Pendellösung effect in photonic crystals

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Abstract: At the exit surface of a photonic crystal, the intensity of the diffracted wave can be periodically modulated, showing a maximum in the "positive" (forward diffracted) or in the "negative" (diffracted) direction, depending on the slab thickness. This thickness dependence is a direct result of the so-called Pendellösung phenomenon, consisting of the periodic exchange inside the crystal of the energy between direct and diffracted beams. We report the experimental observation of this effect in the microwave region at about 14GHz by irradiating 2D photonic crystal slabs of different thickness and detecting the intensity distribution of the electromagnetic field at the exit surface and inside the crystal itself.

OCIS codes: (050.1960) Diffraction theory; (260.2110) Electromagnetic theory; (290.4210) Multiple scattering; (999.9999) Photonic crystal

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1. Introduction

Since the original formulation of the diffraction theory from Ewald [1], the Pendellösung effect was predicted as a periodic exchange of energy between interfering wave-fields. The German term comes from the formal analogy between the mechanical system composed by coupled pendula and the optical problem, where many waves contribute to the optical field. In this formal analogy pendulum is the counterpart of wave whereas the temporal dependence of the mechanical problem corresponds to the spatial dependence in the considered optical problem [2]. Pendellösung is a relatively well known effect of Dynamical Diffraction Theory (DDT), a rigorous formalism accounting for multiple scattering effects that are especially important in X-ray, electron and neutron diffraction from perfect crystals [3]. The requirement of high quality crystals explains why the first experimental observation of the Pendellösung effect has been obtained in 1959 only in X-ray measurements [4], and some years later in neutron diffraction [5, 6]. Recently, using the coherence of third generation synchrotron beams, Pendellösung fringes produced by a plane wave exiting a Si crystal have been recorded [7].

Photonic crystals (PhCs) are artificial periodic structures reproducing natural crystals at different length scale. PhCs can control and manipulate the flow of light in many different ways, since they exhibit a variety of properties, spanning from full photonic band gap to anomalous dispersion phenomena, including superprism and negative refraction effects. The wide range of characteristics shown by PhCs gave rise in the last decade to a multitude of new ideas for opto-electronic integrated devices and systems. Novel concepts of mirrors, waveguides, resonators, and frequency converters based on photonic crystals, to mention a few examples, have been proposed. Indeed, the band theory of the electrons in solids, that is usually the main reference of the PhC theory, is strongly inspired by DDT (as extensively discussed in the pioneering Born's solid state textbook [8]), that represents one of the first example of two-state theory - later became very popular in modern physics - such as up and down spins, electron and hole pairs, etc.

It is not surprising therefore that the Pendellösung effect has been predicted for photonic crystals too. In 2D case, it has been thoroughly studied using both analytical and numerical methods as a function of the PhC contrast index, beam incident angle, and light polarization [9]. Moreover, this study has been extended to opal 3D photonic crystals, where the dependence of diffraction intensity as a function of the layers number has been investigated using a scattering matrix approach [10]. On the experimental side the properties of microwave diffraction in periodic structures have been reported in literature by measuring the pattern of backscattered waves in two dimensional artificial dielectric media [11]. Recently, the Pendellösung effect has been detected also in the optical regime in volume holographic gratings, observing the oscillatory behavior of the angular selectivity of the diffracted light [12].

In this work, we present an accurate theoretical study and precise measurements of the Pendellösung effect in photonic crystals by illuminating with a plane waves beam in the microwave region 2D square lattice PhCs having different number of rows (i.e. slabs with different thicknesses) and detecting the electromagnetic field inside and outside each slab. We show that under particular conditions the intensity of the diffracted wave at the exit surface can be periodically modulated with the slab thickness, presenting a maximum in the "positive" (forward diffracted) or in the "negative" (diffracted) direction. Moreover, we observe that inside the crystal the energy is periodically exchanged between direct and diffracted beams.

2. Theoretical analysis

The Pendellösung effect in PhCs can be understood as a beating phenomenon due to the phase modulation between coexisting plane wave components, propagating in the same direction. The coexistence is possible because such wavevectors are associated to two adjacent bands that are

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overlapped, for a given frequency, in correspondence of suitably chosen PhC parameters.

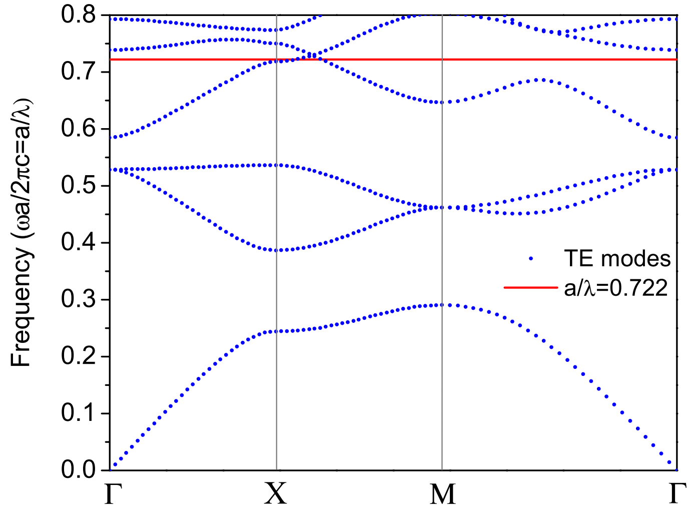

Fig. 1. The band structure of the square-lattice PhC for the TE polarization. The red line represents the normalized frequency $\omega_n = 0.722$ at which the Pendellösung effect takes place (colour online).

In our case the 2D PhC consists of dielectric cylinders in air (dielectric permittivity $\varepsilon_r = 8.6$) arranged in a square geometry and having $r/a = 0.255$, where r is the cylinder radius and a is the lattice constant. If TE polarization (electric field parallel to the rods axis) is considered, an overlap occurs between the forth and the fifth mode for a normalized frequency $\omega_n = va/c = a/\lambda = 0.722$, as shown in Fig. 1. Moreover, the crystal orientation is fixed such that the normal at its surface is along the XM direction. Hence all possible wavevectors excited into the PhC will have the same tangential component lying on XM.

The Pendellösung phenomenon is analyzed in this context for an incident wavevector that satisfies the Bragg law [9, 13]. In Fig. 2 we show in the reciprocal space the first Brillouin zone and the corresponding symmetry points for the square lattice PhC under study. Considering the reciprocal lattice vector that enforces the momentum conservation oriented along ΓX in the first Brillouin zone, the Bragg law is fulfilled when the projection of the incident wavevector coincides with ΓX, so that $\mathbf{k}_h$ is the diffracted wavevector whereas $\mathbf{k}_i$ is the incident one. Using the dispersion surfaces (or Equi-Frequency Surfaces, EFSs), that represent the loci of propagating wavevectors for a fixed frequency, we are then able to evaluate the relevant parameters of the beating effect. The wavevectors inside the PhC are determined by the intersection between each EFS and the XM direction. Amongst the different intersections, only wavevectors having group velocity oriented inside the crystal - in Fig. 2 in opposite direction respect to the external normal to the incident surface - will be effectively excited.

Consider for instance the contribution of the incident wave: there is an interference between two excited components, with the respective wavevectors pointing in two different directions. This produces a spatial periodic modulation along the wavevectors difference vector $\Delta\mathbf{k}$. The modulation distance in the real space along the PhC normal direction is therefore $\Lambda_0 = 2\pi/\Delta k$. The same effect occurs also for the diffracted wave, giving rise to a spatial modulation with the same length but 180° out-of-phase in respect to the previous case.

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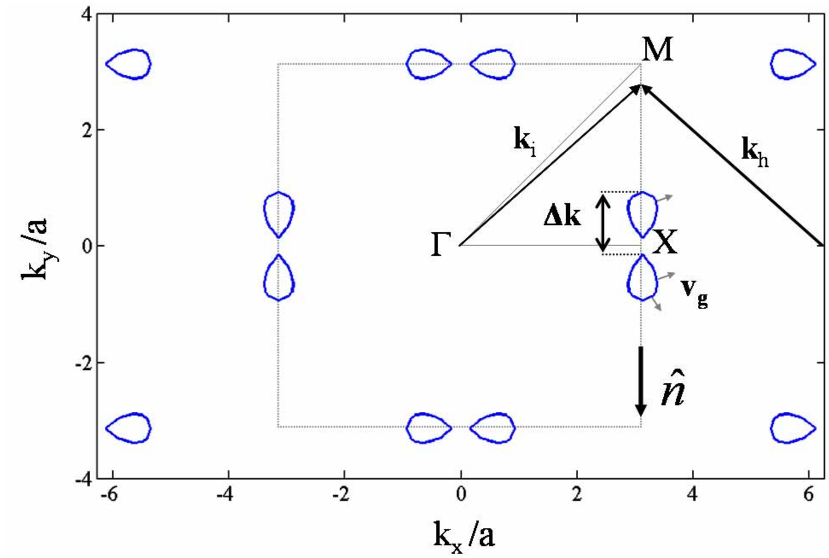

Fig. 2. The reciprocal space with the first Brillouin zone (dotted line) and symmetry points for the square-lattice PhC. The contours for the normalized frequency $\omega_n = 0.722$ are plotted. Arrows indicate the directions of group velocity $\mathbf{v_g}$, whereas $\hat{\mathbf{n}}$ shows the normal to the incident surface (colour online).

As a consequence of the Pendellösung effect, the intensity I at the exit surface is harmonically modulated as a function of the thickness t [9]. When t is an even multiple of half the Pendellösung distance, the beam at the exit surface is parallel to the incident beam, forming a positive angle respect to the PhC normal. On the other hand, when t is an odd multiple of Λ_0 the beam at the exit surface is completely directed along the Bragg diffracted direction, forming a negative angle respect to the PhC normal. Denoting by + and - the two possible directions at the exit surface, this is summarized by:

$$\begin{aligned} t &= \ \ 2m\frac{\Lambda}{2} && \Rightarrow max(I_+) \\ t &= (2m-1)\frac{\Lambda}{2} && \Rightarrow max(I_-) \end{aligned} \tag{1}$$

where $m = 1, 2, \ldots$.

Forcing the Pendellösung distance Λ_0 be an even number of the lattice constant a, Eq. (1) holds for any number n of PhC rows. In particular, $\Lambda_0 = 4a$ ensures that the intensity maxima of the exit waves changes periodically if n is even, and that the energy beam equally splits between positive and negative direction if n is odd.

From the EFSs analysis, assuming a TE polarization, we found that an angle $\theta_i = 43.8^\circ$ and a normalized frequency $\omega_n = 0.722$ for the incident wave satisfy both the Bragg law and the peculiar condition $\Lambda_0 = 4a$.

3. Experimental setup

The experimental results are obtained on 2D PhCs having a different number of rows inserted in a waveguide. First, the electromagnetic wave transmitted by the periodic structure is measured at the exit of the PhC for different crystal thickness and its spatial distribution is shown. Then, the periodic modulation of the intensity of the diffracted waves with respect to n is reported.

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Finally, a comparison along selected directions inside the photonic crystal between the electric field distribution measured and simulated using a Finite Difference Time Domain (FDTD) method is presented.

Measurements are carried out by placing alumina rods with nominal permittivity $\varepsilon_r = 8.6$, radius $r = 0.4cm$ and height $h = 1cm$ in a square geometry with $r/a = 0.255$ ($a = 1.57cm$) sandwiched in an aluminum parallel-plate waveguide terminated with microwave absorbers. Since the loss tangent of alumina is extremely small at the frequency relevant for this work ($\tan\delta < 10^{-4}$), dielectric losses can be neglected. Due to the presence of metallic plates acting as mirrors, current lines that are perpendicular to the plates can be considered as infinitely long, as stated by the well-known mirror theorem. For the same reason the electric fields produced by these currents are constant along the same direction and thus the whole system acts as a 2D structure.

The microwave photonic crystal is built in the shape of a $38.5cm$ wide slab (25 rod columns), with a thickness that can be varied adding or removing rows. A dipole antenna is used as source, oriented to produce an electric field parallel to the rods axis and operating at the frequency of $13.784GHz$, in order to reproduce the same normalized frequency a/λ of the theoretical model. Due to the waveguide characteristics, the TEM mode only can propagate up to $15GHz$. The maps of the real part of the electric field are collected by using a HP8720C Vector Network Analyzer and another dipole antenna as a detector, that moves along the waveguide plane using an x-y step motor. The thickness dependence has been investigated based on the observation of the beams at the exit of the crystal-air interface. We focused our analysis on structures with a number of rows n ranging from 1 to 10.

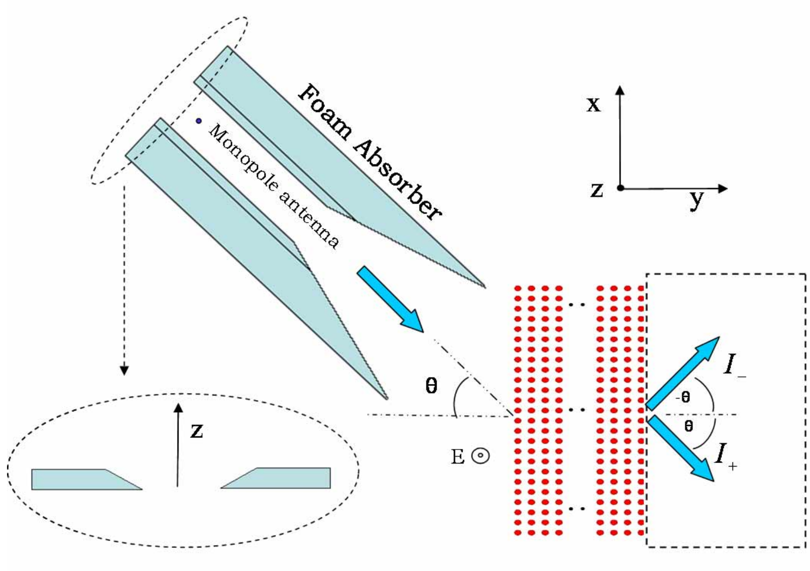

Fig. 3. Schematic layout of the experiment carried out on the square-lattice PhC slab having 25 rods columns and a number of rows n varying from 10 to 1. The dashed line box represents the scanned area during the measurements (colour online).

Figure 3 shows the scheme of the measurement. Particular attention has been paid to the source characteristics. The incident beam has to be as collimated and directive as possible, ideally consisting of a single wavevector only. To realize the experiment, we inserted in the

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parallel plate waveguide two parallel microwave absorber stripes, having the role to ”guide” the electromagnetic wave. The channel is 50*cm* long and 10*cm* wide, with tapered sidewalls in order to ensure a good matching condition at the air-to-absorber interface. The field generated by the dipole source is centered into the absorbers channel. To limit the diffraction at the exit of the waveguide, a phenomenon that strongly reduces the beam directivity, the channel section closer to the PhC interface has been shaped into a triangular profile. All these solutions provide a beam source having transmission properties close to ideal ones.

The incoming beam is then oriented at 43.8° respect to the normal to the PhC interface, whereas the two arrows exiting the surface in both the forward diffracted and diffracted directions represent the signals transmitted through the wave guide and the crystal. Furthermore, the angle that describes the outcoming waves is the same as the source. In the image plane, a tiny dipole antenna (radius $\sim 0.6mm$) scans an area 20*cm* long and 40*cm* wide contiguous to the crystal-air interface, in steps of 4 mm in both x and y direction. As said before, for a fixed frequency and source orientation the intensities I_+ and I_- reach a maximum or a minimum value depending on the difference between the wavevectors inside the crystal and, in turn, on the crystal thickness.

4. Results and discussion

In Figs. 4 (a)-(e) the real part of the electric field experimentally detected in different crystal configurations is mapped in the image plane, using a normalized scale. In Fig. 4(a) the spatial distribution is shown for the case $n = 10$. The maps for the other cases of crystals with an even number of rows ($n = 8, 6, 4, 2$) are presented in Figs. 4(b), (c), (d), (e), respectively. Starting

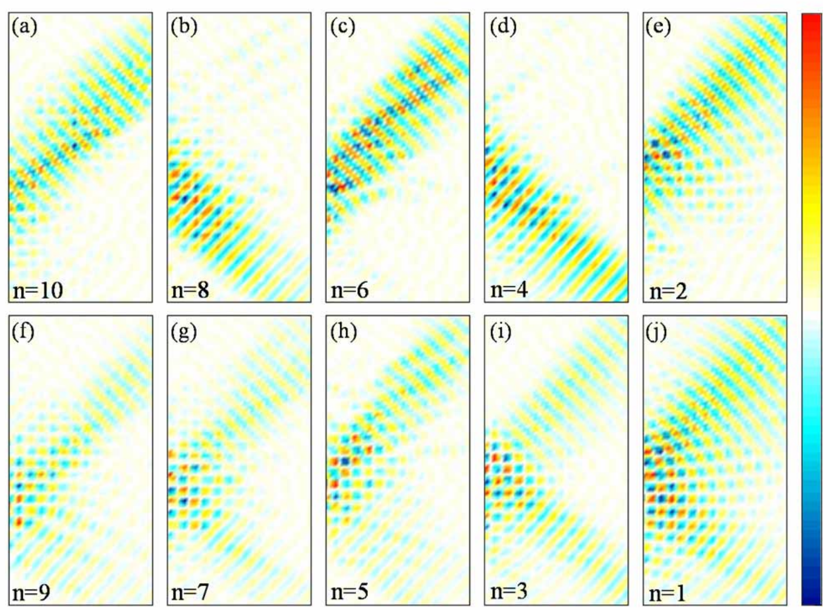

Fig. 4. (a)-(e): mapping of the measured electric field (real part) in a normalized scale for even n; (f)-(j): mapping of the measured electric field (real part) in a normalized scale for odd n (colour online).

the data analysis from the crystal consisting of 10 rows, that is an odd multiple of $\Lambda_0/2$, in this case the beam, as expected, is fully transmitted in the diffracted direction. On the contrary,

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when the PhC consists of 8 rows, the beam exits its surface in the forward diffracted direction (Fig. 4(b)). By reducing the thickness down to 2 rows for any even *n*, the transmitted beam alternatively bends from the negative to the positive direction, as shown in Figs. 4(c)-(e). Other beams related to higher order of diffraction are negligible. Measurements clearly show therefore that for an even number of rows the involved energy is almost entirely concentrated along one exit direction only. It is also clear from the images that the transmitted field propagates along regular and periodic equiphase planes, in agreement with numerical simulations.

Let us now analyze the experimental results for crystals having an odd number of rows. In this case, according to the periodical modulation predicted for the field intensity at the exit surface, the thickness is such that at the crystal-air interface the transmitted energy is equally divided in both positive and negative directions. This is shown in Fig. 4(f)-(j): the electromagnetic beam in the image plane actually splits in two rays having approximately the same intensity, with equiphase planes clearly evident in both directions. It is worth noting that the case with $n = 1$ (Fig. 4(j)) reduces to the well known Bragg grating. The fundamental feature of the Pendellösung effect is the spatial periodic modulation of the transmitted field amplitude with the crystal thickness.

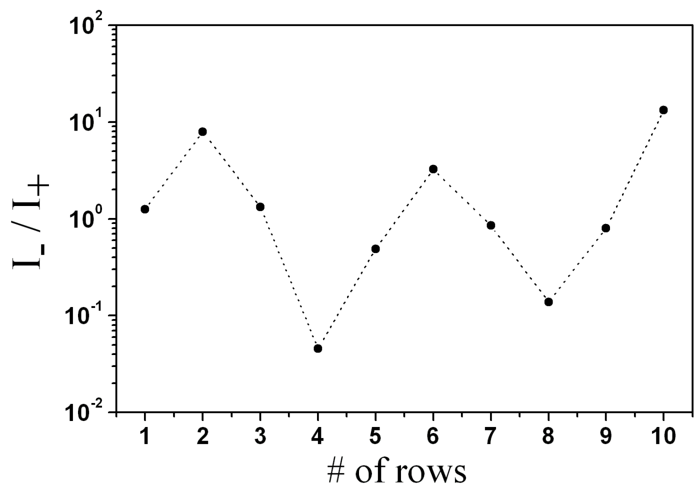

Fig. 5. The measured electric field intensity ratio I_-/I_+ for all the crystal configurations considered. The case of 10 rows corresponds to the maximum thickness $t = (10a + 2r) = 16.4cm$.

We then compared the electric field maximum intensity measured along the two different (positive and negative) transmitted directions as a function of the photonic crystal row number *n* (thickness). As shown in the Fig. 5 using a semi-log scale, the intensity ratio I_-/I_+ changes periodically, being approximately equal to 1 for any odd *n*, and exhibiting pronounced maxima and minima alternatively for any even *n*.

We also evaluated the electromagnetic field distribution inside the PhC slab. FDTD simulations are performed considering a plane monochromatic incident wave having a rectangular transverse profile. The propagation in the slab is of course well different from that in free-space since Bloch modes will be excited and therefore a strong modulation of the electromagnetic field is expected. In particular, in the Pendellösung phenomenon, the positively and negatively refracted components of the incident wave interfere each other inside the crystal and give rise to a periodic exchange of energy. This translates in a spatial modulation of the field intensity, as it can be clearly seen when the dielectric contrast is not very high [13].

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In the case discussed here the strong Bloch modulation makes the visualization of the wave pattern quite difficult. Moreover, when the contrast is high, the intensity maxima inside the rods mask the distribution in the outside region. Therefore, for the sake of clarity, we have suppressed the field inside the dielectrics. Results are shown in Fig. 6(a). It is worth mentioning that the spatial modulation observed in the crystal does not affect the energy direction, which remains normal to the PhC interface. To compare the numerical simulations with experimental data we have then measured the internal field along selected directions normal to the PhC interface. Particular attention has been paid to ensure that the detector antenna moves perfectly parallel to the dielectric rods.

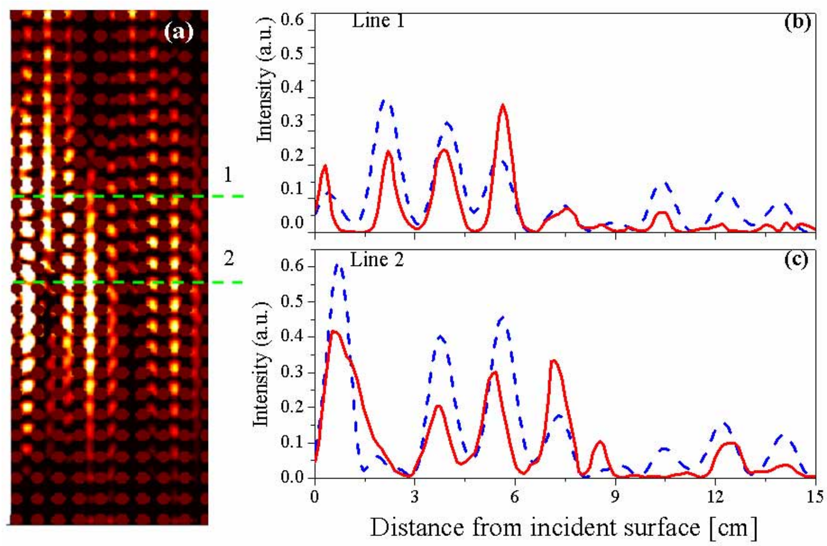

Fig. 6. (a) FDTD simulation of the propagation pattern inside a crystal consisting of 10 rows of the 13.784 GHz plane wave modulated by a rectangular profile and incident at an angle of 43.8° across the XM interface; (b) & (c): electric field intensity distribution (blue dashed lines) along line 1 and 2 respectively compared with the experimental results (red solid lines) (colour online).

Figures 6(b) and 6(c) shows the simulated longitudinal profile of the field intensity along two different lines, (1) and (2) respectively, and the corresponding experimental results properly rescaled. In spite of the strong field modulation, the presence of peaks and valleys centered in different positions along the PhC normal direction and corresponding to different minima and maxima in the wave energy on the two longitudinal lines is evident, as expected by the theory. Besides that, the decrease in the intensity as far as the electromagnetic wave propagates inside the crystal reflects the energy radiated from the finite-size PhC. This can be also observed in Fig. 6(a).

5. Conclusion

We have designed and realized an experiment to measure the Pendellösung effect in PhCs, where two beams are 180° out of phase each other, giving rise to the effect of energy flow swapping back and forth between two different directions into the crystal. Pendellösung can be in principle exploited to design a new kind of photonic crystal based devices. In a previous paper, it has been suggested that this effect can be used to realize a passive polarising beam splitter [13]. In the case presented here, we have shown that adding or removing rows in a finite

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thickness 2D PhC can dramatically change the direction and the features of the electromagnetic wave transmitted through the slab. Data are in a very good agreement with calculations based on the classical approach of the Dynamical Diffraction Theory, showing that this rigorous formalism can be successfully applied to predict some anomalous features exhibited by PhCs. In particular, we underline that the Pendellösung effect has been extensively used for the accurate determination of the structure factors in real crystals [14]. It can be extremely useful in photonic crystals too, allowing precise measurements of some peculiar properties such as the dielectric contrast.

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