

# Testing a quintessence model with CMBR peaks location

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## Abstract

We show that a model of quintessence with exponential potential, which allows to obtain general exact solutions, can generate location of CMBR peaks which are fully compatible with present observational data.

## 1 Introduction

In correspondence of some angular scales, the anisotropy spectrum of the Cosmic Microwave Background Radiation (CMBR) exhibits some peaks of intensity. Standard cosmology succeeds in explaining the origin of these peaks and supplies an analytical procedure for the calculation of their positions. A particularly interesting aspect of this argument is that such locations are very sensitive to the variation of the cosmological parameters. Like other kinds of analysis, this places strong constraints on the cosmological parameters and discriminates among the different models [1].

In this paper we want to test a particular model of quintessence. In order to make this, we first have to calculate, assuming a flat anisotropy spectrum, the theoretical values of the peaks' positions and compare these results with the experimental data.

Peaks are formed during recombination age. The perturbative scales smaller than the value of Hubble's radius at the recombination time  $\eta_{rec}$  re-enter into the horizon before that time and behave like acoustic waves; in other words, in the primordial plasma they generate perturbations comparable to those generated by a sound wave in a fluid [2].

Small perturbations are therefore generated in matter and radiation, giving rise to photon clustering.

The radiation pressure acts against this process. The result of these two competitive processes is the formation of peaks of intensity of radiation (that

is, of peaks of clustering in the distribution of photons) at several scales, characterized from the relation [3]

$$k_n = \frac{n\pi}{r_s(\eta_{rec})}, \quad (1)$$

where  $\eta$  is the conformal time,  $n$  is a positive integer and  $r_s$  is the sound horizon at the instant  $\eta$ , that is,

$$r_s(\eta) = \int_0^\eta c_s d\eta', \quad (2)$$

being  $c_s$  the sound velocity.

Today we observe the CMBR projected on the sky, so that we are interested in the angular positions of the peaks. Once we suppose that the distribution of radiation propagates, from the last scattering up to now, in such a way that it does not modify the position of the peaks, these ones are observed at angular scales corresponding to the three-dimensional scales expressed in Eq. (1). We obtain such values noticing that to a scale  $k_n$  corresponds an angular scale given by

$$l_n \simeq r_\theta(\eta_{rec})k_n, \quad (3)$$

where  $r_\theta$  is the comoving angular distance which in the case of a spatially flat universe has the simple expression

$$r_\theta = \eta_0 - \eta_{rec}. \quad (4)$$

Consequently, we get

$$l_n = \frac{\eta_0 - \eta_{rec}}{\overline{c_s}\eta_{rec}} n\pi = nl_A, \quad (5)$$

where it has been assumed that  $c_s$  is approximately constant between  $\eta$  and  $\eta_{rec}$  and equal to its average value, while  $l_A$  is given by

$$l_A \equiv \frac{\eta_0 - \eta_{rec}}{\overline{c_s}\eta_{rec}} \pi. \quad (6)$$

## 2 Universe with quintessence

In a model of universe without quintessence or a  $\Lambda$ -term the positions of the peaks satisfy relation (5). But the recent observations give data which cannot be deduced from this relation. On the other hand, if we consider a model with quintessence, the presence of this new component modifies Eq. (5). It has been shown that such modifications can be accounted for introducing a discrete series of parameters  $\varphi_n$ , in function of which Eq. (5) is rewritten as [4]

$$l_n = l_A(n - \varphi_n). \quad (7)$$

In [4] the empirical formulas giving the values of the parameters  $\varphi_n$  for a generic model of quintessence are deduced. It is

$$\varphi_n = \bar{\varphi} + \Delta\varphi, \quad (8)$$

with

$$\bar{\varphi} = a_1 (r_{rec})^{a_2} + 0.291\bar{\Omega}_{ls}^\varphi, \quad (9)$$

where

$$a_1 = 0.286 + 0.262(\Omega_b h^2), \quad (10)$$

$$a_2 = 0.1786 - 6.308(\Omega_b h^2) + 174.9(\Omega_b h^2)^2 - 1168(\Omega_b h^2)^3, \quad (11)$$

$$r_{rec} = \frac{\rho_r(z_{rec})}{\rho_m(z_{rec})} = 0.042 (\Omega_b h^2) \frac{z_{rec}}{10^3}, \quad (12)$$

$$\bar{\Omega}_{ls}^\varphi = \eta_{rec}^{-1} \int_0^{\eta_{rec}} \Omega^\varphi(\eta) d\eta. \quad (13)$$

In the last one of these relations, it is assumed that  $\Omega^\varphi(\eta)$  does not change too much rapidly before recombination, so that  $\bar{\Omega}_{ls}^\varphi$  is an effective averaged value of quintessential energy.

In [4] Doran and Lilley obtain the formulas that allow to calculate the theoretical values of the first three peaks.

Therefore, by means of Eq. (5), it is possible to test a given model of quintessence. In fact, once we fix the values of the parameters by which the model is specified, we can calculate the values  $l_n$  of the angular positions of the peaks and compare them with the experimental data. In order to perform this calculation, of course, it is first necessary to find the value of  $l_A$ .

From the definition in Eq. (6), and following [4], we can use the evolution equations for a universe with quintessence and derive the expressions for  $\eta$  and  $\eta_{rec}$  in terms of  $\Omega^\varphi$ . We can also approximate  $\Omega^\varphi$  by means of the constant average  $\bar{\Omega}_{ls}^\varphi$  for the period around last scattering.

In this way, setting  $8\pi G = c = 1$  and denoting with  $\Omega_0^r$  and  $\Omega_0^\varphi$  today's radiation and quintessence components, with  $a_{rec}$  the scale factor at last scattering (having we supposed  $a_0 = 1$ ), and with  $\bar{c}_s$  the average sound speed before last scattering, we obtain

$$3H^2(t) \left(1 - \bar{\Omega}_{ls}^\varphi\right) = \rho^m(t) + \rho^r(t) = \rho_0^m a(t)^{-3} + \rho_0^r a(t)^{-4}. \quad (14)$$

Today, the radiation density is negligible, and we have

$$3H_0^2(t) (1 - \Omega_0^\varphi) = \rho_0^m(t), \quad (15)$$

which we insert in Eq. (14) to obtain

$$\left(\frac{da}{d\eta}\right)^2 = H_0^2 \left(1 - \bar{\Omega}_{ls}^\varphi\right)^{-1} [(1 - \Omega_0^\varphi) a(\eta) + \Omega_0^r]. \quad (16)$$

Integrating this equation by separation of variables, it is possible to obtain  $\eta(a, H_0, \overline{\Omega}_{ls}^\varphi, \Omega_0^\varphi, \Omega_0^r)$ .

In particular, we are interested in  $\eta_{rec}$ , that is,

$$\eta_{rec} = 2H_0^{-1} \sqrt{\frac{1 - \overline{\Omega}_{ls}^\varphi}{1 - \Omega_0^\varphi}} \left\{ \left( a_{rec} + \frac{\Omega_0^r}{1 - \Omega_0^\varphi} \right)^{1/2} - \left( \frac{\Omega_0^r}{1 - \Omega_0^\varphi} \right)^{1/2} \right\}. \quad (17)$$

If we define  $\overline{w}_0$  as the  $\Omega^\varphi$  weighted equation of state of the universe

$$\overline{w}_0 = \frac{\int_0^{\eta_0} \Omega^\varphi(\eta) w(\eta) d\eta}{\int_0^{\eta_0} \Omega^\varphi(\eta) d\eta}, \quad (18)$$

we can integrate the cosmological equation for  $w \simeq \text{const.} = \overline{w}_0$  and rewrite Eq. (16) as

$$\left( \frac{da}{d\eta} \right)^2 = H_0^2 [(1 - \Omega_0^\varphi - \Omega_0^r) a(\eta) + \Omega_0^r + \Omega_0^\varphi a(1 - 3\overline{w}_0)]. \quad (19)$$

This equation gives

$$\eta_0 = 2H_0^{-1} (1 - \Omega_0^\varphi)^{-1/2} F(\Omega_0^\varphi, \overline{w}_0). \quad (20)$$

Thus, we now have  $\eta_0$  and  $\eta_{rec}$ , which we can insert in Eq. (6) for the present time.

The result is [5]

$$l_A = \pi \overline{c}_s^{-1} \left[ \frac{F(\Omega_0^\varphi, \overline{w}_0)}{(1 - \overline{\Omega}_{ls}^\varphi)^{-1/2}} \left\{ \left( a_{rec} + \frac{\Omega_0^r}{1 - \Omega_0^\varphi} \right)^{1/2} - \left( \frac{\Omega_0^r}{1 - \Omega_0^\varphi} \right)^{1/2} \right\}^{-1} - 1 \right], \quad (21)$$

with

$$F(\Omega_0^\varphi, \overline{w}_0) = \frac{1}{2} \int_0^1 dx \left( x + \frac{\Omega_0^\varphi}{1 - \Omega_0^\varphi} x^{(1-3\overline{w}_0)} + \frac{\Omega_0^r(1-x)}{1 - \Omega_0^\varphi} \right)^{-1/2}. \quad (22)$$

### 3 Location of the CMB peaks for an exponential model

Let us then calculate the locations of the CMB peaks in a particular model of quintessence. As it will be seen, through the comparison with the experimental data and the forecasts of the simplest models of quintessence, the model we use is in good agreement with the observations.

The model of quintessence here investigated has been studied by Rubano and Scudellaro [6], starting from a scalar field with a potential

$$V(\varphi) = Be^{-\lambda\varphi} \quad , \quad \lambda = \sqrt{3/2}. \quad (23)$$

This choice leads to the following exact solutions for, respectively, the scale factor, the density and equation of state parameters

$$a = \frac{\tau^2(1 + \tau^2)}{\tau_0^2(1 + \tau_0^2)}, \quad (24)$$

$$\Omega_m = \frac{1 + \tau^2}{(1 + 2\tau^2)^2}, \quad (25)$$

$$w = -\frac{3 + 2\tau^2}{4 + 4\tau^2}, \quad (26)$$

with  $\tau = \omega t$ ,  $\omega^2 = \sigma^2 B^2 = (3/2) B^2$  (this time scale depends on the unknown value of  $B$ , but this does not affect the situation presented here, which depends only on  $\tau$ ). The value of  $\tau$  at present epoch is  $\tau_0$ , so that  $\Omega_{m0} = \Omega_m(\tau_0)$ .

The present values of all the relevant parameters, thus, depend only on the choice of  $\tau_0$ .

Such a model is compatible with the data of the observations of the type-IA supernovae if we choose [7]

$$\begin{aligned} \tau_0 &= [0.82, 1.40] \quad (\text{with the most probable value } t_0 = 1.268), \\ \Omega_{m0} &\in [0.12, 0.30] \quad (\text{with the most probable value } \Omega_{m0} = 0.15). \end{aligned}$$

This last result is smaller than  $\Omega_{m0} = 0.30$ , the commonly accepted one. Indeed, in this model, it is true that  $\Omega_m = 0.15$  is the most probable value, but the distribution statistics of the data gives an approximately equal probability for the other values of the interval. Moreover, some experimental evidences exist that justify a consideration of values of  $\Omega_{m0}$  smaller than the usual one (see [8], for instance).

The model we are testing here has been proved to be compatible also with the data on the peculiar velocities of galaxies [9].

By means of the values given above for  $\tau_0$  and  $\Omega_{m0}$ , it is possible to go back to what is foreseen by this model on the locations of the peaks. In the calculations it has been placed  $\bar{v}_s = 0.52$  and  $\Omega_0^* = 9.89 \cdot 10^{-5}$ .

Choosing  $\Omega_b = 0.05$  and  $h = 0.65$ , we obtain, for  $l_1$ ,  $l_2$ , and  $l_3$ , the values in Table I.

The range of the values comes from the range of  $\tau_0$ , at  $1\sigma$  level.

	Most Probable Value	Range
$l_1$	232	218 – 236
$l_2$	572	527 – 586
$l_3$	884	799 – 909

Table I

On the other hand, Table II shows the values of the angular scales at which CMBR is experimentally observed.

The data in the first column concern the BOOMERANG experiment, while in the second column the data are obtained from a combined analysis of some recent observations.

	BOOMERANG Experiment	Combined Data
$l_1$	$221 \pm 14$	$222 \pm 14$
$l_2$	$524 \pm 35$	$539 \pm 21$
$l_3$	$850 \pm 28$	$851 \pm 31$

Table II

The comparison between the intervals of the values for the single peaks shows that the exponential model for quintessence reproduces the observed data enough faithfully. In particular, it turns out to be more satisfactory when the value of  $\tau_0$  is a little smaller than the most probable value, so corresponding to a value of  $\Omega_{m0}$  greater than 0.15, as it is reasonable with respect to other experimental evidences.

Finally, let us compare our results with the standard theoretical predictions for models with and without a  $\Lambda$ -term, which are showed in Table III.

	Universe with $\bar{w}_0 = \Omega_\varphi = 0$	Universe with $\bar{w}_0 = -1$ and $\Omega_\Lambda = 0.6$
$l_1$	206	220
$l_2$	502	528
$l_3$	711	793

Table III

We see that the model without a  $\Lambda$ -term is out of the observational ranges (as expected), while the other is fully compatible with both the data and our results. Therefore, we find no possibility of distinction.

## 4 Conclusions

Up to now the model presented in [6] has revealed fully compatible with observational data. Even if simple exponential potentials for quintessence do not appear as the preferred ones in the literature, not giving the most known and used results, they anyway still seem to be able to describe some phenomena.

As a matter of fact, we have now also found that, concerning CMBR, the values in Table I are sensibly different from those in Table III.

This gives the hope that more precise measurements of the CMBR spectrum will be sufficient to discriminate among quintessence models. (For a discussion on this point see also [10].)

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