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# An Agent-Based framework for modeling and solving location problems

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**Abstract** The high relevance of location problems in the operations research literature arises from their wide spectrum of real applications, including decision optimization in industrial management, logistics, and territorial planning. Most of these optimization problems fall into the class of *NP*-hard problems, motivating the search for heuristic and approximated algorithms. Currently, a great interest is being devoted to those optimization approaches yielding a concrete integration with spatial analysis instruments (such as Geographical Information Systems) that provide the user with an easy visualization of input data and optimization results.

Agent-Based computing was recently proposed as an alternative to mathematical programming in order to solve problems whose domains are concurrently distributed, complex, and heterogeneous, also thanks to the availability of many commercial and open source codes including graphical interfaces for the elements of the problem.

In this paper we propose a general Agent-Based framework for modeling various location problems. Together with its description, we present some computational results confirming the suitability and the effectiveness of the proposed approach.

**Keywords** Location · Agent-Based models · Simulation · Metaheuristics

**Mathematics Subject Classification (2000)** 90B85 · 90C59

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## 1 Introduction

As suggested by Plastria (2002), a location problem can be characterized by the question "*Where are we going to put things*?" From this two more questions arise:

- *Which places are available*?
- *On what basis do we choose*?

The answer to the first question determines the location space. As location problems are a particular class of optimization problems, the second question requires the definition of the demand space and of an objective function, which can concern, for instance, the minimization of costs, damages or discomfort, or the maximization of profits and quality of services. In some contexts, the objective can be defined by a single criterion, while, in more complex situations, more criteria must be monitored simultaneously.

Starting from Weber (1929), location problems have received an uprising attention related to the increasing demand of decision-making support systems in several application fields. As such problems are characterized by high computational complexity, often belonging to the *NP*-hard class (see Bozkaya et al. 2002 and Drezner and Hamacher 2002), a wide spectrum of heuristic techniques was developed to deal with them.

Besides the aspects related to the quality of the obtainable solutions, in practical applications it is important having flexible tools capable of dealing with problems that are not easy to model and whose data is provided by data management systems. Tools like Geographical Information Systems (GISs) are currently widely used to represent spatial systems.

Brown et al. (2005) point out that a very useful tool to enhance GISs capabilities to solve complex spatial analytic problems, in general, is provided by Agent-Based Models (ABMs). Indeed, the integration between ABMs and GISs can be a useful way to deal with complex spatial problems and provide a visual, flexible, and responsive representation of the problem itself.

In this paper, we propose the general framework for an ABM oriented to the solution of location problems, which represents, to the best of our knowledge, the first attempt to apply ABMs in this context. Our approach is based on the definition of attributes and rules that can be easily adapted to cope with different kinds of location problems and objectives. We present, in particular, an adaptation of the proposed ABM framework to some location problems.

The paper is organized as follows. In Sect. 2, we provide some generalities about ABMs with the indication of some works related to their use to solve optimization problems. In Sect. 3, we shortly describe some classical location problems that we consider in the following. In Sect. 4, we illustrate the general Agent-Based framework to represent location problems. In Sect. 5, the adaptation of this framework to the problems presented in Sect. 3 is depicted. Section 6 shows the implementation of the framework in an Agent-Based simulation environment, while some computational results are delineated in Sect. 7. Section 8 presents a benefit of utilizing the proposed approach in dealing with location problems. Finally, we draw some conclusions and further lines of research in this field.

## 2 Agent-Based models

Agent-Based Models (ABMs) consist of a set of elements (agents) characterized by attributes which interact with each other through the definition of appropriate rules in a given environment. ABMs can be useful to reproduce many systems related to economics and social sciences where the structure can be designed through a network (Billari et al. 2006; Conte et al. 1997). Through ABMs, it is possible implementing an environment with its features, forecasting and exploring its future scenarios, experimenting possible alternative decisions, setting different values for the decision variables, and analyzing the effects of these changes (see Axelrod 1997).

At an aggregated level, the use of ABMs can help in understanding general properties and patterns concerning the whole scenario (Billari et al. 2006) that could be neither deduced nor forecasted by the observation of each agent, due to the complexity of the interactions occurring among the elements of the system.

According to the definition of Wooldridge and Jennings (1995), an agent is a computational system interacting with an environment that can be endowed with the following features:

- *Independence*. Each agent acts without the direct control of human beings or other devices.
- *Social ability*. Interactions occur among entities through a communication language in order to satisfy the design objectives.
- *Re-activeness*. Agents answer in a precise way to signals coming from the environment.
- *Pro-activeness*. Agents are endowed with goal-directed behaviors. They take the initiative in order to satisfy their design objectives.

Furthermore, as it can be derived from Billari et al. (2006) and Weiss (1999), the development of an Agent-Based Model needs a complete description for a set of basic *building blocks* as follows.

- *The object of the simulation*. It has to be specified what is the phenomenon/problem to be reproduced, defining the space where the simulation takes place.
- *The agents' population*. Agents can be grouped in different categories with common characteristics reproducing the various components of the system.
- *The adaptive capability of each agent category*. Agents of each category present a specific adaptive capability, i.e., the degree of re-activeness and pro-activeness.
- *The interaction paradigm among agents*. Each agent can interact with agents of the same or of other categories. In the literature, several interaction paradigms have been defined, such as cooperation, competition, and negotiation (see, for instance, Weiss 1999). On the base of the selected paradigm, the agents evolve in the simulation space in a different way.

Given this peculiarity in dealing with the representation and simulation of complex systems, ABMs have been recently applied to solve optimization problems whose domains present several inter-related components in a distributed and heterogeneous environment (Weiss 1999; Wooldridge 2002), sometimes combined to other optimization techniques.

Davidsson et al. (2003) state that Agent-Based simulation and conventional optimization techniques present complementary characteristics that should be integrated into hybrid approaches to solve complex problems, embedding optimization rules in the behavior of the agents. Cardon et al. (2000) propose an integration of genetic algorithms and multi-agent simulation for solving *NP*-hard scheduling problems. Wei et al. (2005) develop an Agent-Based optimization framework for dynamic resource allocation problems; Desphande and Cagan (2004) introduce an Agent-Based optimization algorithm for solving the process-planning problem that combines stochastic optimization techniques with knowledge-based search. Several applications of Agent-Based optimization to transportation problems can be found in the literature. Bocker et al. (2001) utilize a multi-agent approach to cope with the train coupling and sharing problem on a railway transport system; Fernandez et al. (2004) illustrate multi-agent service architectures for bus fleet management; Mes et al. (2007) develop an Agent-Based approach for real-time transportation problems and compare it to more traditional methods.

## 3 Brief review on continuous location problems with discrete demand

The scientific literature in the field of continuous location problems with demand concentrated in discrete points is wide and rich, and some exhaustive surveys can be found in Klose and Drexl (2005) and Plastria (1996, 2002). We recall some of these problems and their main related work.

*The p-median problem* The *p-Median* problem aims at the minimization of the weighted sum of the distances between $p$ facilities to be opened and a set of demand points. The version of this problem in which the location space is continuous, often indicated as the Multisource Weber Problem (MWP), belongs to the class of *NP*-hard optimization problems, as shown in Megiddo and Supowit (1984). Given a set of demand points $i \in I$, located in $(x_i, y_i)$, and the coordinates $(x_a, y_a) \in S \subset \Re \times \Re$ for a number $p$ of facilities, a possible formulation for the MWP is the following one (Klose and Drexl 2005):

$$\min \sum_{i \in I} \sum_{a=1}^{p} w_i d_i(x_a, y_a) z_{ia} \tag{1}$$

$$\text{subject to} \quad \sum_{a=1}^{p} z_{ia} = 1 \quad \forall i \in I \tag{2}$$

$$z_{ia} \in \{0, 1\} \quad \forall i \in I, a = 1, \dots, p \tag{3}$$

$$x, y \in \Re^p$$

with $d_i = \sqrt{(x_a - x_i)^2 + (y_a - y_i)^2}$ in the case of Euclidean metrics. In this model, $z_{ia}$ equals 1 when a demand point $i$ is assigned to a facility $a$.

Fast heuristic methods to cope with the MWP are considered and compared in Brimberg et al. (2000) and Hansen et al. (1998), while in Aras et al. (2006) the problem is solved using neural networks. As concerns exact algorithms, the first attempt to solve instances of the MWP is proposed by Kuenne and Soland (1972). Later, different approaches are proposed by Rosing (1992), Chen et al. (1998) and Du Merle et al. (1999). A recently developed branch-and-price algorithm (Righini and Zaniboni 2007) permits to find the optimal solution on instances with some thousands of points and some hundreds of sources in less than three hours on a PC.

*The $p$-maximal covering location problem* Location Covering Models are another class of problems, in which the objective is to ensure coverage to given demand points. A demand point is said to be covered by a certain facility if the distance between the two points is lower than a certain threshold or required distance (*RD*). Models of this type generally address the location of urban public facilities, especially emergency facilities. Church and ReVelle (1974) propose the $p$-Maximal Covering Location Problem (MCLP), which seeks to locate $p$ facilities that can cover the maximum amount of demand. Given a set of demand points $i \in I$, located in $(x_i, y_i)$, and the coordinates $(x_a, y_a) \in S \subset \Re \times \Re$ for a number $p$ of facilities, a possible formulation for the $p$-Maximal Covering Location Problem with facility placement on the entire plane can be derived from Mehrez (1983):

$$\max \sum_{i \in I} w_i \zeta_i \tag{4}$$

$$\text{subject to} \quad \sum_{a=1}^{p} z_{ai} \geq \zeta_i, \quad \forall i \in I \tag{5}$$

$$z_{ai} d_i(x_a, y_a) \leq RD \quad \forall i \in I, a = 1, \ldots, p \tag{6}$$

$$z_{ia}, \zeta_i \in \{0, 1\} \quad \forall i \in I, a = 1, \ldots, p$$

$$x, y \in \Re^p$$

The variables $\zeta_i$ and $z_{ai}$ are binary. The variable $\zeta_i$ is equal to 1 if a demand $w_i$ located in $i$ is covered (0 otherwise) and the variable $z_{ai}$ is equal to 1 if the demand concentrated in $i$ is covered by a facility located in $a$. Constraints (5) ensure that a demand point that is considered to be covered has at least one facility within the required distance; constraint (6) ensures that the variable $z_{ai}$ is equal to 1 if the demand located in $i$ can be covered by the service located in $a$ within the required distance *RD* (0 otherwise). The problem is generally complex, and several heuristic methods have been developed to deal with it. A survey on this topic is presented by Galvao et al. (2000).

*The single facility minimum variance location problem* Another kind of problems can be defined when the objective is a measure of "equity" from the demand points to the set of facilities (Eiselt and Laporte 1995). $\mu(x_a, y_a)$ being the average distance among the demand points $i$ and the facility $a$ of coordinates $(x_a, y_a)$ and $\sigma^2(x_a, y_a)$ the variance of the distances, the Single Facility Minimum Variance Location Problem in the Euclidean plane aims at defining the position $(x_a, y_a)$ of the facility which minimizes $\sigma^2(x_a, y_a)$. Drezner and Drezner (2007) solve to optimality large instances of this problem using a *Big Triangle Small Triangle* (BTST) approach.

## 4 Agent-based models and location problems: a general framework

Some of the characteristics of ABMs suggest the possibility to apply this approach to model and solve location problems. The approach appears to be particularly interesting when the location space is a planar region (whose points represent available locations) and the demand can be represented by an enumerable set of discrete points.

Suppose that we have to locate $p$ facilities in a continuous space in which $n$ demand points are positioned. In order to define an Agent-Based framework, in the following, we describe how each block illustrated in Sect. 2 can be specified to represent the problem.

*The object of the simulation* The object of the simulation is to reproduce all the elements of the problem and to define the appropriate rules that agents should follow. The environment of the simulation is represented by the location space, i.e., a portion of plane (for instance, a rectangle of base $b$ and height $h$) where agents are positioned. We assume that distances between elements are defined by a Euclidean metric. Due to the flexibility of the ABMs, the adaptation of the model to different metrics is straightforward.

*The agents' population* We distinguish between two main agent categories (see Fig. 1):

- A set $P$ of "passive" agents representing the demand points with an associated demand $w_i \forall i \in P$.
- A set $A$ of "active" agents representing the facilities to be located.

*The adaptive capability of each agent category* The two agent categories present different adaptive capabilities. Passive agents do not change position, but they interact with the active agents in an autonomous way. They are neither re-active, as they do not react to any signal, nor pro-active, as they do not pursue any objective. On the other hand, the active agents are both re-active, as they answer to the presence of passive agents, and pro-active, as they move in the location space searching for positions according to a given objective.

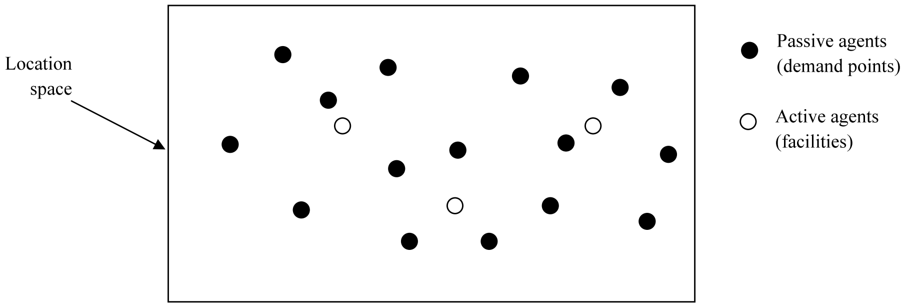



**Fig. 1** Location space and agent categories

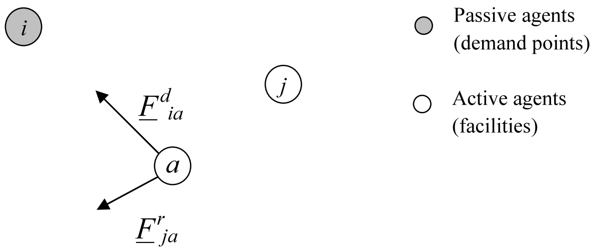



**Fig. 2** Forces operating on an active agent

*The interaction paradigm among agents* As mentioned before, there exist different paradigms to define the interaction among agents. In this context, we adopt the Artificial Potential Fields (APF) paradigm, based on some concepts from physics and biology (see Ferber 1999, Kathib 1986). The paradigm assumes that the agent behavior is regulated by the action of forces. In this context, we suppose that two forces operate on each active agent $a \in A$ (Fig. 2):

- A *demand-driven* force, $\underline{F}^d_{ia}$, due to the presence of a passive agent $i \in P$ which pushes the agent $a$ toward the position of $i$.
- A *repulsive* force, $\underline{F}^r_{ja}$, determined by the presence of an active agent $j \in A$ which pushes the agent $a$ in the opposite direction of $j$.

The intensity of the two forces is a function of the distance between the agents as widely used in spatial interaction models (see, for instance, Fotheringham and O'Kelly 1989, Sen and Smith 1995, and Serra and Colomé 2001).

According to the APF paradigm, we suppose that these forces are significant only within a given distance from the agent $a \in A$. In order to define the forces, the paradigm introduces some calibration parameters expressing the width of the neighborhood within which each force is significant.

In this way we can define a *resulting demand-driven* force (Fig. 3a):

$$\underline{F}^d_{Ra} = \frac{\sum_{i \in P_{rd,a}} \underline{F}^d_{ia}}{|P_{rd,a}|}$$

where $P_{rd,a}$ is the set of passive agents whose distance from $a$ is within a given radius $r_d$. In the same way, the *resulting repulsive* force is given by (Fig. 3b)

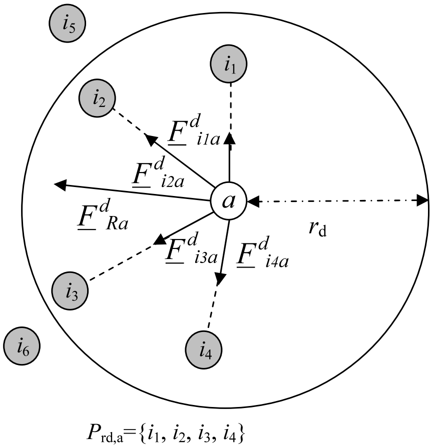
$$\underline{F}^r_{Ra} = \frac{\sum_{j \in A_{rr,a}} \underline{F}^r_{ja}}{|A_{rr,a}|}$$

where $A_{rr,a}$ is the set of active agents whose distance from $a$ is within a given radius $r_r$.

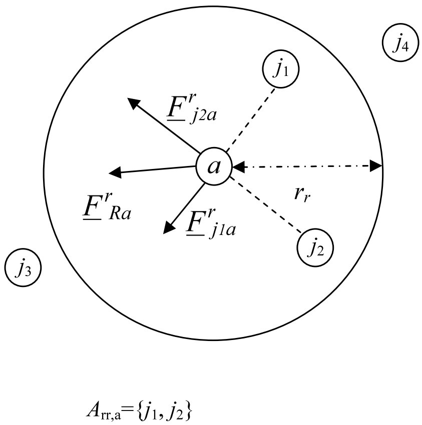
The movement of the agent $a$ is finally determined by the total force $\underline{M}_a$, calculated as a convex combination of the two forces:

$$\underline{M}_a = \alpha \underline{F}^d_{Ra} + (1 - \alpha) \underline{F}^r_{Ra} \tag{7}$$

$\alpha$ being a parameter, $0 \le \alpha \le 1$, expressing the relative weight of each resulting demand-driven and distributive forces.




(a) the resulting demand-driven force




(b) the resulting repulsive force

**Fig. 3** The resulting forces operating on an active agent

## 5 Adaptations of the ABM framework to location problems

The described framework can be particularized to deal with different location problems and to consequently solve them through the development of proper procedures to be implemented in a given environment. In particular, we show how the forces can be specified in relation with the problems illustrated in Sect. 3, according to the specific objective of the problem. In the following, we define the distance vector $\underline{d}_{ba}$ between two agents $b$ and $a$ as the vector applied to the agent $a$ and directed toward the agent $b$ with an intensity equal to the distance $\|\underline{d}_{ba}\|$ between the agents.

*The p-median like problem* We start by describing the adaptation of the proposed ABM framework to solve a class of location problems in which one must minimize an objective function that includes a weighted sum of the distances between $p$ facilities to be opened and a set of demand points. We refer to this as the "$p$-Median like problem."

In this case, the demand-driven force can be expressed by

$$\underline{F}^d_{ia} = w_i \underline{d}_{ia} \quad \forall i \in P_{rd,a} \tag{8}$$

where $w_i$ represents the demand associated to $i$ and $\underline{d}_{ia}$, the above-mentioned distance vector. In practice, we suppose that the influence of a demand point $i$ on the facility $a$ decreases the closer the facility moves towards such a demand point. Indeed, if the active agent reaches exactly the position of the demand point, the demand-driven force becomes zero.

As regards the distributive force, we assume that

$$\underline{F}^r_{ja} = -\frac{\underline{d}_{ja}}{\|\underline{d}_{ja}\|} \frac{1}{\|\underline{d}_{ja}\|} \quad \forall j \in A_{rr,a}$$

The influence of another facility $j$ on the facility $a$ is inversely proportional to the distance $\|\underline{d}_{ja}\|$: the closer the two facilities, the more intense the force $\underline{F}^r_{ja}$ that will tend to push the agent $a$ away from the agent $j$.

The proposed adaptation of the ABM framework to the class of $p$-Median like problems can be applied as a heuristic approach to solve instances of the classical $p$-Median problem described in Sect. 3, since, given any instance of that problem, it provides a feasible solution in finite computational time, whose quality will be experimentally evaluated in the next section, through the comparison with the results arising from the related literature for the $p$-Median problem. According to Drezner (1987), we also observe that, even if the $p$-Median problem does not explicitly consider mutual distances among facilities, the presence of distributive forces allows avoiding facilities overlapping that could yield bad quality objective function values.

The values of the radii $r_d$ and $r_r$ for the determination of $P_{rd}$ and $A_{rr}$ and $\alpha$ are calibration parameters.

The value of $r_d$ can be set as $r_d = \lceil \frac{\min(b,h)}{p+1} \rceil$, $b$ and $h$ being respectively the base and the height of the location space.

As regards the value of $r_r$, a default value of 1 space unit can be considered. In presence of possible constraints on the minimum distance among the facilities, $r_r$ can be fixed according to this aspect.

The value of $\alpha$ can be set equal to 0.5, so the resulting forces are supposed to have the same relative weight.

*The p-maximal covering-like problem* We consider now the adaptation of the ABM framework to the class of $p$-Maximal Covering-like problems, in which the objective is to ensure the coverage to some demand points under threshold constraints. In this case, the demand-driven force is expressed as follows:

$$\underline{F}^d_{ia} = \frac{w_i}{p_i} \underline{d}_{ia} \quad \forall i \in P_{rd,a}$$

where $p_i$ is the number of active agents covering the demand point $i$, i.e., within the distance *RD* from $i$. In this way, we suppose that if a demand point $i$ is covered by more than one facility, its demand-driven force is equally shared among those facilities.

In this case, the values of the radii $r_d$ and $r_r$ can be fixed equal to *RD* and $\alpha = 0.5$.

*The single facility minimum variance-like problem* In the adaptation of the ABM framework to the Single Facility Minimum Variance-like problem, since we deal with a single facility location, the repulsive forces are not present. The expression of the demand-driven force is calculated as in (8).

Due to the absence of repulsive forces, $\alpha = 1$ and $r_r = 0$, the only parameter to be calibrated is $r_d$, whose value can be fixed as already shown for the $p$-Median-like case.

A summary of the adaptations of the Agent-Based framework to the illustrated location problems is reported in Table 1.

**Table 1** Summary of the expressions of the forces for the illustrated location problems

| Problem | Demand-driven force | Repulsive force | Calibration parameters |
|---|---|---|---|
| $p$-median-like problem | $\underline{F}_{ia}^{d} = w_i \underline{d}_{ia}\ \forall i \in P_{rd,a}$ | $\underline{F}_{ja}^{r} = -\frac{\underline{d}_{ja}}{\Vert\underline{d}_{ja}\Vert} \frac{1}{\Vert\underline{d}_{ja}\Vert}\ \forall j \in A_{rr,a}$ | $\alpha, r_d, r_r$ |
| $p$-maximal covering-like problem | $\underline{F}_{ia}^{d} = \frac{w_i}{p_i} \underline{d}_{ia}\ \forall i \in P_{rd,a}$ | $\underline{F}_{ja}^{r} = -\frac{\underline{d}_{ja}}{\Vert\underline{d}_{ja}\Vert} \frac{1}{\Vert\underline{d}_{ja}\Vert}\ \forall j \in A_{rr,a}$ | $\alpha, r_d, r_r$ |
| Single facility minimum variance-like problem | $\underline{F}_{ia}^{d} = w_i \underline{d}_{ia}\ \forall i \in P_{rd,a}$ | – | $r_d$ |

## 6 Implementation of the framework

The illustrated framework has been implemented within the *NetLogo* Agent-Based simulation environment (http://ccl.northwestern.edu/netlogo) using the proprietary programming language and its Java architecture. *NetLogo* allows reproducing the two agent categories introduced above. In particular, passive agents are represented by cells in a grid network, each cell being identified by a couple of integer coordinates.

In the implemented procedure, whose scheme is represented in Fig. 4, it is possible to distinguish the following steps.

1. *Initialization*. The parameters of the problem (number of facilities $p$, values of the radii $r_d$ and $r_r$, $\alpha$, objective function, expression of the forces) are defined.
2. *Individuation of the initial solution*. The position of $p$ active agents in the location space is randomly determined according to a uniform distribution with values ranging within the extreme coordinates of the location space.
3. *Evolution of the current solution*. For each active agent $a$ located in the current positions, the total force $\underline{M}_a$ is calculated according to (7) so that the active agents change position on the base of this force and the solution assumes a new objective function value.
4. *Diversification*. If a diversification criterion (defined in terms of number of nonimproving iterations, fixed a priori as a parameter) is satisfied, a diversification move is enacted, and the procedure goes back to Step 2; otherwise, it goes to Step 5.
5. *Stopping criterion*. If a stopping criterion is satisfied, the procedure ends; otherwise it goes back to Step 3.

The procedure behaves as a metaheuristic searching for better solutions thanks to an evolutionary mechanism which is performed until a diversification or a stopping criterion are satisfied. On the base of a diversification criterion, the procedure restarts from a new initial solution.

Possible stopping criteria are represented by a given total number of evolution iterations or a fixed running time, to be defined as parameters.

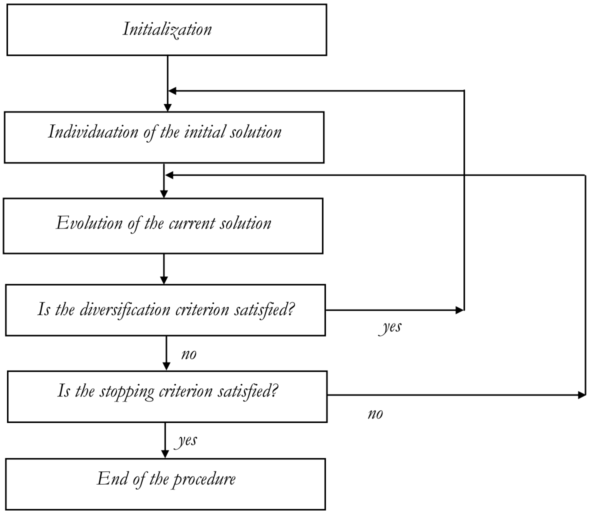



**Fig. 4** The scheme of the implemented procedure

## 7 Computational experiences

We illustrate some examples of application of the Agent-Based framework to the location problems introduced and described in Sects. 3 and 5, in order to show the capability of the proposed approach to solve these problems and to analyze the provided performances in terms of computational times and quality of the solution. The procedure was run on a PC with a *Dual-Core T2250* 2.0 GHz CPU and 2 GB of RAM. In all the experiments, the calibration parameters were set according to the criteria illustrated in Sect. 5.

As stopping criterion, we fixed a number of 150 iterations, while, to start the diversification, we considered 10 nonimproving iterations.

In order to evaluate the quality of the provided solution, we calculated the gap from the known optimal solution as

$$\text{Gap} = \left(\frac{\text{ABM Best Solution} - \text{Optimal Solution}}{\text{Optimal Solution}}\right) \times 100$$

*Solving $p$-median problem instances* We applied the proposed framework to solve one of the benchmark problems (*Bongartz*287) available for $p$-Median problem (http://ina2.eivd.ch/Collaborateurs/etd/problemes.dir/location.html). This problem is characterized by 287 demand points with variable demand values, whose coordinates assume integer values in the range $[0, 50]$. We used a $100 \times 100$ grid of passive agents; thus, each grid point can be associated or not to a demand point. We solved the problem for $p$ varying from 2 to 10.

Results reported in Table 2 show that ABM finds near optimal solutions in limited computing times.

*Solving $p$-maximal covering problem instances* In absence of benchmark instances for this version of the problem, we generated instances in a $100 \times 100$ location space

**Table 2** Computational results on the *Bongartz*287 instance

| $p$ | Optimal solution | ABM best solution gap | ABM runtime (sec) |
|---|---|---|---|
| 2 | 14427.593010 | 0.00% | 0.50 |
| 3 | 12095.442160 | 0.00% | 1.50 |
| 4 | 10661.476590 | 0.00% | 1.50 |
| 5 | 9715.627471 | 0.10% | 2.30 |
| 6 | 8787.556817 | 0.23% | 4.10 |
| 7 | 8160.320284 | 0.53% | 4.20 |
| 8 | 7564.294907 | 0.22% | 5.40 |
| 9 | 7088.128333 | 0.38% | 6.00 |
| 10 | 6705.035556 | 1.32% | 6.00 |

(for a total number of 10000 demand points) with known optimal solutions according to the following criteria. Once fixed the distance threshold $RD$ (we assume $RD = 4$ space units) and given the number $p$ of facilities to be opened, each instance is produced through the random generation of 4 sets of $p$ circles of radius $R \geq RD$ in the location space. The coordinates of the center of each circle were chosen according to a uniform distribution. For each set $s$ ($s = 1..4$), we assigned the same demand value $w_s$ to the points internal to each circle. In particular, we fixed $w_1 = 1$, $w_2 = 1/2$, $w_3 = 1/4$, $w_4 = 1/8$. Points belonging to the intersection of more circles were given the maximum demand value. This way the optimal solution of the $p$-Maximal Covering problem on such instances is known in advance, as it can be obtained locating the $p$ facilities exactly in the center of the $p$ circles with unitary demand values.

For some combination of values $(R, RD)$ and for each value of $p$ varying from 1 to 10, we generated 5 different instances. For each instance, the procedure was run 10 times.

The results indicate the frequency with which the optimal solution is found (calculated as [number of times]/50) in the case $R = RD$ and in the case $R = 2RD$ (Table 3).

As the procedure always finds the optimal solution, the average running times to find the optimal solution are reported for each value of $p$. The $R$ value does not seem to affect the results in terms of final solution, but there is a slight variation in the computational times.

However, the results appear interesting, and the optimal solutions are detected in limited computing times.

*Solving single facility minimum variance problem instances* The adaptation of the ABM framework to the Single Facility Minimum Variance-like problem was applied to some instances contained in Drezner and Drezner (2007). These instances consider a continuous location space and a discrete demand space with demand points of equal demand values distributed on the Euclidean plane.

In order to solve the instances, we used a $100 \times 100$ grid of agents, i.e., 10000 passive agents. As, in general, the position of an original discrete demand point did not coincide with any grid points, an adaptation of the instances demand data was performed, associating each demand point to the closest grid point; thus, each grid point has been weighted with a demand value equal to the number of associated

**Table 3** Computational results on the $p$-Maximal Covering randomly generated instances

| $p$ | $R = RD$ | | $R = 2RD$ | |
|---|---|---|---|---|
| | Optimal solution frequency | Average runtime (sec) | Optimal solution frequency | Average runtime (sec) |
| 1 | 100.00% | 2.1 | 100.00% | 2.1 |
| 2 | 100.00% | 3.2 | 100.00% | 3.4 |
| 3 | 100.00% | 4.0 | 100.00% | 4.3 |
| 4 | 100.00% | 5.4 | 100.00% | 6.1 |
| 5 | 100.00% | 7.2 | 100.00% | 8.3 |
| 6 | 100.00% | 10.4 | 100.00% | 11.6 |
| 7 | 100.00% | 14.9 | 100.00% | 17.9 |
| 8 | 100.00% | 18.3 | 100.00% | 20.8 |
| 9 | 100.00% | 22.2 | 100.00% | 23.4 |
| 10 | 100.00% | 26.3 | 100.00% | 30.5 |

**Table 4** Computational results on the instances in Drezner and Drezner (2007) for the Single Facility Minimum Variance problem

| Demand points | Optimal solution | ABM best solution gap | ABM runtime (sec) |
|---|---|---|---|
| 2000 | 0.0204669774 | 0.96% | 3.1 |
| 5000 | 0.0203239336 | 0.08% | 3.1 |
| 10000 | 0.0205132773 | 0.19% | 4.3 |

demand points. The ABM provides the coordinates of the facility to be located with a ten-digit precision in the continuous location space. Then, the objective function value was computed as the variance of the distances of the original demand points from the located facility. This way, the objective function value includes the effects of the aggregation operation and, thus, associated errors (Plastria 2000).

Table 4 shows the capability of the proposed approach to find good results in reasonable computational times.

## 8 Benefits of the proposed approach

The computational results presented in the previous section showed the effectiveness of the proposed ABM approach to solve the considered continuous location problems with discrete demand. Even if the approach appears competitive with other heuristics in terms of computational performances, the interest in using ABMs for location problems goes far beyond the computational efficiency.

The proposed approach can be viewed as a sort of metaheuristic in which some steps are performed through agent-based computation. Even if the approach could be implemented in a "traditional" way, the use of an ABM framework provides several additional benefits.

First of all, the current availability of open source environments for ABMs implementation (i.e., *NetLogo*, *JAS*, *SWARM*, *REPAST*) with dedicated libraries let modeling such heuristics easy to perform even for nonspecialist users. The presence, within each of the cited toolkits, of integrated Graphical User Interfaces (GUIs) allows the immediate graphical representation of the elements of the problem together with a visual indication of the evolution of the solution.

These aspects could help users, even not particularly skilled in implementation aspects, in the search of adoptable practical solutions, especially in presence of constraints which cannot be easily modeled in a mathematical way, such as forbidden regions, obstacles, and minimum distance constraints among facilities.

The framework presents a significant flexibility that matches the huge variety of problems arising in the context of location studies. As previously illustrated, versions of the problems with variations in the objective function and/or constraints can be tackled through proper modifications of the elements of the framework (i.e., expressions of forces, calibration parameters) or introducing new characteristics in the paradigm of the model.

Among the benefits, it should be also mentioned the possibility of an easy and effective integration of ABM tools with Geographical Information Systems (GIS), as deeply shown by Brown et al. (2005), Parker (2005) and Guo et al. (2008). The starting phase of a location decisional process is often the representation and the analysis of the problem by means of maps, datasets, and geo-statistical analysis tools of GIS software. A straightforward data import process and an interactive visualization and contextualization of the output and parameter setting of the optimization process is therefore a relevant requirement for a decision support system in the field of location analysis. The use of ABM tools allows users to solve problems through continuous interactions between optimization framework and GIS applications. Currently, these processes are simplified by the presence of a growing number of ABM toolkits that permits a direct access to GIS vector and raster datasets. For instance, the *NetLogo* platform we adopted in this paper offers GIS extensions, developed thanks to the contribution of the Center for Connected Learning (CCL) and Computer-Based Modeling of the Northwestern University, Chicago, and available at the Internet address http://ccl.northwestern.edu/netlogo/docs/gis.html.

## 9 Conclusions

In recent years, Agent-Based modeling is becoming more and more frequently used as an approach for solving complex optimization problems. In this work, an Agent-Based framework for modeling location problems was proposed and illustrated. The original contribution of the work consists mainly in the proposal of an approach that, compared to the other heuristic methods in the literature, is easy to implement and appears particularly suitable for the integration with GIS-based data.

Moreover, it presents characteristics of flexibility as the general framework can be applied, with slight modifications, to solve different kind of locations problems (i.e., $p$-Median-like problem, $p$-Maximal Covering-like problem, Single Facility Minimum Variance-like problem). The features of the model suggest also the possibility of immediately adapting the approach to take into account constraints (for instance, minimum distance constraints among facilities, presence of obstacles or forbidden areas in the location space) whose formulation makes the problem hard to solve using mathematical programming-based methods.

The preliminary computational experiences appear encouraging and indicate that the approach provides reasonable quality solutions within limited running times.

Future researches will include an extensive computational experimentation to test the scalability of the proposed ABM approach on very large scale instances of the considered problems. Moreover, there will be studied the adaptation of the framework to other classes of location problems such as the anti-$p$-Median problem (Erkut and Neuman 1989; Cappanera et al. 2003) and the $p$-Minimal Covering problem with distance constraints (Berman and Huang 2008).

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