



Punching Shear Strength Prediction for Reinforced Concrete Flat Slabs without Shear Reinforcement

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Abstract

Failure of flat slabs usually occurs by punching shear mode. Current structural codes provide an experience-based design provision for punching shear strength which is often associated with high bias and variance. This paper investigates the effect of adding a horizontal reinforcement mesh at the top of the slab-column connection zone on punching the shear strength of flat slabs. A new equation considering the effect of adding this mesh was proposed to determine the punching shear strength. The proposed equation is based on the Critical Shear Crack Theory combined with the analysis of results extracted from previous experimental and theoretical studies. Moreover, the equation of load-rotation curves for different steel ratios together with the failure criterion curves were evaluated to get the design points. The investigated parameters were the slab thicknesses and dimensions, concrete strengths, size of the supporting column, and steel ratios. The model was validated using a new set of specimens and the results were also compared with the predictions of different international design codes (ACI318, BS8110, AS3600, and Eurocode 2). Statistical analysis provides that the proposed equation can predict the punching shear strength with a level of high accuracy (Mean Square Error =2.5%, Standard Deviation =0.104, Mean=1.0) and over a wide range of reinforcement ratios and compressive strengths of concrete. Most of the predictions were conservative with an underestimation rate of 12%.

Keywords: Flat Slabs; Steel Reinforcement; Punching Shear; Load-Rotation.

1. Introduction

Flat slabs are reinforced concrete one-way and two-ways slabs in which beams or girders are not provided, and their loads are directly transformed to the supporting columns. Flat slabs are common in reinforced concrete structures due to the ease and relatively low cost of construction, and their advantages are more than mentioned in this context. In this structural system, the slab shear loads are being concentrated on supporting reinforced concrete columns, which may increase the tendency of the supporting columns to punch the slab and limit the overall load capacity of such a system. Several solutions are available to override this problem including the construction of drop panels, column capitals or heads, or a combination of column capitals and drop panels. Figure 1 shows the load-deflection diagrams for five slab-column connections to compare the strength and behavior of using different methods i.e., drop panel, column capital, and shear reinforcement either stirrups or shear studs. For practical engineering, there are many limitations on drop panels and column capitals for architectural purposes and mechanical engineering installations. Furthermore, using shear reinforcement in the vicinity of the slab-column connection causes steel congestion. However, using flexural reinforcement increase the punching shear strength.

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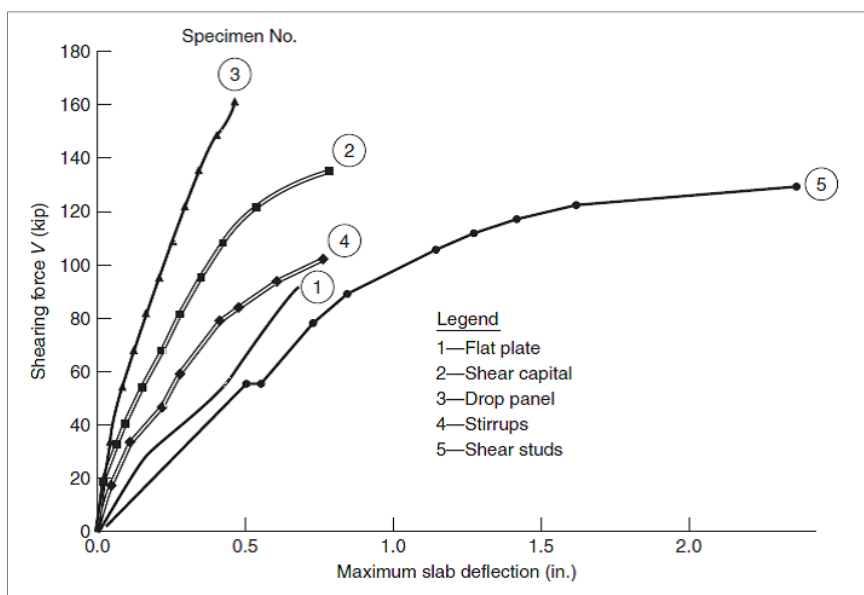


Figure 1. Load deflection curves for slabs strengthened against two-way shear failure [1]

The big fruit of the flat slab structural system can be best achieved if the designers could avoid the use of drop panels or columns capitals. In such a case, the maximum floor height and flat soffit could be maintained. To do so, the designer should employ the slab thickness to its optimum level. The analysis and design of flat slabs to resist the two-way shear (punching) are based on several parameters including geometric properties like slab thickness, column dimensions, area of steel reinforcement, and material mechanical properties such as concrete and steel strengths. To promote the total capacity of the flat slab while maintaining optimum thickness, the punching shear strength of the slab can be improved by introducing shear studs, shear stirrups, and bent-up bars. Moreover, some of the building codes take into consideration the steel reinforcement effect in punching shear capacity calculations, like EU2 [2] and BS8110 [3].

The development of the Critical Shear Crack Theory (CSCT) was suggested by Muttoni and other researchers [4-7] based on extensive experimental work. Figure 2 shows the nature of failure and the cracking pattern of punching shear failure for a flat slab as represented by Guandalini and Muttoni [6]. They measured the radial strains at the slab soffit as a load function. The figure shows the development of a concrete strut across the critical path of the shear crack. In research conducted by Muttoni and Schwartz [5], it was proved that the opening width of the critical shear crack will reduce the strength of the inclined concrete compression strut that contributes to the slab shear capacity and eventually leads to a punching shear failure. According to Muttoni [7], the width of the critical crack can be assumed to be a function of the slab rotation angle (ψ) and its effective depth (d).

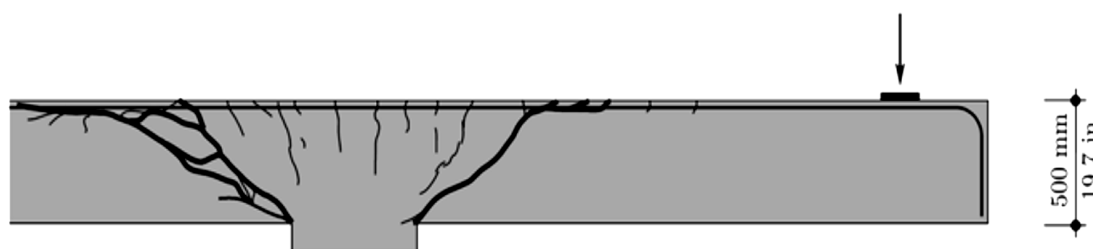


Figure 2. Punching shear failure mode and cracking pattern of flat slab [6]

Guandalini et al. [8], carried out an experimental test on slabs constructed with a low reinforcement ratio. The study showed that the ACI-318 code [9] equation for calculating the punching shear strength of flat slab could provide conservative predictions as compared with the experimental results; meanwhile, it provides satisfactory results when compared to the predictions by EC2 and the equations of the critical shear crack theory. An experimental investigation was conducted by Kinnunen and Nylander [10] reported that the shear strength value of circular flat slabs could achieve higher values for the higher reinforcement ratios. Increasing the reinforcement ratio from 0.5 to 2.1 is reported to increase the shear strength by more than 35%. It was also noted that flat slabs reinforced with smaller reinforcement ratios showed a more ductile failure manner as compared with the ones reinforced using higher reinforcement ratios. The punching shear strength is also affected by the concrete compressive strength. It was reported that for a high concrete strength up to 130 MPa the increment in the punching shear strength can reach up to 43% as compared to normal concrete strength [11]. Also, the reinforcement ratio will have a similar effect as reported by Kinnunen and Nylander [10]; however, the increase in reinforcement ratio has less effect in the case of high-strength concrete. The ACI318 and EC2 equations

were reported to provide good predictions in the case of high-strength concrete [11]. Lightweight concrete is known to show a lower shear strength by about 15% as compared to normal weight concrete [12]. Osman et al. [13], investigated the punching shear of lightweight high-strength concrete flat slabs. The findings revealed that the reinforcement ratio has a greater effect on punching shear strength as compared with normal-weight concrete. Also, a reduction factor of 0.95 was suggested for lightweight high-strength concrete rather than 0.8 and 0.85 required by ACI318 [9] and BS8110 [3] codes, respectively. The aforementioned codes were described to provide conservative values. A new model for both normal and high strength concrete was proposed by Annash et al. [14]. This model considered the effect of critical section position, concrete compressive strength, slab effective depth, reinforcement ratio, and dimensions of the column and its shape. Their model proved to give closer predictions of the as compared with different codes.

An interesting comparative study by Safiee and Ashour [15] was conducted to investigate the accuracy of existing models for punching shear strength as described by several codes. The study considered the experimental results of 218 flat slabs. Their study revealed that the models of ACI 318-11 and CSA-A23.03-04 codes tend to overestimate the punching shear strength as compared to the models specified by CEB-FIP-90 and BS8110 codes. The most conservative predictions were obtained by the CEB-FIP-90 model but also at a high scattering level while the BS8110 model provided predictions with the smallest mean square error. More detailed results were provided by Bartolac et al. [16] where they stated that all models provided by EC2, ACI318-11, and fib Model Code 2010 tend to underestimate the punching shear strength in the case of flat slabs without shear reinforcement and this was also true for flat slabs with shear reinforcement except for EC2 models which tend to overestimate the results in this case.

The use of shear reinforcement is also found to affect the punching shear strength of flat slabs, where incorporation of close loop stirrups can significantly increase the shear capacity [17]. Moreover, it was noted that the contribution of concrete to punching shear strength is a function of the amount of shear reinforcement [18]. Similarly, the flexural reinforcement can also increase the punching shear capacity of the slab-column connection for flat slabs with or without shear reinforcement [19]. Other factors that may affect the punching shear strength may include the presence of openings and their shape, size, and location [20]. The punching shear capacity deteriorates if the opening is located within the critical perimeter around the column. The type of concrete also contributes to punching shear strength where the use of self-compacting concrete is found to enhance the capacity of flat slabs [21], and the use of high strength concrete led to a significant increase in punching shear capacity as compared to normal strength concrete [22]. Issa and Ismail [23], carried out a comparative study between the international building codes and collected experimental tests on punching strength. ACI-318-19 and Egyptian building codes don't take into consideration the flexural reinforcement effect on punching shear strength; however, they can provide a good correlation with the data from experiments.

In this paper, a new method for increasing the punching shear strength of flat slabs was proposed. The proposed method has the privilege of ease of implementation and promoting the flexural and punching shear resistance. The contribution of the proposed horizontal mesh of reinforcement at the top of the slab-column connection zone on punching shear strength was analyzed in detail by comparing the experimental results extracted from different published work with the current structural design codes. The study included a wide range of concrete compressive strength and reinforcement ratios. A new equation was derived to consider the effect of this mesh on the punching shear capacity. The application of such reinforcement mesh may help the designers to use flat slabs for longer spans while maintaining optimum slab thickness. The proposed formula may also form the base for predicting the punching shear capacity of such flat slabs.

2. Provision of Design Codes

Different design codes have formulated different equations for the calculation of punching shear strength. These equations have considered several influencing parameters and also different limits for some of these parameters. However, most of the design codes consider the shear capacity as the product of the shear critical area by the shear stress. The shear stress is usually taken as a function of the compressive strength of concrete. The critical shear area is considered as a multiplication of the slab depth by the critical shear perimeter which has different limits for the different codes. Some of the international design codes like the American Concrete Institute Code (ACI 318-19) [9] and the Australian code (AS3600) [24] do not account for the steel reinforcement ratio of the slabs. While other codes like CEP-FIP [25] Eurocode 2 [2] and the Swiss Code [26] do.

The ACI 318-19 [9], as well as the Australian code AS3600 [24] design formulas for calculating the punching shear of normal-weight concrete flat slabs, are given by equations 1 through 4. These equations relate the nominal punching shear strength (V_n) to the effective slab depth (d), average perimeter (b_o) of the critical section which should be calculated at a distance of $d/2$ from the column face, and concrete cylindrical compressive strength (f'_c).

$$V_n = v_n b_o d \quad (1)$$

where the value of the shear stress (v_n) is determined as the minimum value calculated from the following equations:

$$v_n = \frac{1}{3} \sqrt{f'_c} \quad (2)$$

$$v_n = \left(1 + \frac{2}{\beta_c}\right) \frac{\sqrt{f'_c}}{6} \quad (3)$$

$$v_n = \left(\frac{\alpha_s d}{b_o} + 2\right) \frac{\sqrt{f'_c}}{12} \quad (4)$$

where d and b_o are the effective depth and critical shear perimeter calculated at a distance of $d/2$ from the face of the column, respectively in (mm), $\alpha_s = 40, 30,$ and 20 for interior, edge, and corner columns, respectively, β_c is the ratio of the longest to the shortest column dimension. The equations for calculating the design punching shear resistance (V_R) as described by the CEP-FIP [25] Eurocode 2 [2] are as follows:

$$V_R = C_{RD} k (100 \rho_1 f_{ck})^{\frac{1}{3}} u_1 d \leq v_{Rmax} u_o d \quad (5)$$

where C_{RD} is a calibration factor equals to 0.18 in EC2 and 0.12 in CEP-FIP-90, $k = 1 + \sqrt{(200/d)} \leq 2$ and represent the size factor, $\rho_1 = \sqrt{\rho_{1x} \rho_{1y}} \leq 0.02$ in which ρ_{1x} and ρ_{1y} are the tension steel ratios in x and y directions, respectively, u_1 is the perimeter in (mm) of the critical section calculated at $2d$ from the column face, u_o is the column perimeter in (mm), d is the slab effective depth in (mm), f_{ck} is the characteristic cubic compression strength of concrete in (MPa), and $v_{Rmax} = 0.24(1 + 250/f_{ck})f_{ck}$. Based on the British code BS 8110 [3], the nominal shear force is given by equation 6 as follows:

$$V_n = \tau_c b_o d \quad (6)$$

where $\tau_c = \frac{0.79}{\gamma_m} \left(100 \rho \frac{f_{cu}}{25}\right)^{\frac{1}{3}} (k)^{\frac{1}{4}}$, $k = 400/d \geq 1$, γ_m is the partial material factor of safety of concrete taken as 1.5, ρ is the reinforcement ratio $\leq 3\%$ and calculated using A_s taken as the area of longitudinal tension reinforcement which continues for a distance $\geq d$ beyond the section being considered, d is the effective depth of the section in (mm), b_o is the critical perimeter taken at $1.5d$ from the face of the column in (mm), and f_{cu} is the cubic compressive strength of concrete in (MPa) and should be ≤ 40 MPa.

The Swiss Code for the design of structural concrete SIA 262 [26] expresses the shear strength as a function of the load-rotation relationship. Also, it considers the effect of the maximum aggregate size since it will affect the amount of shear developed along the critical path of the shear crack by affecting the surface roughness. The Code adopted the following equations to predict the slab punching shear strength:

$$\psi = 0.33 \frac{L f_y}{d E_s} \left(\frac{V_d}{8m_R}\right)^{1.5} \quad (7)$$

$$\frac{V}{b_o d \sqrt{f'_c}} = \frac{2}{3\gamma_c} \frac{1}{1 + 20 \frac{\psi d}{d_{g0} + d_g}} \quad (8)$$

where L is the slab span in (m), m_R is the slab flexural capacity in the column region, V_d is the factored shear force, γ_c is the concrete partial factor of safety equals 1.5, ψ is the rotation angle in radians, d_g is the maximum size of aggregate in (mm), and d_{g0} represent a reference aggregate size equal to 16 mm.

3. Proposed Model

The critical shear crack theory has formulated the semi-empirical failure criterion as explained by Muttoni and Schwartz [5], together with the load-rotation relationship to develop the design punching shear equation. The adopted failure pattern and the used terms are illustrated in Figure 3.

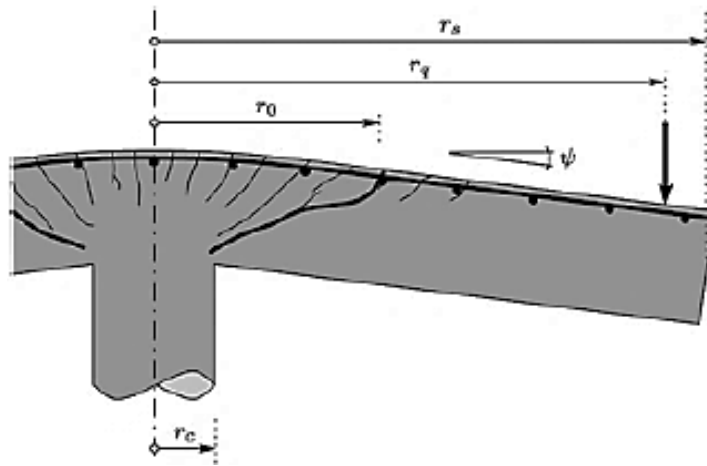


Figure 3. Failure pattern and definition of the used terms as by CSCT [7]

The failure criterion equation and the load-rotation equation, respectively are given by:

$$V_R = \frac{0.75b_o d \sqrt{f'_c}}{1 + \frac{15\psi d}{d_{go} + d_g}} \tag{9}$$

$$\psi = 0.33 \frac{L f_y}{d E_s} \left(\frac{V_d}{8m_{Rd}} \right)^{3/2} \tag{10}$$

Solving Equations 9 and 10 simultaneously yields the proposed design criteria for the punching shear, as illustrated in Figure 4. The points of intersections are determined graphically to determine the optimized design shear capacity which takes into consideration the participation of slab reinforcement in addition to the concrete capacity. A flowchart showing the research methodology is depicted in Figure 5.

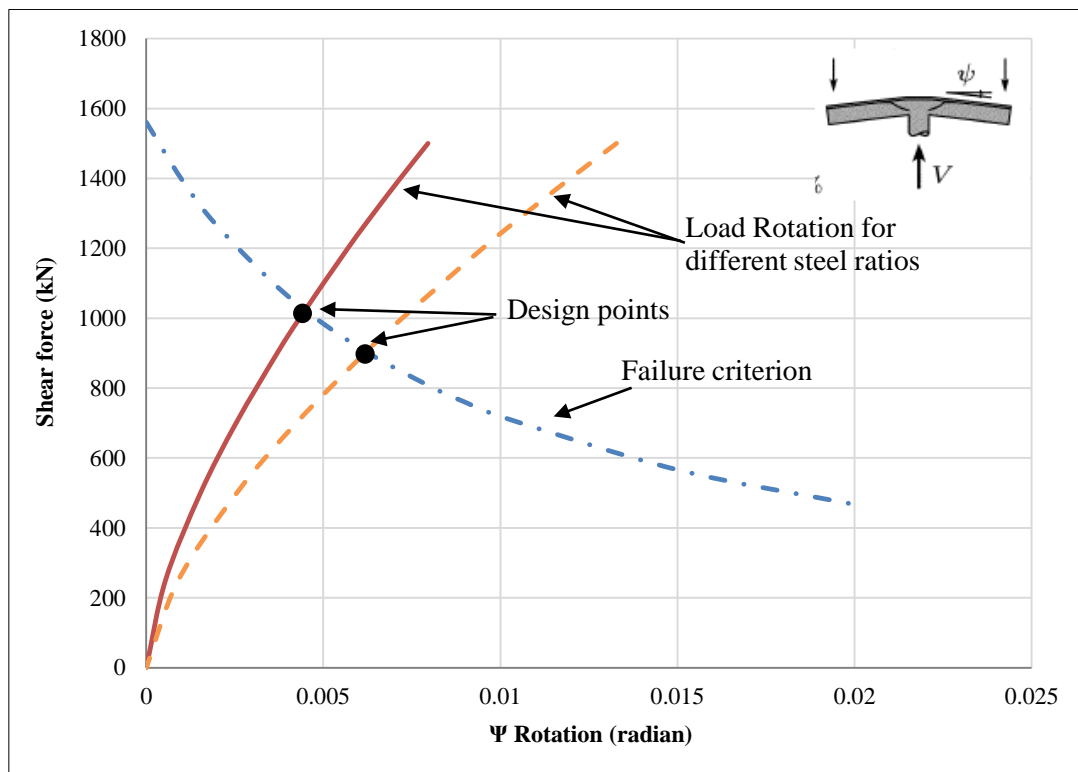


Figure 4. Load-rotation curve with a failure criterion

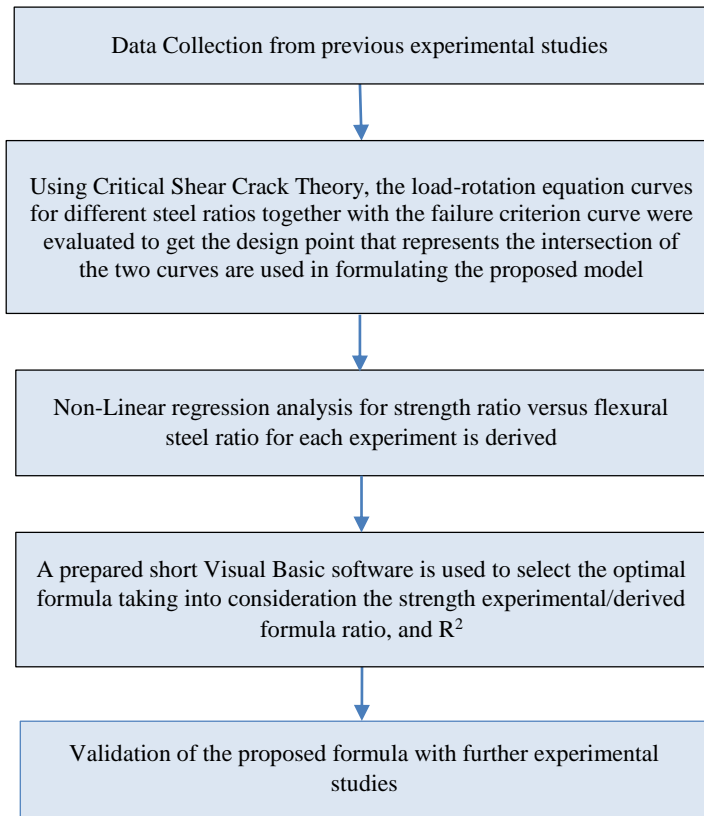


Figure 5. Research Methodology Flowchart

To derive an explicit relationship between punching shear force (V) and steel ratio (ρ), the load-rotation (as given by Equation 10) is drawn for different ρ values with the failure criteria equation (as given by Equation 9) as shown in Figure 6. Then, the optimum design points (intersection points) are located and plotted as shown in Figure 7. This procedure was repeated for all the experimental results obtained from several published studies as presented in Table 1. A nonlinear regression analysis using Equation 11 was carried out for all the graphs shown in Figure 7. The output equations are summarized in Table 2 where x is the reinforcement ratio and y is the shear strength ratio. A prepared short Visual Basic software is used to select the optimal formula. The best fit curve is found to be best expressed in a form of a logarithmic equation as:

$$\frac{V}{b_o d \sqrt{f_c}} = 0.0627 \ln(\rho) + 0.6446 \tag{11}$$

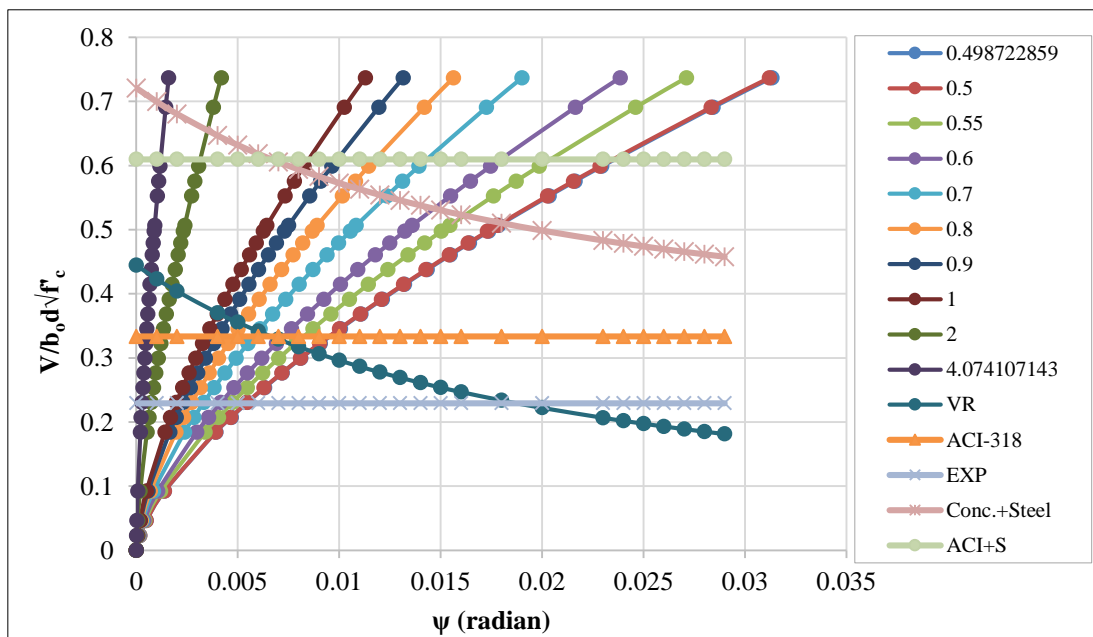


Figure 5. Shear strength ratio versus rotation angle for different steel ratios

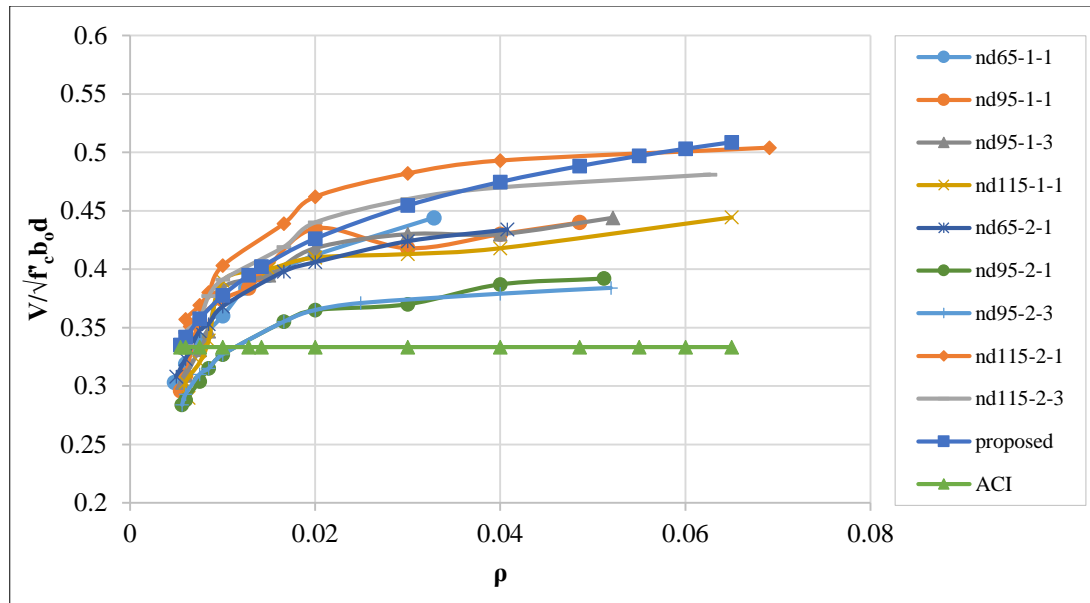


Figure 6. Shear strength ratio versus reinforcement ratio for different test samples and proposed formula

Table 1. Data and results from previous studies

Specimen ID	Type	Diameter or Width (mm)	f _c ' (MPa)	Column Diameter or Width(mm)	Slab Depth (mm)	ρ (%)	V _{Exp.} (kN)	Ref.
slab 5	Circular	1372	54.4	150	125	0.58	190	[27]
slab 12			60.4		125	1.28	319	
slab 15			68.4		125	1.28	276	
slab 16			99.2		125	1.28	362	
slab 22			84.2		125	1.28	405	
slab 23			56.4		125	0.87	341	
HSC0	Circular	2400	90.3	250	240	0.8	965	[28]
HSC2			85.7		240	0.8	889	
HSC4			91.6		240	1.2	1041	
HSC6			108.8		240	0.6	960	
HS2	Square	1500	70.2	150	120	0.84	249	[29]
HS7			73.8		120	1.19	356	
HS3			69.1		120	1.47	356	
HS4			65.8		120	2.37	418	
HS5			68.1		150	0.64	365	
HS12			75	90	1.52	258		
HS13			68	90	2	267		
HS14			72	120	1.47	498		
HS15			71	300	1.47	560		
nd65-1-1			Square	2500	64.3	200	320	
nd95-1-1	83.7	320			1.42		2250	
nd95-1-3	89.9	320			2.43		2400	
nd115-1-1	112	320		1.42	2450			
nd65-2-1	70.2	240		1.66	1200			
nd95-2-1	88.2	240		1.66	1100			
nd95-2-3	89.5	240		2.49	1250			
nd115-2-1	119	240		1.66	1400			
nd115-2-3	108.1	240		2.49	1550			
nd95-3-1	1100	120		1.72	330			

IA30d-32			25.8		123	0.5	258	
IA30a-25			24.6		124	1.1	408	
IA30e-34	Circular	1840	26.9	300	120	1.0	332	[10]
IA30c-31			29.2		119	2.1	540	
IA30c-30			29.5		120	2.1	491	
IA30e-35			24.3		122	1.0	332	

Table 2. Derived regression equations

Equation	R ²	V _{exp} /V _{Eq}
$y = 0.0689\ln(x) + 0.6699$	0.963	0.991
$y = 0.0882\ln(x) + 0.7658$	0.918	0.809
$y = 0.07\ln(x) + 0.669$	0.944	0.996
$y = 0.0689\ln(x) + 0.6699$	0.963	0.914
$y = 0.0882\ln(x) + 0.7658$	0.916	0.753
$y = 0.07\ln(x) + 0.669$	0.944	0.649
$y = 0.074\ln(x) + 0.6475$	0.975	1.035
$y = 0.0742\ln(x) + 0.6521$	0.978	0.979
$y = 0.0777\ln(x) + 0.6711$	0.949	0.975
$y = 0.0676\ln(x) + 0.6073$	0.976	1.04
$y = 0.0646\ln(x) + 0.6709$	0.865	0.85
$y = 0.06\ln(x) + 0.6334$	0.979	1.062
$y = 0.0748\ln(x) + 0.7048$	0.993	1.211
$y = 0.0627\ln(x) + 0.6446$	0.908	1.041
$y = 0.0597\ln(x) + 0.6238$	0.828	1.097
$y = 0.0599\ln(x) + 0.6356$	0.973	1.155
$y = 0.0492\ln(x) + 0.5469$	0.968	0.945
$y = 0.0455\ln(x) + 0.5313$	0.944	0.96
$y = 0.0685\ln(x) + 0.7106$	0.952	1.035
$y = 0.0636\ln(x) + 0.6746$	0.961	1.083
$y = 0.0138\ln(x) + 0.1686$	0.950	1.013

4. Results and Discussion

A total of 35 test results for the punching shear strength of flat reinforced concrete slabs were collected from literature [10, 27-30]. All the collected data was for flat slabs without any drop panels or column capitals. The collected data were representative and covered a wide range of the included variables. The compressive strength of the collected data ranged from 25 to 120 MPa, reinforcement ratio ranged from 0.5-2.5%, and slab depth ranged from 90 to 320mm. The statistical analysis of the collected data is given in Table 3. The data also included columns having circular and square cross-sections.

Table 3. Statistical analysis of collected data variables

Parameters	Minimum	Maximum	Mean	Mode
Compressive strength (MPa)	25.6	119	71.7	70, 30
Effective depth (mm)	90	320	173.9	120
Reinforcement ratio (%)	0.5	2.5	1.35	1.28

The collected data is initially used to derive the proposed model and then it was used to assess the other models as adopted by different design codes including the Australian AS3600, Eurocode 2, ACI318, and BS8110 codes. The assessment was carried out by comparing the ratio of the experimental punching shear force results of the collected data and the value predicted by the equations of these design codes. The comparison for each specimen is presented in Table 4.

Table 4. The ratio of experimental/ predicted punching shear force value as calculated using different design codes equations and the proposed model

Specimen ID	f'c	Exp/ AS3600	Exp/ EC2	Exp/ ACI318	Exp/ BS8110	Exp/Proposed model
slab 5	54.4	0.70	0.59	0.72	0.86	1.21
slab 12	60.4	1.12	0.73	1.14	1.40	0.92
slab 15	68.4	0.91	0.61	0.93	1.16	1.14
slab 16	99.2	0.99	0.70	1.01	1.35	1.04
slab 22	84.2	1.20	0.83	1.23	1.59	0.86
slab 23	56.4	1.24	0.91	1.26	1.53	0.76
HSC0	90.3	0.81	0.72	0.82	1.05	0.88
HSC2	85.7	0.76	0.67	0.78	0.98	0.83
HSC4	91.6	0.87	0.68	0.88	1.12	0.85
HSC6	108.8	0.73	0.74	0.75	0.98	0.86
HS2	70.2	0.67	1.09	0.69	0.88	0.86
HS7	73.8	0.94	1.37	0.96	1.23	0.72
HS3	69.1	0.97	1.30	0.99	1.26	0.93
HS4	65.8	1.17	1.33	1.19	1.50	0.91
HS5	68.1	0.72	1.25	0.74	0.88	0.99
HS12	75	1.01	1.41	1.03	1.44	0.83
HS13	68	1.10	1.37	1.12	1.54	0.94
HS14	72	1.06	1.49	1.08	1.53	0.97
HS15	71	0.97	1.41	0.99	1.52	0.99
nd65-1-1	64.3	1.13	1.57	1.15	1.23	0.91
nd95-1-1	83.7	1.09	1.58	1.11	1.23	1.07
nd95-1-3	89.9	1.12	1.37	1.14	1.28	1.03
nd115-1-1	112	1.02	1.56	1.04	1.22	0.94
nd65-2-1	70.2	1.13	1.41	1.15	1.24	0.97
nd95-2-1	88.2	0.92	1.20	0.94	1.05	1.03
nd95-2-3	89.5	1.04	1.18	1.06	1.19	0.84
nd115-2-1	119	1.01	1.38	1.03	1.21	0.87
nd115-2-3	108.1	1.17	1.38	1.19	1.39	0.92
nd95-3-1	85.1	1.00	1.26	1.02	1.21	0.98
IA30d-32	25.8	0.83	0.95	0.71	1.06	0.88
IA30a-25	24.3	0.76	0.93	0.66	1.03	1.15
IA30e-34	26.9	0.89	1.09	0.81	1.49	0.96
IA30c-31	29.2	0.61	0.88	0.52	1.1	1.28
IA30c-30	29.2	1.03	0.97	0.93	1.61	1.15
IA30e-35	24.3	0.87	1.09	0.79	1.35	0.99

The proposed equation to estimate the punching shear strength of flat slabs is very simple and applicable for use. Figure 7 shows that the experimental values versus the values calculated using the proposed model and different design codes. The figure shows that the proposed model can predict the punching shear force with a good accuracy whereas the values are very close to the 1 to 1 diagonal line. In general, the proposed model tends to underestimate the punching shear capacity of the flat slabs whereas most of the points are located above the 1 to 1 diagonal line as shown in Figure 7. This means that the proposed model will provide safe conservative predictions, the average underestimation percentage of the values is about 12% which is acceptable for shear estimations.

As comparing the predictions of this model to the ones estimated using the different design codes, it can be seen that this model provides relatively better or similar predictions as can be extracted from Figure 7 and Table 5. Figure 7 shows that ACI318 and EC2 codes tend to underestimate the values as compared with the experimental values by an average of 10% and 26%, respectively. While the AS3600 and BS8110 tend to underestimate the values for the slabs having lower shear strength and overestimate the values for the slabs having high shear strength. This means that the shear capacity predicted by these models may need to be multiplied by a reduction factor for slabs designed to resist high punching shear force. The statistical analysis presented in Table 5 shows that the proposed model has the lowest prediction mean square error of 2.5% which is similar to the AS3600 model while it was more than 10% as predicted by EC2 and BS8110. The variance for the proposed model was the lowest as over all other codes, this means that this model will provide the lowest risk of predictions where the predicted results are close to the measured value. A similar conclusion can be extracted by looking at the mean value and other statistical parameters.

Table 5. Results of statistical analysis of the ratio of experimental / predictions by proposed and design codes models

Model	Minimum	Maximum	Mean	Variance	Standard deviation	Mean square error
Proposed model	0.65	1.21	1.00	0.014	0.104	2.5%
AS3600	0.67	1.24	0.99	0.024	0.156	2.5%
EC2	0.59	1.58	1.14	0.108	0.329	12.8%
ACI318	0.69	1.26	1.01	0.025	0.159	2.6%
BS8110	0.87	1.59	1.24	0.044	0.209	10.3%

To validate the proposed model, a new set of data including 15 flat slabs was collected from the published research work [8, 11] and their results were compared with the predicted values using the proposed model. The compressive strength of the concrete for the new data set ranged from 50 to 120 MPa. The validation process was conducted in order to check the accuracy of the proposed model for predicting new results other than that the ones used in the model development. The experimental versus predicted values are illustrated in Figure 8 for the graph of the validation data set. The figure shows that the model can predict new values with good accuracy and still provides conservative results. The mean square error for the new predicted data was only 5.4%.

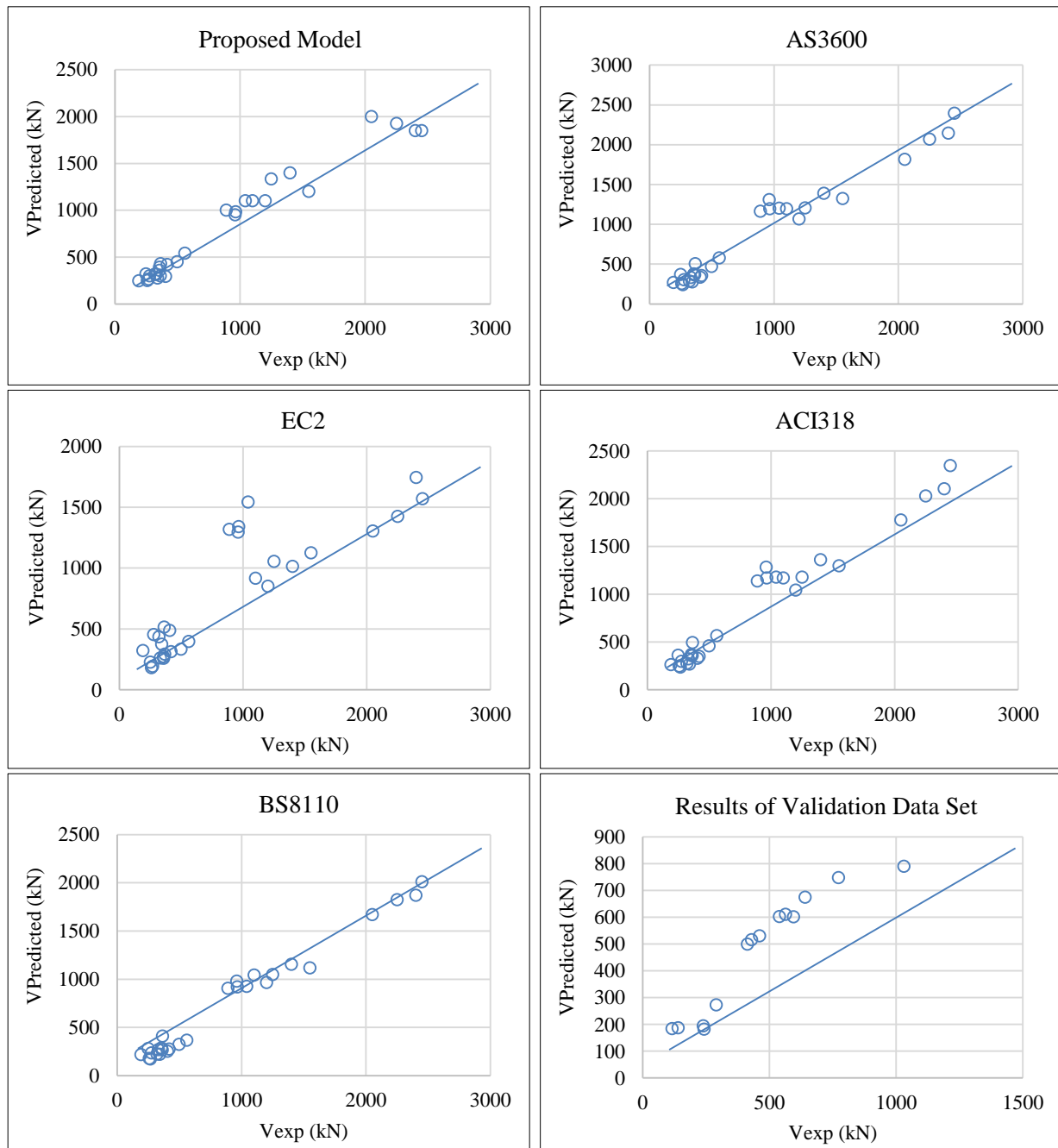


Figure 7. Experimental punching shear force versus prediction value as calculated using the proposed model and equations of design codes

The proposed model takes into consideration the steel ratio and the compressive strength of concrete. To illustrate the effect of these two parameters on the predicted values as calculated by the proposed model as well as the existing design codes equations, the compressive strength and reinforcement ratio was plotted against the experimental/ predicted ratio as displayed in Figures 9 and 10, respectively. Figure 9 shows that the variability of predictions by the proposed model was the lowest over the entire range of compressive strength; yet, it tends to have higher variability at a lower strength ($f'c \leq 50$ MPa). Similar behavior can be seen for the effect of reinforcement ratio as given in Figure 10. However, the variability was almost the same for the whole range of reinforcement ratios. In the case of AS3600 and ACI318, the predictions also have low variability, but it is noted that the predictions at a lower reinforcement ratio provide an experimental/ predicted ratio less than 1 while it is more than 1 for the higher reinforcement ratios. To illustrate the effect of the steel reinforcement ratio on the shear strength capacity of flat slabs, the proposed model was used to calculate the shear strength for flat slabs at the minimum and maximum steel ratios as calculated by the ACI318 code equations, the shear strength capacity was found to be increased by only 2% for the minimum steel ratio, while an increase of 33% was obtained at the maximum steel ratio.

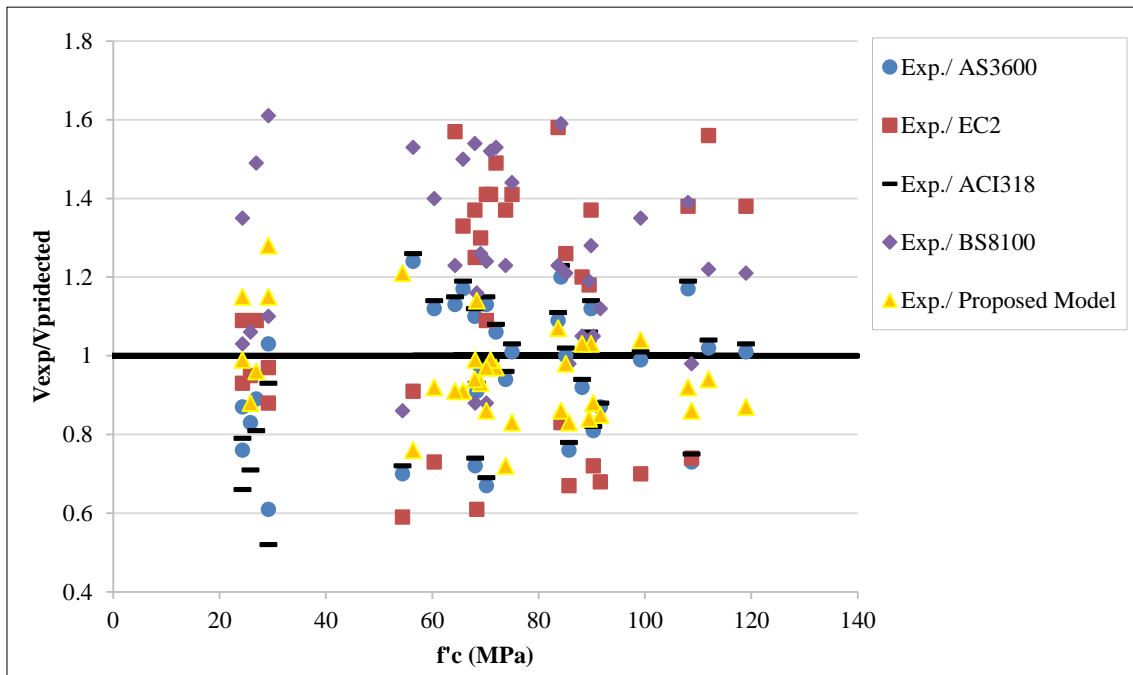


Figure 8. Compressive strength versus the ratio of experimental/ predicted shear force

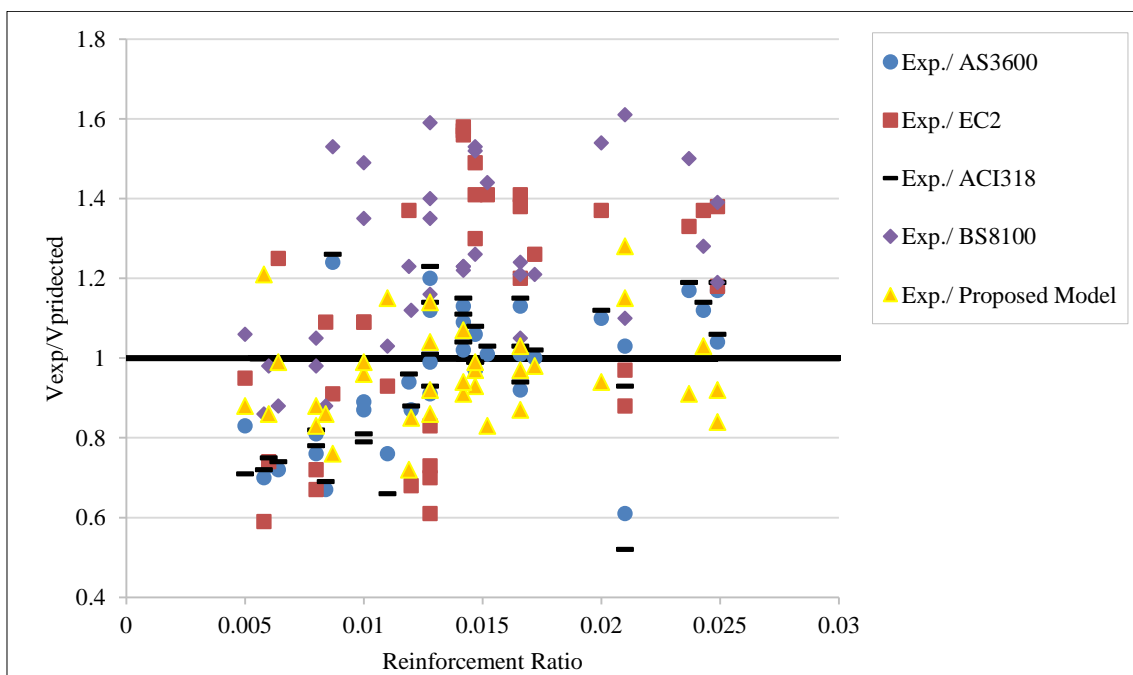


Figure 9. Reinforcement ratio versus the ratio of experimental/ predicted shear force

Moreover, Figure 11 shows the load-rotation curves for different steel ratios (0.8% and 0.57%) and the failure criteria curve. As an example, it was found that the value of the experimental shear force (965 kN) is very close to the intersection design point as estimated by the proposed model (which is 950 kN). While some design codes overestimate the shear force (1310 kN and 1170 kN for EC 2 and ACI-318, respectively). To investigate the effect of shear reinforcement, another curve is drawn in Figure 11. It is clear that the use of shear reinforcement increases the punching shear capacity by 38% compared with the shear strength of flat slabs without shear reinforcement.

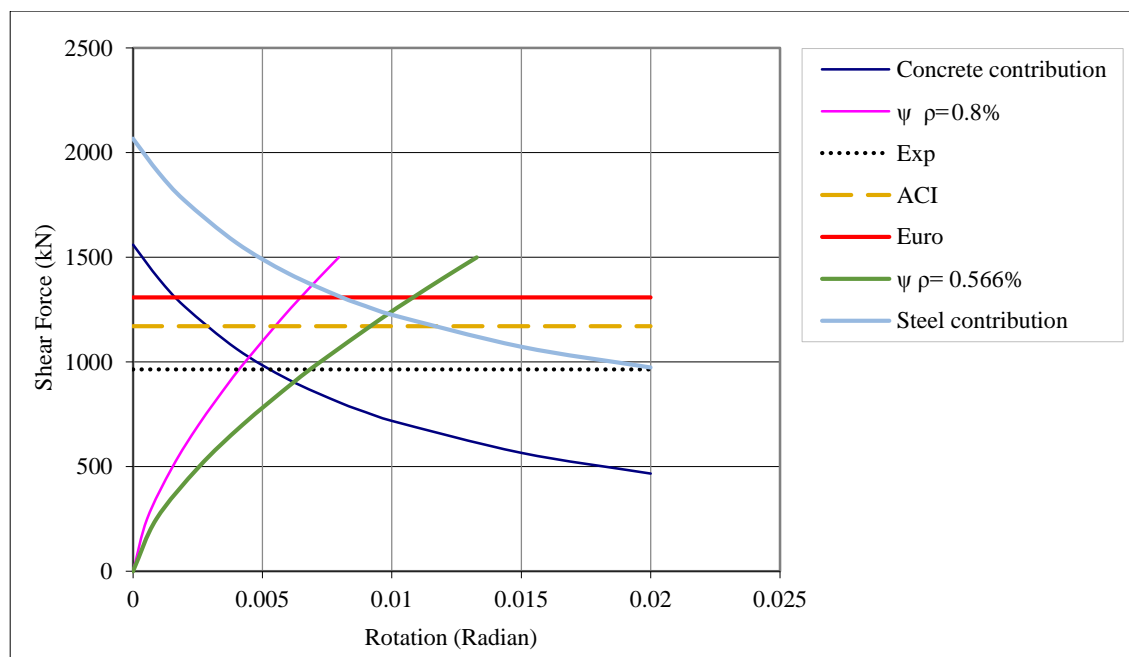


Figure 10. Load-rotation for different steel ratios and failure criterion comparison for concrete contribution, steel contribution, experimental, ACI-318, and EU-2 codes values

5. Conclusion

A mathematical model to predict the shear strength of flat slabs was proposed based on the results of previous experimental studies and CSCT theory. The load-rotation equations for different steel ratios together with the failure criterion curve were evaluated to get the design point that represents the intersection of the two curves. A comparative study was carried out between the results obtained using the proposed equation and those obtained using ACI-318, BS8110, AS3600, EC 2 codes equations, and those obtained from previous experimental studies. Statistical analysis results of the ratio of experimental / predictions by proposed formula proved to predict the punching shear strength at high accuracy (Mean Square Error =2.5%, Standard Deviation =0.104, Mean=1.0) over a wide range of concrete compressive strengths ranging from 25 to 120 MPa and reinforcement ratio ranged from 0.5-2.5%. It was found that values of shear force obtained using the proposed equation are very close to those obtained from experimental shear force and they provide conservative values. As compared with the existing design codes formulation, ACI-318 and EC2 codes tend to underestimate the values as compared with the experimental values. While the AS3600 and BS8110 tend to underestimate the values for the slabs having lower shear strength and overestimate the values for the slabs having high shear strength. The reinforcement ratio can affect the shear capacity as large as 33% at the maximum reinforcement ratio while the utilization of shear reinforcement can increase the capacity a greater percentage.

6. Declarations

6.1. Author Contributions

The conceptualization and the methodology of the research were discussed and decided by all authors; formal analysis, H.Q.; data curation, A.Y. and M.R.; resources, A.M. and M.R.; validation, H.Q. and A.M.; writing the first draft, H.Q.; review and editing by all authors. All authors have read and agreed to the published version of the manuscript.

6.2. Data Availability Statement

The data presented in this study are available in article.

6.3. Funding

The authors received no financial support for the research, authorship, and/or publication of this article.

6.4. Conflicts of Interest

The authors declare no conflict of interest.

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