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A problem of emphasizing features of a surface roughness by means the Discrete Wavelet Transform

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Abstract

When we are interested to the detection of the roughness features by means of the 3D reconstruction, based on photometric stereo techniques, an important problem is the elimination of the brightness variation due to different light conditions which can alter the response.

This paper will concentrate on presenting results of a new method for eliminating this problem.

Every pixel of a picture gives only one number: the brightness of the corresponding point on the object, whereas the surface orientation is described by a normal vector that has two degrees of freedom. The level of brightness depends on many factors as well as the homogeneity of reflection properties of the material or its physical continuity and the surface smoothness or roughness.

In this work we will show how the application of the Discrete Wavelet Transform (DWT) to the processing of some images, captured on different light conditions, permits to solve the problem of emphasizing roughness features of a metallic surface. Wavelet transforms can model irregular data patterns such as sharp changes, better than the Fourier transforms and standard statistical procedures (e.g., parametric and non-parametric regressions) and provide a multiresolution approximation to the data.

Here we propose, also, a non-parametric method, based on the wavelet theory, for the estimation of the threshold level of a gray levels distribution, obtained from the intensity image matrix.

Keywords: Image processing; Surface roughness; Wavelet transform; Optimum threshold

1. Introduction

The study of criteria for evaluating the surface roughness represents, to day, one of the most important problem for the production of some critical mechanical organs in order to confer them some specific and functional characteristics. For that reason many authors consider the roughness as the fourth dimension of the design.

Since, in many cases, the roughness determines the level of brightness of a metallic surface, its reflectance can be studied by a specific model, based on some hypotheses and solving the image irradiance equation using some algorithms [1,2].

In this paper we will show a new method of image processing for emphasizing the features of a surface roughness, captured on different light conditions. It is based on the properties of the Wavelet Transform to detect the presence of details which, usually, are lost when we apply to the signal a filtering process in order to reduce the noise.

The Wavelet Transform gives a time-frequency representation of a signal that has two main advantages over usual methods: an optimal resolution even in the time and frequency domains and lack of the requirements of stationarity of the signal. It is defined as the convolution between the signal $x(t)$ and the wavelet functions $\psi_{a,b}(t)$which are dilated or contracted and shifted versions of a unique wavelet function $\psi(t)$.

Contracted versions of the wavelet function will match the high frequency components of the original signal and on the other hand, the dilated versions will match low frequency oscillations. Then, by correlating the original signal with wavelet functions of different sizes, we can obtain the details of the signal at different scales. These correlations with the different wavelet functions can be arranged in a hierarchical scheme called multiresolution decomposition [3].

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The multiresolution decomposition separates the signal into “details” at different scales, the remaining part, being a coarser representation of the signal, is called “approximation”. Moreover, it was shown [3] that each detail (D_j) and approximation signal (A_j) can be obtained from the previous approximation A_{j-1} via a convolution with high-pass and low-pass filters, respectively.

When wavelets are used to encode two-dimensional signals (i.e., pictures or images), often this is done by using “separable products” of a one-dimensional wavelet and a one-dimensional scaling function. This makes it possible to use the Wavelet Transform and in this case each new wavelet measures variations in the image along three different directions: horizontal, vertical and diagonal edges. The Discrete Wavelet Transform (DWT) of images is then calculated essentially by applying the one-dimensional Wavelet Transform along the rows and the columns of the image [4–6]. Therefore, an algorithm similar to the one-dimensional case is possible for two-dimensional wavelets and scaling functions obtained from one-dimensional ones by tensor product [3,7,8].

In mathematics, a singularity is a point at which a function is not differentiable although it is differentiable in a neighbourhood of that point. Singularities record large signal changes over very short time changes. Crests of roughness may be though of as a smoother version of singularities where large signal changes occur over slightly broader time changes.

A theory for examining the singularities of functions using the Wavelet Transform was developed [9], which has been applied here to identify and characterise the crests that make up the surface roughness of a metallic test piece on different light conditions.

2. A threshold estimator based on Haar Wavelet

We follow an approach similar to [10].

Let $X_1, X_2, \ldots, X_N$ be N independent identically distributed random variables whose density (unknown) is $f(x)$. In this case, a wavelet estimator of f is given by [11,12]

$$\hat{f} = \sum_k \hat{a}_{j_0}\varphi_{j_0k}(x) + \sum_{j=j_0}^{j_1}\sum_k \hat{b}_{jk}\psi_{jk}(x) \quad (1)$$

where $j_0, j_1 \in \mathbf{Z}$

$$\hat{a}_{j_0k} = \frac{1}{N}\sum_{i=1}^{N}\varphi_{j_0k}(X_i), \qquad \hat{b}_{jk} = \frac{1}{N}\sum_{i=1}^{N}\psi_{jk}(X_i)$$

$$\varphi_{jk}(x) = 2^{j/2}\varphi\left(2^j xk\right), \quad k \in \mathbf{Z}$$

and

$$\psi_{jk}(x) = 2^{j/2}\psi(2^j xk), \quad k \in \mathbf{Z}$$

Finally, $\psi(x)$ is a function (mother wavelet) whose first $h(h \in \mathbf{N})$ moments are zero and $\varphi(x)$ is a function (father wavelet) orthonormal to $\psi(x)$, according to the L^2 norm. In our work we choose:

$$\varphi(x) = \begin{cases} 1, x \in (0, 1] \\ 0, x \notin (0, 1] \end{cases} \quad \text{and} \quad \psi(x) = \begin{cases} -1, x \in \left[0, \frac{1}{2}\right] \\ 1, x \in \left(\frac{1}{2}, 1\right] \\ 0, x \notin [0, 1] \end{cases}$$

They are called Haar Wavelets. It is easy to prove that, if $\psi(x)$ is a mother wavelet, then also $\psi_{j,k}$ is a mother wavelet.

Moreover, in this case, the systems of functions

$$\{\{\varphi_{j_0k}\}, \{\psi_{jk}\}, k \in \mathbf{Z}, j \in [j_0, j_1] \cap \mathbf{Z}\}$$

is an orthonormal system in $L^2(\mathbf{R})$. The definition of the estimator (1) is based on the Parseval Theorem. In fact, according to this result, any $L^2(\mathbf{R})$ can be represented as a convergent series

$$h(x) = \sum_k a_{j_0k}\varphi_{j_0k}(x) + \sum_{j=j_0}^{j_1}\sum_k b_{jk}\psi_{jk}(x) \quad (2)$$

where

$$a_{j_0k} = \int_{t=-\infty}^{+\infty} h(t)\varphi_{j_0k}(t)\,\mathrm{d}t \quad \text{and}$$

$$b_{jk} = \int_{t=-\infty}^{+\infty} h(t)\psi_{jk}(t)\,\mathrm{d}t$$

Now assume that φ and ψ are compactly supported. Then, following [11], by utilising the relation below

$$\sum_k \varphi_{jk}(x)\cdot\varphi_{jk}(X_i) + \sum_k \psi_{jk}(x)\cdot(X_i) = \sum_k \varphi_{j+1k}(x)\cdot\varphi_{j+1k}(X_i)$$

it follows that the estimator (1) can be written in the equivalent form

$$\hat{f}_{j_1}(x) = \frac{1}{N}\sum_{i=1}^{N}\sum_k \varphi_{j_1+1k}(x)\varphi_{j_1+1k}(X_i) \quad (1')$$

The choice of j_1 is discussed in statistic literature. The choice of j_1 is made in order to minimize the mean integrated squared error

$$\text{MISE} = E||f - \hat{f}_{j_1}||_2^2 = E||\hat{f}_{j_1} - E(\hat{f}_{j_1})||_2^2 + E||f - E(\hat{f}_{j_1})||_2^2$$

where $E(\cdot)$ is the expected value of the observations and $||\cdot||_2$ denotes the usual L^2 norm. Note that

$$E(\hat{f}_{j_1}(x)) = \int_{y=-\infty}^{+\infty} \varphi_{j_1+1k}(x)\varphi_{j_1+1k}(y)f(y)\,\mathrm{d}y$$

Now, let

$$q(x, y) = \sum_{k=-\infty}^{+\infty} \phi(x-k)\phi(y-k)$$

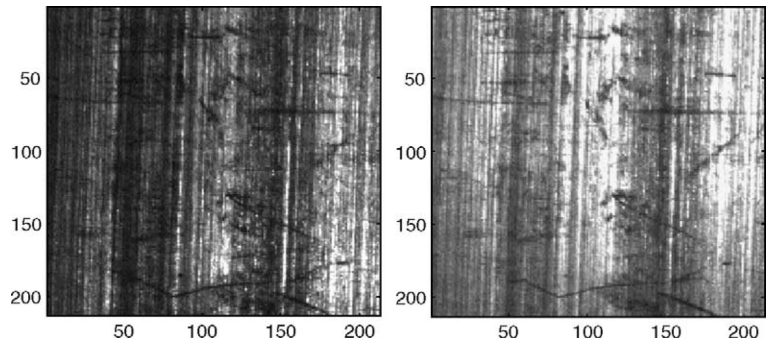

Fig. 1. Image of surface roughness of the same metallic test piece obtained on different light conditions.

and $q(x, y) = 2^n q(2^n x, 2^n y) < 2^n$. It follows that

$$E\|\hat{f} - E(\hat{f}_{j1})\|_2^2 \leq \frac{1}{N}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} q_{j_{1+1}}^2(y, x) f(y)\,\mathrm{d}x\,\mathrm{d}y$$

$$= \int_{-\infty}^{+\infty} q_{j_{1+1}}(y, y) f(y)\,\mathrm{d}y \leq \frac{2^{j_{1+1}}}{N}$$

Now, let W_n be the space spanned by the orthonormal basis $\{2^{n/2}\psi(2^{n/2}x - k),\ k \in \mathbf{Z}\}$. Furthermore, let V_0 be the space spanned by the orthonormal basis $\{\psi(x - k),\ k \in \mathbf{Z}\}$ and let $V_n = V_{n-1} + W_{n-1}$ ($i \in \mathbf{N}$). By construction, $\cup_{n \in \mathbf{N}_0} V_n$ is dense in $L^2(\mathbf{R})$.

Since $E(f_{j_1})$ is the projection of f on the space V_{j_1}, we can deduce that $E\|f - E(\hat{f}_{j_1})\|_2^2$ converges to zero as j_1 tends to infinity, for any $f \in L^2(\mathbf{R})$.

An alternative measure of error is the mean square error

$$\mathrm{MSQ} = E[f(x) - \hat{f}_{j_1}(x)]^2$$
$$= E[E(\hat{f}_{j_1}(x) - \hat{f}_{j_1}(x)]^2 + E[f(x) - E\hat{f}_{j_1}(x)]^2$$

However, in a regression estimation (i.e., when $X_i - X_{i-1} = 1/N = \text{const.}$) it is accepted that $j_1 = \log_2 N - \ln(\log_2 N)$. Therefore, in our work we have chosen

$$j_1 = \log_2\left(\frac{1}{\mathrm{median}(x_i - x_{i-1})}\right) - \ln\left(\log_2\left(\frac{1}{\mathrm{median}(x_i - x_{i-1})}\right)\right) \quad (3)$$

where

$$x_i = \frac{X_i}{\sqrt{\sum_{i=1}^{N} X_i}}$$

The determination of an optimal threshold is a compromise of many factors.

The $\hat{f}(x)$ can be calculated by utilising (1) or (1′). Then, for any $i \in \{1, 2, \ldots, N\}$, we evaluate the $\hat{F}(X_i)$ by numerical quadrature of $\hat{f}(x)$ by setting $\int_0^{X_i} \hat{f}(x)\,\mathrm{d}x := \hat{F}(X_i)$. Now suppose that $\{X_i\}$ are sorted in ascending order. Therefore, for any $u \in [X_1, X_N]$, we can estimate $E(X - u, X > u)$ by the position

$$\int_u^s \hat{f}(x)(x - u)\,\mathrm{d}x =: \hat{E}(X - u, X > u)$$

where s is given by $s = \sup\{x: F(x) < 1\}$.

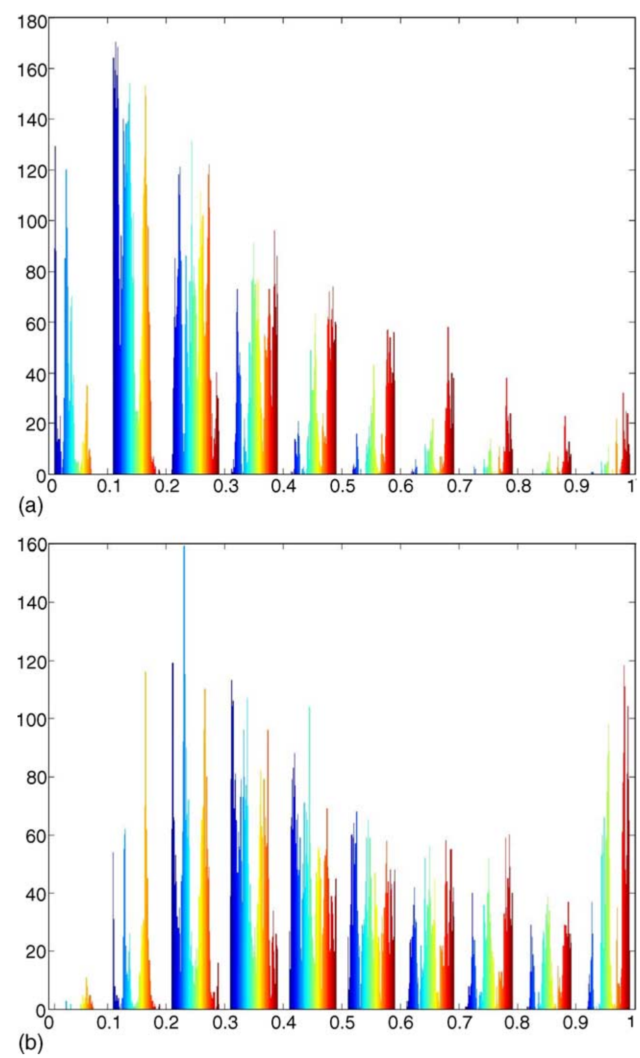

Fig. 2. (a) Gray levels histogram referred to the first image (left). (b) Gray levels histogram referred to the second image (right).

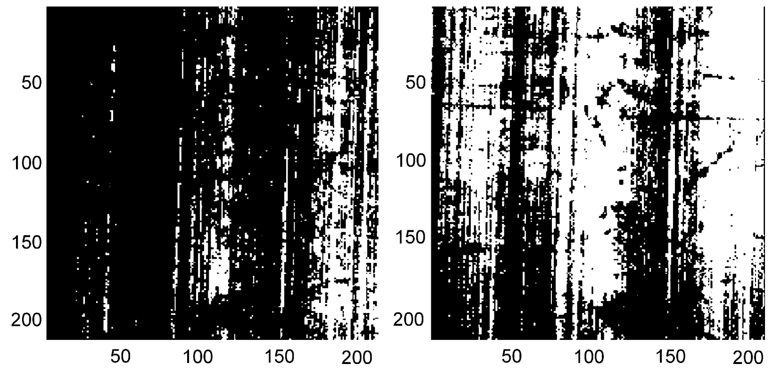

Fig. 3. An example of binary images obtained by applying usual thresholding method.

Finally, we evaluate $E(X-u|X>u)$ by setting

$$\hat{E}(X-u)|X>u):=\frac{\hat{E}(X-u,X>u)}{1-\hat{F}(x)}$$

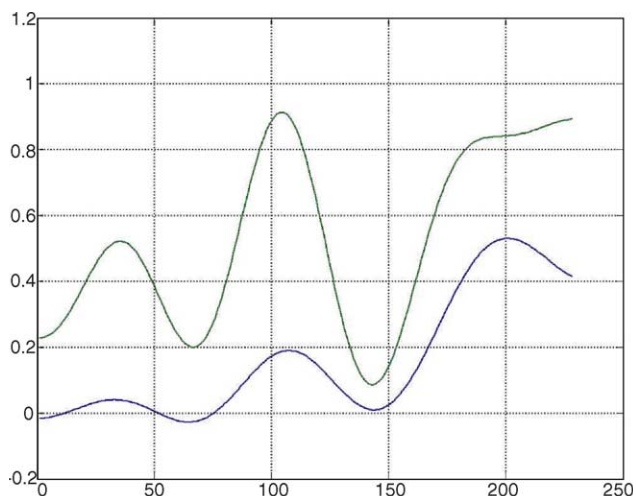

Fig. 4. Grand average of the approximation coefficients calculated on images of Fig. 3.

3. Application of DWT and results

Fig. 1 shows the images of the same metallic test piece captured on different light conditions. It is evident the difference of brightness caused by the position of the lamp. The images are converted into intensity matrix containing the values in the range 0 (black) to 1 (full intensity or white) (Fig. 2a and b).

The evidence of prevalence of darkness on the first image (left) with respect to the second one (right) is illustrated by comparing their histograms bar plot of gray levels.

The importance of converting the intensity image to binary image by an optimum thresholding is illustrated in Fig. 3 obtained by applying to the gray levels histogram the usual method [13–15]. The output binary image has values of 0 (black) for all pixels in the input image with luminance less than threshold parameter and 1 (white) for all other pixels. Note that we specify the threshold in the range [0,1], regardless of the class of the input image.

Since exists a large difference in brightness level for the same image it would be difficult to base a quantitative measure of surface roughness on such data by means of the 3D reconstruction by photometric stereo techniques.

We have verified, also, that the threshold obtained by applying the DWT to the gray levels histogram provides the best response in terms of emphasizing the surface roughness features.

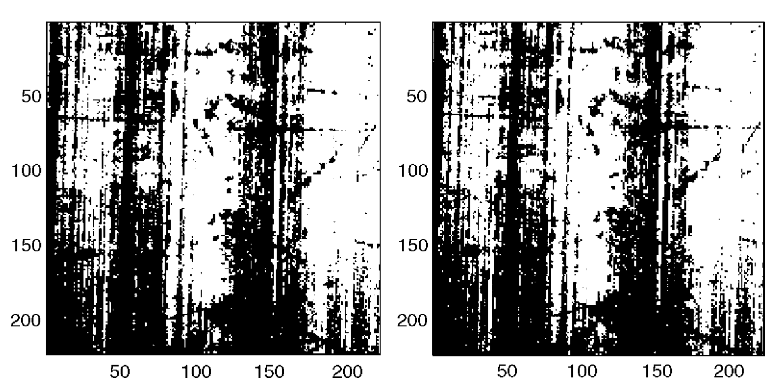

Fig. 5. An example of the better performance obtained with DWT.

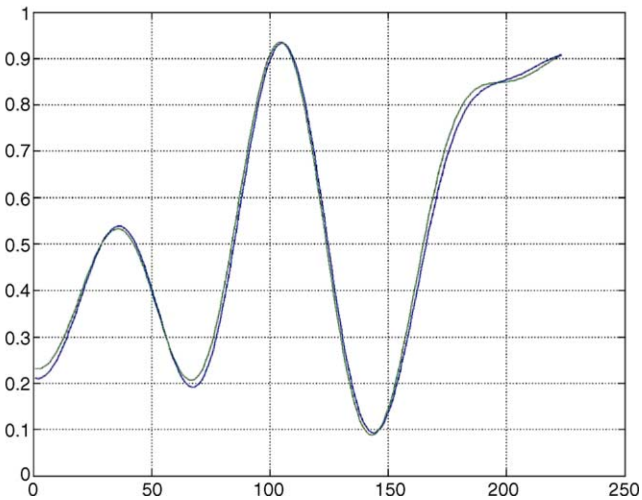

Fig. 6. Grand average of the approximation coefficients calculated on images of Fig. 5.

The output of the wavelet transform is a set of coefficients which give a measure of how the properties of the chosen wavelet "fit" the function at the particular translation and scale.

The results by applying the DWT to the original images are presented in Fig. 4, in which are presented the mean distribution of the approximation wavelet coefficients.

Note that the shape of the curves, in terms of qualitative response, is quite similar, while, in terms of entropy, a common concept in many fields, mainly in signal processing [16], is more different: 24.0990 and 54.9523 respectively for the first (left) and the second (right) image.

The improvement of the response obtained by choosing a good thresholding level is illustrated in Figs. 5 and 6, where the value of the entropy is of 48.5104 and 48.3762 respectively for the first (left) and the second (right) image.

The data presentation shown by Fig. 6 appears to be the best way of displaying the ability shown by DWT for eliminating the error due to brightness influence.

4. Conclusion

As a general remark we can state that with wavelets a better resolution and localization of the features of the image is achieved.

There are many wavelet packet libraries. They differ from each other by their generating filters φ and ψ, the shape of the basic waveforms and the frequency content. It was important for our investigation to have refined frequency resolution. Therefore, we have chosen the wavelet packets derived from the splines of the eighth order.

It was discovered that the brightness of a metallic surface produces disturbances of two types. The first type consists of the high-frequency oscillations which determine the main features of the processed image. The second type consists of relatively low-frequency oscillations which are associated with the energy content of the image as well as his entropy. Both the information are useful for detecting the resolution in terms of denoising and enhancement of the image.

Future research shall identify correlations between individual features across a given sample set. This will allow a more selective averaging to take place with the effect of reducing the required number of trials necessary to obtain the features of a surface roughness. By understanding the underlying mechanism that generate the brightness variation and reducing the number of trials necessary to calculate it, the overall objective is to develop a method to extract the features from a single trial.

The preliminary results demonstrate that the signals, obtained from the images, can be decomposed into a finite set of distinct crests without any loss of overall information, when averaged across all signals. The crest detection method has generated a set of peaks that are characterised by position, scale and amplitude. This allows them to be analysed in more refined ways than just the usual methods and, of course, it assumes that the method used in this paper represents the first step in order to eliminate the error due to the brightness influence to measure the surface roughness.

In this manner the progress of metal machining, grinding and polishing operations can be monitored, in real time, until the desired surface finish is achieved.

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