A General Solution to the Aircraft Trim Problem

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Trim defines conditions for both design and analysis based on aircraft models. In fact, we often define these *analysis points* more broadly than the conditions normally associated with *trim conditions* to facilitate that analysis or design. In simulations, these analysis points establish initial conditions comparable to flight conditions. Based on aerodynamic and propulsion systems models of an aircraft, trim analysis can be used to provide the data needed to define the operating envelope or the performance characteristics. Linear models are typically derived at trim points. Control systems are designed and evaluated at points defined by trim conditions. And these trim conditions provide us a starting point for comparing one model against another, one implementation of a model against another implementation of the same model, and the model to flight-derived data.

In this paper we define what we mean by trim, examine a variety of trim conditions that have proved useful and derive the equations defining those trim conditions. Finally we present a general approach to trim through constrained minimization of a cost function based on the nonlinear, six-degree-of freedom state equations coupled with the aerodynamic and propulsion system models. We provide an example of how a trim algorithm is used with a simulation by showing an example from JSBSim.

List of Symbols

$[\,\cdot\,]$	a generic 3×3 matrix.
$[\,(\tilde{\cdot})\,]_{\mathrm{F}}$	the anti-symmetric matrix that expresses the cross product by vector $(\vec{\cdot})$ in frame F, i.e. $\vec{\Omega} \times \vec{V}$ becomes $[\tilde{\Omega}]_{\mathrm{W}} \cdot \{V\}_{\mathrm{W}}$ in the wind frame (*tilde operator*).
$[M] \cdot \{c\}$	standard matrix product, row-by-column, of matrix $[M]$ times column matrix $\{c\}$.
$[C_{\mathrm{F}_2 \leftarrow \mathrm{F}_1}]$	transformation matrix from frame F_1 to frame F_2, i.e. $\{V\}_{\mathrm{F}_2} = [C_{\mathrm{F}_2 \leftarrow \mathrm{F}_1}] \cdot \{V\}_{\mathrm{F}_1}$.
$[I]_{\mathrm{B}}$	aircraft inertia tensor in the body-axis frame (mass·length^2).
$[I^r]_{\mathrm{B}}$	inertia tensor of a generic rotating subpart, e.g. engine rotor or propeller, in a reference frame with axes parallel to the main body axes and origin somewhere on the part's axis of rotation (mass·length^2).
δ_e, δ_a, δ_r	angular deflections of elevator, ailerons, and rudder (radians).
δ_T	throttle setting (mapped in the range $[0,1]$).
γ	flight path angle (radians).
$\boldsymbol{g}$, $\boldsymbol{f}$	vector valued functions representing implicit and explicit, respectively, aircraft state equations.
$\boldsymbol{u}$	n_c-dimensional vector of aircraft control inputs.

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