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OPTIMAL INBOUND/OUTBOUND PRICING MODEL FOR REMANUFACTURING IN A CLOSED-LOOP SUPPLY CHAIN

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The paper presents a model for optimizing inbound and outbound pricing for closed-loop supply chains that remanufacture reusable products. Remanufacturers create reusable products from returned used products and sell the products "as new" to manufacturers or consumers. By implementing a return subsidy, remanufacturers can encourage the consumer to return used products. Demand for the as-new components often depends on the selling price and inventory. The available inventory increases as the subsidy increases and as the price decreases. Our model can determine the optimal subsidy and selling price for used and remanufactured products, respectively. Our model uses the Karush–Kuhn–Tucker conditions to solve its nonlinear problem. Sensitivity analysis reveals how different parameters affect profit under model-optimized conditions.

Keywords: Reusable products; Remanufacturing; Return subsidy; Price and stock sensitivity

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1. INTRODUCTION

Environmental consciousness and consumer pressure have caused many businesses to attempt to mitigate their ecological damage by reusing, remanufacturing, and recycling products. Many products, such as car batteries, printer cartridges, and computers, are reused through remanufacturing. Simultaneously, companies are reducing costs through product or component reuse. These companies implement reverse logistics in their businesses. For example, HP has developed a business model based on selling remanufactured printer cartridges (Francie, 2015). Eastman Kodak also remanufactured recyclable cameras with great success, collecting single-use cameras after the film had developed (Guide and Van Wassenhove, 2002).

Remanufacturing transforms used products into "as-new" products (Thierry, 1995). Remanufacturing enterprises have been in business for over 70 years, replacing dilapidated and deteriorating components. Remanufacturing fulfills both green and financial objectives, especially in the US automobile industry. Not only do the savings in component costs help to reduce disposal and energy costs, but they also generate significant additional earnings. Specifically, the United States International Trade Commission (2012) reported that US companies earned total revenues from the sales of remanufactured products of US$43 billion in 2011.

At VivaTech 2021, Apple CEO Tim Cook announced a new environmental goal to produce all Apple products from recyclable components by 2030. Recycling products and green consumption are trends in the consumer market that encourage companies to actively develop green technologies to gain the favor of green consumers and reduce environmental damage. However, uncertainty regarding market demand, remanufactured product quality, and recycled component availability also affect manufacturing costs. We present a model that simplifies reverse logistics manufacturing for a supply chain with feedback. The model considers remanufacturing production and subsidies for consumer returns. This inbound/outbound pricing model can be used to optimize such subsidies and the selling price of remanufactured goods to maximize profit. The

model and analysis explore factors related to the optimal return subsidy and the relationships within the supply chain among cost components through feedback. Our research conclusions can help to improve the business strategies of such remanufacturing firms.

Section 2 details our proposed problem and presents an evaluation of related literature. Section 3 details the model development. Section 4 provides a mathematical example, results, and discussion. Section 5 describes a sensitivity analysis revealing the effects of various parameters on our proposed closed-loop remanufacturing system. Finally, Section 6 presents our study's contributions, limitations, and future research directions.

2. LITERATURE SURVEY

Studies have extensively researched product remanufacturing and reuse, including practical case studies. Considerable research has addressed closed-loop supply chain problems. Mitra (2007) modeled the maximum expected price and revenue from recovered products. Choi *et al.* (2007) devised a strategy for recycling products to meet stationary demand. Kim *et al.* (2006) investigated reverse logistics for reusable components and developed a mixed-integer program to maximize total savings. Such manufacturers have two choices for supply: ordering required components from other suppliers or remanufacturing "as-new" products. Patel *et al.* (2022) also used mixed-integer programming to optimize distribution center locations, production, storage, and scheduling for an integrated supply model.

Geyer *et al*. (2007) investigated a closed-loop chain from a system perspective by considering capacity planning and remanufacturing. Rubio and Corominas (2008) optimized manufacturing-remanufacturing policies for a lean production environment. Wang *et al.* (2011) optimized a hybrid manufacturing/remanufacturing system under uncertain demand. Shi *et al.* (2011) optimized manufacturing and pricing decisions for a closed-loop system. Yu *et al.* (2019) considered an integrated manufacturer-retailer closed-loop system with price-sensitive return and demand. Nahr *et al.* (2020) developed a multi-objective, multi-product, multi-period, closed-loop green supply chain under uncertainty and discounts. Suzanne *et al.* (2020) considered circular economy in production planning, and Konstantaras *et al.* (2021) optimized inventory decisions for a closed-loop supply chain under a carbon tax regulatory mechanism.

Zanoni *et al*. (2012) developed an optimal lot scheduling policy for multiple remanufactured products. Jayant *et al.* (2012) surveyed reverse logistics and identified research gaps. Huang *et al.* (2014) investigated a unified inventory policy for the secondary market. Hong *et al.* (2016) devised an equilibrium model for electronic scraps in a four-stage network. Wang *et al.* (2016) investigated a system with a feedback loop. In general, the number of literature focusing on remanufacturing is increasing (Dwicahyani *et al*., 2017; Zouad *et al.*, 2018; Shu *et al.*, 2018; Turki *et al.*, 2018; Taleizadeh *et al.*, 2019).

Sana and Goyal (2014) investigated a (Q, r, L) inventory policy for lead-time-dependent demand. Sana *et al.* (2014) developed a triple-tiered supply chain model for multiple products and multiple players. Sana (2016) developed a dual-stage inventory model for manufacturers and retailers to use to maximize profits by determining the optimal lot size/production quantity and reorder point. Yan and Cao (2017) empirically analyzed the benefits of cooperation between the vendors and buyers of recycled products to address a research gap concerning scenarios featuring information asymmetry. Li *et al.* (2018) applied the Stackelberg policy to investigate the system. Mohammed *et al.* (2018) investigated robust optimization for a closed-loop supply chain with carbon footprint considerations. Berk *et al.* (2021) presented a fuzzy design problem with multiple objectives and a fuzzy environment. Several other studies have addressed integration issues in the forward and reverse supply chains (Zhao *et al*., 2016; Zhao *et al*., 2016; Zhao *et al*., 2019; Taha *et al*., 2020; Seyda and Saliha, 2021).

Another consideration of our model is stock-dependent demand. Datta *et al.* (1998) researched stock-dependent demand with promotions under varying situations. Urban (2005) proposed a periodic model of demand that is affected by stock level. Dye and Ouyang (2005) established an economic order quantity (EOQ) model for perishable products in which stock affected demand. Chang *et al.* (2006) improved upon that study by developing a general model for stock-dependent demand. Wu *et al.* (2006) evaluated a model with partial backordering. Goyal and Chang (2009) constructed a model with stock-dependent demand for optimizing ordering and transfer policy. Sarkar *et al.* (2010) used several methodologies to investigate a supplier with stock-dependent demand. Subsequently, Sarkar (2012) proposed a model of EOQ policy with delayed payment, stock-dependent demand, and imperfect products. Kabirian (2012) examined linear/exponential price-sensitive demand for a manufacturing system. Georgiadis and Athanasiou (2013) investigated a closed-loop supply chain under capacity constraints. Finally, Yan *et al.* (2017) proposed a forecasting method using artificial intelligence.

Finally, we review the literature on financial incentive problems. Klausner and Hendrickson (2000) argued that financial incentives in buy-back campaigns affect the number of returns. They suggested that an appropriate incentive is crucial to attracting a sufficient number of used products for remanufacturing. Although several studies have acknowledged that effective incentives are crucial for remanufacturing enterprises (Guide and Van Wassenhove, 2001; Yan and Cao, 2017; Shu *et al.*, 2018), few analytical models have been developed.

Several studies have investigated product return problems. Fleischmann *et al.* (1997) discussed mathematical models of remanufacturing with feedback. Savaskan *et al.* (2004) investigated a system with returned products: used products were

classified as acquired from (i) customer returns, (ii) retailers, or (iii) recycling agents, in which replenishment depended on the product price and incentives for returns. Their study illustrated how supply chain structures affect the collection and return rate of used products. According to the literature, optimal incentive allocation is a common research problem that presents numerous operational challenges. Kaya (2010) proposed three reverse logistics models for remanufacturing wherein products are returned through the supply chain. Acar *et al.* (2015) proposed a methodology based on integer programming; return numbers varied with the incentives provided by the manufacturer. Chan *et al.* (2020) synchronized a random demand model of a single supplier and multiple buyers. The researchers created a mathematical model for developing coordination strategies under conditions of inventory shortage and surplus, as well as a solution algorithm for their model.

To delineate the contributions of our study, we compare the problem presented herein with those in the literature, as shown in Table 1.

Table 1. Manufacturing models in the literature and our proposed model

Authors (year)	Closed-loop supply chains	Remanufacturing	Demand	Return-subsidy policy
Dwicahyani *et al.* (2017).	Yes	Yes	Deterministic	No
Yan & Cao (2017)	No	No	Subsidy & price-dependent	Yes
Yan *et al.* (2017)	No	No	Uncertainty	No
Li *et al.* (2018)	Yes	No	Deterministic	No
Mohammed *et al.* (2018)	Yes	No	Uncertainty	No
Shu *et al.* (2018)	Yes	Yes	Subsidy & carbon tax-dependent	Yes
Turki *et al.* (2018)	Yes	Yes	Time-dependent	No
Zouadi *et al.* (2018)	Yes	Yes	Deterministic	No
Yu *et al.* (2019).	Yes	Yes	Price-dependent	No
Taleizadeh *et al.* (2019).	Yes	Yes	Stochastic demand	No
Zhao *et al.* (2019)	No	No	Deterministic	No
Suzanne *et al.* (2020).	Yes	Yes	Overview	Overview
Nahr *et al.* (2020)	Yes	Yes	Uncertainty	No
Chan *et al.* (2020)	Yes	Yes	Deterministic	No
Taha *et al.* (2020)	No	No	Deterministic	No
Berk *et al.* (2021)	Yes	No	Uncertainty	No
Konstantaras *et al.* (2021)	Yes	Yes	Time-dependent	No
Seyda and Saliha (2021)	No	No	Uncertainty	No
Proposed model	Yes	Yes	Subsidy-dependent	Yes

Our model differs from those of other studies in the following ways. Instead of using constant demand and traditional production policies, our model employs a non-constant demand, an incentive policy (return subsidy), and remanufacturing. Such assumptions in the model regarding demand, reasonable incentives, and remanufacturing are reasonable because collected returns often contain poor-quality products. Herein, we model the effect of a subsidy mechanism for incentivizing product returns and how changing parameters affect costs.

3. MODEL DESIGN

Figure 1 illustrates a reverse logistics production system with feedback. At a remanufacturing plant, returned and used products are collected, inspected, tested, and classified into two groups: products that qualify for remanufacturing, and defective products, which are discarded. The components are inspected, organized, replaced or reassembled at the reproduction stage, and then tested for conformity to specifications. Remanufacturers offer subsidies to promote recycling; the stock of returned products increases linearly with the return subsidy.

For remanufactured products, the sales rate decreases as price and stock increase. The notation used in our problem definition, that is, profit maximization based on the cost of remanufacturing and subsidy paid per returned unit, is presented as follows:

We use the following notation for modeling:

ρ = selling price of a remanufactured product (\$/unit)
S = subsidy for product returns (\$/unit), $0 \leq S < \rho$
P = maximum external reference price for new products
Q = available stock of returned products
C = annual manufacturing capacity of the remanufacturer
ξ = the acceptable quality level for order quantity Q
m = direct cost of the recycling process (\$/unit)
r = salvaged value (\$/unit), $r < S$
Rev = revenue from sold remanufactured products
ETP = expected net profit from remanufactured products

Our model is based on the following assumptions:

(1) The unit subsidy S is paid regardless of the quality of the returned product.
(2) A single product is produced.
(3) A single period is considered.
(4) The available stock Q increases as follows with the subsidy: $Q = Q(S) = a + b \cdot S$ (Abad and Jaggi, 2003). Here, a and b ($b > 0$) are scale parameters that are constants specified by the manufacturer. The relationship between stock and subsidy is illustrated in Figure 2. The capacity of the remanufacturer is limited as $\xi \cdot Q \leq C$.
(5) The demand D for a remanufactured product is sensitive to stock and the external reference price of a corresponding new product (Mitra, 2007).

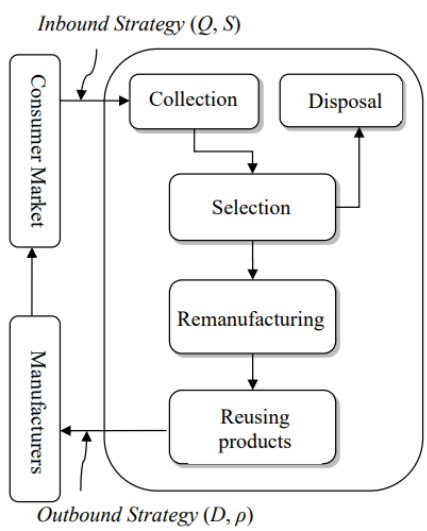

Figure 1. Closed-loop remanufacturing system

(6) The direct cost comprises the expenses directly related to the recycling process (including collection, screening, transportation, storage, handling, and remanufacturing costs). The direct cost increases proportionally with output.
(7) Some returns are defective and are equally probable over the interval $[\alpha, \beta]$. Thus, the acceptable quality level ξ is uniformly distributed over $[\alpha, \beta]$, in which the upper and lower bounds of this interval are 1 and 0, respectively. The probability density of continuous uniform distribution follows the format (Mohr *et al.*, 2021):

$$f(\xi) = \begin{cases} \dfrac{1}{b-a} & a \leq \xi \leq b \\ 0, & \text{otherwise} \end{cases}$$

When $a = 0$, and $b = 1$, it simplifies to:

$$f(\xi) = \begin{cases} 1, & 0 \leq \xi \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Note: in which if $\xi = 0$, no products are accepted for sale, and all are returned and remanufactured, and if $\xi = 1$, all products are accepted for sale, and none are returned or remanufactured.

(8) The return subsidy is less than the price of a remanufactured product; specifically, $S < \rho$, $0 \le \rho$, and $0 \le S$.

(9) The probability of selling a remanufactured product decreases with price and stock.

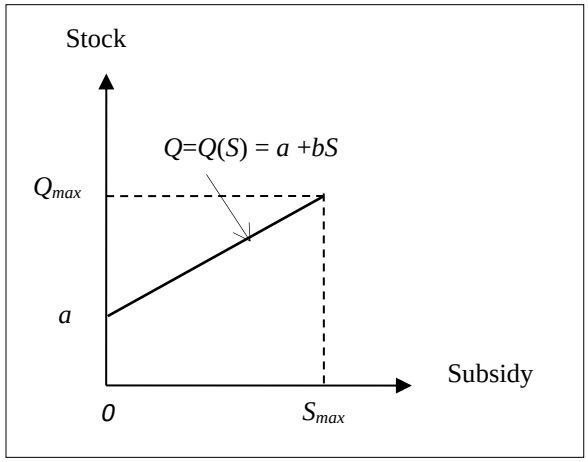

Figure 2. Returns stock quantity shown to increase with subsidy level

In this model, the available stock of collected returns Q is determined by the formula $Q = Q(S) = a + b \cdot S$. The expression shows that the recycled stock increases linearly with the subsidy. The minimum available stock is a, when $S = 0$; the maximum available stock is Q_{max}, when $S = S_{max}$. The demand for remanufactured products is assumed to depend on price and stock. The probability of selling a remanufactured product is $(1 - \frac{\rho}{P}) \cdot (1 - \frac{\xi \cdot Q}{C})$, where P is the maximum external reference price for a corresponding new product. The expected sales can be calculated as follows:

$$D(\rho, S) = (1 - \frac{\rho}{P}) \cdot (1 - \frac{\xi \cdot Q}{C}) \cdot \xi \cdot Q \tag{1}$$

The revenue from sold remanufactured products is as follows:

$$\text{Revenue} = \left(1 - \frac{\rho}{P}\right) \cdot \left(1 - \frac{\xi \cdot Q}{C}\right) \cdot \xi \cdot Q \cdot \rho \tag{2}$$

Total profit TP is given by the following formula:

TP = total revenue + total salvage value − total direct cost − total return subsidy.

We use direct cost to simplify complex costs and facilitate mathematical modeling. The direct cost increases proportionally with output.

Through computation of the individual components for the total revenue, salvage value, direct cost, and total return subsidy, TP can be obtained as

$$TP(\rho, S) = (1 - \frac{\rho}{P})(1 - \frac{\xi \cdot Q}{C}) \cdot \xi \cdot Q \cdot \rho + (1 - \xi) \cdot Q \cdot r - \xi \cdot Q \cdot m - S \cdot Q \tag{3}$$

The expected value of $TP(\rho, S)$ is $ETP(\rho, S)$:

$$\begin{aligned} ETP(\rho, S) &= (1 - \frac{\rho}{P})(E[\xi] \cdot Q \cdot \rho - \frac{E[\xi^2] \cdot Q^2 \cdot \rho}{C}) + (1 - E[\xi])Q \cdot r - E[\xi] \cdot Q \cdot m - S \cdot Q \\ &= \left(1 - \frac{\rho}{P}\right)\left\{\left(\frac{\alpha+\beta}{2}\right) \cdot (a + b \cdot S) \cdot \rho - \left(\frac{(\alpha-\beta)^2}{12} + \frac{(\alpha+\beta)^2}{4}\right) \cdot \frac{(a+b \cdot S)^2}{C} \cdot \rho\right\} \\ &\quad + r \cdot (a + b \cdot S) - (\frac{\alpha+\beta}{2})(r + m)(a + b \cdot S) - S \cdot (a + b \cdot S) \end{aligned} \tag{4}$$

where $E[\xi] = \mu = \frac{(\alpha+\beta)}{2}$, $V[\xi] = \sigma^2 = \frac{(\alpha-\beta)^2}{12}$, and $E[\xi^2] = V[\xi] + [E[\xi]]^2 = \sigma^2 + \mu^2 = \frac{(\alpha-\beta)^2}{12} + \frac{(\alpha+\beta)^2}{4}$ for ξ uniformly distributed over [α, β].

The optimization problem can be represented as

$$\begin{aligned} Max \quad & ETP(\rho, S) \\ & = (1-\tfrac{\rho}{P})(E[\xi]\cdot Q\cdot\rho - \tfrac{E[\xi^2]\cdot Q^2\cdot\rho}{C}) + (1-E[\xi])Q\cdot r - E[\xi]\cdot Q\cdot m - S\cdot Q \\ & = (1-\frac{\rho}{P})\left\{\left(\frac{\alpha+\beta}{2}\right)\cdot(a+b\cdot S)\cdot\rho - \left(\frac{(\alpha-\beta)^2}{12} + \frac{(\alpha+\beta)^2}{4}\right)\cdot\frac{(a+b\cdot S)^2}{C}\cdot\rho\right\} \\ & \quad + r\cdot(a+b\cdot S) - (\frac{\alpha+\beta}{2})(r+m)(a+b\cdot S) - S\cdot(a+b\cdot S) \end{aligned} \tag{5}$$

subject to

$$\rho \le P \tag{6}$$

$$\xi\cdot Q \le C, \text{ and} \tag{7}$$

$$0 \le r < S < \rho. \tag{8}$$

The Karush–Kuhn–Tucker (KKT) conditions are used in optimization problems with constraints such as bounds and inequalities. To solve a price-constrained problem with a return subsidy, we apply the KKT method, as illustrated in Appendix A. We use the Hessian matrix to prove optimality and demonstrate that the remanufacturer's expected profit is concave in ρ and S.

Lemma 1. *The remanufacturer's anticipated profit* ETP *under the return subsidy policy is concave in* ρ. *Thus, the concavity holds under the following conditions:*
Let $0 \le S \le S_{max}$. Here, S_{max} is the upper bound of the return subsidy:

$$S_{max} = \frac{\left(\frac{\alpha+\beta}{2}\right)\cdot C}{\left(\frac{(\alpha-\beta)^2}{12} + \frac{(\alpha+\beta)^2}{4}\right)\cdot b} - \frac{a}{b}$$

Proof.
To demonstrate concavity, the sufficient condition $\frac{\partial^2 ETP}{\partial\rho^2} \le 0$ validates

$$\left(\frac{\alpha+\beta}{2}\right)(a+bS) - \left(\frac{1}{C}\right)\left(\frac{(\alpha-\beta)^2}{12} + \frac{(\alpha+\beta)^2}{4}\right)(a+bS)^2 \ge 0 \tag{9}$$

thus,

$$S \le \frac{\left(\frac{\alpha+\beta}{2}\right)\cdot C}{\left(\frac{(\alpha-\beta)^2}{12} + \frac{(\alpha+\beta)^2}{4}\right)\cdot b} - \frac{a}{b} \tag{10}$$

Let the maximum return subsidy be S_{max}.

$$S\frac{\left(\frac{\alpha+\beta}{2}\right)\cdot C}{\left(\frac{(\alpha-\beta)^2}{12} + \frac{(\alpha+\beta)^2}{4}\right)\cdot b}\frac{a}{b}_{max} \tag{11}$$

Here, S_{max} is the upper bound of the return subsidy. With the sufficient condition $\frac{\partial^2 ETP}{\partial\rho^2} \le 0$, when $S \le S_{max}$, concavity property holds.

When $S \le S_{max}$, the concavity property holds.

Lemma 2. *The remanufacturer's expected profit* ETP *under the return subsidy policy is concave in* S.

Proof. The second-order derivative of (4), with respect to *S*, yields

$$\frac{\partial^2 ETP}{\partial S^2} = -\frac{2}{C}\left(\left(1-\frac{\rho}{P}\right)\left(\frac{(\alpha-\beta)^2}{12}+\frac{(\alpha+\beta)^2}{4}\right)\cdot\rho\cdot b^2\right) - 2b \tag{12}$$

Because $\frac{1}{C}\left(\left(1-\frac{\rho}{P}\right)\left(\frac{(\alpha-\beta)^2}{12}+\frac{(\alpha+\beta)^2}{4}\right)\cdot\rho\cdot b^2\right) > 0$, and $b > 0$, the second-order derivative of (4) is negative; hence, the total expected profit of the remanufacturer *ETP* is concave in *S*.

Proposition 1. *Equation (4) has the optimal solution* ρ* *and* S*, *which achieves global maximum profit for the remanufacturer; the concavity and Hessian matrix* $(\partial^2 ETP/\partial\rho^2)\ \ (\partial^2 ETP/\partial S^2) - (\partial^2 ETP/\partial S\partial\rho)^2 > 0$ are fulfilled.

Proof. Refer to Appendix B.

From (B1) and (B2), (B5) can be confirmed, thus demonstrating the concavity of $ETP(\rho, S)$ with the optimal values ρ^* and S^*.

Lemma 3. *If the return subsidy is not applied, the remanufacturer's expected total profit* ETP *is concave only for* ρ*; a closed-form solution* ρ^* *must exist that maximizes* ETP.

Proof. Let *S* = 0. From (4),

$$\begin{aligned} ETP(\rho) &= (1-\tfrac{\rho}{P})(E[\xi]\cdot a\cdot\rho - \tfrac{E[\xi^2]\cdot a^2\cdot\rho}{C}) + (1-E[\xi])\cdot a\cdot r - E[\xi]\cdot a\cdot m \\ &= (1-\tfrac{\rho}{P})\left\{\left(\tfrac{\alpha+\beta}{2}\right)\cdot a\cdot\rho - \left(\tfrac{(\alpha-\beta)^2}{12}+\tfrac{(\alpha+\beta)^2}{4}\right)\cdot\tfrac{a^2}{C}\cdot\rho\right\} + r\cdot a - (\tfrac{\alpha+\beta}{2})(r+m)\cdot a \end{aligned} \tag{13}$$

When we equate the first derivative of (13), with respect to ρ, to zero as $\frac{dETC}{d\rho} = 0$, we obtain the closed-form solution $\rho^* = P/2$.

4. RESULTS AND DISCUSSION

We select the base-case values of some critical parameters, referring to Mitra (2007). A detailed sensitivity analysis is described in Section 5. The parameters are as follows:

Maximum unit price of a remanufactured product, *P*	= \$100/unit
Annual production capacity of the remanufacturer, *C*	= 300 units
Total direct cost for the recycling process, *m*	= \$50/unit
Salvage value per defective product, *r*	= \$1/unit
Scale parameter, *a*	= 100
Scale parameter, *b*	= 50

The acceptable quality level ξ can be any value in the range [α, β] (where α = 0 and β = 1). Ξ is assumed to be uniformly distributed, with its probability density function being

$$f(\xi) = \begin{cases} 1, & 0 \le \xi \le 1 \\ 0, & \text{otherwise} \end{cases}$$

From (5) under the KKT conditions, we derive the optimal solution ρ^*= \$50/unit and S^*= \$1.32/unit. The number of collected returns is Q^* = 128 units. By applying the return subsidy policy, the manufacturer can expect a total profit of ETP^* = \$560/year. Without applying the return subsidy, the manufacturer can expect a total profit of ETP^* = \$476/year. The percentage of profit increase *PPI* is 17.65%. The expected net profit is 36.65%. The optimal solutions and the subsidy policy are summarized in Table 2. The expected total profit is plotted in three dimensions in Figure 3.

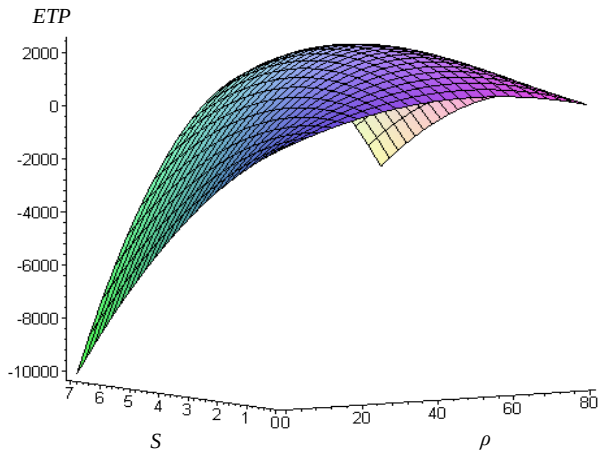

Figure 3. Graphical representation of convex *ETP* (where ρ^*= 50, S^*= 1.32)

Table 2. Optimal solutions with and without return subsidy (a = 100)

Description	Without Return- Subsidy Policy	With return-subsidy policy										
		b ≈ 0	b = 10	b = 20	b = 30	b = 40	b = {50}	b = 60	b = 70	b = 80	b = 90	b = 100
ρ^*	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0
S^*	N.A.	1.06	1.06	1.06	1.13	1.29	1.32	1.31	1.28	1.23	1.18	1.14
Q^*	88	88	97	106	116	131	128	153	162	170	176	182
Rev	1361	1361	1481	1597	1727	1911	2088	2177	2275	2357	2426	2486
ETP	476	476	488	492	505	534	560	584	605	623	639	654
PPI (%)	N.A.	0	2.52	3.36	6.09	12.18	17.65	22.69	27.10	30.88	34.24	37.39
ROI (%)	53.77	53.77	49.15	44.51	41.33	38.78	36.65	36.67	36.24	35.93	35.75	35.69

{ }: base column; $PPI = (ETP^*_{with} - ETP^*_{without})/ETP^*_{without}$
ROI: return on investment = (*revenue – total cost*)/*total cost*

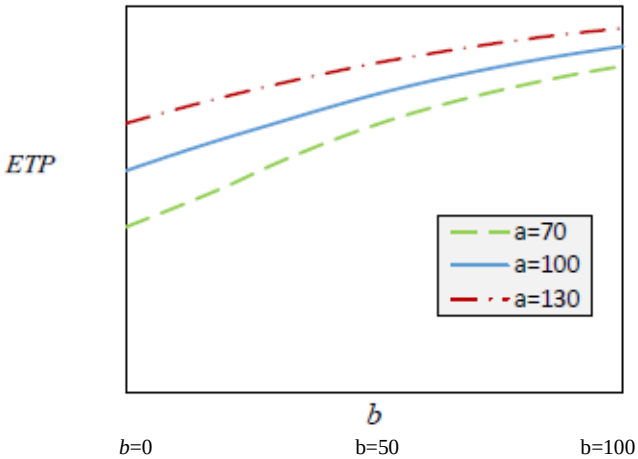

Figure 4. Expected annual profit with various scale parameters a as the scale parameter b increases

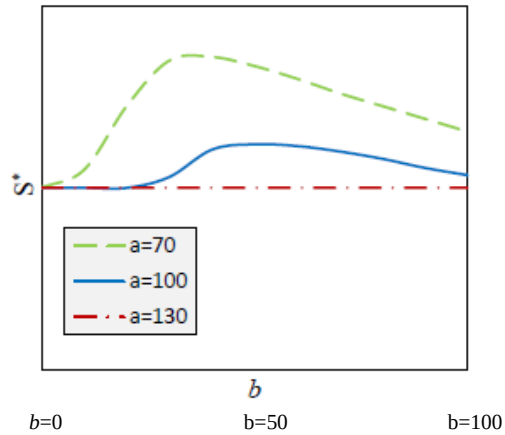

Figure 5. Optimal return subsidy S^* with various scale parameters a, as the scale parameter b increases

The key conclusions are as follows:

(1) As illustrated in Table 2, the manufacturer earns more profit by applying the return subsidy than by not applying it when a = 100 and b > 20. Thus, the lower bound of b is $b_{a=100}^{min}$. For a = 70 and a = 130, the lower bounds $b_{a=70}^{min}$ and $b_{a=130}^{min}$ can be derived from (4) and (C3), respectively.

(2) As depicted in Figure 4, *ETP* increases with b under various scale parameters a. $ETP_{a=130}$ is always higher than $ETP_{a=100}$ and $ETP_{a=70}$, and $ETP_{a=100}$ >$ETP_{a=70}$. The expected total profit increases as the scale parameter a increases.

(3) Figure 4 shows that *ETP* converges as b increases for all scale parameters.

(4) The scale parameters a and b can be used to develop a marketing index for the remanufactured product. Tables 4, 5, and 6 demonstrate that these parameters affect revenue significantly.

(5) The *PPI* increases with the increasing b value but decreases with the increasing a value. The range of *PPI* is from -12% to 74% for a = 70, -13% to 36% for a = 100 and -14% to 17% for a = 130, respectively.

(6) As b approaches 0, *PPI* approaches zero. The corresponding *ETP* for a = 70, a = 100, and a = 130 is less than the *ETP* without the return-subsidy policy until b exceeds 10, 20, and 30, respectively. As b increases, the changes in *PPI* with the scale parameters tend to be larger.

(7) As depicted in Figure 5, as the scale parameter b increases, the optimal subsidies $S_{a=70}^{*}$, $S_{a=100}^{*}$, and $S_{a=130}^{*}$ increase from \$1.06/unit to their peak at \$1.14/unit, and when b approaches a bigger number, S^* drops to \$1.06/unit; the same pattern occurs when b approaches 0. It is noted that $S_{a=70}^{*}$>$S_{a=100}^{*}$>$S_{a=130}^{*}$.

The proof of the optimal solution without the subsidy policy is given in Appendix C.

5. SENSITIVITY ANALYSIS

This section examines the effect of critical closed-loop model parameters through sensitivity and statistical analysis. Our model of a return subsidy policy can optimize the decision variables ρ and S and dependent variables Q, *Rev*, *TC*, and *ETP* for predetermined parameter variables in set $\Phi = \{P, C, m, r, a, b, \alpha, \beta\}$; these are denoted by ρ^*, S^*, Q, *Rev*, *TC*, and *ETP*. The changes in ρ, S, Q, *Rev*, *TC*, and *ETP* are analyzed for various values of the parameters in set Φ. Table 3 presents the results of a sensitivity analysis performed by increasing or decreasing the parameters in set Φ by 30%. For parameters a and b, the sensitivity analysis results with different C values are shown in Tables 4, 5, and 6. The conclusions are as follows:

(1) As indicated in Table 3, except for m, all of the Φ parameters (i.e., P, C, r, a, b, α, and β) have positive correlations with the percentage of incremental profit change $PIPC = \left\{ETP(\rho^*, S^*) - \underset{default}{ETP}(\rho^*, S^*)\right\} / \underset{default}{ETP}(\rho^*, S^*)$.

(2) *PIPC* is the most sensitive to P among these parameters. When P decreases or increases by 30%, *PIPC* decreases by approximately 60% and increases by 80%, respectively. The next most influential parameter is m. When m decreases or increases by 30%, *PIPC* decreases by approximately 5555% and increases by 6060%, respectively. *PIPC* is also slightly sensitive to C, a, and b.

(3) As shown in Table 3, *PIPC* is least sensitive to r; when r decreases or increases by 30%, *PIPC* decreases by 0.2% and increases by 44%, respectively.

(4) As indicated in Table 4, when C = 300, a borderline occurs between a = 200 and a = 250, below which the subsidy is not feasible.

(5) Additionally, when C = 300, *ETP* is negative when a and b are large, as depicted in Table 4.

(6) As shown in Table 6, when C = 1000, because Q << C, all returned products are beneficial for the return subsidy policy and various a and b values. However, when C < Q, not all returned products are beneficial because these returned products are affected by the key factors a and b.

Table 3. Results of sensitivity analysis

Parameter	Changed (%)	ρ*	*S**	*Q*	*Rev*	*TC*	*ETP*	*PIPC* (%)	*ROI* (%)
	(Default)	50.0	1.32	128	2088	1528	560	NA	36.65
P	-30%	35.0	1.00	150	1596	1365	231	-58.75	16.92
	+30%	65.0	2.02	201	3098	2085	1013	80.86	48.57
C	-30%	50.0	1.00	150	1823	1365	458	-18.21	33.55
	+30%	50.0	1.60	180	2396	1780	616	10.05	34.63
m	-30%	50.0	1.76	188	2358	1455	903	61.30	62.10
	+30%	50.0	1.00	150	1969	1716	253	-54.82	14.74
	-30%	50.0	1.22	161	2063	1504	559	-0.20	37.16

	+30%	50.0	1.30	165	2148	1567	582	3.84	37.12
	-30%	50.0	1.68	154	2009	1540	469	-16.25	30.45
a	+30%	50.0	1.00	180	2281	1638	643	14.82	39.26
	-30%	50.0	1.14	140	1862	1303	559	-0.11	42.94
b	+30%	50.0	1.22	179	2273	1681	592	5.73	35.23
	-30%	50.0	1.24	162	1892	1369	523	-6.59	38.21
α	+30%	50.0	1.20	160	2262	1648	614	9.57	37.22
	-30%	50.0	1.28	164	1841	1333	509	-9.20	38.16
β	+30%	50.0	1.12	156	2276	1655	621	10.91	37.53

PIPC: percentage of incremental profit change = $\left\{ETP(\rho^*, S^*) - \underset{default}{ETP}(\rho^*, S^*)\right\} / \underset{default}{ETP}(\rho^*, S^*)$
ROI: return on investment = $(ETP - TC)/TC$

Table 4. Sensitivity of *ETP* to *a* and *b* (*C* = 300)

a \ *b*	10	30	50	70	90	110
0	119	283	390	466	522	565
50	279	392	477	539	585	621
100	459	518	560	618	652	680
150	593	634	666	691	709	720
200	680	701	715	722	721	712
250	720	722	718	706	686	658
300	712	696	673	642	603	557
350	658	624	582	532	474	410
400	557	504	443	374	298	215
450	410	337	257	170	76	(27)
500	215	123	25	(81)	(195)	(316)

( ) indicates negative profit, and the gray background represents a downward trend in *ETP*.

Table 5. Sensitivity of *ETP* to *a* and *b* (*C* = 600)

a \ *b*	10	30	50	70	90	110
0	144	378	561	708	828	928
50	337	524	687	819	928	1020
100	582	694	825	939	1035	1117
150	805	887	976	1067	1147	1217
200	1001	1072	1139	1203	1266	1322
250	1170	1231	1287	1339	1387	1430
300	1314	1364	1409	1450	1488	1520
350	1430	1469	1504	1535	1562	1583
400	1520	1548	1573	1593	1609	1621
450	1583	1602	1616	1625	1630	1631
500	1621	1628	1631	1630	1625	1615

The gray background represents a downward trend in *ETP*.

Table 6. Sensitivity of *ETP* to *a* and *b* (*C* = 1000)

a \ *b*	10	30	50	70	90	110
0	155	416	626	797	941	1064
50	365	577	766	923	1056	1169
100	634	764	920	1058	1177	1265
150	884	977	1088	1202	1305	1394
200	1109	1192	1271	1356	1439	1515
250	1309	1382	1452	1518	1580	1640
300	1485	1549	1609	1665	1717	1765
350	1637	1691	1742	1788	1830	1868
400	1765	1809	1850	1886	1919	1947
450	1868	1903	1934	1961	1983	2002
500	2327	2382	2435	2484	2531	2576

6. SUMMARY AND CONCLUSIONS

On January 7, 2003, both the European Parliament and the Council of the European Union issued Directive 2002/96/EC on waste electrical and electronic equipment (WEEE 1.0), which encouraged the recycling of waste electrical and electronic equipment. On July 24, 2012, the European Commission published European Parliament and Council Regulation

2012/19/EU as an amendment of the former directive (WEEE 2.0). After a further revision came into effect on August 15, 2018, the WEEE directive covered nearly all electrical and electronic devices. With the implementation of a hazardous substances ban and waste recycling regulations, green products have become integral to this industry's future. Therefore, we propose a generalized pricing model for product reuse considering a return subsidy policy, the price and stock sensitivity of demand, and the imperfect nature of recycled products. Through closed-loop remanufacturing, manufacturers can sell reusable products on the secondary market. Our inbound/outbound pricing model provides the optimal value of a return subsidy and the optimal price for selling reused products. Our result indicates a PPI of 17.24% under such a return subsidy policy. The sensitivity analysis revealed that profit is most affected by the maximum sale price for remanufactured products and least affected by the salvage value.

Our research provides several managerial insights: (i) Customers always expect a price difference between a remanufactured product and a new one; thus, the price of a new product affects the revenue from corresponding remanufactured products significantly. If the external reference price of a new product is higher, remanufacturing yields greater profit. (ii) Greater profits are achieved by reducing remanufacturing costs. (iii) The subsidy price also significantly affects remanufacturing revenue. Managers can use scale factors a and b to determine optimal pricing for return subsidies, especially when the manufacturing capacity is low. (iv) Because profit is least sensitive to the salvage value, managers should not spend resources improving it. Our proposed model has potential applications to in revenue management for product reuse in a green remanufacturing supply chain. Therefore, this study can enable remanufacturing to realize financial goals in addition to environmental ones. In future research, our model can be extended to other reverse logistics systems to account for multiple products and market environments.

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