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Three-way decomposition of weighted log-odds ratio for customer satisfaction analysis

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Abstract

In literature several methods have been proposed for the service quality assessment. A large number of models have been proposed to evaluate Service Quality (Servqual, Normed Quality, Servperf etc.). Starting from the SERVPERF paradigm, in this paper we propose to use Odds Ratio analysis to evaluate Customer Satisfaction. In particular the data has been collected in three-way contingency tables in which the crossed variables are perception evaluations, importance evaluations and dimensions. For each slice we computed the Odds Ratio. Thus a weighted version of log-Odds Ratio Analysis for three-way is proposed and analyzed by the Parafac/Candecomp algorithm. A case study on Patient Satisfaction (PS) survey that was carried out at a Neapolitan government hospital is presented in the last part of the paper in order to show the proposed methods.

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Keywords: Odds ratio, Three-way contingency table, Parafac/Candecomp method, Customer Satisfaction

1. Introduction

During the last twenty years, the strategy of firms has gradually shifted from marketing to Total Quality Management to Customer Satisfaction (CS). Particularly, for a company, the knowledge of the customer evaluation

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of a given service represents an important starting point for every business strategy. In fact CS matters not only to the customer, but even more so to the business because it directly impacts a company's bottom line profits. Furthermore, it is one of the most important components of a company's positive brand image. In literature several methods have been proposed for the service quality assessment and many of them are based on the Gap Theory of Service Quality, which was proposed for the for-profit sector by Parasuraman et al. (1994). Cronin and Taylor (1992) were the first to offer a theoretical justification for discarding the expectations from the Servqual and consider only the performance, and this model is known as Servperf. The Servperf model considers twenty-two items and five quality dimensions, and such dimensions are: (1) the reliability of the service provider, (2) the responsiveness of the service provider, (3) the tangible aspects of the service, (4) the assurance provided by the service staff, and (5) the empathy shown to consumers.

Starting from the Servperf paradigm, in this paper we propose to use Odds Ratio analysis to evaluate CS. In particular the data has been collected in three way contingency tables in which the crossed variables are the evaluation of the perception and importance for each dimension. Thus, Odds Ratio of perception and importance for each dimension are arranged by rows and columns in IxJ slices of a three-way table $IxJxK$.

The odds ratio (OR) is one of the main measures of association in $2x2$ contingency tables. Also for IxJ tables the ORs are commonly used to describe the relationship between the row and column variables. Starting from the selected data table, several OR methods have been proposed. For example, Aitchison (1990) and Greenacre (2009) proposed to analyze contingency tables. On the other hand, De Roiij and Anderson (2007) proposed to analyze two-way tables with the ORs, which has a total number of ORs equivalent to $[I(I-1)/2]x[J(J-1)/2$. Nevertheless the number of ORs needed to capture the association structure may still be too large for one to gain insight into the nature of the relationship between the variables. Also a general framework for connecting all these methods has been proposed by D'Ambra and al. (2013).

In the statistical literature, the analysis of association for variables placed in IxJ two-way contingency tables is a topic widely discussed. On the other hand, the analysis of the association in a three-way contingency table by ORs is rather limited. Several variants of the Parafac/Candecomp method (CP - Harshman, 1970; Caroll and Chang, 1970) has been proposed for the ORs by De Rooij and Anderson (2007). Following this approach, we focused our attention on the OR as association measure (Agresti and Coullb, 2002) and proposed to use a weighted log-odds ratio. Finally, to show that this new approach give richer results a full interpretation of a case study is presented in the last part of the paper.

2. The association in a two-way contingency table through Odds Ratio

Let $\mathbf{N} = (\boldsymbol{n_{ij}})$ be a two-way contingency table that cross-classifies n units according to I row categories and J column categories of two crossed variables. Let X_i and Y_j be the i-th and j-th categories of X and Y. The matrix of proportions is denoted by $\mathbf{P} = \boldsymbol{n}^{-1}\mathbf{N}$ with general term p_{ij}. The marginal relative frequencies of the i-th row and j-th column of $\mathbf{P}$ are $p_{i\circ}$ and $p_{\circ j}$ and they may be represented in vector form, particularly the vector **r** (resp. **c**) consists of $p_{i\circ}$ (resp. $p_{\circ j}$), for $i = 1, 2, \dots, I$ (resp. $j = 1, 2, \dots, J$).
Let $OR_{ii'jj'} = n_{ij}\, n_{i'j'}$ $(1 \le i \le i' \le I, 1 \le j \le j' \le J)$ be the *OR*, the complete set of *OR*s for table **N** can be placed in a two-way table, called $\mathbf{S} = [\boldsymbol{s}_{\tilde{\imath}\tilde{\jmath}}]$, of dimension $\tilde{I}\ x\ \tilde{J}$, where $\tilde{I} = I(I-1)/2$ and $\tilde{J} = J(J-1)/2$.
Starting from the data tables **N** and **S** two different statistical methods have been developed. These methods are linked between them and with Altham association measure. Moreover for improving the performance a weighing system can be considered. The main characteristics of these methods are summarized in table 1.
The first is the unweighted Log Ratio Analysis (LRA), which is proposed by Aitchison (1990). It starts from the logarithms of the matrix **N**, called $L(\mathbf{N})$, and 0.5 is added when an element of **N** is equal to 0. Then, let **1** be a vector of ones of appropriate order in each case, an SVD of the following matrix is performed

$$\mathbf{Z}^U = \mathbf{D}_r^{1/2}(\mathbf{I} - \mathbf{1}\mathbf{1}^t/I)L(\mathbf{N})(\mathbf{I} - \mathbf{1}\mathbf{1}^t/J)\mathbf{D}_c^{1/2} \quad (1)$$

$\mathbf{Z}^U$ is a double-centered matrix respect to the geometric mean and with uniform weights.

Let $L(\mathbf{S})$ be the logarithm transformation of $\mathbf{S}$, in this table the rows (resp. columns) are formed by all pairs of categories of X (resp. Y). Performing an Uncentered Generalized Principal Component Analysis (UGPCA) (Cadima and Jolliffe, 2009) of $L(\mathbf{S})$ with weight matrices $\mathbf{B}$ and $\mathbf{D}$, we obtain a factorial representation in which pairs of categories of X and Y are drawn.
The unweighted LRA is linked with UGPCA, in fact the sum of squares of $\mathbf{Z}^U$ (inertia) is equal to the trace of the matrix $\mathbf{B}^{1/2}L(\mathbf{S})\,\mathbf{D}L(\mathbf{S}^t)\mathbf{B}^{1/2}$ and, consequently, it is joined with the association measure proposed by Altham (1970). This model can be improved through the introduction of a set of marginal weights for the rows and the columns. Given N_{ij}^* independent random variables, with a Poisson distribution and parameter τ_{ij}, then $E(N_{ij}^*) = Var(N_{ij}^*) = \tau_{ij}$. When τ_{ij} has a very large value it is preferred to consider the Poisson random variable $L_{ij} = log(N_{ij}^*)$ with parameters $E(L_{ij}) \cong log(\tau_{ij})$ and $Var(L_{ij}) \cong log(1/\tau_{ij})$, so it is appropriate to apply the logarithm transformation. Moreover, under the independence hypothesis, τ_{ij} can be estimated by $np_{i\circ}p_{\circ j}$, which justifies the weighting system based on row and column marginal totals of $\mathbf{P}$. For these reasons the model proposed is

$$\mathbf{Z}^U = \mathbf{D}_r^{1/2}(\mathbf{I} - \mathbf{1}\mathbf{1}^t/I)L(\mathbf{N})(\mathbf{I} - \mathbf{1}\mathbf{1}^t/J)\mathbf{D}_c^{1/2} = \mathbf{D}_r^{1/2}(\mathbf{I} - \mathbf{1}\boldsymbol{r}^t)L(\mathbf{N})(\mathbf{I} - \mathbf{1}\boldsymbol{c}^t)\mathbf{D}_c^{1/2} = \mathbf{D}_r^{1/2}\mathbf{A}\mathbf{D}_c^{1/2} \qquad (2)$$

where $\mathbf{Z}$ is a double-centered with respect to $p_{i\circ}$ and $p_{\circ j}$, and the matrix $\mathbf{A}$ is the same used for estimating the bilinear part in the RC(M) association model (Goodman, 1985), when the least square method is used for parameter estimations (D'Ambra, 1988). Moreover, the SVD of $\mathbf{Z}$ gives the weighted LRA proposed by Greenacre (2009): $\mathbf{Z} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{V}^t = \sum_{m=1}^{M} \mathbf{u}_m \lambda_m \mathbf{v}_m^t$, where M is rank of $\mathbf{Z}$, $\boldsymbol{\Lambda}$ is a diagonal matrix with singular value λ_m, $\mathbf{u}_m$ and $\mathbf{v}_m$ are the m-th column of $\mathbf{U}$ and $\mathbf{V}$, respectively.

Table 1.The methods for association in a two-way contingency table through Odds Ratio

The association between the categories of X and Y variables through the analysis of the table $L(N)$		
Version	Unweighted (Atchinson, 1990)	Weighted (Greenacre, 2007; D'Ambra et al. 2013)
Matrix	$\mathbf{Z}^U = \mathbf{D}_r^{1/2}(\mathbf{I} - \mathbf{1}\mathbf{1}^t/I)L(\mathbf{N})(\mathbf{I} - \mathbf{1}\mathbf{1}^t/J)\mathbf{D}_c^{1/2}$	$\mathbf{Z} = \mathbf{D}_r^{1/2}(\mathbf{I} - \mathbf{1}\boldsymbol{r}^t)L(\mathbf{N})(\mathbf{I} - \mathbf{1}\boldsymbol{c}^t)\mathbf{D}_c^{1/2} = \mathbf{D}_r^{1/2}\mathbf{A}\mathbf{D}_c^{1/2}$
Weighting System	$\mathbf{D}_r = (1/I)\mathbf{I}$ $\mathbf{D}_c = (1/J)\mathbf{I}$	$\mathbf{D}_r$ diagonal matrix with general term $p_{i\circ}$ $\mathbf{D}_c$ diagonal matrix with general term $p_{\circ j}$
Standard Coordinates	$\mathbf{F} = \mathbf{D}_r^{-1/2}\mathbf{U}\boldsymbol{\Lambda}$ $\mathbf{G} = \mathbf{D}_c^{-1/2}\mathbf{V}\boldsymbol{\Lambda}$	$\mathbf{F} = \mathbf{D}_r^{-1/2}\mathbf{U}\boldsymbol{\Lambda}$ $\mathbf{G} = \mathbf{D}_c^{-1/2}\mathbf{V}\boldsymbol{\Lambda}$
Principal Coordinates	$\tilde{\mathbf{F}} = \mathbf{D}_r^{-1/2}\mathbf{U}$ $\tilde{\mathbf{G}} = \mathbf{D}_c^{-1/2}\mathbf{V}$	$\tilde{\mathbf{F}} = \mathbf{D}_r^{-1/2}\mathbf{U}$ $\tilde{\mathbf{G}} = \mathbf{D}_c^{-1/2}\mathbf{V}$
Factorial representation of OR	Not available	$OR_{ii'jj'} = exp\left(\sum_{m=1}^{M} \lambda_m(\tilde{f}_{im} - \tilde{f}_{i'm})(\tilde{g}_{jm} - \tilde{g}_{j'm})\right)$ $OR_{ii'jj'} = exp\begin{pmatrix} \frac{1}{2}d^2(\mathbf{f}_{i'},\mathbf{g}_j) + \frac{1}{2}d^2(\mathbf{f}_i,\mathbf{g}_{j'}) + \\ -\frac{1}{2}d^2(\mathbf{f}_i,\mathbf{g}_j) - \frac{1}{2}d^2(\mathbf{f}_{i'},\mathbf{g}_{j'}) \end{pmatrix}$
Association measure	The inertia of $\mathbf{Z}^U$ is equal to Altham measure.	The inertia of $\mathbf{Z}$ is equal to weighted Altham measure
$L(\mathbf{S})$ two-way table of dimension $\tilde{I}\ x\ \tilde{J}$ containing the complete set of log ORs.		
	Un-weighted version (De Rooij and Anderson, 2007)	Weighted version (D'Ambra et al., 2013)
Matrix	$\mathbf{B}^{1/2}L(\mathbf{S})\mathbf{D}L(\mathbf{S}^t)\mathbf{B}^{1/2}$	$\mathbf{B}^{1/2}L(\mathbf{S})\mathbf{D}L(\mathbf{S}^t)\mathbf{B}^{1/2}$
Weighted system	$\mathbf{B} = (1/I)\mathbf{I}$ $\mathbf{D} = (1/\tilde{J})\mathbf{I}$	$\tilde{\mathbf{B}}$ diagonal matrices of dimensions $\tilde{I}$ with general term $p_{i\circ}p_{i'\circ}$ $\tilde{\mathbf{D}}$ diagonal matrices of dimensions $\tilde{J}$ with general term $p_{\circ j}p_{\circ j'}$.
Association measure	The UGPCA is linked with Altham's measure for $Q = 2$, in fact: $tr(\mathbf{B}^{1/2}L(\mathbf{S})\mathbf{D}L(\mathbf{S}^t)\mathbf{B}^{1/2}) = \sum_{\tilde{\imath}}\sum_{\tilde{\jmath}}[log s_{\tilde{\imath}\tilde{\jmath}}]^2/\tilde{I}\tilde{J}$	In this case we show that: $tr(\tilde{\mathbf{B}}^{1/2}L(\mathbf{S})\tilde{\mathbf{D}}L(\mathbf{S}^t)\tilde{\mathbf{B}}^{1/2}) = \sum\sum_{i<i'}\sum\sum_{j<j'} p_{i\circ}p_{i'\circ}p_{\circ j}p_{\circ j'}[logOR_{ii'jj'}]^2$ WUGPCA decomposes a synthetic measure of the log ORs. It could be a weighted version of Altham's measure

In the same way the weighted system can be used for the analysis of $L(\mathbf{S})$. It is possible to show that performing the UGPCA of $L(\mathbf{S})$ with weights $\widetilde{\mathbf{B}}$ and $\widetilde{\mathbf{D}}$ gives the weighted analysis of the log *OR* matrix (WUGPCA). In this case the trace of $\widetilde{\mathbf{B}}^{1/2}L(\mathbf{S})\widetilde{\mathbf{D}}L(\mathbf{S}^t)\widetilde{\mathbf{B}}^{1/2}$ is equal to $\sum_{i<i'}\sum\sum_{j<j'}\sum p_{i\circ}p_{i'\circ}p_{\circ j}p_{\circ j'}\left[logOR_{ii'jj'}\right]^2$. Therefore, the WUGPCA decomposes a synthetic measure of the log *OR*s, which is a weighted version of Altham's measure. The weighted LRA is linked with WUGPCA; in fact, the total variance of $\mathbf{Z}$ is equal to the trace of the matrix $\widetilde{\mathbf{B}}^{1/2}L(\mathbf{S})\widetilde{\mathbf{D}}L(\mathbf{S}^t)\widetilde{\mathbf{B}}^{1/2}$. Starting from weighted LRA, it is possible to have a direct and an indirect factorial representation of log-*OR*s as proposed by D'Ambra et al. (2013).

3. The association in a three-way contingency table through Odds Ratio

Let $L(\underline{\mathbf{S}})$ be the $\tilde{I}\ x\ \tilde{J}\ x\ K$ three-way contingency table that can be sliced so as to get the k frontal $\tilde{I}\ x\ \tilde{J}$ table $L(\mathbf{S})_k$. These slices can be concatenated between them, and consequently, one obtains the following matricizing of the three-way table as $L(\mathbf{S})_A$ of dimensions $\tilde{I}\ x\ \tilde{J}K$.

Defining $\mathbf{A}$, $\mathbf{H}$ and $\mathbf{C}$ of dimensions $\tilde{I}\ x\ F$, $\tilde{J}\ x\ F$ and $K\ x\ F$ respectively, the loadings for the first, second and third mode, the CP model for log-odds ratio in a slice-wise form can be written as:

$$L(\mathbf{S})_k = \mathbf{A}\mathbf{Q}_k\mathbf{H}^t + \mathbf{E}_k \tag{3}$$

where $\mathbf{Q}_k$ is a diagonal table containing the k-th row of $\mathbf{C}$, and $\mathbf{E}_k$ is the k-th frontal slice of residual three-way table. The loading matrices of the CP model are estimated by minimizing the objective function

$$\sum_{k=1}^{K}(L(\mathbf{S})_k - \mathbf{A}\mathbf{Q}_k\mathbf{H}^t)^2 \tag{4}$$

To take into account the weight structures of each frontal slice $\mathbf{\Delta}_k^l$ the objective function can be written as:

$$\sum_{k=1}^{K}\mathbf{B}_k^{1/2}(L(\mathbf{S})_k - \mathbf{A}\mathbf{Q}_k\mathbf{H}^t)^2\,\mathbf{D}_k^{1/2} \tag{5}$$

where $\mathbf{B}_k$ and $\mathbf{D}_k$ of dimensions $\tilde{I}\ x\ \tilde{I}$ and $\tilde{J}\ x\ \tilde{J}$ respectively, are diagonal tables with rows and columns weight of frontal slice $L(\mathbf{S})_k$.

The CP is only one of the models used to analyse multi tables, a more general approach is to analyse each frontal slice $L(\mathbf{S})_k$ by singular value decomposition (SVD)

$$L(\mathbf{S})_k = \mathbf{A}_k\mathbf{Q}_k\mathbf{H}_k^t + \mathbf{E}_k \tag{6}$$

where $\mathbf{Q}_k$ is the matrix of singular values, $\mathbf{A}_k$ and $\mathbf{H}_k$ are the left and right singular vectors. This approach has little to recommend it because the analysis of each frontal slice is independently related to that of another frontal slice. However, it is possible to observe that the equation (3) is given by the equation (6) with restrictions of loadings for the first and second mode, thus CP can be considered as a special case of SVD on each frontal slice. Moreover, restrictions can be imposed only on the left or right singular vectors, or on singular value, so several models can be considered.

It is well known that a CP model gives the best low-rank approximation of a three-way table in a least squares sense (Bro et al, 2001), but to investigate the structure of a three-way data set we consider the approach that assures the best mix of parsimony, interpretability and goodness to fit. Algorithms based on alternating least squares can be used to estimate the loading matrices of CP models (Smilde et al., 2005), however in order to fit models to multiple tables with various combinations of restriction on $\mathbf{A}_k$, $\mathbf{H}_k$ and $\mathbf{L}_k$ the algorithm proposed by De Rooij and Anderson (2007) is implemented in R 2.15.2.

4. Application and conclusions

This study concerns a Patient Satisfaction (PS) survey that was carried out at a Neapolitan government hospital. One thousand two hundred questionnaire forms were delivered to the patients during hospitalization. The study could help the hospital management to improve service quality. The questionnaires were delivered to the patients in the hospital in order to collect data from April 8th through April 30th, 2012; while the interviews were held from April 30th to May 7th, 2012. The questionnaire comprises in particular five attribute-importance measurements and five corresponding performance measurements, each defined on a seven-points Likert scale. PS studies quality attributes on two dimensions: their performance level (satisfaction) and their importance to the patient, and that importance is defined by the Servperf dimensions.

The nature of data should be considered before we carry out a multidimensional analysis. Particularly, perception evaluation and, in this case, also importance evaluation, both have an ordinal scale. This scale establishes an explicit rank, but not all arithmetic transformations are significant because the distances between points on an ordinal scale are not significant. Due to the non metric nature of this data, different approaches are proposed to quantify the ordinal data on a continuous scale. This transformation is necessary to perform quantitative multivariate analysis. In this research we overcome the problem of quantification by summarizing the collected data in a three-way contingency table (row, column and tube), specifically for each dimension (tube) we construct a two-way contingency table cross-classifying the importance (column) and performance (row) measurements.

In the analysis we classified the importance levels [I = 3; low (1); medium (2); high (3)] against the performance levels [J = 3; low (1); medium (2); high (3)] for each Servperf dimension [K = 5; 'Tangibles' (1), 'Reliability' (2), 'Responsiveness' (3), 'Assurance' (4), 'Empathy' (5)]. The question is whether the importance and perception levels are associated, and whether the association between importance and perception is different for all the dimensions. We hypothesized that the association between importance and performance measurements is a direct measure of scarce financial and human resources that the management of the hospital has in order to improve the patient satisfaction. Thus, the managment prioritize the aspects considered as the most important. The complete set of log-ORs for each dimension has been computed and the used 3x3x5 table, and it is shown as an unfolder table (table 2)

Table 2.complete set of log-ORs for each dimension

	Dimension 1 -Tangibles			*Dimension 2 -Reliability*			*Dimension 3 -Responsiveness*			*Dimension 4 - Assurance*			*Dimension 5 - Empathy*		
	W12	W13	W23	W12	W13	W23	W12	W13	W23	W12	W13	W23	W12	W13	W23
P12	-0,025	-0,718	-0,693	-0,280	0,442	0,722	-0,780	-0,336	0,443	1,609	1,587	-0,022	0,762	-0,128	-0,890
P13	-0,268	-0,208	0,061	-1,204	-0,057	1,147	0,705	0,000	-0,705	1,792	0,981	-0,811	-0,372	-0,022	0,349
P23	-0,243	0,511	0,754	-0,924	-0,499	0,425	1,485	0,336	-1,148	0,182	-0,606	-0,788	-1,134	0,105	1,240

The row (X) and column (Y) categories are P12, P13, P23 and W12, W13, W23 respectively; the tube categories are 1, 2, 3, 4 and 5. Since the variables importance and perception evaluation are ordinal, we analyzed the table of ORs for each slice.

Based on the loss values presented in table 3, the choice of model to apply is not clear-cut. Thus, we can point out that each weighted model is better than their unweighted version. If the loss values are plotted against the number of parameters, it is possible to show that the models with two dimensions have better of these with only one. Considering the goodness-of-fit of the model to data, parsimony, and interpretation, the models $\mathbf{X}\mathbf{D}_k\mathbf{Y}$ and $\mathbf{X}\mathbf{D}_k\mathbf{Y}_k$ stand out as good representations of the data.

Table 3.Loss values for each method

Model	Weighted	Unweighted
$\mathbf{X}_k\mathbf{D}_k\mathbf{Y}_k$	0.0000	0.0000
$\mathbf{X}\mathbf{D}_k\mathbf{Y}_k$	0.0392	0.9373
$\mathbf{X}_k\mathbf{D}_k\mathbf{Y}$	0.3016	2.9268
$\mathbf{X}\mathbf{D}_k\mathbf{Y}$	0.1253	0.9427
$\mathbf{X}_k\mathbf{D}\mathbf{Y}_k$	0.2665	2.7763
$\mathbf{X}\mathbf{D}\mathbf{Y}_k$	0.1628	1.5155
$\mathbf{X}_k\mathbf{D}\mathbf{Y}$	0.2051	2.2500
$\mathbf{X}\mathbf{D}\mathbf{Y}$	0.2810	2.9828

The interpretation of the first one is that there is a different association between perception and importance for each dimension, but the perception and importance levels have the same interpretation for all the dimensions. The interpretation of the second one is that the amount of association is different for each dimension, and the perception levels have different interpretations in the five dimensions. On the other hand, the importance levels have identical interpretations in each dimension. Here the first model will be examined in depth. It is possible to observe that the perception and importance categories have the same meaning for each of the five dimensions, although the amount of association is different (table 4).

Table 4: Measures of association for dimensions

Dimensions	Association
Tangibles	0,02740
Reliability	0,05378
Responsiveness	0,06216
Assurance	0,10100
Empathy	0,04717

In figure 1 one can see that for the ‘Assurance’ and ‘Responsiveness’ dimensions the amount of association is much bigger than that for the others three dimensions. A factorial plan for D_k has been performed in order to confirm this association structure. The two-dimensional plot reveals that for the ‘Assurance’ and ‘Responsiveness’ dimensions the association between the perception and importance levels is strong but with a different structure (i.e. for different pairs of modalities). On the other hand, for the ‘Tangibles’ and ‘Reliability’ dimensions the association is poor.

FIGURE 1 - HERE

The graphical display with XD_k versus Y and X versus YD_k are shown in figure 1a and 1b, respectively. These displays confirm that the association is strongest for the ‘Assurance’ and ‘Responsiveness’ dimensions since the vectors representing these slices are relatively long. In figure 1a, the vectors P12_4, P13_4, P23_3, P23_4, P13_3 and P12_3 belonging to the ‘Assurance’ and ‘Responsiveness’ dimensions than those of the other dimensions. Similarly, in figure 1b the longer vectors are W12_4, W13_4, W23_3, W23_4, W13_3 and W12_3. By contrast, the association between the ‘Reliability’ and ‘Tangibles’ dimensions is not very strong. Regarding the direction of the association we have that when the angle is smaller than 90^o a positive relationships exists, so there is not a main direction of the association. In other words, we have both positive and negative associations and both positive and negative log-ORs for each dimension. In figure 1a the vectors P12_4 and P12_3 have a negative association, which means that the order of corresponding ORs is diametrically opposite, therefore P12W12 > P12W13 >P12W23 for the ‘Assurance’ dimension and P12W12 < P12W13 <P12W23 for the ‘Responsiveness’ dimension.

FIGURE 2 - HERE

In figure 1b we point out that the pairs of vectors W23_3-W23_4, W12_4-W13_4 and W12_3-W13_4 have a positive association, i.e. the order of their corresponding ORs is the same. The opposite happens as for the pairs of vectors W12_4-W12_3 and W13_4-W13_3, where the association is negative.

In order to verify the relationship between the association and the log-ORs for each table we can project, for example, the points W12 and W13 onto the vector P12_4, and see that the point W12 projects higher, that is, it has a larger value of the log-OR (figure 1a). In the same way in figure 1b, we can project the points W12_3 and W13_3 onto the vector P23, and see that the point W12_3 projects higher, which means that it has a larger value of the log-OR.

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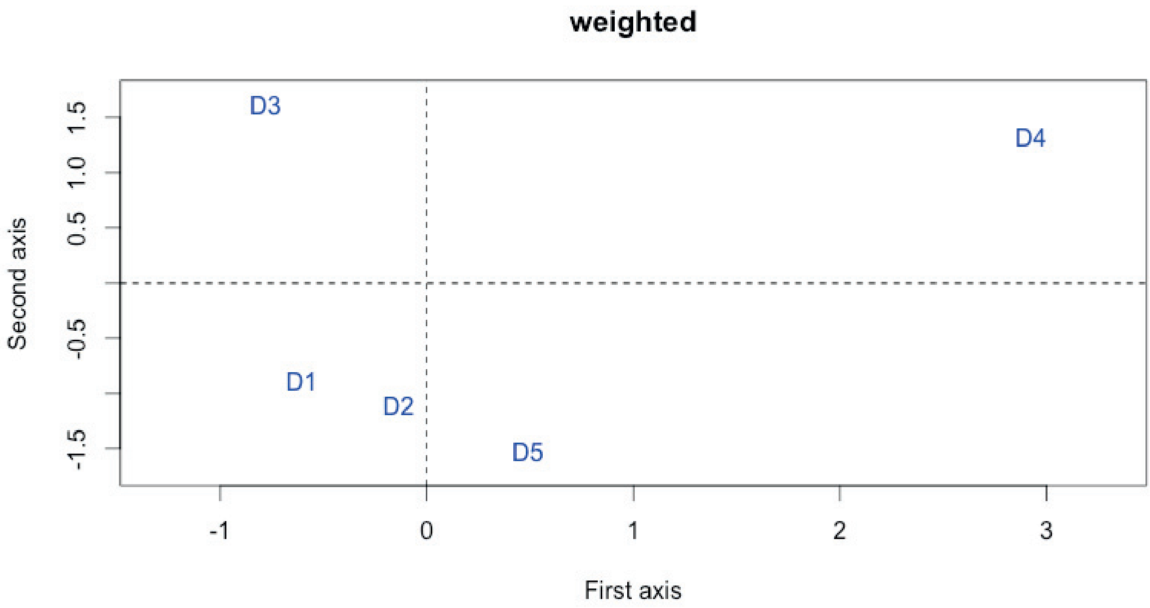

Fig. 1. Two dimensional plot of customer satisfaction dimensions: 'Tangibles' (D1), 'Reliability' (D2), 'Responsiveness' (D3), 'Assurance' (D4), 'Empathy' (D5).

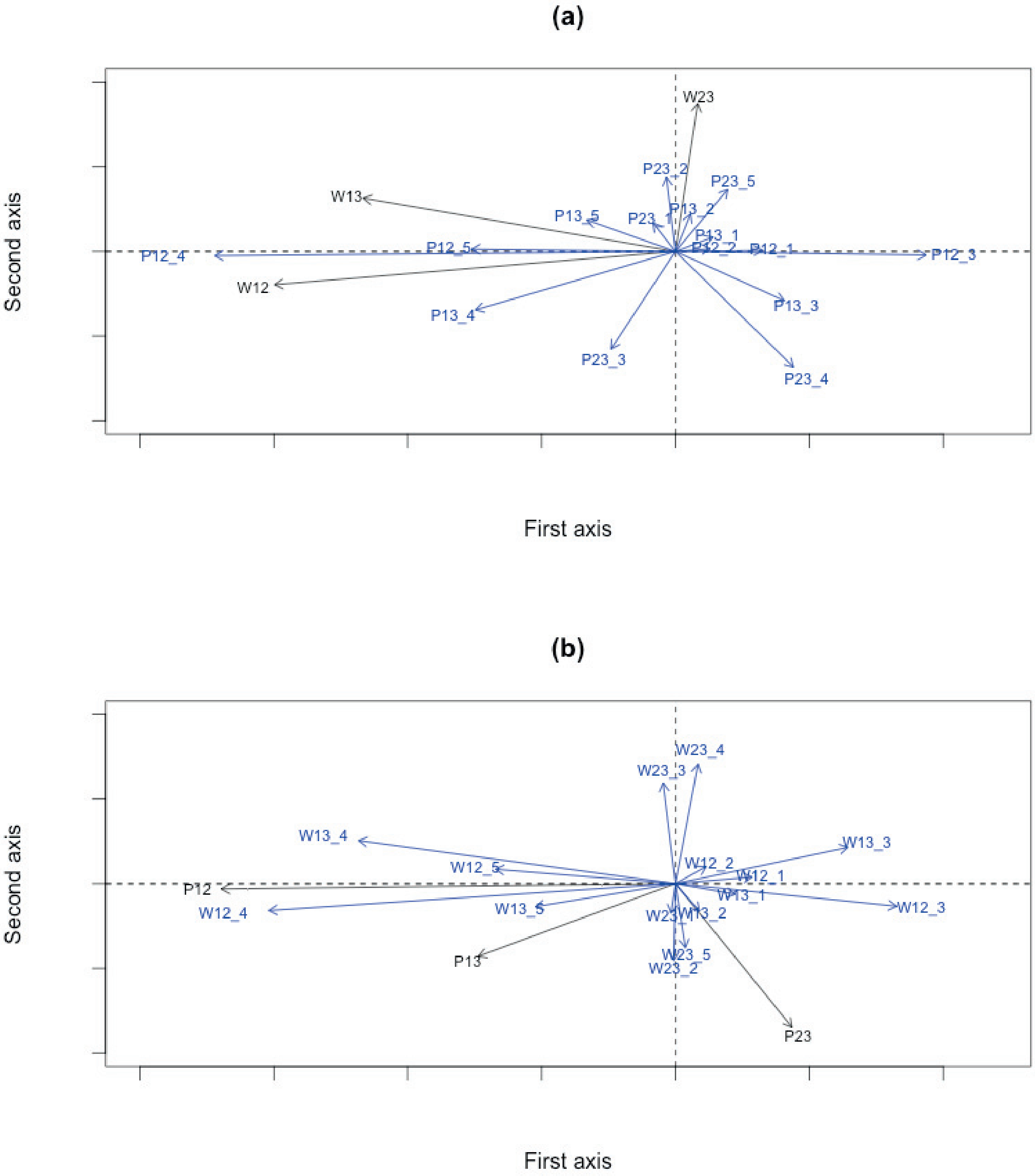

Fig. 2. (a) Two dimensional plot of perception for each dimension *vs.* fixed importance; (b) Two dimensional plot of importance for each dimension *vs.* fixed perception.