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ONE-TIME ORDER INVENTORY MODEL FOR DETERIORATING AND SHORT MARKET LIFE ITEMS WITH TRAPEZOIDAL TYPE DEMAND RATE

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Determining the end of the sales period for a one-time order inventory policy for technology products that see rapid innovation and improvement, such as smartphones, is a vital decision. While the market life cycle is short, with long lead times and expensive deliveries. Such situations can force the number of orders to be few or even only once. Products with the latest technology consist of many components that allow for deterioration from the start. This study discusses the effect of the market life cycle, as indicated by the trapezoidal demand rate, on deteriorating item inventory policies. This study will provide new insights into inventory policy. Mathematical models with a non-linear generalized reduced gradient approach can find the optimal end of the selling period and the order size to achieve maximum profit. A sensitivity analysis showed several findings that provide insight for management.

Keywords: Inventory Model, Deteriorating Item, Trapezoidal Type, Ramp Type, Market Life Cycle, Product Life Cycle.

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1. INTRODUCTION

The market life cycle of advanced technological products is getting shorter, for example, smartphone products. This reduction in the life cycle is related to competition and market saturation. The saturation of the smartphone market causes vendors to search for various technological innovations (Kwak, 2020). Therefore inventory management faces the challenges of a reduced product life cycle (Aviles-Sacoto *et al*., 2019). Historically, the development of smartphones has seen new products launched almost every year (Carton *et al*., 2018); it then changed to about every half year, and even in recent years there are only about three months between new products. GSMArena.com (Access by January 2021) estimated one smartphone brand's launch time and prices and the estimation for low and medium classes that new products launch every three months. The market life cycle of the products with new technology cannot be predicted by standard seasonality, but rather it depends on the release date (Carton *et al*., 2018). The many features of a smartphone allow this technology product to develop incredibly fast. This rapid development forces the launch of new products that continually get faster. In the end, this reduces the demand for old products or the experience of obsolescence (Goyal and Giri, 2001). The demand rate influenced by the market life cycle can be seen in Figure 1. There are three phases: the first phase is the increasing demand rate (IDR) phase; the second phase is the mature phase, which is the constant demand rate (CDR); and the third phase is the decreasing demand rate (DDR).

Supply chains for technology products that have rapid innovation and improvement can involve multi-nationalities; for example, a smartphone distributor can order products from manufacturers in different countries (Carton *et al.*, 2018). Shipping products between countries can take a long time or have expensive delivery costs and large lot sizes. Alipour *et al.* (2020) saw that product life limits the delivery time. Although this is related to fresh products, with the shorter market life cycle of product technology, marketing time can be affected by the length of delivery time. A short market life cycle results in the opportunity to place very few orders or only even once. A one-time order is usually supported by a survey or pre-order from the customer to determine the market potential and estimate the demand for a new product. Order size is a very important decision that must be made by taking into account the market potential and also the market life cycle to obtain maximum profit.

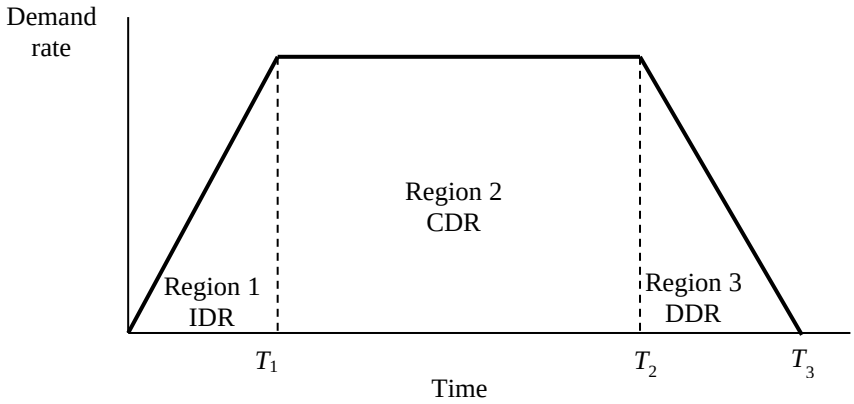

Figure 1. Trapezoidal demand rate

Product technology has experience deterioration since the beginning. Like smartphones have many components that can deteriorate, especially the battery component. Product deterioration can also occur during handling and storage. Negligence in handling, such as dropping products, can damage the product. Storage in humid or places that are too hot can damage the product. The primary distributor of a smartphone can store many products, and the effect of deterioration needs to be considered in the inventory policy.

From the situation above, this study sees the importance of the role of one-time orders by considering the market life cycle and product deterioration. Rapid technological developments encourage the market life cycle of a product to become shorter. A trapezoidal demand rate represents this life cycle that has three phases: growth, maturity, and declining phases. The shorter cycle period can trigger a one-time order item, where the future order is no longer for the same product. A new product will replace the role of the current product, which means the product experienced obsolescence. This obsolescence affects the optimal length of the sales period to obtain maximum profit. Besides the obsolescence factor, optimal condition is influenced by deterioration and holding cost. Technology products can experience deterioration before they become obsolete and are usually expensive. High prices lead to high holding costs. The management needs to see the effect of product obsolescence, deterioration, and holding costs so that the prepared inventory through order quantity provides maximum profit. If the influence of product deterioration and holding cost is dominant, then optimal sales can end before the product experiences obsolescence (DDR). On the other hand, the management can provide the sale period until obsolescence occurs with a demand rate close to zero if the effect of obsolescence is dominant. If the importance of these factors is in balance, then optimal sales can end when obsolescence (DDR) has occurred with the demand rate still significant or not close to zero. Therefore the search for the best end of the selling period is an essential thing to do.

This article consists of six sections, and the rest are as follows. Section 2 discusses the literature review to see the contribution of the current study. Section 3 describes the problem statement and assumptions to see the link of essential variables involved in this study. Section 4 is a model development process that presents the detail of the mathematical modeling process and a method for solving problems. Section 5 is a numerical analysis using an instance that can test the developed model and solution search method, followed by sensitivity analysis to provide insight for management. Finally, Section 6 provides the conclusions that summarize the study results and potential future research ideas.

2. LITERATURE REVIEW

Many studies on inventory deteriorating items have been carried out. Ghare and Schrader (1963) introduced exponential deterioration with a constant deterioration rate. The constant deteriorating rates have been popular since the 1990s (Amiri *et al.*, 2020; Bardhan *et al.*, 2019; Bhunia *et al.*, 2018; M.-C. Cheng *et al.*, 2020; Chung and Tsai, 1997; Goswami and Chaudhuri, 1991; He *et al.*, 2020; Nagare *et al.*, 2020; Wee, 1995; Widyadana and Irohara, 2019). The constant deteriorating rate is the basis for providing insight into the incidence of deterioration. Moreover, this rate is sometimes combined with various other factors that can affect inventory policies, such as demand rates.

The study of the demand rate in the inventory model is another important consideration. Several studies have modeled demand depending on other factors such as selling price (Otrodi *et al.*, 2019; Shaikh *et al.*, 2019), stock (Bardhan *et al.*, 2019), and so on. Additionally, some models depend on the time or time-varying demand rates. The time-varying demand can model the market life cycle.

The most basic time-varying demand is the linear demand rate. The study of the linear demand rate is very mature. Donaldson (1977) proposed an analytical solution to find the optimal replenishment times for a positive linear trend in demand. Silver (1979) confirmed that the Silver-Meal heuristic (Silver and Meal, 1973) produces a simple policy in a linear trend demand problem. Mitra *et al.* (1984) proposed a simple method to determine order points and order quantities for a linear trend demand pattern. Hariga (1993) generated an optimal inventory replenishment schedule in growing and declining markets. Hariga (1993) assumed that the demand rate linearly increased or decreased. Goswami and Chaudhuri (1991) formulated an economic order quantity (EOQ) model for linear trend demand, and it was complemented by Chung and Tsai (1997), who provided a concrete algorithm. Bose *et al.* (1995) and Chakrabarti and Chaudhuri (1997) modeled the inventory system with a linearly increasing demand rate for deteriorating items by considering the inflation in inventory-related costs. However, the static linear demand rate has not been able to show the characteristics of the market change.

One type of demand rate that can represent the market change is the ramp-type demand rate. Mandal and Pal (1998) introduced the EOQ model with a constant deterioration rate and a ramp-type demand rate covering both increasing demand rate and stable phases. K. Wu and Ouyang (2000) and Deng *et al.* (2007) completed different solution procedures and improved some of the weaknesses in previous methods. Skouri *et al.* (2009) modeled the general ramp-type demand rate by considering the Weibull deterioration. Several other studies have added various related factors, such as partial backlogging (Ahmed *et al.*, 2013), inflation under fuzziness (Pal *et al.*, 2015), maximum lifetime (Sett *et al.*, 2016), and delay payment strategy (Shi *et al.*, 2019).

The ramp-type demand rate usually represents only two phases, but there is another type of demand rate that can represent all phases (growth, maturity, and decline phases) in the market life cycle, namely the trapezoidal type demand rate. Panda *et al.* (2008) modeled inventory deteriorating items with three phases of demand rate, consisting of increasing, constant, and decreasing. Panda *et al.* (2008) modeled the demand rate for seasonal products, and their approach shows that the demand rate is influenced by the season. Orders can be placed within that season and can occur at different market life phases so that orders can occur more than once in a season. M.B. Cheng and Wang (2009) modeled the trapezoidal demand rate with linear IDR, CDR, and DDR with fixed cycle times. Reordering is done at predetermined intervals so that this model does not seek the optimal ordering interval. It was improved by S.W. Lin (2011), Chung (2012), and K.P. Lin (2013). M.B. Cheng *et al.* (2011) modeled the general trapezoidal demand rate for fixed order intervals, which was then improved by J. Lin *et al.* (2014). S.W. Lin (2011) modeled multiple replenishment cycles in one planning horizon and used the hide-and-seek simulated annealing approach. Zhao (2014) and Zhao (2016) applied Weibull-distributed deteriorating items in the trapezoidal demand rate inventory model. Vandana and Srivastava (2017) considered inflation and time discounting in their model. Several studies used profit as a performance measure to produce a solid model by considering income and purchasing costs, whereas previously, only total costs were used (Saha *et al.*, 2021; J. Wu *et al.*, 2017; J. Wu *et al.*, 2016; J. Wu *et al.*, 2018). The research gap summary (see Appendix) shows that the inventory model of deteriorating items with a trapezoidal type demand rate mostly uses a given fixed order cycle period. Several studies that look for optimal order cycles, such as Panda *et al.* (2008) and Sun *et al.* (2020), have focused on multiple orders in one market life cycle. Only Nagare *et al.* (2020) focused on finding the optimal timing from the end of the sale. This study contributes to an inventory model of deteriorating items with a trapezoidal-type demand rate that focuses on finding the optimal time from the end of the sale. The end of the sales period is similar to the time when the inventory runs out, but it is not followed by a new cycle because a new type of product will own the new cycle.

The model developed by Nagare *et al.* (2020) is the closest model to the current study. This study has addressed three things that enrich the point of view of the inventory model in such a situation. First, they assumed that deterioration occurs when entering a period of decreasing demand rate (obsolescence). In reality, many types of products deteriorate before a decline in market demand. Second, the management needs to know the difference between the effects of obsolescence and deterioration. In reality, technology products such as smartphones experience obsolescence not only due to product deterioration but also due to some other factors. Sometimes obsolescence is more dominant, sometimes deteriorated items and holding cost, or maybe balance between them. These effects cannot be explained by their model. Third, they used an enumerative search method to find when the sale ends after it has deteriorated. The enumerative search method is exhaustive, whereas, in the applied model, one needs an efficient solving method.

This study proposes a one-time order inventory model for deteriorating and short-market life items with a trapezoidal demand rate. This type of demand rate is the simplest and most complete to express the growth, maturity, and decline phases of the market life cycle. Thus this study can give the basic concepts that are essential for knowledge. The main difference between the current study and previous research is that most of the previous research looked at the sales period that kept repeating and determined the optimal order time, whereas, in this study, there was only one order at the beginning of the period, and looked for when the sale ended which could generate maximum profit. Only the model of Nagare *et al.* (2020) is close to the current study. However, Nagare *et al.* (2020) assumed that the deterioration occurred during the declining phase,

whereas this study considers product deterioration occurs from the beginning. Thus it can better reflect the characteristics of technology products and analyze the different effects of the market life cycle and product deterioration. The market life cycle may represent the time when obsolescence occurs. The projected revenue will increase due to longer market life; on the other hand, the risks and costs of inventory will increase due to product deterioration and holding costs. Determining which of these factors affects the total profit and vital for management. Further, this study used a non-linear generalized reduced gradient (GRG) to find the end of the selling period by determining the total profit as an objective function and several constraints related to the end of the sales period, rather than using an enumerative search like in Nagare *et al.* (2020).

3. PROBLEM STATEMENT AND ASSUMPTIONS

According to the problem background described in Section 1, the current study sees the need for an inventory model with one-time order to handle the deteriorating and short-market life items. The decrease in the demand rate in the market life cycle can describe the situation of obsolescence. In addition, technology products can deteriorate from the beginning. Management needs to know when is the right time to stop selling a type of product and switch to a new substitute product. Referring to the literature review in Section 2, the study to find the end of the sales period is rare; there is a need for the deterioration that occurred from the beginning to represent technology products, and also there is important to see the significance of the influence of market life cycle, product deterioration, and holding costs on the profit generated.

This study considers the inventory level decrease due to the demand and deteriorated items. It will be the function of the inventory declining rate. The demand rate function has a different formula for each region and follows a trapezoidal shape to express the growth, maturity, and decline period in the market life cycle. Finding the general solution from the inventory declining rate will generate the inventory level equations. The constant of the general solutions will be solved by equating the inventory level equations from two consecutive phases or regions in the market life cycle so that the inventory level function is the continuous function along with all market life cycle phases.

The decision variable, the end of the selling period, replaces the time variable in the last region equation, and the inventory level is equal to zero. The number of inventory in each region is the finite integration of the inventory level function and is used to calculate the holding cost. In addition, the number of deteriorated items is the proportion of the number of inventory in each region following the constant deteriorating rate and is used to calculate the additional cost of deterioration, excluding the unit cost. Total inventory cost summarizes the holding cost, deterioration cost, and one-time ordering cost.

The objective function, the total profit, consists of revenue, acquisition cost, and total inventory cost. Revenue is calculated based on the number of demands collected from the beginning to the end of the sales period and the selling price. Acquisition cost is calculated based on the number of orders and the purchase price. The order quantity as a practical decision variable is calculated based on the desired end of the sales period.

In addition, it uses several constraints related to the domain of the decision variable, non-negative demand rate, monotonically decreasing inventory level, and positive order quantity. The best total profit becomes the solution with the optimal end of the selling period. This modeling process is described in the model building subsection. Follows by the solution search method uses GRG non-linear to find the optimal solution to this problem efficiently.

Instead of the problem statement, the current study uses five assumptions to make it more focused and clear to present the basic concepts needed. The assumptions used in the development of the current model are the following:

- There is only one order for a product type. Panda *et al.* (2008) described a season as a life cycle so that a season can have more than one order, in this study, a one-time order is a challenge in the short market life cycle era. One-time orders limit one type of product to one order.
- Delivery occurs only once. The number of products delivered is equal to the number ordered, which will be the initial inventory.
- Shortage costs that occur only have an impact on reduced sales. Giri *et al.* (2020) found that increasing the degree of substitution of a product will increase profit. It can be applied to smartphone products that have tight competition so that substitution by other brands or other types is very likely to occur. In this study, seeing a shortage of a product when supplies run out is not seen as a loss because substitute products can replace it. Therefore this study does not use the newsvendor inventory model that considers shortage as a loss. This study is expected to make the inventory model more complete, especially for short-market life products with rapid technology innovation and improvement.
- IDR, CDR, and DDR are linear. Dynamic linear demand rate is the simplest model that can describe the growth, maturity, and decline phases of a market life cycle. Thus it can be used as the basis for the concept in this study.
- The deterioration rate is constant. Constant deterioration rate is the simplest model that can describe product deterioration. Therefore it also can be used as the basis for the concept in this study.

4. MODEL DEVELOPMENT

This section consists of notations and model building. The notations explain the symbols used in the mathematical model. Further, the model building subsection describes the modeling process in this study.

4.1 Notations

This study uses mathematical formula involving symbols that represent the various variables used. There are three decision variables to express the end of the sales period (T_1^*, T_2^*, and T_3^*), and one decision variable expresses the order quantity (Q_0). Figure 2 shows T_1^*, T_2^*, and T_3^* are the end of the sales period in the first, second, and third regions. T_1^* has a domain from time zero to T_1, while T_2^* has from T_1 to T_2, and T_3^* has from T_2 to T_3. The best end of the sales period is used to calculate the required order quantity (Q_0) as a practical decision variable. Q_0 is the size of the inventory at the beginning. The longer the desired sales time, the greater the order quantity required. The notations used in this study are as follows:

T_1^*	= end of selling period (region 1).
T_2^*	= end of selling period (region 2).
T_3^*	= end of selling period (region 3).
Q_0	= the order size.
$TP*$	= the best of the total profit.
TP_i	= total profit if the end of the selling period falls on the region i.
α	= a multiplier at IDR.
β	= an initial level of demand rate.
γ	= a multiplier at DDR.
θ	= a deteriorating rate.
T_1	= the time when IDR changes to CDR.
T_2	= the time when CDR changes to DDR.
T_3	= the time when the demand rate is equal to zero.
p	= price.
s	= purchasing unit cost.
c	= ordering/setup cost.
h	= holding cost.
m	= deteriorating costs excluding purchasing unit costs.
$R(t)$	= a demand rate at time t.
TR_i	= total demand if the end of the selling period falls on the region i.
$Q(t)$	= inventory level at time t.
I	= inventory within a cycle.
I_i	= inventory within the region i.
C_1	= a general solution constant for period before T_1.
C_2	= a general solution constant for T_1 to T_2.
C_3	= a general solution constant for T_2 to T_3.
D	= the number of deteriorating items.
D_i	= the number of deteriorating items in the region i.
TC	= total inventory cost.
TC_i	= total inventory cost if the end of the selling period falls on the region i.

There are five figures to express the notations used in the model. Figure 2 shows three inventory level curves ending in three different regions. Case 1 indicates that sales end in the first region. It involves only one region with $Q_1(t)$ as the inventory level curve. I_1 shows the number of inventory in the first region. Case 2 indicates that sales end in the second region. It involves two curves, namely $Q_1(t)$ and $Q_2(t)$. I_1 and I_2 are the numbers of inventory in the first and second regions. Case 3 indicates that sales end in the third region. It involves three curves, namely $Q_1(t)$, $Q_2(t)$, and $Q_3(t)$. I_1, I_2, and I_3 are the total inventory in the first, second, and third regions. Figure 3 shows two inventory level curves, $Q(t)$, with deterioration rate (θ = 0.1) and without deterioration (θ = 0). The inventory level decreases due to demand and deterioration. With deterioration, inventory runs out at time B faster than without deterioration ending at time A. Figure 4 shows the demand rate curve, $R(t)$, that changes over time, t, where T_1 is the boundary between the first and second regions, and T_2 is between the second and

third regions. Furthermore, T_3 is when the demand rate is equal to zero. The parameters of the demand rate are α, β, and γ. They are the increasing slope, initial, and decreasing slope of the demand rate, respectively. Figure 4 also shows the total demand calculated from the beginning to the end of the selling period. The selling time can end in region 1 (TR_1), region 2 (TR_2), or region 3 (TR_3). Figure 5 shows the number of items that have deteriorated, D_1, D_2, and D_3 for regions 1, 2, and 3, respectively. The number of deteriorated items is the area under the $\theta Q(t)$ curve. Figure 6 shows the revenue curve ($p \times TR_i$), the cost curve ($s \times Q_0 + TC_i$), and the profit curve (TP_i) depending on the end of the sales period.

All units of time are in years, and the unit value of money is US dollars. The deterioration rate (θ) is a proportion that will be multiplied by the inventory level to obtain the amount of deterioration. Ordering cost (c) is in dollars per order. Holding cost (h) is in dollars per unit per year. Additional deterioration cost (m) is in dollars per unit (excluding purchasing costs). The selling price (p) and purchasing price (s) are in dollars per unit.

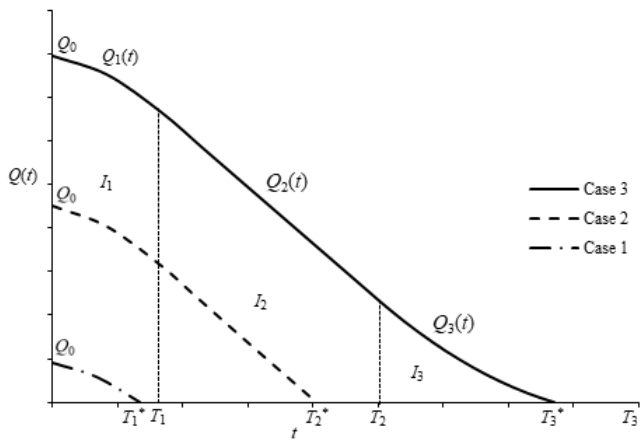

Figure 2. Three cases are related to when the selling period ends

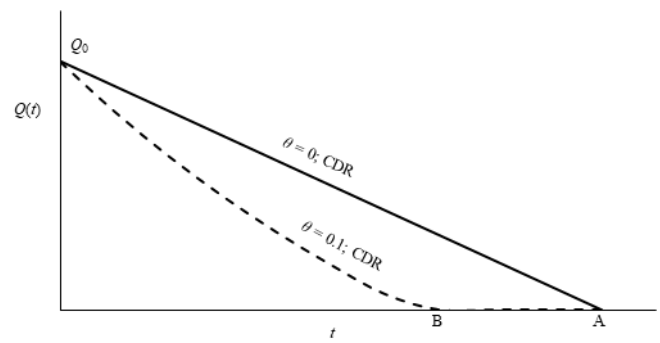

Figure 3. Constant deterioration rate (θ)

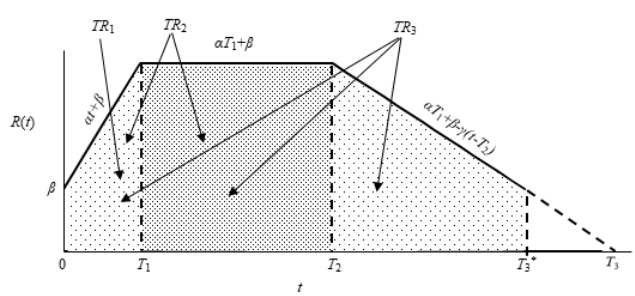

Figure 4. The dynamics of the demand rate $R(t)$

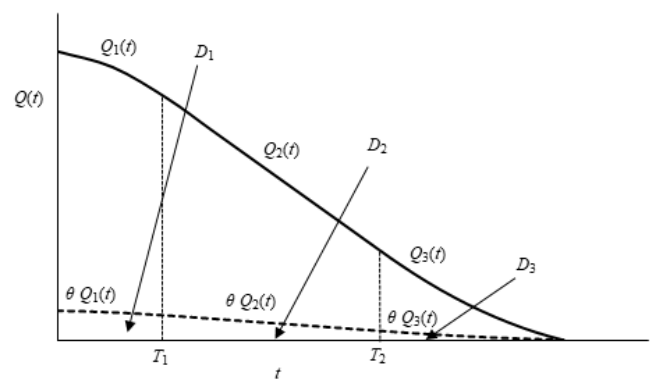

Figure 5. Number of items that deteriorated (D_i)

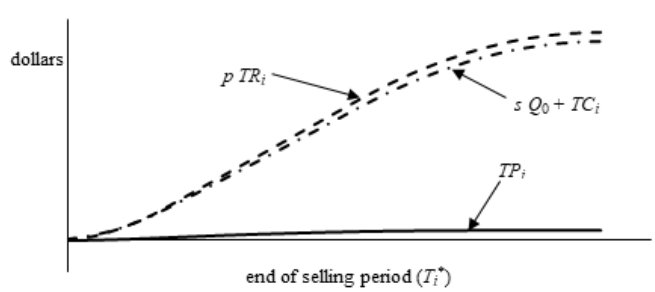

Figure 6. Revenue, costs, and profit

4.2 Model building

This subsection describes the model development process and solution search method. Figure 7 shows the research methodology flowchart in twelve steps, which begins with identifying the background of the problem as the first step. The introduction section describes the one-time order policy, considering the item deterioration and the market life of a technology product are essential. The second step, review the literature, describing a literature study on inventory policies with deteriorating items, inventory with a trapezoidal demand rate, one-time order inventory, and gaps that are investigated in the current study. In the third step, problem statements and assumptions summarize the contribution of this research and explain the relationship of the essential variables in the developed model. The fourth step is the model development involving all market life cycle phases by setting the end of the sale to be in the third phase to describe the trapezoidal demand rate. This step ensures the continuity of the demand rate function through all phases. The fifth step is to adjust the model if the sales end in the first or second phases. The best end of sales may occur before the decreasing demand rate (DDR) period. The sixth step is to create a model to compare the optimal total profit in each phase and obtain the best total profit overall. The seventh step is the starting point to find a solution using a GRG non-linear in all market life cycle phases. The eighth step is to calculate the best profit and the optimal end of the selling period. The ninth step is to calculate the order quantity to reach the desired end of sales. The management will find it easier to apply the order quantity as an inventory policy rather than the end of the selling period. The tenth step is output analysis which presents analysis related to total profit and optimal sales period. The eleventh is sensitivity analysis for some parameters to provide insight for management. The last step is the conclusion which summarizes the results and shares potential research for the future.

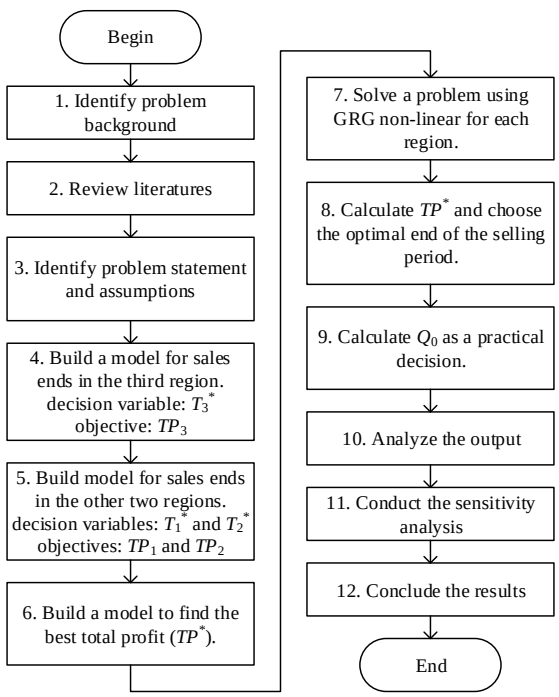

Figure 7. Research methodology flowchart

Modeling starts from the declining rate of the inventory level in Eq. (1), as inspired by Ghare and Schrader (1963). The minus $\theta Q(t)$ describes a decrease in the inventory level caused by deterioration. Another factor that reduces inventory is the demand rate, $R(t)$, which can change over time.

$$\frac{dQ(t)}{dt} = -\theta Q(t) - R(t) \tag{1}$$

By following the market life cycle, the demand rate has three conditions as shown in Eq. (2): the beginning period (from zero to T_1) experiences an IDR; the period between T_1 and T_2 has a CDR; the period between T_2 and T_3 has a DDR. This study models the demand rate as linear. Figure 4 shows the dynamics of the demand rate $R(t)$ that forms a trapezoidal pattern. In the first region, the demand rate increases linearly over time with slope α, and intercept β is the initial demand rate. This

period lasts from the beginning ($t = 0$) to T_1. Next, the second region has a constant demand rate ($\alpha T_1 + \beta$) where t is substituted by constant T_1. This period (from T_1 to T_2) is the peak of the demand rate. In the third region, the demand rate decreases linearly with a slope minus γ, and this period starts from T_2 and ends at the end of sales (T_3^*). If there is an excess inventory, the third region can end when the demand rate is zero (T_3).

$$R(t) = \begin{cases} \alpha t + \beta; 0 \le t \le T_1 \\ \alpha T_1 + \beta; T_1 \le t \le T_2 \\ \alpha T_1 + \beta - \gamma(t - T_2); T_2 \le t \le T_3 \end{cases} \tag{2}$$

Figure 2 shows three cases that may occur related to when the selling period will end in the first, second, or third regions. Firsts, the authors model the complete stages as the end of the selling period fall in region 3 (DDR region). The first period (IDR) with $R(t) = \alpha t + \beta$. The rate of decline in inventory becomes Eq. (3).

$$\frac{dQ_1(t)}{dt} = -\theta Q_1(t) - (\alpha t + \beta); 0 \le t \le T_1 \tag{3}$$

The corresponding solution is derived in Eq. (4).

$$\begin{aligned} & e^{\theta t}\frac{dQ_1(t)}{dt} + e^{\theta t}\theta Q_1(t) = -e^{\theta t}(\alpha t + \beta) \\ & \frac{d(e^{\theta t}Q_1(t))}{dt} = -e^{\theta t}(\alpha t + \beta) \\ & e^{\theta t}Q_1(t) = -\int e^{\theta t}(\alpha t + \beta)dt \\ & Q_1(t) = -e^{\theta t}\left(\frac{\alpha t}{\theta}e^{\theta t} - \frac{\alpha}{\theta^2}e^{\theta t} + \frac{\beta}{\theta}e^{\theta t} - C_1\right) \\ & Q_1(t) = \frac{\alpha}{\theta^2} - \frac{\alpha t}{\theta} - \frac{\beta}{\theta} + C_1 e^{-\theta t}; 0 \le t \le T_1 \end{aligned} \tag{4}$$

The second period with the CDR, $R(t) = \alpha T_1 + \beta$, and Eq. (1) becomes Eq. (5).

$$\frac{dQ_2(t)}{dt} = -\theta Q_2(t) - (\alpha T_1 + \beta); T_1 \le t \le T_2 \tag{5}$$

Then, the corresponding solution is derived in Eq. (6).

$$\begin{aligned} & e^{\theta t}\frac{dQ_2(t)}{dt} + e^{\theta t}\theta Q_2(t) = -e^{\theta t}(\alpha T_1 + \beta) \\ & \frac{d(e^{\theta t}Q_2(t))}{dt} = -e^{\theta t}(\alpha T_1 + \beta) \\ & e^{\theta t}Q_2(t) = -\int e^{\theta t}(\alpha T_1 + \beta)dt \\ & e^{\theta t}Q_2(t) = -\frac{(\alpha T_1 + \beta)}{\theta}e^{\theta t} + C_2 \\ & Q_2(t) = -\frac{(\alpha T_1 + \beta)}{\theta} + C_2 e^{-\theta t}; T_1 \le t \le T_2 \end{aligned} \tag{6}$$

The third period with the DDR lasts from T_2 to T_3^*. This decrease is also modeled linearly by $R(t) = \alpha T_1 + \beta - \gamma (t - T_2)$. Therefore, Eq. (1) becomes Eq. (7).

$$\frac{dQ_3(t)}{dt} = -\theta Q_3(t) - \left(\alpha T_1 + \beta - \gamma(t - T_2)\right); T_2 \le t \le T_3^* \tag{7}$$

The corresponding solution is derived in Eq. (8).

$$e^{\theta t}\frac{dQ_3(t)}{dt} + e^{\theta t}\theta Q_3(t) = -e^{\theta t}(\alpha T_1 + \beta - \gamma(t - T_2))$$

$$\frac{d(e^{\theta t}Q_3(t))}{dt} = -e^{\theta t}(\alpha T_1 + \beta - \gamma(t - T_2))$$
$$e^{\theta t}Q_3(t) = -\int e^{\theta t}(\alpha T_1 + \beta - \gamma(t - T_2))dt$$
$$e^{\theta t}Q_3(t) = -\frac{\alpha T_1+\beta+\gamma T_2}{\theta}e^{\theta t} + \frac{\gamma t}{\theta}e^{\theta t} - \frac{\gamma}{\theta^2}e^{\theta t} + C_3$$
$$Q_3(t) = -\frac{\alpha T_1+\beta+\gamma T_2}{\theta} + \frac{\gamma t}{\theta} - \frac{\gamma}{\theta^2} + C_3e^{-\theta t}; T_2 \le t \le T_3^* \tag{8}$$

The amount of inventory over the period 0 to T_3^* is the integral $Q(t)$, which in each successive region is I_1, I_2, and I_3.

$$I_1 = \int_0^{T_1} Q_1(t)dt$$
$$I_1 = \int_0^{T_1}\left(\frac{\alpha}{\theta^2} - \frac{\alpha t}{\theta} - \frac{\beta}{\theta} + C_1e^{-\theta t}\right)dt$$
$$I_1 = \left(\frac{\alpha}{\theta^2} - \frac{\beta}{\theta}\right)[t]_0^{T_1} - \frac{\alpha}{2\theta}[t^2]_0^{T_1} - \frac{C_1}{\theta}\left[e^{-\theta t}\right]_0^{T_1}$$
$$I_1 = \left(\frac{\alpha}{\theta^2} - \frac{\beta}{\theta}\right)T_1 - \frac{\alpha}{2\theta}T_1^2 - \frac{C_1}{\theta}\left(e^{-\theta T_1} - 1\right) \tag{9}$$
$$I_2 = \int_{T_1}^{T_2} Q_2(t)dt$$
$$I_2 = \int_{T_1}^{T_2}\left(-\frac{(\alpha T_1+\beta)}{\theta} + C_2e^{-\theta t}\right)dt$$
$$I_2 = -\frac{(\alpha T_1+\beta)}{\theta}(T_2 - T_1) - \frac{C_2}{\theta}\left(e^{-\theta T_2} - e^{-\theta T_1}\right) \tag{10}$$
$$I_3 = \int_{T_2}^{T_3^*} Q_3(t)dt$$
$$I_3 = \int_{T_2}^{T_3^*}\left(-\frac{\alpha T_1+\beta+\gamma T_2}{\theta} + \frac{\gamma t}{\theta} - \frac{\gamma}{\theta^2} + C_3e^{-\theta t}\right)dt$$
$$I_3 = \left(-\frac{\gamma}{\theta^2} - \frac{\alpha T_1+\beta+\gamma T_2}{\theta}\right)(T_3^* - T_2) + \frac{\gamma}{2\theta}\left(T_3^{*2} - T_2^2\right) - \frac{C_3}{\theta}\left(e^{-\theta T_3^*} - e^{-\theta T_2}\right) \tag{11}$$

T_3^* describes the end of the selling period when inventory level = 0. The value of C_3 must be expressed in T_3^* to find the optimal value of T_3^*. The equation can be derived from $Q_3(T_3^*) = 0$.

$$-\frac{\alpha T_1+\beta+\gamma T_2}{\theta} + \frac{\gamma T_3^*}{\theta} - \frac{\gamma}{\theta^2} + C_3e^{-\theta T_3^*} = 0$$
$$C_3 = \left(\frac{\alpha T_1+\beta+\gamma T_2}{\theta} - \frac{\gamma T_3^*}{\theta} + \frac{\gamma}{\theta^2}\right)e^{\theta T_3^*} \tag{12}$$

The value of C_2 can be found by using Eqs. (6) and (8). It refers to the continuity of function $Q(t)$, where both equations have the same value at T_2.

$$Q_2(T_2) = Q_3(T_2)$$
$$-\frac{(\alpha T_1+\beta)}{\theta} + C_2e^{-\theta T_2} = -\frac{\alpha T_1+\beta+\gamma T_2}{\theta} + \frac{\gamma T_2}{\theta} - \frac{\gamma}{\theta^2} + C_3e^{-\theta T_2}$$
$$C_2e^{-\theta T_2} = -\frac{\gamma}{\theta^2} + C_3e^{-\theta T_2}$$
$$C_2 = -\frac{\gamma}{\theta^2}e^{\theta T_2} + C_3 \tag{13}$$

The substitution of Eq. (12) into Eq. (13) and Eq. (14) can be obtained.

$$C_2 = -\frac{\gamma}{\theta^2}e^{\theta T_2} + \left(\frac{\alpha T_1+\beta+\gamma T_2}{\theta} - \frac{\gamma T_3^*}{\theta} + \frac{\gamma}{\theta^2}\right)e^{\theta T_3^*} \tag{14}$$

Next, the C_1 is formulated using Eqs. (4) and (6). Where both have the same value at T_1.

$$Q_1(T_1) = Q_2(T_1)$$
$$\frac{\alpha}{\theta^2} - \frac{\alpha T_1}{\theta} - \frac{\beta}{\theta} + C_1 e^{-\theta T_1} = -\frac{(\alpha T_1 + \beta)}{\theta} + C_2 e^{-\theta T_1}$$
$$C_1 = C_2 - \frac{\alpha}{\theta^2} e^{\theta T_1} \tag{15}$$

Eq. (14) is substituted into Eq. (15), and Eq. (16) can be obtained.

$$C_1 = -\frac{\gamma}{\theta^2} e^{\theta T_2} + \left(\frac{\alpha T_1 + \beta + \gamma T_2}{\theta} - \frac{\gamma T_3^*}{\theta} + \frac{\gamma}{\theta^2}\right) e^{\theta T_3^*} - \frac{\alpha}{\theta^2} e^{\theta T_1} \tag{16}$$

Eqs. (17), (18), and (19) show the number of items that deteriorated (D_i) as regions below curve $\theta Q_i(t)$. It can be formulated over the period 0 to T_3^*.

$$D_1 = \int_0^{T_1} \theta Q_1(t) dt$$
$$D_1 = \theta I_1 \tag{17}$$

This also applies to D_2 and D_3.

$$D_2 = \theta I_2 \tag{18}$$

$$D_3 = \theta I_3 \tag{19}$$

The total inventory cost when the end of the selling period falls in the DDR region (TC_3) consists of ordering costs, holding costs, and deterioration costs.

$$TC_3 = c + hI + mD \tag{20}$$

By substituting Eqs. (17), (18), and (19) into Eq. (20), Eq. (21) is obtained.

$$TC_3 = c + h(I_1 + I_2 + I_3) + m\theta(I_1 + I_2 + I_3)$$
$$TC_3 = c + (h + m\theta)(I_1 + I_2 + I_3) \tag{21}$$

The total inventory in each region I_1, I_2, and I_3 are affected by T_3^*. Substituting Eqs. (12), (14), and (16) into Eqs. (9), (10), and (11), then substituting Eq. (21) will form an equation with a decision variable T_3^*. This formula is shown in Eq. (22).

$$TC_3 = c + (h + m\theta)\left(\begin{array}{l} \left(\frac{\alpha}{\theta^2} - \frac{\beta}{\theta}\right) T_1 - \frac{\alpha}{2\theta} T_1^2 \\ - \frac{-\frac{\gamma}{\theta^2} e^{\theta T_2} + \left(\frac{\alpha T_1 + \beta + \gamma T_2}{\theta} - \frac{\gamma T_3^*}{\theta} + \frac{\gamma}{\theta^2}\right) e^{\theta T_3^*} - \frac{\alpha}{\theta^2} e^{\theta T_1}}{\theta} \left(e^{-\theta T_1} - 1\right) \\ - \frac{(\alpha T_1 + \beta)}{\theta} (T_2 - T_1) \\ - \frac{-\frac{\gamma}{\theta^2} e^{\theta T_2} + \left(\frac{\alpha T_1 + \beta + \gamma T_2}{\theta} - \frac{\gamma T_3^*}{\theta} + \frac{\gamma}{\theta^2}\right) e^{\theta T_3^*}}{\theta} \left(e^{-\theta T_2} - e^{-\theta T_1}\right) \\ + \left(-\frac{\gamma}{\theta^2} - \frac{\alpha T_1 + \beta + \gamma T_2}{\theta}\right) (T_3^* - T_2) + \frac{\gamma}{2\theta} (T_3^{*2} - T_2^2) \\ - \frac{\left(\frac{\alpha T_1 + \beta + \gamma T_2}{\theta} - \frac{\gamma T_3^*}{\theta} + \frac{\gamma}{\theta^2}\right) e^{\theta T_3^*}}{\theta} \left(e^{-\theta T_3^*} - e^{-\theta T_2}\right) \end{array}\right) \tag{22}$$

T_3^* calculations can be done optimally by a non-linear generalized reduced gradient (GRG) using Excel Solver. Eq. (23) shows the objective function of this problem, which is to maximize the total profit (TP_3). It is income (price × total sales) minus purchase value (purchase unit cost × order quantity) and total inventory cost. Some constraints ensure a feasible solution and guarantee the optimal value of the GRG non-linear method, where $T_3^* \geq T_2$ (see Eq. (24)), and the function $Q(t)$ is monotonically decreasing from T_2 to T_3^* (see Eq. (25)) and $Q(t) \geq 0$, which is supported by the non-negative demand rate (see Eq. (26)) and $Q_0 \geq 0$ (see Eq. (27)).

$$TP_3 = p \times TR_3 - s \times Q_0 - TC_3 \tag{23}$$

where the demand for all three regions is

$$TR_3 = (0.5\alpha T_1^2 + \beta T_1) + \big((\alpha T_1 + \beta)(T_2 - T_1)\big) + \big((\alpha T_1 + \beta + \gamma T_2)(T_3^* - T_2) - 0.5\gamma(T_3^{*2} - T_2^2)\big)$$

Subject to:

$$T_3^* \geq T_2 \tag{24}$$

$$T_3^* \leq -\frac{1}{\theta}\ln\left(\frac{\gamma}{\theta^2 C_3}\right) \tag{25}$$

$$T_3^* \leq \frac{\alpha T_1 + \beta}{\gamma} + T_2 \tag{26}$$

$$Q_0 \geq 0 \tag{27}$$

Figure 2 shows the end of the selling period also can fall in region 1 (IDR) or region 2 (CDR). In these cases, adjustments to the model are required.

Region 1:

T_1^* is the optimal end of the selling period in the first region.

Modification of Eq. (21) becomes Eq. (28).

$$TC_1 = c + (h + m\theta)I_1 \tag{28}$$

Eq. (9) becomes Eq. (29).

$$I_1 = \left(\frac{\alpha}{\theta^2} - \frac{\beta}{\theta}\right)T_1^* - \frac{\alpha}{2\theta}(T_1^*)^2 - \frac{C_1}{\theta}\left(e^{-\theta T_1^*} - 1\right) \tag{29}$$

Eq. (16) becomes Eq. (30).

$$C_1 = \left(\frac{\alpha T_1^* + \beta}{\theta} - \frac{\alpha}{\theta^2}\right)e^{\theta T_1^*} \tag{30}$$

The objective function is to maximize the total profit in the first region (see Eq. (31)).

$$TP_1 = p \times TR_1 - s \times Q_0 - TC_1 \tag{31}$$

where the demand in the first region is

$$TR_1 = (0.5\alpha(T_1^*)^2 + \beta T_1^*)$$

Subject to Eq. (27) and the domain of T_1^* between 0 and T_1.

Region 2:

T_2^* is the optimal end of the selling period in the second region.
Modification of Eq. (21) becomes Eq. (32).

$$TC_2 = c + (h + m\theta)(I_1 + I_2) \tag{32}$$

Eq. (10) becomes Eq. (33).

$$I_2 = -\frac{(\alpha T_1 + \beta)}{\theta}(T_2^* - T_1) - \frac{C_2}{\theta}\left(e^{-\theta T_2^*} - e^{-\theta T_1}\right) \tag{33}$$

Eq. (14) becomes Eq. (34).

$$C_2 = \left(\frac{\alpha T_1 + \beta}{\theta}\right) e^{\theta T_2^*} \tag{34}$$

Eq. (16) becomes Eq. (35).

$$C_1 = \left(\frac{\alpha T_1 + \beta}{\theta}\right) e^{\theta T_2^*} - \frac{\alpha}{\theta^2} e^{\theta T_1} \tag{35}$$

The objective function is to maximize the total profit in the first and second regions (see Eq. (36)).

$$TP_2 = p \times TR_2 - s \times Q_0 - TC_2 \tag{36}$$

where the demand in the first and second regions is

$$TR_2 = (0.5\alpha T_1^2 + \beta T_1) + \left((\alpha T_1 + \beta)(T_2^* - T_1)\right)$$

Subject to Eq. (27) and the domain of T_2^* between T_1 and T_2.

The global optimal of total profit (TP^*) can be found by comparing all regions and choosing the maximum value (see Eq. (37)).

$$TP^* = \max_{i=1,2,3} TP_i \tag{37}$$

The order size (Q_0) is calculated by Eq. (38).

$$Q_0 = Q_1(0) = \frac{\alpha}{\theta^2} - \frac{\beta}{\theta} + C_1 \tag{38}$$

where C_1 is calculated depending on the region where the global optimal end of the selling period lies.

5. NUMERICAL ANALYSIS

For instance, the current case has the parameters α = 50,000, β = 1,000, γ = 25,000, θ = 0.01, c = 1,000, h = 50, m = 5, T_1 = 0.083, T_2 = 0.25, p = 200, and s = 180. Demand rate (unit per year) has an IDR from the period 0 to 0.083 years (for example, a smartphone needs almost a month for introduction and growth stages, and 0.083 years equals one month), then CDR until the period of 0.25 years (3 months), and DDR until the inventory runs out or until demand rate reaches zero.

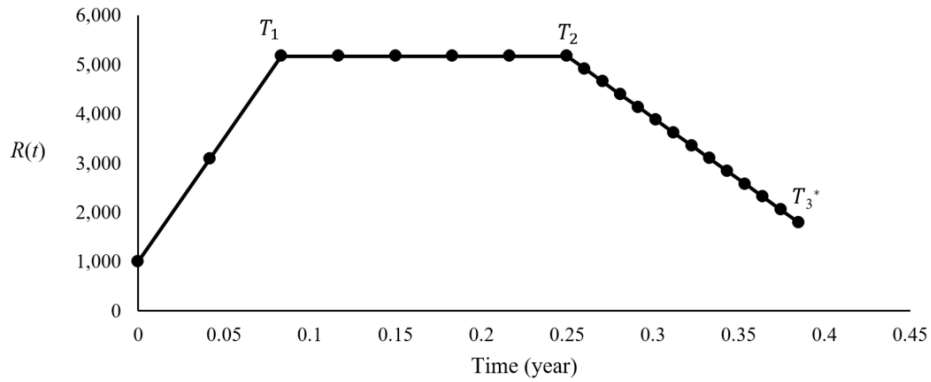

Figure 8. Demand rate with α = 50,000, β = 1,000, and γ = 25,000

5.1 Output analysis

Figure 8 shows that IDR occurs from the beginning to T_1, then continue with CDR until T_2, and finally DDR until T_3^*. T_3^* is the time when supplies run out and signals the end of the selling period. Continuing to sell the product makes the demand rate go to zero at some point. There is a constraint that the demand rate must be a non-negative value ($R(t) \geq 0$). This is different from several studies that have used an exponential DDR. They showed that the demand rate never reaches zero because it is asymptotic to the time axis (Nagare *et al.*, 2020). Management often dislikes the assumption of a never-ending demand rate. They want a worst-case scenario with demand ending at a certain point in time. This can be modeled with a linear decreasing demand rate. Figure 9 shows that the slopes of decreasing inventory are influenced by the deterioration and demand rate. The deterioration occurs more in the beginning than in the end because of $\theta Q(t)$. The value of θ is constant, while the value of $Q(t)$ decreases over time. Furthermore, the demand rate has the highest value when in the CDR, from T_1 to T_2. Hence, Figure 9 shows a steeper decline in CDR than in IDR or DDR.

The total profit in the first region (TP_1), the second region (TP_2), and the third region (TP_3) is 3,459 dollars, 13,233 dollars, and 15,148 dollars, respectively. Thus, the global optimum lies in the third region with the end of the selling period (T_3^*) equal to 0.385 years. Hence the order size (Q_0) is equal to 1,591 units. The demand rate at 0.385 years is 1,792 units per year. This demand rate is still far from zero (see Figure 8). Hence the effect of obsolescence compared to the deteriorated items and holding cost is in balance. There is no dominant factor between obsolescence, item deterioration, and holding cost. Thus the management should be concerned with all these factors.

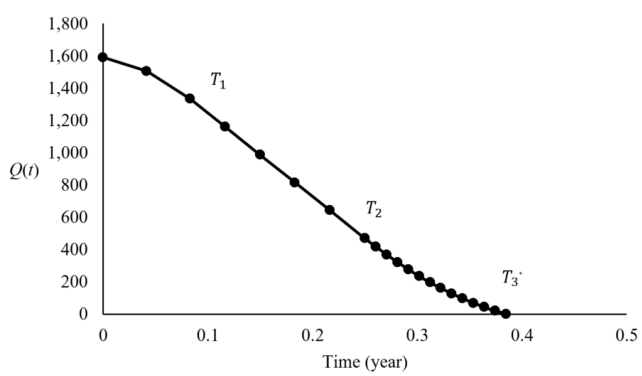

Figure 9. Inventory level for the current case.

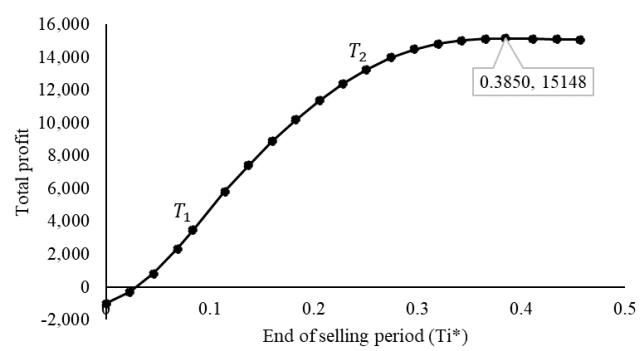

Figure 10. Graph of the optimal end of the selling period based on the total profit.

Figure 10 shows the dynamics of the total profit due to the shift in the sale period ends. The negative profit occurs when the sales period ends too soon (close to zero) because the revenue generated is lower than the costs incurred. In this case, the profit increases from time 0 to T_1, and continues to increase to T_2, with the optimal solution obtained when T_3^* = 0.385 years

with TP = 15,148 dollars. In this case, after T_2, the total profit increases starting from 0.25 years and then decreases after 0.385 years. The optimal solution for the end of the selling period for this product type lies in the third region.

5.2 Sensitivity analysis

The effects of the deterioration occur from the beginning of the life cycle (see Figure 11). The higher deterioration rate will shift the optimal end of the selling period to the earlier time with a lower total profit. Furthermore, it can shift the optimal end time to sell to the earlier phase of the market life cycle. This behavior cannot be represented by Nagare *et al.* (2020), in which the effect of the deterioration rate only occurs in DDR. Thus, in their model, the change in deterioration rate cannot shift the optimal end of the selling period to a different phase. Additional deteriorating costs, excluding purchase costs, have little effect on changes in the optimal end of the selling period and total profit (see Figure 12).

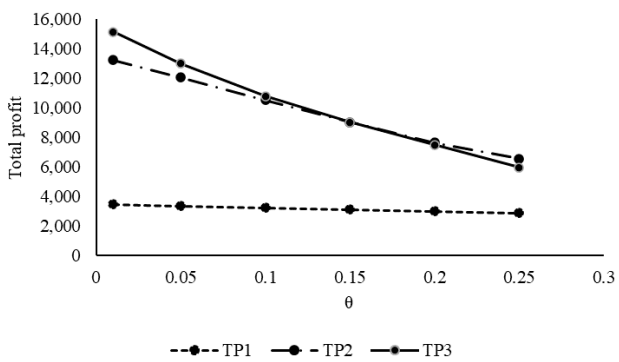

Figure 11. The effects of θ.

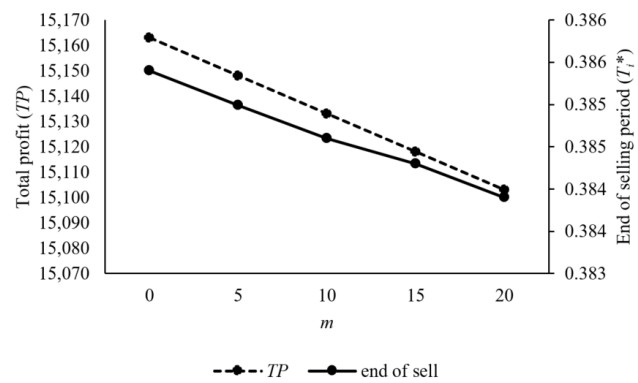

Figure 12. The effects of m.

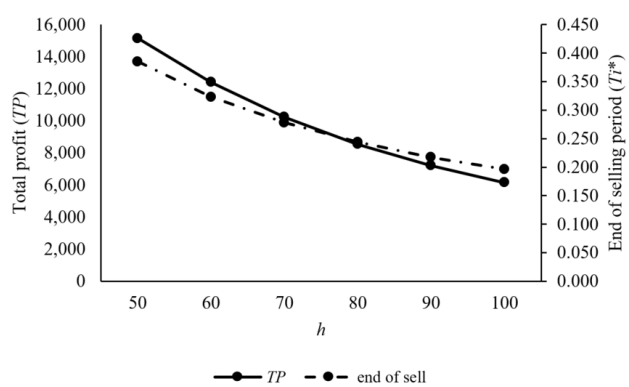

Figure 13. The effects of holding cost.

Holding costs have the effect of shortening the optimal end of the selling period such that it can move from the third region to the second region or an earlier region while reducing total profit (see Figure 13). The effect of holding cost is in line with the deteriorating rate. This finding confirms Nagare *et al.* (2020) model that can shift the optimal end of the selling period between regions or phases using the ratio of the selling price minus the purchasing price to the holding cost as the indicator. The initial demand rate only affects profit but not the end of the selling period (see Figure 14).

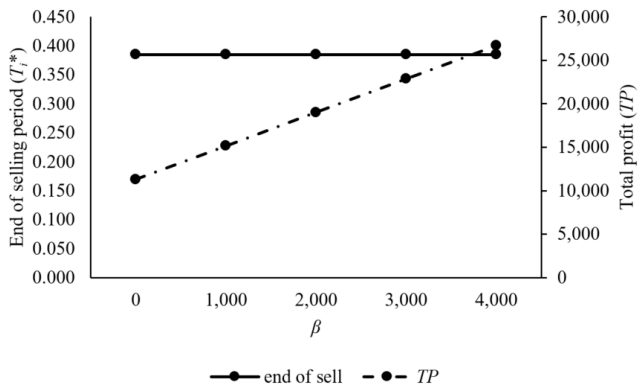

Figure 14. The effects of initial demand rate.

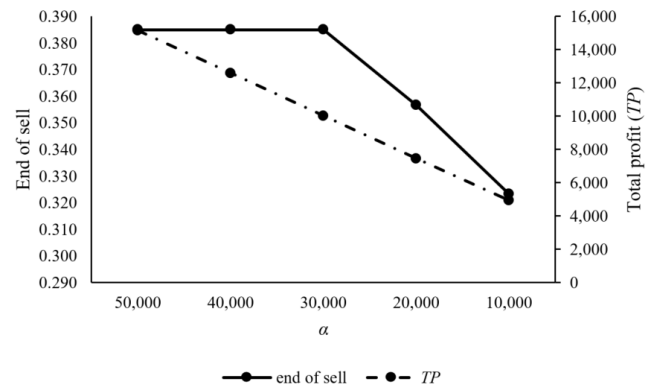

Figure 15. The effects of the IDR slope.

The flattening of the IDR slope does not affect the end of the selling period, with the condition that the demand rate is still positive ($R(T_3) > 0$). However, this decreases the profit (see Figure 15). Steepening the DDR slope does not affect the end of the selling period when the demand rate is still positive. However, profit decreases with less steep (see Figure 16).

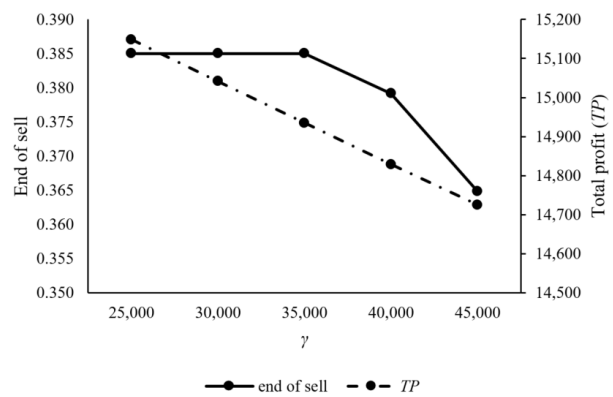

Figure 16. The effects of the DDR slope.

A sensitivity analysis of T_1 is related to the length of the growth period (IDR). There was an increase in profit along with an increase in T_1. Prolonged growth periods lead to higher demand rates and profits, even though the end of the selling period remains the same (see Figure 17). Sensitivity analysis T_2 relates to the obsolescence period of a product type. There are three different conditions in this (see Figure 18). First, the shorter maturity period makes the profit smaller and the selling period shorter. When T_2 = 1 month (0.08 years), it reaches a demand rate equal to zero faster than T_2 = 2 months (0.17 years). An important finding from this phenomenon is that when a period of obsolescence occurs too fast, it has more effects on the optimal end of the selling period and the profit than the effect of holding costs and deterioration. Second, when T_2 = 3 months (0.25 years), when the profit is smaller than in T_2 = 4 months (0.33 years), but the selling period is not different, the demand rate after the end of the selling period (0.385 years) has not reached zero. An important finding in this condition is that obsolescence has a balanced power of influence with the holding cost and the deterioration. The difference in profit where the same end of the selling period shows the effect of obsolescence. Besides, the same end of the selling period at the different levels of obsolescence (T_2) shows the effect of holding costs and deterioration. Third, for T_2 = 5 months (0.42 years) and T_2 = 6 months (0.5 years), the end of the selling period occurs in the CDR phase or occurs at a shorter time than T_2 so that the profit is still the same. An important finding from this phenomenon is when the end of the selling period occurs at CDR ($T_2 \geq T_2^*$), there will be no effect for the longer mature period on the length of the selling period and will lead to the same profit. In this case, the management should consider the holding costs and deterioration despite the maturity period.

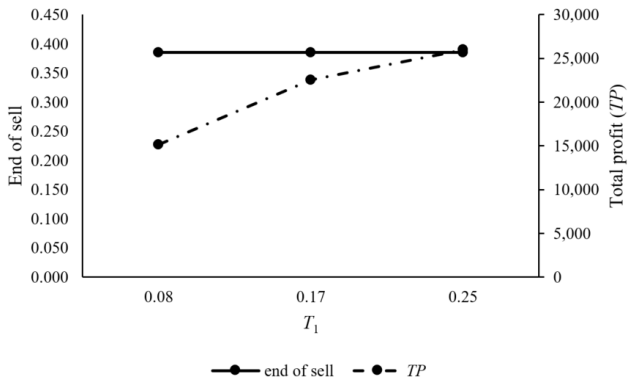

Figure 17. The effects of the end of the IDR period.

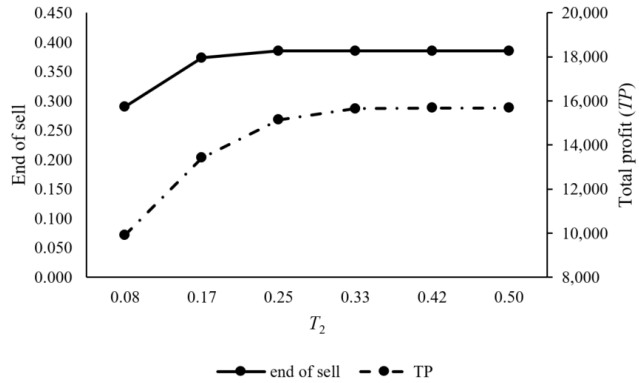

Figure 18. The effects of the end of the CDR period.

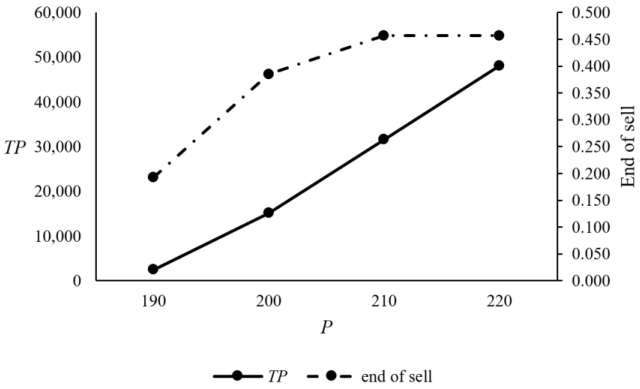

Figure 19. The effects of the selling price on the profit and the end of selling.

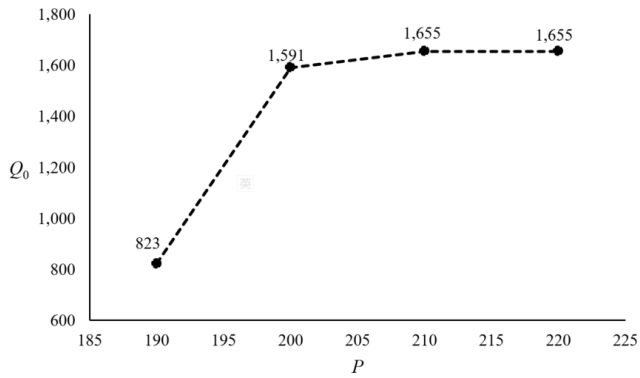

Figure 20. The effects of the selling price on the order size.

An increasing selling price will increase the total profit, which in this model assumes there is no influence of the selling price on the demand rate. A sensitivity analysis of the selling price indicates the effect of price on the decisions. If the selling price is lower, it will accelerate the end of the selling period and even move to the earlier phases. If the selling price is higher, then it can extend the sales period as long as there is still demand (see Figure 19 and Figure 20). If there is no more demand, a higher selling price only increases profit and does not extend the sales period or create larger order sizes (Q_0). The lower selling price will have lower profit and even threaten feasibility.

A sensitivity analysis of ordering costs was not performed because this study examined one-time orders. However, accommodating it in the model can help to ensure the feasibility of the decisions while maintaining a positive total profit. A sensitivity analysis was also not applied to the purchasing costs because, in general, purchasing costs greatly affect holding costs and profit margins. A higher purchasing cost will result in a smaller profit margin and a larger holding cost, and both have a parallel effect. It shows the magnitude of purchasing costs' influence on performance and model decisions in line with the holding cost and inversely proportional to the selling price.

6. CONCLUSIONS

This study provides findings that combine the effect of product obsolescence, holding cost, and deterioration. The steepness of decreasing inventory levels that varies from each stage of the market life cycle verifies the importance of considering the market life cycle in the inventory model and also confirms the importance of estimating potential demand with precision. This study also shows that the optimal end of a sales period can occur in any region and indicates that management can determine the optimal end of the sell period target in a mature period and even during growth. This shows the importance of the effect of holding cost and deterioration. The decision to determine the end of a sales period and the order size to obtain maximum profit is a challenge in industries with advanced technological products that have rapid innovation and

improvement timelines, such as smartphone products. This decision is also vital for international delivery, where the manufacturers and distributors are from different countries.

A sensitivity analysis shows a significant influence on inventory policy by deterioration rate, holding cost, the slope of IDR and DDR, the end of the CDR period (T_2), and selling price. However, the initial demand rate (beta) and the end of the IDR period (T_1) do not influence the decisions and only affect profit performance. In previous studies that stated the independence of inventory policy from demand rate (Hung, 2011; S.W. Lin, 2011), Zhao (2014) and Zhao (2016) were interested in the effect of many parameters except the demand parameters. The present study is consistent with the results of those studies in several conditions; however, the present study is different in the linear decreasing demand rate model with the profit as the performance measure in that it can make the demand rate equal to zero at a specific time so that it can limit the sales period and limit the order size, and finally affects the profit. It is necessary to consider the deterioration rate in all phases of the market life cycle because advanced technological products experience deterioration not only in the decreasing phase. The deterioration rate and the holding cost collaboration effect can shift the optimal end of the selling period to an earlier stage than the DDR phase before obsolescence occurs. This is different from Nagare *et al.* (2020), where deterioration can occur only in the DDR phase. Meanwhile, the additional deteriorating cost has a small effect, and it confirms the previous finding by M.B. Cheng *et al.* (2011).

Future model development may look at a broader view by considering the multi-echelon inventory policy. Good collaboration can lead to better overall performance. Further, involving different product types and the inventory policy for two or more types of products with sequential launching dates to capture the technological innovation or improvement.

ACKNOWLEDGEMENT

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APPENDIX - THE RESEARCH GAP SUMMARY

Author(S)	Ramp Type or Trapezoidal Type Demand Rate	Include IDR, CDR, and DDR	One-Time Order	Deterioration Can Occur in All Phase	Order Cycle is Not A Given Parameter	Profit Performance	End of Selling Period Decision
Mandal and Pal (1998)	yes	no	no	yes	no	no	no
Panda *et al.* (2008)	yes	yes	no	yes	yes	no	no
M.B. Cheng and Wang (2009)	yes	yes	no	yes	no	no	no
Skouri *et al.* (2009)	yes	no	no	yes	no	no	no
M.B. Cheng *et al.* (2011)	yes	yes	no	yes	no	no	no
S.W. Lin (2011)	yes	yes	no	yes	no	no	no
Ahmed *et al.* (2013)	yes	no	no	yes	no	no	no
K.P. Lin (2013)	yes	yes	no	yes	no	no	no
Zhao (2014)	yes	yes	no	yes	no	no	no
Pal *et al.* (2015)	yes	no	no	yes	no	no	no
Zhao (2016)	yes	yes	no	yes	no	no	no
Sett *et al.* (2016)	yes	no	no	yes	no	no	no
J. Wu *et al.* (2016)	yes	yes	no	yes	no	yes	no
Vandana and Srivastava (2017)	yes	yes	no	yes	no	no	no
J. Wu *et al.* (2017)	yes	yes	no	yes	no	yes	no
J. Wu *et al.* (2018)	yes	yes	no	yes	no	yes	no
Otrodi *et al.* (2019)	no	no	no	-	yes	yes	no
Shaikh *et al.* (2019)	no	no	no	-	yes	yes	no
Shi *et al.* (2019)	yes	no	no	yes	yes	no	no
Bardhan *et al.* (2019)	no	no	no	-	yes	yes	no
Sun *et al.* (2020)	yes	yes	no	yes	yes	no	no
Nagare *et al.* (2020)	yes	yes	yes	no	yes	yes	yes
Saha *et al.* (2021)	yes	yes	no	yes	no	yes	no
This paper	yes	yes	yes	yes	yes	yes	yes