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An Introduction to Monte Carlo-Tree Method

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Abstract

The article aims to introduce concepts in option pricing and risk management. Pricing and risk management is one of the fundamental problems in financial mathematics. Then readers may explore further to understand how to use mathematical models in pricing and risk management. More specifically, our research introduces a new method called Monte Carlo-Tree (MC-Tree), for option pricing and risk management with high accuracy.

Keywords: binomial trees, Monte Carlo method, credit valuation adjustment, European options, American options, counterparty credit risk.

Introduction to Monte Carlo-Tree Method

Just as you insure your properties, such as cars or houses, through buying an insurance policy, we use options as a special form of insurance against such price movements. Options contracts allow an investor to protect against big loss in the financial markets. An option is like buying or selling a stock at a pre-defined price at a predetermined future date. That pre-defined price is called a strike price, and that predetermined date is called the expiry date. Two types of options contract are call and put. Figure 1 shows the characteristics (call/put, expiry date, strike price) of an option contract on Apple stock on the market.

A put option means you agree to sell a stock on a particular future day at a particular price, whereas a call option holder has the right to buy a stock at a predefined price on a predefined day in the future. Profits can be accrued when options allow you to buy cheaply or sell at high levels relative to the future market prices.

Option contracts usually represent groups of 100 shares. These contracts give holders the right, but not the obligation, to exercise the option at, or before, expiry. Therefore, an option costs $2 per share, meaning you spend $200 to buy the option contract. For example, today, you buy a call option on an Apple stock with a strike price of $100 and an expiry date of tomorrow at the cost of $2 per share (i.e., you purchase an option at the cost $2 per share which allows

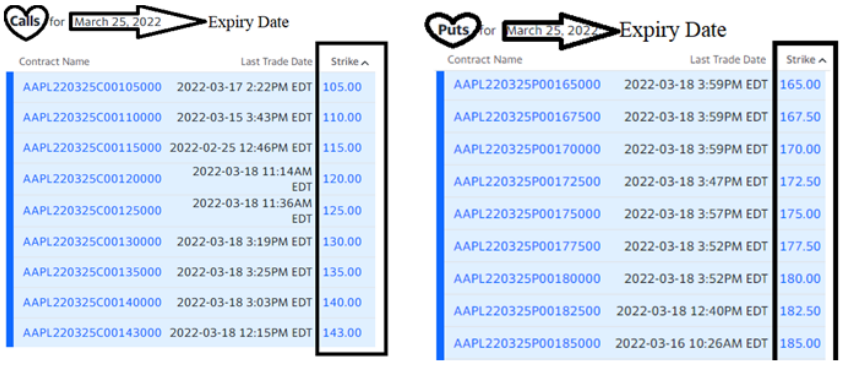

Figure 1: An example of call and put option contracts on Apple stock. Source: https://finance.yahoo.com/quote/AAPL/options

you to buy Apple stock tomorrow at $100 per share). If the price of Apple stock rises to $110 per share tomorrow, you make a profit at $110 - ($100 + $2) = $8 per share ($8 x 100 shares = $800 in total) because you buy these shares at $100, and can sell them at $110 each. If the price of Apple stock drops below $100 per share, the strike price on the option contract is higher than the market price, so you will let the contract expire. Hence, you lose the option's cost at $200, assuming you bought one option contract. When an option expires, it becomes worthless, meaning you lose $2 per share and subsequently $200 for each option contract you bought.

There are various kinds of options on the market. Here we mention two popular options: the European option and the American option. The European option holder has the right (but not the obligation) to exercise the option only at expiry. Unlike a European option, an American option gives the holder the right (but not the obligation) to exercise at any time before the expiration date.

The natural question arises from the above example "how can the option writer price the option at $2 per share and subsequently $200 for each option contract ?" One can develop and implement financial models for pricing certain financial products, especially options. Classical methods, such as the binomial tree and MC method, was employed to price these options contracts. There is a need for more accurate and efficient option pricing calculations. Our research aims to improve the existing method of pricing option contracts, such as the binomial model, through the incorporation of Monte-Carlo Simulations.

Pricing is used in calculating risk, such as counterparty risk. Every trade on the market requires at least two parties, so a counterparty is a party on the other side of a transaction. Counterparty risk happens when the counterparty can not meet obligations on a financial contract, leading to a loss to another party (like a bank). Credit Valuation Adjustment (CVA) is used to capture the counterparty risk. Modelling CVA for American options is complicated due to the complexity of CVA and the characteristics of American options.

An option payoff is the net Profit/Loss made by the option holders. Present value is the value of a sum of money in the present. The Monte-Carlo (MC) Method simulates hypothetical

scenarios and the corresponding payoffs, then takes the average payoffs relative to the present value. In this case, it simulates hypothetical increases and decreases in stock price movements, and calculates the average profit.

Conclusion

Stock Prices can go up, down, or stay the same. Traditional mathematical models, such as binomial models, use probability trees. Probability trees basically describe the probability of stock prices rising/declining over time relative to previous movements. We aim to improve the accuracy of these tree models using Monte-Carlo simulations on the probability.

We successfully improve upon existing methods by developing a more efficient and accurate pricing method. We do this successfully for multiple kinds of option contracts. Apart from pricing, we develop and implement a new algorithm for calculations of the average of the immediate loss if the counterparty defaults in the formula of the unilateral CVA for the American put option using the MC-Tree method.

Declaration of Interest

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