Секція 1
ПЕРСПЕКТИВНІ МОДЕЛІ І МЕТОДИ МЕХАНІКИ ДЕФОРМІВНОГО ТВЕРДОГО ТІЛА

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MODELLING AND ANALYSIS OF FILTRATION IN THE MEDIA OF MULTIDIMENSIONAL NANOPOROUS PARTICLES

Abstract. Mathematical models of two-level transport "filtration-consolidation" in the system "interparticle space - nanoporous particles" are considered, which take into account, along with the flow of liquid in the skeleton, the internal flow of liquid from particles.

We consider the nanoporous particles [1, 2] containing liquid as a porous layer subjected to unidimensional pressing (Fig. 1). The liquid flowing occurs inside the particles, outside the nanoporous particles and between these two spaces. The nanoporous particles are separated by the porous network. The layer of particles is considered as a double-porosity media. Fig. 1 illustrates two levels of the considered elementary volume: level 1(a)) for the system of macropores in ***interparticle spaces*** and level 2 (b and c)) for the system of nanopores in ***intraparticle spaces***, which includes two subspaces of particles of different sizes: ***intraparticle spaces***1 – subspace of nanoporous particles with a radius of at least R_1 and ***intraparticle spaces***2 – a subspace of nanoporous particles with a radius of at least R_2 (R_1>R_2).

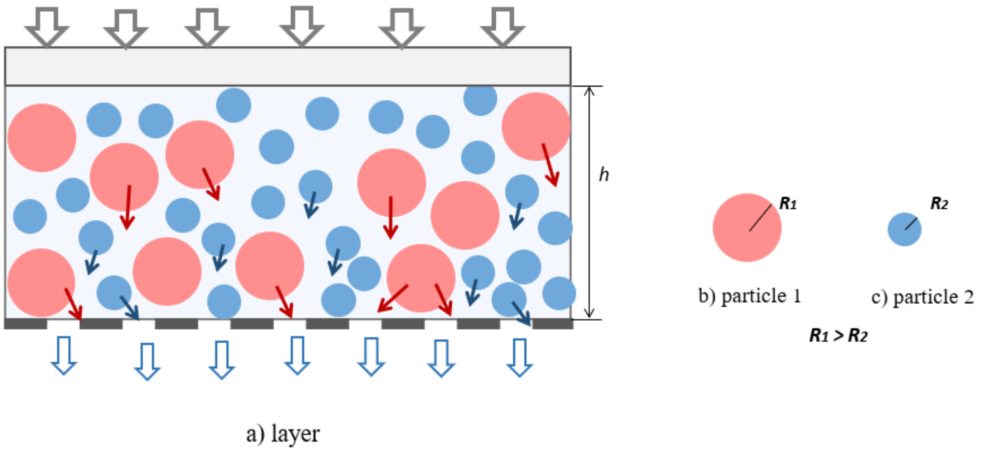

Fig. 1. Schematization of mass transfer in a two-level system of pores

The mathematical model of the considered transfer, taking into account the specified physical factors, can be described in the form of the following system of boundary value problems for partial differential equations:

Problem A: to find a limited solution of the consolidation equation for a layer of nanoporous partiles media in the domain $D_1 = \{(t,z): t>0,\ \ 0<z<h\}$:

$$\frac{\partial P_1(t,z)}{\partial t} = b_1 \frac{\partial^2 P_1}{\partial z^2} - \beta_1 \frac{\varepsilon}{R_1}\frac{\partial}{\partial t}\int_0^{R_1} P_2(t,x,z)dx - \beta_2 \frac{1-\varepsilon}{R_2}\frac{\partial}{\partial t}\int_0^{R_2} P_3(t,x,z)dx \qquad (1)$$

with the initial condition:

$$P_1(t,z)\big|_{t=0} = P_E\,, \tag{2}$$

the boundary conditions (for variable z)

$$P_1(t,z)\big|_{z=0} = 0\,; \qquad \frac{\partial P_1}{\partial z}\Big|_{z=h} = 0 \qquad \text{(impermeability condition);} \tag{3}$$

Problems B1,2: to find the limited solutions of the consolidation equations for the nanoporous partiles (radius R_i) in the domains:

$$\frac{\partial P_i}{\partial t} = b_i \frac{\partial^2 P_i}{\partial x_i^2},\ \ i=\overline{1,2} \tag{4}$$

with the initial conditions:

$$P_i\big|_{t=0} = P_E(z),\ \ i=\overline{1,2} \tag{5}$$

the boundary conditions (for radial variable x) :

$$\frac{\partial P_i}{\partial x}\Big|_{x=0} = 0\,; \qquad P_i(t,x,z)_{|x=R_i} = P_1(t,z)\,. \tag{6}$$

Nomenclature:

P_1 - liquid pressure in interparticle space, $P_2,\ P_3$ - liquid pressure in intraparticle space1 and intraparticle space2 (interier of spherical particles 1 and 2) in accordance, b_1 - is a consolidation coefficient in interparticle space, $b_2,\ b_3$ - consolidation coefficients in intraparticle space1 and intraparticle space2, $\beta_1,\ \beta_2$ - is the elasticity factor of the particles 1 and 2 in accordance, h - is layer thickness, R_1, R_2 - radius of particles 1 and 2.

The analytical solution of the model: pressure profiles in ***interparticle spaces*** and ***intraparticle spaces*1** and ***intraparticle spaces*2**. The analytical solution of the problem is found using the operational Heaviside's method, Laplace integral and Fourier integral transformations. Applying the finite integral Fourier transform (cos) [3, 4]: we obtain the solutions of the problems B1, B2 :

$$P_2(t,x,z) = P_E(z)\frac{2}{R_1}\sum_{m_1=0}^{\infty}\frac{(-1)^{m_1}}{\eta_{m_1}}e^{-b_2\eta_{m_1}^2 t}\cos\eta_{m_1}x + \frac{2}{R_1}\sum_{m_1=0}^{\infty}(-1)^{m_1}b_2\eta_{m_1}\int_0^t e^{-b_2\eta_{m_1}^2(t-\tau)}P_1(\tau,z)\,dz\cos\eta_{m_1}x$$

$$P_3(t,x,z) = P_E(z)\frac{2}{R_2}\sum_{m_2=0}^{\infty}\frac{(-1)^{m_2}}{\eta_{m_2}}e^{-b_3\eta_{m_2}^2 t}\cos\eta_{m_2}x + \frac{2}{R_3}\sum_{m_2=0}^{\infty}(-1)^{m_2}b_3\eta_{m_2}\int_0^t e^{-b_3\eta_{m_2}^2(t-\tau)}P_1(\tau,z)\,dz\cos\eta_{m_2}x \tag{7}$$

Substituting the expressions (7) into the consolidation equation (1), after a series of transformations and successive application to the problem (1)-(3) of the integral Laplace transform [3] and the finite integral Fourier transform [4, 5], we obtain

$$P_{1_n}^*(s) = \left(b_1\lambda^n + s + \beta_1\varepsilon\frac{\sqrt{b_2}}{R_1}\sqrt{s}\cdot th\left(\sqrt{\frac{s}{b_2}}R_1\right) + \beta_2(1-\varepsilon)\frac{\sqrt{b_3}}{R_2}\sqrt{s}\cdot th\left(\sqrt{\frac{s}{b_3}}R_2\right)\right)^{-1}\cdot$$
$$\cdot\left(2 + \frac{\beta_1\varepsilon}{R_1}\sqrt{\frac{b_2}{s}}th\left(\sqrt{\frac{s}{b_2}}R_1\right) + \frac{\beta_2(1-\varepsilon)}{R_2}\sqrt{\frac{b_3}{s}}th\left(\sqrt{\frac{s}{b_3}}R_2\right)\right)P_E\frac{1}{\lambda_n} \tag{8}$$

Applying the integral operator of the inverse integral Laplace transformation to expression (8) and using the Heaviside theorem on the root expansion of the denominator of Laplace-images expressions of and performing the inverse Fourier integral transition on the variables z, we finally obtain an analytical expression for pressure distributions in the ***interparticle space*** [3, 5]:

$$P_1(t,z)=P_E\frac{2}{h}\sum_{n=0}^{\infty}\sum_{j=1}^{\infty}\frac{e^{-\nu_{jn}^2 t}}{\Phi(\nu_{jn})}\left[1-\beta_1\varepsilon\frac{2}{R_1^2}\sum_{k=0}^{\infty}\frac{1-e^{-b_2\left(\eta_k^2-\frac{\nu_{jn}^2}{b_2}\right)t}}{\left(\eta_k^2-\frac{\nu_{jn}^2}{b_2}\right)}-\beta_2(1-\varepsilon)\frac{2}{R_2^2}\sum_{k=0}^{\infty}\frac{1-e^{b_3\left(\mu_k^2-\frac{\nu_{jn}^2}{b_3}\right)t}}{\left(\mu_k^2-\frac{\nu_{jn}^2}{b_3}\right)}\right]\frac{s\sin\lambda_n z}{\lambda_n}, \qquad (9)$$

where ν_{jn}, $j=\overline{1,\infty}$; $n=\overline{0,\infty}$ – the roots of transcendental equation (10).

$$\nu^2-b_1\lambda_n^2-\beta_1\varepsilon\nu\frac{\sqrt{b_2}}{R_1}tg\left(\frac{\nu R_1}{\sqrt{b_2}}\right)-\beta_2(1-\varepsilon)\nu\frac{\sqrt{b_3}}{R_2}tg\left(\frac{\nu R_2}{\sqrt{b_3}}\right)=0, \qquad (10)$$

here

$$\Phi(\nu_{jn})=1+\beta_1\varepsilon\frac{\sqrt{b_2}}{2R_1}\left(\frac{1}{\nu_{jn}}tg\left(\nu_{jn}\frac{R_1}{\sqrt{b_2}}\right)+\frac{R_1}{\sqrt{b_2}}\frac{1}{\cos^2\left(\nu_{jn}\frac{R}{\sqrt{b_2}}\right)}\right)+\beta_2(1-\varepsilon)\frac{\sqrt{b_3}}{2R_2}\left(\frac{1}{\nu_{jn}}tg\left(\nu_{jn}\frac{R_2}{\sqrt{b_3}}\right)+\frac{R_2}{\sqrt{b_3}}\frac{1}{\cos^2\left(\nu_{jn}\frac{R_2}{\sqrt{b_3}}\right)}\right),$$

$\eta_k=\frac{(2k+1)\pi}{2R_1}$, $k=\overline{0,\infty}$ – are the roots of equation $ch\left(\sqrt{\frac{s}{b_2}}R_1\right)=0$, *(s=iη, i-* imaginary unit),

$\mu_k=\frac{(2k+1)\pi}{2R_2}$, $k=\overline{0,\infty}$ – are the roots of equation $ch\left(\sqrt{\frac{s}{b_3}}R_2\right)=0$, ($s=i\mu$)

$\lambda_n=\frac{2n+1}{2h}\pi$ – are the spectral numbers of integral Fourier transformation (Sin-Fourier).

Substituting into formulas (7) the analytical expression of pressure distributions in the ***interparticle space*** P1(t,z), calculated according to (9), we obtain the final expressions for determining the time-space distributions of pressures $P_2(t,x,z)$ and $P_3(t,x,z)$ in the spaces of nanoporous particles: ***intraparticle spase*2** and ***intraparticle spase*3** in accordance.

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