negative for $\beta > 0$, and

$$\left(\frac{\partial^2 \pi_{r,3}}{\partial^2 p_3}\right)\left(\frac{\partial^2 \pi_{m,3}}{\partial^2 w_3}\right) - \left(\frac{\partial^2 \pi_{r,3}}{\partial p_3 \partial w_3}\right)\left(\frac{\partial^2 \pi_{m,3}}{\partial w_3 \partial p_3}\right) = 2\beta^2 > 0.$$

The profit function is jointly concave in w_3 and p_3, respectively. Therefore, we can conclude the solution of the first-order necessary condition is the unique optimal solution. □

Proof of Proposition 3:

The proof can be completed by proving the results of the first derivative of the decision variables p^*_3, w^*_3, q^*_3, $\pi^*_{m,3}$, and $\pi^*_{r,3}$ subject to the parameter are absolutely negative or positive:

$$\frac{\partial p^*_3}{\partial c^*_{m,3}} = \frac{1}{4} > 0, \frac{\partial p^*_3}{\partial c_r} = \frac{1}{4} > 0, \frac{\partial p^*_3}{\partial \alpha} = \frac{3}{4\beta} > 0,$$
$$\frac{\partial w^*_3}{\partial c^*_{m,3}} = \frac{1}{2} > 0, \frac{\partial w^*_3}{\partial \alpha} = \frac{1}{2\beta} > 0, \frac{\partial w^*_3}{\partial c_r} = -\frac{1}{2} < 0,$$
$$\frac{\partial q^*_3}{\partial \alpha} = \frac{1}{4} > 0, \frac{\partial q^*_3}{\partial c^*_{m,3}} = -\frac{\beta}{4} < 0, \frac{\partial q^*_3}{\partial c_r} = -\frac{\beta}{4} < 0,$$
$$\frac{\partial \pi^*_{m,3}}{\partial \delta} = -\theta(A - Bln(c^*_{m,3})) < 0,$$

$\frac{\partial \pi_{r,3}^*}{\partial \delta} = \Theta(A - Bln(c_{m,3}^*)) > 0$. □

APPENDIX D

Proof of Lemma 6:

It is easy to see that $\pi_{r,4}(p_4)$ is concave in p_4. We have $\frac{\partial^2 \pi_{r,4}}{\partial^2 p_4} = -2\beta < 0$. Generating the first derivative of Equation (37b) subject to p_4 and setting the result equal to zero, we have:

$$\frac{\partial \pi_{r,4}(p_4)}{\partial p_4} = \alpha - p_4\beta + (c_r - p_4 + w_4)\beta = 0. \quad (55)$$

Solving Equation (55), we can find $p_4(w_4)$ in Lemma 6.

Proof of Lemma 7:

Taking the first-order derivative of Equation (41), we have $\frac{\partial \pi_{r,4}}{\partial c_{m,4}} = \frac{B\Theta}{c_{m,4}} + \frac{1}{8}(-\alpha + (c_{m,4} + c_r)\beta)$, leading to the following two cases:

(I) When $B \geq \frac{(c_r\beta-\alpha)^2}{32\Theta\beta}$, we have $\frac{\partial \pi_{m,4}}{\partial c_{m,4}} \geq 0$, and thus $c_{m,4}^* = c_0$.

(II) When $B < \frac{(c_r\beta-\alpha)^2}{32\Theta\beta}$, we have two roots, $c_4 = \frac{\alpha-c_r\beta-\sqrt{-32B\Theta\beta+(-\alpha+c_r\beta)^2}}{2\beta}$, $c_4' = \frac{\alpha-c_r\beta+\sqrt{-32B\Theta\beta\delta+(-\alpha+c_r\beta)^2}}{2\beta}$, leading to the following two subcases.

 (i) When $c_0 \leq \frac{\alpha}{2\beta} - \frac{c_r}{2}$, we have $c_0 \leq c_4'$, and thus π_r is increasing in $c_{m,4}\, \epsilon\, [0, c_1]$ and decreasing in $c_{m,4}\, \epsilon\, [c_4, c_4']$. It then follows that $c_{m,4}^* = \min(c_0, c_4)$. By comparing the value of c_0 and $c_4 = \frac{\alpha-c_r\beta-\sqrt{-32B\Theta\beta+(-\alpha+c_r\beta)^2}}{2\beta}$, we have $c_{m,4}^* = \begin{cases} c_4 \; if \; B < \frac{c_0(\alpha-c_r\beta-\beta c_0)}{8\Theta}, \\ c_0, \quad otherwise. \end{cases}$

 (ii) When $c_0 > \frac{\alpha}{2\beta} - \frac{c_r}{2}$, with $B < \frac{c_0(\alpha-c_r\beta-\beta c_0)}{8\Theta}$, we have $c_4 \leq \frac{1}{2} < c_0 \leq c_4'$ and thus $\pi_{r,4}$ is increasing in $c_{m,4}\, \epsilon\, [0, c_4]$ and decreasing in $c_{m,4}\, \epsilon\, [c_4, c_0]$, which yields $c_{m,4}^* = c_4$; with $\frac{c_0(\alpha-c_r\beta-\beta c_0)}{8\Theta} \leq B \leq \frac{(c_r\beta-\alpha)^2}{32\Theta\beta}$, we have $c_4 \leq \frac{1}{2} \leq c_4' \leq c_0$, which yields $c_{m,4}^* = \begin{cases} c_4 \; if \; B < \frac{(c_0-c_4)(\beta(c_4+c_0+2c_r)-2\alpha)}{16\,\Theta\, ln(\frac{c_0}{c_4})}, \\ c_0, \quad otherwise, \end{cases}$ after comparing the values of $\pi_{r,4}(c_0)$ and $\pi_{r,4}(c_4)$. We have the result given in Lemma 7 by combining the above cases of (I) and (II). □

Proof of Theorem 4:

Substituting $c_{m,4}^*$ from Lemma 7 and $p_4(w_4)$ from Lemma 6 into Equation (37b), we have $\pi_{m,4}(w_4)$. Generating the first derivative of $\pi_{m,4}(w_4)$ subject to w_4 and setting the result equal to zero, we have:

$$\frac{\partial \pi_{m,4}(w_4)}{\partial w_4} = \frac{1}{2}(c_{m,4} - w_4)\beta + \frac{1}{2}(\alpha - (c_r + w_4)\beta) = 0. \quad (56)$$

The unique optimal wholesale price of the manufacturer in Equation (44) is obtained by solving Equation (56). The unique optimal selling price of the retailer shown in Equation (45) is obtained by substituting Equation (44) into Equation (39).

To prove the uniqueness of the equilibrium prices, we take the second derivatives of (37b) and (38) subject to p_4 and w_4:

$$\frac{\partial^2 \pi_{r,4}}{\partial^2 p_4} = -2\beta < 0$$
$$\frac{\partial^2 \pi_{m,4}}{\partial^2 w_4} = -\beta < 0$$

which is absolutely negative for $\beta > 0$, and

$$\left(\frac{\partial^2 \pi_{r,4}}{\partial^2 p_4}\right)\left(\frac{\partial^2 \pi_{m,4}}{\partial^2 w_4}\right)-\left(\frac{\partial^2 \pi_{r,4}}{\partial p_4 \partial w_4}\right)\left(\frac{\partial^2 \pi_{m,4}}{\partial w_4 \partial p_4}\right)=2\beta^2>0.$$

The profit function is jointly concave in w_4 and p_4, therefore, the solution of the first-order necessary condition is the unique optimal solution. □

Proof of Proposition 4:

The proof can be completed by proving the results of the first derivative of the decision variables p_4^*, w_4^*, and q_4^* subject to the parameter are absolutely negative or positive:

$$\frac{\partial p_4^*}{\partial c_{m,4}^*}=\frac{1}{4}>0, \frac{\partial p_4^*}{\partial c_r}=\frac{1}{4}>0, \frac{\partial p_4^*}{\partial \alpha}=\frac{3}{4\beta}>0,$$
$$\frac{\partial w_4^*}{\partial c_{m,4}^*}=\frac{1}{2}>0, \frac{\partial w_4^*}{\partial \alpha}=\frac{1}{2\beta}>0, \frac{\partial w_4^*}{\partial c_r}=-\frac{1}{2}<0,$$
$$\frac{\partial q_4^*}{\partial \alpha}=\frac{1}{4}>0, \frac{\partial q_4^*}{\partial c_{m,4}^*}=-\frac{\beta}{4}<0, \frac{\partial q_4^*}{\partial c_r}=-\frac{\beta}{4}<0.$$

APPENDIX E

Proof of Proposition 5:

Comparing c_0, $c_{m,2}^*$, $c_{m,3}^*$, and $c_{m,4}^*$, we have $c_{m,3}^* \leq c_{m,2}^* \leq c_{m,4}^* \leq c_0$. From that, comparing $w_1^*, w_2^*, w_3^*, w_4^*$ provided in (8), (20), (32), and (44), respectively, and comparing $p_1^*, p_2^*, p_3^*, p_4^*$ provided in (9), (21), (33), and (45), respectively, we have Proposition 5. □

Proof of Proposition 6:

To prove Proposition 6, we compare the profit of the manufacturer under four systems pair by pair.
Comparing (47) and (11), it is straightforward that $\pi_{m,4}^* \geq \pi_{m,1}^*$ since $c_{m,4}^* \leq c_0$.
Comparing (35) and (11), we have:

$$\pi_{m,3}^* - \pi_{m,1}^* = -\frac{1}{8}(c_0-c_3)(-2\alpha+(c_0+c_3+2c_r)\beta)-A\delta\theta+B\delta\theta ln(c_3). \quad (57)$$

Solve $\pi_{m,3}^* - \pi_{m,1}^* \geq 0$, we have (ii) in Proposition 6.
Similarly, comparing (23) and (11), we have:

$$\pi_{m,2}^* - \pi_{m,1}^* = -\frac{1}{8}(c_0-c_2)(-2\alpha+(c_0+c_2+2c_r)\beta)-A\theta+B\theta ln(c_2). \quad (58)$$

Solve $\pi_{m,2}^* - \pi_{m,1}^* \geq 0$, we have (iii) in Proposition 6.
Comparing (35) and (47), we have:

$$\pi_{m,3}^* - \pi_{m,4}^* = \frac{1}{8}(c_3-c_4)(-2\alpha+(c_3+c_4+2c_r)\beta)-A\delta\theta+B\delta\theta ln(c_3). \quad (59)$$

Solve $\pi_{m,3}^* - \pi_{m,4}^* \geq 0$, we have (iv) in Proposition 6.
Comparing (23) and (47), we have:

$$\pi_{m,2}^* - \pi_{m,4}^* = \frac{1}{8}(c_2-c_4)(-2\alpha+(c_2+c_4+2c_r)\beta)-A\theta+B\theta ln(c_2). \quad (60)$$

Solve $\pi_{m,2}^* - \pi_{m,4}^* \geq 0$, we have (v) in Proposition 6.
Comparing (35) and (23), we have:

$$\pi_{m,3}^* - \pi_{m,2}^* = -\frac{1}{8}(c_2-c_3)(-2\alpha+(c_2+c_3+2c_r)\beta)-A(-1+\delta)\theta-B\theta ln(c_2)+B\delta\theta ln(c_3). \quad (61)$$

Solve $\pi_{m,2}^* - \pi_{m,4}^* \geq 0$, we have (v) in Proposition 6. □
The proof of Propositions 7 and 8 are analogous to that of Proposition 6 and thus omitted.