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Resonances in alpha-nuclei interaction

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Abstract
Tunnelling of α particles through the Coulomb barrier is considered. The main attention is given to the effect of sharp peaks arising in the case of coincidence of the α energy with that of a quasistaionary state within the barrier. The question of the α-nucleus potential is discussed in this light. The method is applied to the α decay of a compound nucleus of ^{135}Pr. The appearance of the peaks in the spectrum of emitted particles is predicted. They can give rise to 'anomalous' properties of some neutron resonances. The peaks can also be observed in the incoming α-nucleus channel. Observation of the peaks would give unique information about the α-nucleus potential.

1. Introduction

Classical α-decay (e.g. [1]), as well as α-decay from the compound nuclei formed in fusion–fission reactions [2–5] and neutron resonances [7, 8], comprises a significant element of nuclear study. In the fusion–fission reaction, prefission emission of alphas and other light charged particles provides us with a source of information about the timescale of the process [2]. However, some details of the theoretical description still remain inadequately elucidated in the literature. Thus, emission of alphas and light charged particles from hot compound sources produced in heavy-ion or energetic proton–nucleus collisions is usually made in terms of the inverse cross section [9]. Such an approach assuming the time reversibility of the process leaves out of scope a possibility of its profound experimental check. This is in contrast with a number of suggestions that violations of the reversibility may arise, e.g., due to the back-transparency of the inner slope of the potential barrier in the incoming channel [3], or different response of the nuclear surface on the interaction with the emitted or an incoming particle of the same type. Temperature effects on the barrier distribution also can influence the α spectrum in the exit channel [4], whereas in the entrance channel an experimental fit of the optical model parameters is only possible for cold nuclei [5]. Note also an original study in [10].

Moreover, traditional decay calculations usually deal with tunnelling through a barrier of a particle which is in the quasistationary state inside the potential well. Encountering the inner wall, it has a chance to tunnel through the barrier out of the well. The tunnelling probability is then given by the classical action, which equals the ratio of the squares of the wavefunction outside and inside the barrier [11]. But how to proceed if the wavefunction *increases* under the barrier? The latter situation should be considered yet as a normal case, as the potential parameters in the exit channel, together with the quasistationary energies, are determined, in principle, by the nucleon–nucleon interactions, whereas the energies of the emitted particles are determined by the nuclear masses, and these are quite not bound to be in correlation. Thus, the parameters of the α-nucleus potential are fitted in the way that the energy of the emitted particle exactly coincides with a quasistationary state inside the barrier. A complete investigation of this kind is presented in [6]. A method has been developed for determining potential parameters from the energy and width of an α decay. It is clear that such a procedure only can be conducted for the particular energy of a given α line. This procedure evidently looses its reasons, e.g., in the case of α-decay of a certain neutron resonance to a number of final states [7, 8]. At the same time, it is hardly reasonable to ascribe different parameters to the α–nucleus potential depending on the energy of the emitted α particle. Furthermore, some properties of the α-decay of 'anomalous' neutron resonances could be a consequence of an *accidental* coincidence of the energies of the emitted particles with the quasistationary energies [8, 12]. The same situation arises in important cases when a pre-emission virtual α particle or cluster is between the quasistationary states, as in alpha decay from compound systems formed in fusion reactions [2–5]. The emission spectrum is continuous in this case.

Such quasistationary states are subject to utmost fragmentation (e.g., [13]). We want to stress that this just produces the conditions when such 'eigenvalue' structure will reveal itself most distinctively, due to the factor of the barrier penetration. Emission of 'resonance' alpha particles will be enhanced, as the penetration is always calculated for the actual energy of the emitted particle (e.g., [14] and references therein). Therefore, the resonances under consideration are not duly related to the real collective states of α clusters, though they can be virtually formed on the nuclear surface with a considerable probability (e.g., [15]). Correspondingly, similar effects take place in the surface-α-cluster model [16].

Our approach allows one to calculate the decay width at any energy of the emitted particle. Strong resonance effects are, specifically, predicted in alpha spectra from subbarrier decay of compound systems. Note that the resonances can be probably even easier observed in the incoming channel in the subbarrier alpha–nucleus reactions. Already in [6] it was demonstrated that using simultaneously data for α decay and α scattering allows one to obtain maximal information about the α-nucleus potential. Such resonances were observed in proton–nucleus reaction [17, 18]. However, in the case of α-nucleus reactions, to the best of our knowledge, we cannot refer to any data. In our present interpretation, for the purpose of clarity, we shall mainly follow the physical sense and first principles of quantum mechanics, rather than a strict mathematical approach. The same results can be derived in a consecutive mathematical treatment.

2. Theory

2.1. α decay widths

Let $\Psi(\mathbf{r}_1, \ldots, \mathbf{r}_A)$ be a wavefunction of the source nucleus with the mass number A. First, it can be expressed in terms of the channel wavefunction basis as products of the wavefunction of the daughter nucleus $\varphi_n(\mathbf{r}_1, \ldots, \mathbf{r}_{A-4})$ and the α particle w.f. $\chi_k^{(L)}(\mathbf{r}_{A-3}, \ldots, \mathbf{r}_A)$:

$$\Psi = \sum_{L=0}^{L_0} \sum_{nk} C_{nkL} \varphi_n(\mathbf{r}_1, \ldots, \mathbf{r}_{A-4}) \chi_k^{(L)}(\mathbf{r}_{A-3}, \ldots, \mathbf{r}_A),$$
$$\equiv \sum_{L=0}^{L_0} \sum_{n} C_{nL} \varphi_n(\mathbf{r}_1, \ldots, \mathbf{r}_{A-4}) \eta_L^{(f)}(\mathbf{R}; \mathbf{r}_{A-3}, \ldots, \mathbf{r}_A), \tag{1}$$

where we selected the angular momentum L of the relative motion of the α particle in the nucleus. $\eta_L^{(f)}$ may be treated as a wavefunction of the relative motion of the α cluster in the mother nucleus which evidently turns out to depend on the relative coordinate $\mathbf{R}$. In simple cases of pure configuration the expansion coefficients C_{nkL} are reduced to genealogical coefficients.

Let then the nucleus make a transition $i \to f$. As a result, in the exit channel we observe the system in a state which is described by a wavefunction as a superposition of the plane wave and incoming spherical wave [19] at large α-nucleus distances R:

$$\psi_{f\mathbf{p}}(\mathbf{r}_1, \ldots, \mathbf{r}_A) \underset{R\to\infty}{\sim} \varphi_f(\mathbf{r}_1, \ldots, \mathbf{r}_{A-4}) g_{\mathbf{p}}(\mathbf{r}_{A-3}, \ldots, \mathbf{r}_A), \tag{2}$$

$$g_p(\mathbf{r}_{A-3}, \ldots, \mathbf{r}_A) \underset{R\to\infty}{\sim} \left[\mathrm{e}^{\mathrm{i}\mathbf{pR}} + \frac{A(\vartheta, \varphi)}{R} \, \mathrm{e}^{-\mathrm{i}pR} \right] \xi(\mathbf{r}_{A-3}, \ldots, \mathbf{r}_A), \tag{3}$$

$$\mathcal{F}_{\mathbf{p}}(\mathbf{R}) \equiv \left[\mathrm{e}^{\mathrm{i}\mathbf{pR}} + \frac{A(\vartheta, \varphi)}{R} \, \mathrm{e}^{-\mathrm{i}pR} \right].$$

In equation (3), $g_{\mathbf{p}}$ is the channel wavefunction, which is the eigen function of the α-nucleus Hamiltonian with an appropriate mean-field single particle potential $U_\alpha(R)$:

$$(H - \varepsilon_{\mathbf{p}}) \mathcal{F}_{\mathbf{p}} = 0, \tag{4}$$

$$\varepsilon_p = p^2/2M_\alpha. \tag{5}$$

Furthermore, taking into account the asymptotics (3), the wavefunction $\mathcal{F}_{\mathbf{p}}(\mathbf{R})$ can be expressed in terms of the spherical harmonics in a usual way:

$$\mathcal{F}_p(\mathbf{R}) = \sum_{\ell=0}^{\infty} i^\ell (2\ell + 1) \, \mathrm{e}^{\mathrm{i}\delta_\ell} R_{p\ell}(R) Y_{\ell m}(\theta, \varphi). \tag{6}$$

To find a transition amplitude, one has to change to the coordinate system of the exit channel $|f\mathbf{p}\rangle$. The transformation of equation (1) then conventionally reads as

$$\Psi = \sum_{\mathbf{p}} \langle \psi_{f\mathbf{p}} | \Psi \rangle \psi_{f\mathbf{p}}, \tag{7}$$

the re-expansion coefficients giving the transition amplitude under consideration. This way is similar to that found by Migdal when solving his classical problem of shake of an atom in β decay [19]. Substituting equations (1) together with (2), (3) and (6) into equation (7), we arrive at the following expression for the transition amplitude:

$$M_{f\mathbf{p}} = \sum_{\ell=0}^{L_0} C_{f\ell} i^\ell \, \mathrm{e}^{\mathrm{i}\delta_\ell} Y_{\ell m}(\theta, \varphi)$$
$$\times \left\langle R_{p\ell}(R) Y_{\ell m}(\theta, \varphi) \xi(\mathbf{r}_{A-3} - \mathbf{R}_1, \ldots, \mathbf{r}_A - \mathbf{R}) \middle| \eta_\ell^{(f)}(\mathbf{r}_{A-3}, \ldots, \mathbf{r}_A) \right\rangle$$
$$\equiv \sum_{\ell=0}^{L_0} C_{f\ell} \langle \mathbf{p}\xi | f\xi \rangle_\ell i^\ell \, \mathrm{e}^{\mathrm{i}\delta_\ell} Y_{\ell m}(\theta, \varphi). \tag{8}$$

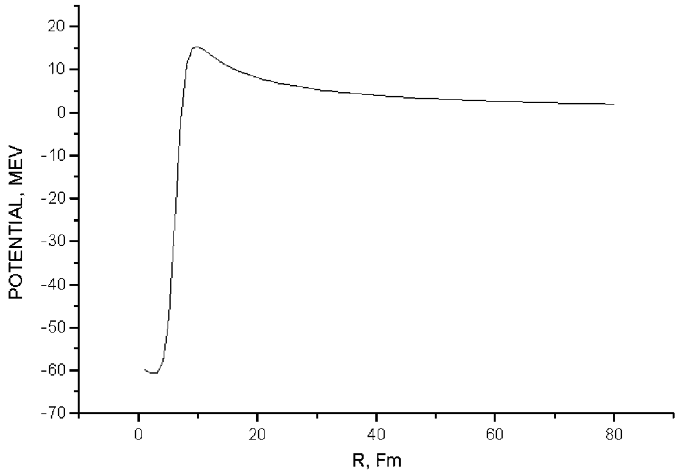

Figure 1. α-nucleus potential for the system of ^{131}La + α with a zero angular momentum.

Taking into account that the wavefunctions $\mathcal{F}_{\mathbf{p}}$ are normalized at one particle in a unit volume, with the flux $\boldsymbol{v} \equiv \mathbf{p}/M_\alpha$, we obtain from (8) the following expression for the decay probability per unit time:

$$\Gamma_{\mathbf{p}} \equiv \frac{\mathrm{d}^3 W}{\mathrm{d}^3 p} = |M_{f\mathbf{p}}|^2 v. \tag{9}$$

Inserting $M_{f\mathbf{p}}$ from equation (8) into equation (9) and integrating over all the angles of emission within 4π, we arrive at the following final expression for the decay width:

$$\Gamma_\alpha \equiv \frac{\mathrm{d}N}{\mathrm{d}\varepsilon_\alpha} = 4\pi M_\alpha^2 v \sum_k \sum_{\ell=0}^{L_0} |C_{f\ell}|^2 |\langle p|f\rangle_\ell|^2. \tag{10}$$

Supposition of the utmost fragmentation [13] of the eigen α levels means that

$$|C_{f\ell}|^2 \approx \text{const}. \tag{11}$$

Then the probability of emission is determined by the next factor, $|\langle p|f\rangle_\ell|^2$, which gives the penetration probability. Let us therefore study this factor in the next section.

3. Method of numerical solution. Eigenvalues

The α-nucleus potential is characterized by a Coulomb barrier, which is high enough, to form quasi-bound states inside the barrier (figure 1).

These would be usual eigenstates, if the barrier were infinitely broad. The values, however, go over the resonances on the continuum background, whenever the penetrability of the barrier is taken into account. Coupling to the continuum causes the energy shift and broadening of the eigenstates. Affected eigenvalues can be determined as follows.

The Schrödinger equation for an α particle in the field of a nucleus reads as follows:

$$\left\{-\frac{1}{2m}\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} - \frac{L(L+1)}{r^2}\right] + V(r)\right\}\Psi = E\Psi, \tag{12}$$

with the potential

$$V(r) = V_{SW}(r) + V_C(r), \tag{13}$$

$$V_{SW} = \frac{-V_0}{1 + \exp\left(\frac{r-c}{a}\right)}, \tag{14}$$

and the Coulomb potential was taken into account as due to the sharp-edge charge distribution:

$$V_C(r) = \begin{cases} \dfrac{\alpha Z}{2R_0}\left[3 - \left(\dfrac{r}{R_0}\right)^2\right] & \text{for } r < R_0, \\ \dfrac{\alpha Z}{r} & \text{for } r \geqslant R_0, \end{cases} \tag{15}$$

with R_0 being the nuclear radius.

On the radius segment between the origin $r = 0$ and the first turning point R_{c1}: $0 < r \leqslant R_{c1}$, equation (1) was integrated numerically with the initial condition

$$\Psi(r) \underset{r\to\infty}{\sim} r^L. \tag{16}$$

The general solution of the Schrödinger equation under the barrier is a linear combination of the two linearly independent solutions. One of them exponentially vanishes, and the other exponentially increases with increasing R. The coefficients can be obtained by sewing the functions at the internal turning point. The eigenvalues may be obtained from a condition that the coefficient for the exponentially increasing solution vanishes. In principle, this can also be achieved by sewing to the asymptotic regular solution under the barrier given by the Airi function [19]. We checked that both ways lead to essentially the same result, taking into account the numerical accuracy of the method. Actually, the eigen solutions were obtained by numerical integration from the external turning point R_{c2} towards the internal one, with somewhat an arbitrary initial condition

$$y(R_{c2}) = 1, \qquad y'(R_{c2}) = -0.25. \tag{17}$$

The derivative in equation (17) is negative, as the solution is assumed to exponentially decrease under the barrier. In the course of integration, only the right solution survives, which exponentially increases with decreasing R under the barrier. The other exponentially vanishes, in so far that the eigenvalue obtained practically very weakly depends on the concrete numbers in equation (17).

In the general case, the fundamental set was obtained by numerical integration from R_{c2} to R_{c1} with two different initial conditions:

$$y(R_{c2}) = 1, \qquad y'(R_{c2}) = \pm 1. \tag{18}$$

For $r = R_{c1}$, the resulting solution increases under the barrier (see figures 3 and 4) if not an eigenstate, in contrast with the behaviour of each of the fundamental solutions. This demonstrates mathematical correctness of the method. For the numerical integration, the Runge–Kutta–Nyström method was used. The Shtermer method was also tried, with the same results. Behind the barrier, the both solutions oscillate.

4. Numerical results

Calculations were performed with the Saxon–Woods potential (14), with the parameters $V_0 = 100$ MeV, $s = 2.3$ Fm, $c = 1.2A^{1/3}$ Fm. The imaginary part of the potential is usually

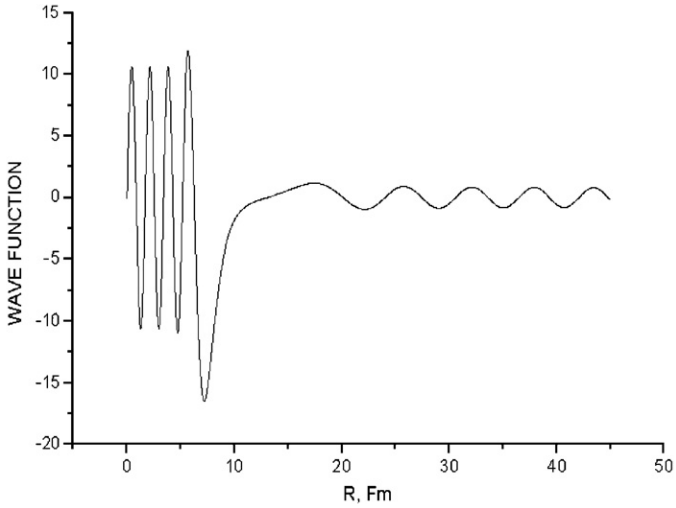

Figure 2. α wavefunction for the system of ^{131}La + α with an α energy of 10.79 MeV.

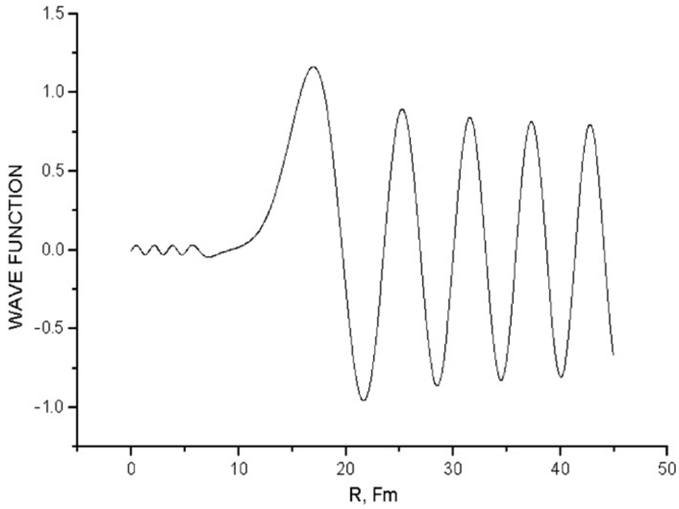

Figure 3. α wavefunction for the system of ^{131}La + α, with an α energy of 11 MeV.

omitted for the energies below 12 MeV. Calculated wavefunctions for various energies are presented in figures 2–4 for the system α + ^{131}La, $L = 0$.

Figure 2 shows the wavefunction for $E_\alpha = 10.79$ MeV. The corresponding wavefunction has a maximal amplitude inside the barrier [6]. That is why the overlapping integral $\langle p|f\rangle_\ell$ in (10) is expected to be large in this case.

In figures 3 and 4, we present the wavefunctions outside the resonance, for energies of 11 and 14 MeV, respectively. The wavefunctions are normalized at $\delta(p - p')$. These figures are in dramatic contrast with the resonance one, presented in figure 2. The amplitude of the wavefunction within the barrier is much smaller than outside. As a result, the overlapping integral $\langle p|f\rangle_\ell$ in (10) is expected to be small in the nonresonance case, thus depressing the

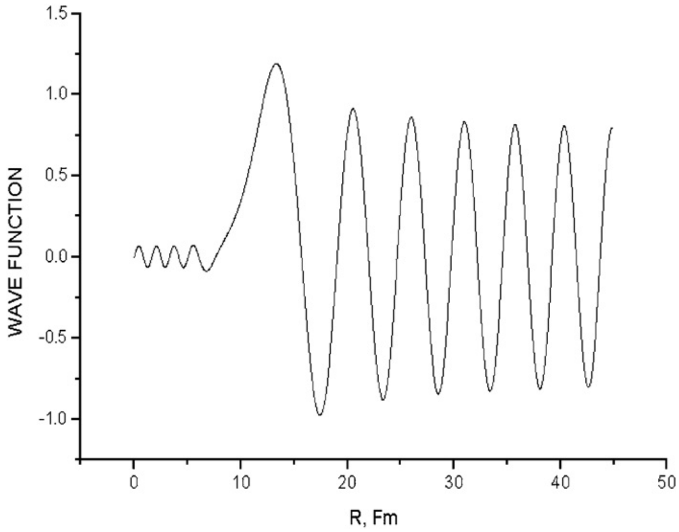

Figure 4. α wavefunction for the system of ^{131}La + α, with an α energy of 14 MeV.

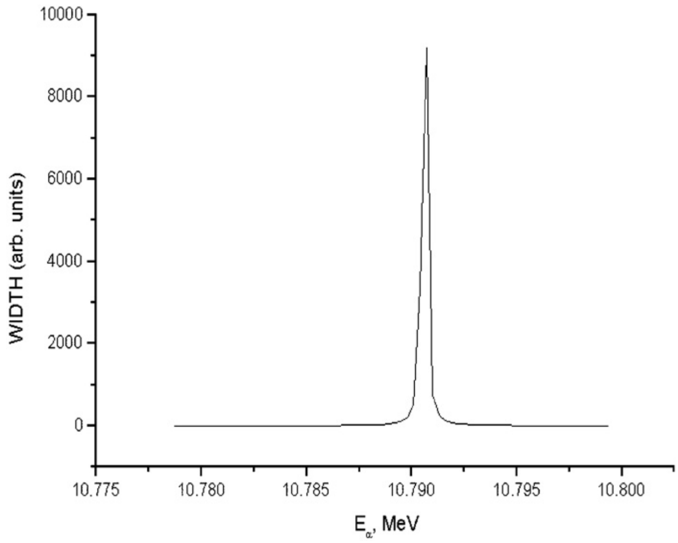

Figure 5. Profile of the α decay line in an anticipated subbarrier spectrum, half-width being around 200 eV. (The system is ^{131}La + α, for the alphas emitted with L = 0.)

nonresonance α decay. Finally, in figure 5 we present the calculated profile $\Gamma(E)$. That has a typical resonance shape with a half-width of around 200 eV.

5. Conclusion

Therefore, the spectrum of the subthreshold α particles turns out to be modulated, directly indicating the resonance states inside the barrier. Observation of this effect would really mean discovering new physics. Detection of the peaks would result in unique information obtained about the α-nucleus potential. On the one hand, the absence of the experimental data on the peaks in α-decay, as discussed in the introduction, or in the incoming α-nucleus channel is in favour of (11). On the other hand, it can be due to a very narrow width of the resonances.

In heavy-ion collisions, this effect may be smoothed by mixed multipolarities. The effect must also manifest itself in the usual α decay of neutron resonances, or nuclei far from the drip line. In this case, the set of the allowed L values is usually not wide. Moreover, a partial wave with a certain L may make the predominant, or even unique contribution, like for the decay of neutron resonance $4^+ \rightarrow 0^+$ in Sm^{148} [8, 12]. This can be exploited for an experimental test of the theory.

The decay width can also be calculated on the basis of a phase approach:

$$\Gamma(E) \sim \left(\frac{\mathrm{d}\phi(E)}{\mathrm{d}E}\right)^{-1}, \tag{19}$$

where $\phi(E)$ is the phase shift. Exactly the same peaks, with essentially the same amplitudes and half-widths, are obtained in this way.

Note that the quasi-collective resonance as discussed previously appears at certain transition energy, being actually *independent* of the energies of the initial and final nuclear states, which can vary. An analogous picture is observed experimentally in the giant dipole resonance (GDR) phenomenon. The latter manifests itself in the γ spectrum of highly excited nuclei as a characteristic bump with exactly the same parameters as in reverse processes of photonuclear reactions. The spectrum depends on the transition energy, and not on the initial nuclear energy as a consequence of the Axel–Brink hypothesis. The analogous picture is predicted in the present approach for the α decay. Furthermore, it is suggested that experimentally it can be easier to find the resonance in the reverse process of merging α with the nuclei.

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