IOP Publishing European Journal of Physics
Eur. J. Phys. **35** (2014) 045025 (12pp) doi:10.1088/0143-0807/35/4/045025

Check the Lambert–Beer–Bouguer law: a simple trick to boost the confidence of students toward both exponential laws and the discrete approach to experimental physics

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Received 21 February 2014, revised 23 April 2014
Accepted for publication 12 May 2014
Published 11 June 2014

Abstract
Exponential decay is a prototypical functional behaviour for many physical phenomena, and therefore it deserves great attention in physics courses at an academic level. The absorption of the electromagnetic radiation that propagates in a dissipative medium provides an example of the decay of light intensity, as stated by the law of Lambert–Beer–Bourguer. We devised a very simple experiment to check this law. The experimental setup, its realization, and the data analysis of the experiment are definitely simple. Our main goal was to create an experiment that is accessible to all students, including those in their first year of academic courses and those with poorly equipped laboratories. As illustrated in this paper, our proposal allowed us to develop a deep discussion about some general mathematical and numerical features of exponential decay. Furthermore, the special setup of the absorbing medium (sliced in finite thickness slabs) and the experimental outcomes allow students to understand the transition from the discrete to the continuum approach in experimental physics.

Keywords: preparation of didactic experiences, measurements in optics, Lambert–Beer–Bouguer law, exponential decay

(Some figures may appear in colour only in the online journal)

1. Introduction

The Lambert–Beer–Bourguer (LBB) law describes the absorption of a beam of electromagnetic radiation (typically, visible light) which propagates through a dissipative medium, establishing a relationship between the intensity of the radiation and the optical path within the medium [1]. According to the original work of Mach [2], the first applications of the law appeared separately in 1760 in both the 'Traité d'optique' by Bourguer [3] and the 'Photometria' by Lambert [4]. The two are acknowledged as the founders of 'photometry,' while Beer's contribution dates back to late 1852 [5]. Briefly, the law correlates the radiant power in a beam of electromagnetic radiation (e.g. ordinary light) to both the length of the path of the beam in an absorbing medium and to its concentration [6].

Therefore, despite its age, the LBB law is a firm foundation of a methodology, and in some sense we can speak of it in terms of second-order linearity. The law remains a milestone both in experimental research (as shown from the large number of papers in international didactical [7], general physics [8], optics [9], surface science [10], chemistry [11], and biology [12] journals), and education (as we see from the many web pages in which examples of didactic experiments are discussed).

In its basic version, the law is formulated as follows:

Let x be the coordinate (geometrical length) along the propagation direction of a radiation beam, with I_0 being the intensity of the incident beam at the boundary of the medium. If we indicate the boundary of the medium with $x = 0$ (i.e. the medium occupies the region between $x = 0$ and $x > 0$), then I_0 is the intensity at $x = 0$ (the intensity by which the beam enters the medium). According to the LBB law, the intensity of the beam in a position x within the medium (i.e. in the interval $[0, \infty[$) is given by:

$$I(x) = I_0 e^{-\alpha x}. \tag{1}$$

In equation (1), α acts as the 'light absorption factor' per unit length of the medium; its physical dimensions are reciprocal of the length. In principle, the 'light absorption factor' ought to be described by an xyz tensor. However, if we restrict the analysis to an optically isotropic and homogeneous medium and use a beam made by plane waves, we are allowed to assume α as a constant.

Equation (1) states that inside a dissipative medium, the relationship between the intensity of light and the crossed thickness is simple exponential decay. Therefore, in the case of easier test conditions described above (plane waves with the wave front perpendicular to the propagation direction and a homogeneous and isotropic optical medium), the constant α can be easily estimated by measuring the incident light intensity and the emerging intensity if the thickness of the medium is known.

In this paper, we suggest a didactic experience to check the LBB law. We illustrate its usefulness in teaching as well as its didactical potential, not only for physical content, but also for some general considerations it suggests about the general features of an exponential function like equation (1), from the points of view of applied mathematics and the most appropriate procedure of experimental data collection.

2. Basic theoretical considerations

2.1. The classical derivation of the LBB law

It should be emphasized that the assumption of spatial isotropy of α does not imply its constancy if the wavelength of the incident radiation changes. Therefore, when using a simplified experimental setup as we propose, the estimation of the absorption coefficient of a medium should be performed by a monochromatic light source, which does not entail a loss of the didactical content.

To get equation (1) and to gain a proper comprehension of its conceptual bases, we assume that a portion of medium having infinitesimal thickness, dx, extends between the positions x and $x + \mathrm{d}x$. Let us also assume that the absorbed light is proportional to the thickness and to the incident intensity. We can write:

$$I\left(x + \mathrm{d}x\right) - I\left(x\right) \equiv \mathrm{d}I = -aI\left(x\right)\mathrm{d}x. \tag{2}$$

The minus takes into account that the intensity variation corresponds to the amount of absorbed intensity; we have a decrease of I. In equation (2), α represents the proportionality coefficient describing our assumption about the absorption. Equation (2) results in a differential equation and an associated Cauchy problem, whose solution is the LBB law:

$$\begin{cases} \dfrac{\mathrm{d}I}{\mathrm{d}x} = -\alpha I \\ I\left(0\right) = I_0 \end{cases} \Rightarrow \quad I\left(x\right) = I_0 e^{-\alpha x}.$$

The above derivation shows how the LBB law models the simplest mechanism of light absorption by an opaque dissipative medium. Namely, the relative intensity of absorbed light, dI/I, is proportional to the length of the geometrical path, dx. Based on these remarks, an experiment aimed to verify equation (1) can equivalently be considered a validation of these assumptions.

2.2. A discrete version of the derivation of the LBB law

An appealing alternative approach to the problem discussed in the previous section lies in its representation through a discretized process. We must pay attention to the discrete version of the derivation of the LBB law since it suggests an obvious experimental approach: The empirical verification of equation (1) needs the measures of the light intensity at a finite number of thicknesses.

Let us suppose that we split a bar of a dispersive medium in thin slices, with every slice of the same thickness, S. The output surface of the jth slice therefore has the position $j \cdot S$. Considering those slices as 'elementary portions,' $I_{\mathrm{in},j}$ and $I_{\mathrm{out},j}$ being the input and output light intensity of the jth slice, respectively, for this slice equation (2) reads

$$I_{\mathrm{out},j} - I_{\mathrm{in},j} \equiv \Delta I_j = -aI_{\mathrm{in},j}S, \tag{3}$$

which leads to

$$-\frac{\Delta I_j}{I_{\mathrm{in},j}S} = \alpha. \tag{4}$$

Equation (4) assures us that the relative intensity loss (i.e. absorption coefficient) per unit length does not depend on the particular slice. This result is also independent of the thickness, S, of the slices, and the hypothesis of equally thick slices might be weakened.

Let D be the length of the whole bar, and let us divide it into N slices. By the iterative application of equation (3), the final output intensity, I_D, results in

$$I_D \equiv I_{\text{out}.N} = I_{\text{in}.N}\left(1 - aS\right) = I_{\text{out}.N-1}\left(1 - aS\right) = I_{\text{in}.N-1}\left(1 - aS\right)^2 = \dots = I_{\text{in}1}\left(1 - aS\right)^N \equiv I_0\left(1 - aS\right)^N.$$

This expression restores the LBB law if we consider the limit toward a continuing partition of the bar (i.e. the limit of infinitely thin slices). To evaluate this limit, we observe that $S = D/N$, and the limit is given by $N \to \infty$. The final result is

$$\lim_{N\to\infty} I_0 \left(1 - \frac{\alpha D}{N}\right)^N = I_0 e^{-\alpha D},$$

which corresponds to equation (1) for $x = D$.

It is crucial to include the meaning of this limit from the experimental point of view when the discrete approach is used, meaning the experiment is actually performed through discrete steps. Note that we will not know the thickness of each slice or how many slices there are when the bar is split to accomplish the approach described in this section. Actually, these variables are not needed. Just observe that $N \to \infty$ implies $\alpha D/N \to 0$, which means, in an empirical sense made only by finite quantities, $\alpha D/N \ll 1$, or $\alpha S \ll 1$. We learned that an absolute, 'universal' thickness which identifies and defines the quasi-continuous regime cannot be acknowledged. Given a dissipative medium, we can assert that the thickness of a slice fulfils our requirements if the product between its thickness and its absorption coefficient, which is a dimensionless number, is much lower than 1.

In equations (3) or (4), this condition is equivalent to having a relative intensity loss in the slice much smaller than 1 ($\alpha S = \Delta I_j/I_j$). In the following section, we will point out how this condition is needed in order to get the correct estimate of the absorption coefficient defined by the 'continuous version' of the LBB law derivation through a 'single-slice measurement.' We will also show that the α coefficient defined by the finite ratio of equation (4) is the same as that defined in equation (1), and by the continuous approach based on equation (2).

3. The experiment

Our version of the experiment can be performed by using commercial instrumentation and very common materials. We developed the experiment in our classrooms with minimal instrumentation, which includes a commercial 20 W halogen lamp with 12 V alternating current power supply; an achromatic doublet to produce plane waves, with the light source placed in the focus of the doublet; a standard digital lux-meter, chosen to fit the spectrum of the light source; cubes of optical Plexiglas coupled to the lens by air; and an appropriate optical guide. The purpose of the achromatic doublet is to produce, at the same time, plane waves without chromatic aberration; we reported this in the general design of the experiment, but we verified that even with a commercial halogen lamp the chromatic aberration has negligible effects, and the experimental setup can be further simplified, avoiding the usage of the doublet.

The lamp and the lux-meter are aligned with the opposite ends of the rail that hold the Plexiglas bar, and the Plexiglas cubes are lined up between them, as shown in figure 2. The lux-meter measures the luminous intensity emitted through the last Plexiglas cube. The measurements can be repeated for several thicknesses of the medium. This thickness changes simply by changing the number of cubes.

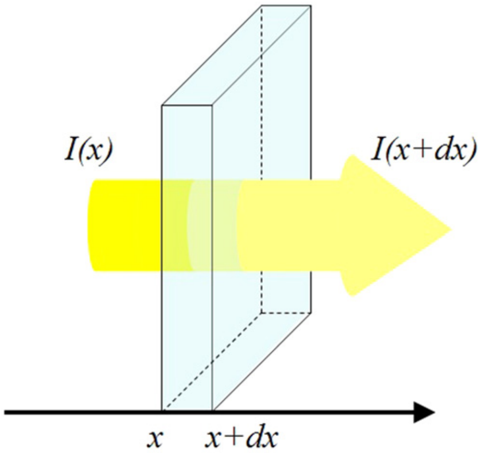

Figure 1. An elementary portion of the absorbing medium of infinitesimal thickness, dx.

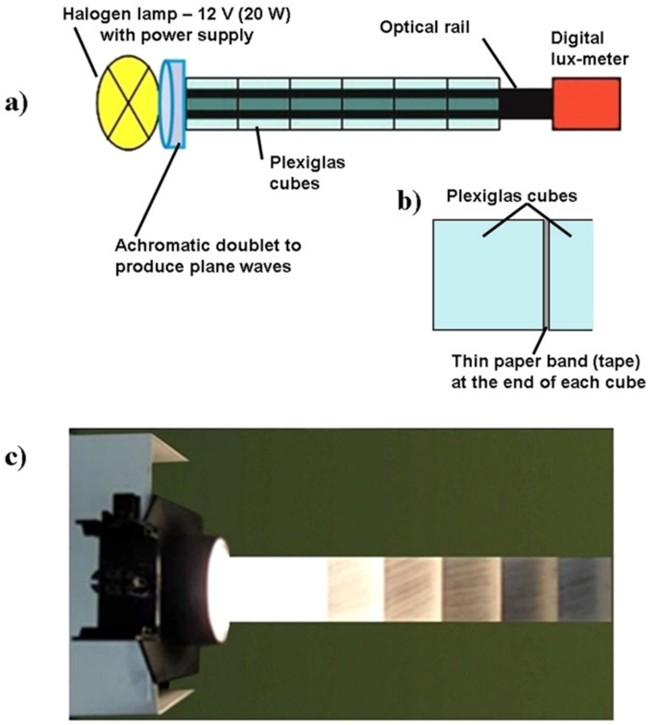

Figure 2. (a) Schematic representation of the experimental apparatus; (b) drawing of the cube and tape system, which 'artificially' increases the global absorption of a single cube to a detectable level; (c) illustration of the experimental apparatus, in which the decreasing intensity of light is clearly visible along the cubes.

Before starting the experiment, we measured the emitted intensity with the lux-meter at different distances from the lamp, without the Plexiglas cubes, to confirm that the influence of the intensity loss with distance (Kepler's effect) was negligible. In our experimental design, this was true within the experimental errors of the measurements, up to the maximum distance reached with the Plexiglas cubes, even when the commercial lamp was placed without any optical device before the cubes.

The Plexiglas used in our experiment has a low absorption coefficient, so the intensity absorption of a single cube was hardly detectable. To manage this challenge, we covered the output surface of each cube with a thin paper band, which allowed us to increase the absorption of each cube to a detectable level, while simultaneously allowing the absorption to be low enough that the light was still measurable after a certain number of cubes, thus maintaining the realization of the didactical experience. This allowed us to select a more appropriate 'effective' absorption coefficient for each cube-band couple. The final result of the experimental setup is shown in figure 2.

This procedure represents the practical realization of the previously described discretization. The evaluated physical intensity loss is a measure of the 'effective' absorption coefficient, which is not, of course, that of the Plexiglas. The effective α of each cube results from the combination of Plexiglas and the paper band. Each single taped cube acts as an elementary piece of the 'artificial material,' and the effective overall α manifests itself only at the end of each piece. In other words, this system can be viewed as a simulation of what it happens when the discretization is actually realized.

In the next subsection, we will return to the question of the 'too-low absorbed light per single cube,' and we will introduce further information on the considerations developed about the 'discrete derivation' of the LBB law and on the meaning of the condition, $\alpha S \ll 1$.

The experiment can proceed over two mainstreams:

(1) If the cubes are the same length, the students can measure the intensity of output light as a function of the thickness of the bar (i.e., as a function of the number of cubes, whose length has the role of length unit), plot the data, verify the exponential decay according to equation (1), and infer from the curve the 'effective' absorption coefficient of the built system.
(2) Students can estimate for each single cube the ratio at the left side of equation (4). A further discussion on this point is developed in the following subsection.

In figure 3(a), some data for $I(x)$ are plotted as an example. The experimental points have equally spaced abscissas, since they are recorded by adding one cube after another. The spacing between the abscissas is 3 cm, which is equal to the thickness of the cubes. I_0 has been measured by the lux-meter directly from the halogen lamp, with no cubes in the middle.

The exponential decay is particularly evident when the natural logarithm of I/I_0 is plotted instead of I:

$$I\,(x) = I_0 e^{-ax} \Rightarrow ln\left[I\,(x)/I_0\right] = -ax. \tag{5}$$

The plot in figure 3(b) clearly shows the linear behaviour of $ln[I/I_0]$ versus x. This representation is the easiest way to infer the absorption coefficient, α, from the data: a linear fit on the logarithmic data, in which α is the opposite of the slope, while the intercept is expected to be consistent with zero. From the reported data, $\alpha = (15.0 \pm 0.5)\ m^{-1}$ can be estimated. It is interesting to note that the effective observation of neat exponential decay for $I(x)$ also confirms that we are using plane waves, which do not suffer from substantial losses along x apart from the absorption from the medium (the previously mentioned Kepler's effect).

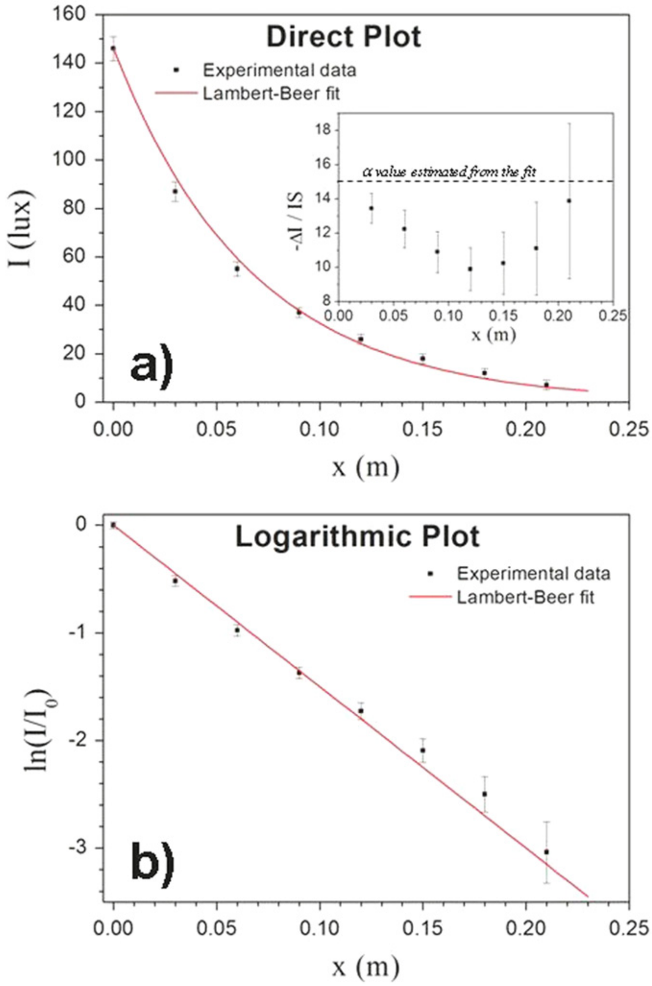

Figure 3. (a) Experimental data of light intensity collected in the apparatus described in the text. The resulting fit performed with the Lambert–Beer law is overlapped in the data. In the inset: values of the ratio—$\Delta I/I \cdot S$ on the single cubes. (b) The same data are plotted as $ln(I/I_0)$ vs x, so that the LBB decay assumes a linear behaviour. This plot has been used for a linear fit, and the resulting fit line overlaps the data.

4. The numerical consequences of exponential decay

4.1. The discrete approach and its intrinsic approximation

Let us now operate according to equation (4), which considers the discrete finite ratio $-\Delta I/(I{\cdot}S)$ for each single cube. This ratio is easily calculated from the data, and the results are reported in the inset of figure 3(a), which also shows the value of the absorption coefficient as estimated above. As we can see, the ratio values are barely constant within the errors, but their values are well (systematically) below the experimental α. At first glance, this could appear to be a consequence of experimental errors and inaccuracy. However, it is not so simple: a crucial role is played by the discretization itself, and by the difference between finite incremental ratios and their limits for zero-increment. To understand this point, and to provide a

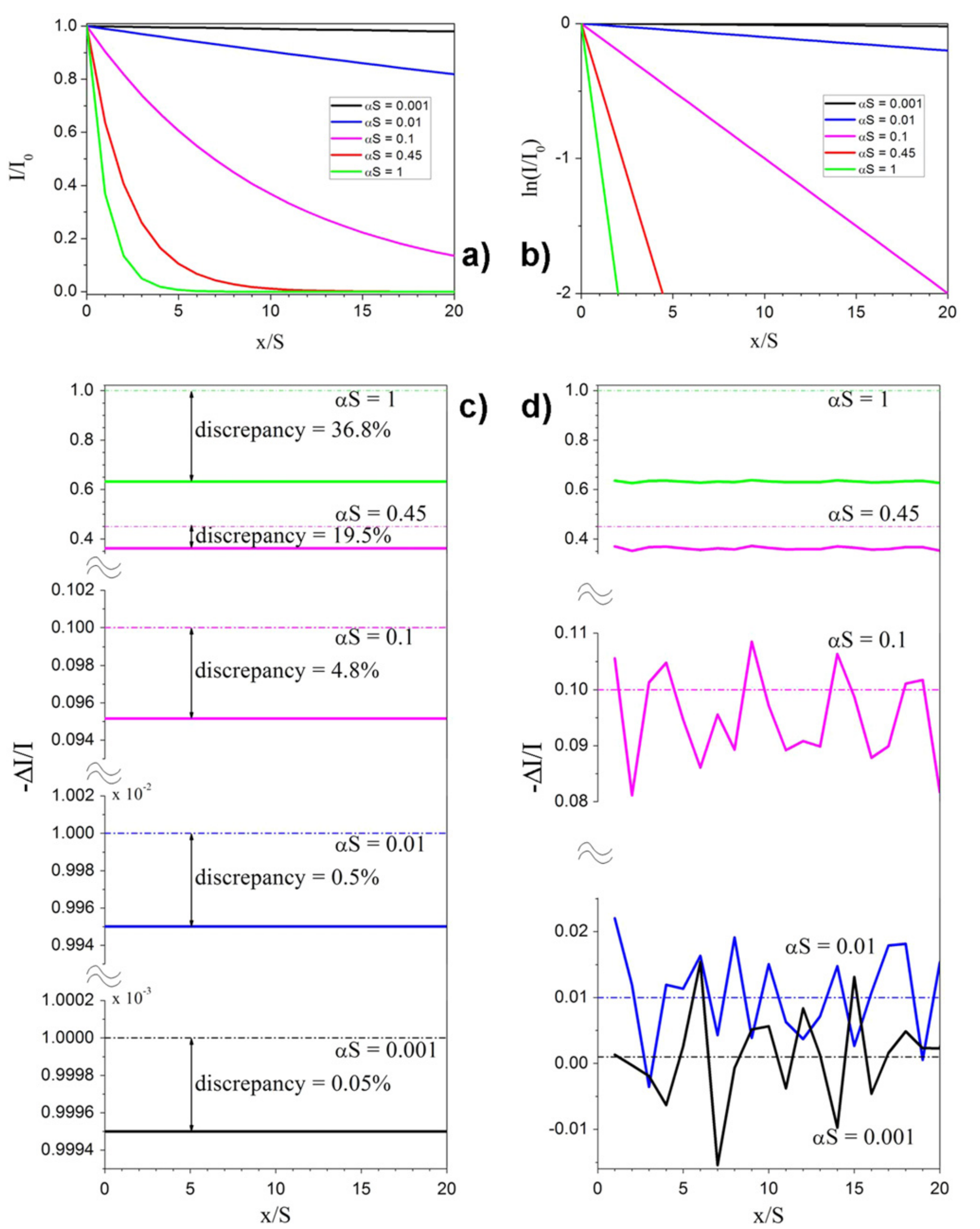

Figure 4. (a) Calculated exponential decaying light intensity for some values (indicated in the figure) of the product $\alpha \cdot S$. (b) The same simulated data plotted as $ln(I/I_0)$ vs x. (c) $-\Delta I/\mathrm{I}$ vs x for each exponential curve; the plots have been reported with different vertical scales, because of their very different vertical ranges, to magnify the discrepancy between this ratio and the 'true' α. (d) The same operation as in (c), performed after a noise (noise-to-signal ratio = 1%) has been added to the exponential curves.

didactically effective illustration for students, we prefer to proceed through numerical simulations of analytic and empirical functions, rather than with mathematical demonstration.

Therefore, let us consider again the LBB law in equation (1). Figure 4(a) shows curves calculated from that law, for different values of the product $\alpha \cdot S$. *Calculated* means that the y values are *exactly* what the exponential function in equation (1) gives for the corresponding x values; the curves are not affected by any experimental error or fluctuation. Both the vertical and horizontal scales have been normalized, to I_0 and S respectively, in order to have general plots. Figure 4(b) reports the same curves on a logarithmic scale, on which they are linear.

In figure 4(c), we plot the results for $-\Delta I/I$ versus x for each curve, which, according to equation (4), should be constant over x and equal to the product, $\alpha \cdot S$. The plots show that the calculated ratio is actually constant over x, and therefore potential x-dependence in an experimental dataset would be an effect of experimental fluctuations. However, such a constant value is systematically lower than the 'true' $\alpha \cdot S$ value (i.e., the one that we used in equation (1) to generate the curves themselves, which is the one coming from the continued derivation). We also show the relative discrepancy between $\alpha \cdot S$ given by both the discrete ratio and the 'true' ratio, which increases as $\alpha \cdot S$ approaches 1.

Now, we can assert that the α coefficient introduced in equations (3) and (4) coincides with the one defined by the continuous derivation in equations (1) and (2) if and only if $\alpha \cdot S \ll 1$.

We point out that one of the $\alpha \cdot S$ values we considered, 0.45, corresponds to the value we have from our data: $S = 3 \cdot 10^{-2}$ m, $\alpha = 15$ m^{-1}. Our simulation says that in this case, the discrete evaluation of a single block gives, in the ideal case, an underestimation of about 19–20%. Referring to the data in the inset of figure 3(a), the average value for α from the discrete ratio computation is about 12, which is actually lower than the absorption coefficient estimated through the fit of 20%.

What can a student learn from these results? These considerations have, in our opinion, an impressive didactical impact which overcomes the experimental exercise from which we started. They illustrate how a discrete experimental calculation on single constituting units can produce reliable results only if the mathematical continuity limit is approached. In our experiment, this translates to the condition $\alpha \cdot S \ll 1$, which is also very simple to understand from the point of view of its derivation, making the exponential LBB law very suitable to illustrate these concepts. The effectiveness of the exponential decay in showing the subtle differences between a discrete and a continuous approach also lies in the fact that it can be tested with quantities that are both easy to evaluate (the ratios $\Delta I/I$, or in general $\Delta y/y$) and are expected to have a constant value, allowing ease of comparison and discrepancy calculation.

But there is an additional aspect of the exponential decay that can more effectively illustrate the origin of the approximations arising from the discrete versus continuous approach. Plotting the exponential law for $I(x)$ on a logarithmic vertical scale leads to the linear plot in figure 4(b), described by equation (5), which uses the linear fit proposed to students. In equation (5), and therefore in the logarithmic plot, the absorption coefficient is the opposite of the angular coefficient of the line. Obviously, for a line, the evaluation of the angular coefficient gives the same result independently from the chosen interval. In other words, if we apply a discrete computation to evaluate α from the line in the logarithmic plot, the result is not affected by the fact that we are using a finite interval, and it will equal the 'true' absorption coefficient. In formulas, with the slice thickness, S, representing the elementary Δx, from the linear relation of equation (5) we have for the jth slice:

$$\alpha = -\frac{ln\left[I_{\text{out}.j}/I_0\right] - ln\left[I_{\text{in}.j}/I_0\right]}{S} = -\frac{ln\left[I_{\text{out}.j}\right] - ln\left[I_{\text{in}.j}\right]}{S} = -\frac{\Delta ln\left[I_j\right]}{S}. \tag{6}$$

As we said, for the elementary properties of a linear function the α value evaluated in this way is equal to that obtained with the correct continuous approach. Comparing equations (6) with (5), and this assertion with the above consideration about the 'wrong' result of the discrete computation on the exponential form, we can infer the primary reason for the discrepancies: the 'true' absorption coefficient, working with finite differences, is equal to $(\Delta ln(I))/S$ and not to $(\Delta I/I)/S$, because $\Delta I/I = \Delta ln(I)$ only when $\Delta I \to 0$.

4.2. Experimental fluctuations and their role in the discrete approach

The information presented in the previous section clarifies why there is a systematic discrepancy between the decay rate estimations through the continuous interpolation, which are operated by the fit procedure and the discrete calculation of single steps. The discrepancy arises for intrinsic reasons, not because of experimental errors. At the same time, we see that in the 'exact' exponential decay, with no fluctuations, the ratio $(\Delta I/I)/S$ underestimates the α value, but it is the same for all the slices. As a consequence, although the experimental fluctuations do not affect the α underestimation, they must be responsible for its dependence on the interval on which equation (4) is applied.

We would like to briefly discuss this point, which is not completely disconnected from the previous one. Once again, for didactical purposes, we start from the ideal discrete curves reported in figure 4(a). We artificially overlapped a noise onto these simulated ideal data; the noise was made by random fluctuations δI of maximum amplitude 1% of the ideal value I_{true} ($-10^{-2} < \delta I/I_{\text{true}} < 10^{-2}$). On such new 'dirty' data, we reapplied the procedure of equation (4), obtaining the results summarized in figure 4(d).

As we can see, the consequent relative fluctuations in the $(\Delta I/I)/S$ values are extremely dependent on the $\alpha \cdot S$ values, with an opposite trend with respect to the accuracy of the α estimation: the smaller $\alpha \cdot S$ is, the larger the fluctuations of the discrete evaluation are. For the chosen noise amplitude, figure 4(d) shows that the computation of single steps would be meaningless for the first two curves ($\alpha \cdot S = 0.001$ and $\alpha \cdot S = 0.01$), while it makes sense for greater values of $\alpha \cdot S$, with fluctuations decreasing as $\alpha \cdot S$ increases. If we constrain the experimental noise to a relative amplitude of 0.1%, the curve at $\alpha \cdot S = 0.01$ produces reasonable oscillations for $(\Delta I/I)/S$, while the curve at $\alpha \cdot S = 0.001$ still gives meaningless results. As a general criterion, we can state that the oscillations in the $(\Delta I/I)/S$ estimation give reliable results when $\alpha \cdot S \gg \delta I/I_{\text{true}}$.

The opposite trends for the two desirable conditions in the discrete process—the accuracy of the estimation and a reasonable distribution width of the calculated values over an experimental dataset—establish the need for a compromise. Small $\alpha \cdot S$ values favour an accurate estimation of the absorption coefficient through the discrete approach; however, this condition can give extremely fluctuating values around this accurate 'ideal' value if it is not supported by a good noise-to-signal ratio, which must be much smaller than $\alpha \cdot S$. So, to overcome the fitting procedure of the nonlinear exponential law through the linearization on the logarithmic scale, a student might think, after reading the previous subsection, that using thinner Plexiglas slices (smaller S) or avoiding use of the tape (smaller α) would reduce the product $\alpha \cdot S$. However, it would be useless since reducing this a-dimensional product below the noise-to-signal ratio would produce unreliable fluctuating results.

5. Conclusions

In this paper, we proposed a simple optical experience that can be performed with very basic experimental instrumentation and materials. The simple empirical work and analysis allow students to consider more general questions concerning the features of exponential decay behaviour. The exponential decay describes a large number of physical phenomena; as discussed, it arises anytime a physical quantity decreases with relative losses proportional to a second physical quantity. In our experience this is a geometrical length, but it could be a time or any other quantity.

The LBB law for light absorption provides one of the easiest and fastest ways to manage an example of such an important prototypical function in the experimental setup described in this paper. Furthermore, having built the absorbing medium through discrete steps, the conceived experiment also allows students to understand the computational aspects and numerical problems of finite differences approaches versus continuous ones. This is recognized as a primary issue in many fields of computational physics and technology, and, as a consequence, it also deserves academic exploration for its wide application in discrete dynamics [13][1].

The peculiar features of the exponential function make it very suitable to introduce this topic to undergraduate students; they can study the problems and understand the results as we described without introducing any formalism, simply by looking at the numerical outputs. This discussion can also be regarded as a milestone for deeper considerations about numerical discrete computation in experimental physics, such as numerical derivative problems. Indeed, the basic considerations we developed are the real base of the difficulties in numerical derivatives. Overly sparse data can produce uncorrected absolute values, which is a problem when an absolute estimation, rather than a normalized trend, is needed. However, having denser experimental points may be useless, since a good noise-to-signal ratio is always the ultimate requirement to get both accurate estimations and manageable fluctuations.

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[1] As an example, cfr. the entire work of Professor Francisco Balibrea Gallego at Universidad of Murcia, advisor of numerous *PhD theses* on this topic, whose career was celebrated by the *Czech–Slovak–Spanish Workshop on Discrete Dynamical Systems* (*La Manga del Mar Menor, Spain,* 20–24 September 2010) organized in his honour and with a special issue in *Journal of Difference Equations and Applications* vol 18, No. 4, April 2012. In particular, he also gave a PhD class on *Discrete Dynamical Systems* (see details on www.doqma.um.es/2005-2007/uk/documentos/discrete_dynamical_systems.pdf).

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