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On the distributional properties of household consumption expenditures: the case of Italy

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Abstract In this paper we explore the statistical properties of the distributions of consumption expenditures for a large sample of Italian households in the period 1989–2004. Goodness-of-fit tests show that household aggregate (and age-conditioned) consumption distributions are not log-normal. Rather, their logs can be invariably characterized by asymmetric exponential-power densities. Departures from log-normality are mainly due to the presence of thick lower tails coexisting with upper tails thinner than Gaussian ones. The emergence of this irreducible heterogeneity in statistical patterns casts some doubts on the attempts to explain log-normality of household consumption patterns by means of simple models based on Gibrat's Law applied to permanent income and marginal utility.

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1 Introduction

In the last years, considerable effort has been devoted to the study of the distributional properties of key microeconomic variables and indicators. For example, a huge amount of contributions has explored the statistical properties of wealth and personal income distributions, both across years and countries (see, e.g.,Chatterjee et al. 2005, and references therein). Similarly, in the field of industrial dynamics, a large body of literature has successfully characterized the shape of cross-section firm size and growth-rate distributions, and their evolution over time (cf. among others Axtell 2001; Bottazzi and Secchi 2006a).

These studies show that, despite the turbulence typically detected at the microeconomic level (e.g., entry and exit of firms; positive and negative persistent shocks to personal income; etc.), there exists an incredible high level of regularity in the shape of microeconomic cross-section distributions, both across years and countries. For instance, personal income distributions appear to be characterized by a log-normal body with a Pareto upper tail in the majority of cases (Clementi and Gallegati 2005a; Souma 2001). Furthermore, as far as growth-rate distributions are concerned, there seems to emerge a sort of universality feature: the same family of distributions[1] is indeed able to fit growth rates for firms in different sectors, industries and even countries (both cross-sectionally and along the time-series dimension; cf. Lee et al. 1998; and Fagiolo et al. 2008).

Notwithstanding such successful results, the above line of research has not been extensively applied, so far, to other key microeconomic variables for which detailed cross-section data are available, namely household consumption expenditures (HCEs). This is somewhat surprising for two related reasons (Attanasio 1999). First, understanding consumption is crucial to both micro- and macro-economists, as it accounts for about two thirds of GDP and it decisively determines social welfare. Second, while we know a lot about the statistical properties of aggregate consumption time-series and microeconomic life-cycle profiles, our knowledge about the distributional properties of cross-section HCEs is rather poor, as we almost always limit ourselves to the first and second moments thereof.

The only exception to this trend is a recent contribution by Battistin et al. (2007). They employ expenditure and income data from U.K. and U.S. surveys and show that HCE distributions are, within cohorts, well approximated by log-normal distributions (or, as they put it, are "more log normal than income" distributions).[2] Battistin et al. (2007) show that this evidence can be accommodated by assuming that a sort of

[1] That is, the exponential-power family of densities, originally introduced by Subbotin (1923). More on this below.

[2] Log-normality of HCE distributions in U.K. is confirmed by another early study in the econophysics domain, see Hohnisch et al. (2002).

Gibrat's Law of Proportionate Effects (Gibrat 1931; Kalecki 1945) holds for permanent income and, through intertemporal utility maximization, for household consumption (Hall 1978).[3]

This is a nice empirical result, because it seems to establish a stylized fact holding across cohorts and, possibly, countries, i.e. the distribution of the logarithms of HCEs is Gaussian. In turn, it implies that all the moments of the distribution exist but we only need a two-parameter density to characterize the large majority of observed HCE patterns. The main empirical message of the paper is therefore that, as far as HCE distributions are concerned, there is no need to look at higher moments. Indeed, skewness, kurtosis, etc., of logged HCE distributions would mimic the correspondent Gaussian moments independently of cohorts, years and age classes. Furthermore, log-normality of HCE distributions implies that one can explain them by means of simple multiplicative growth models building upon the idea that consumption results from "the cumulation of random shocks to income and other variables that affect utility" (Battistin et al. 2007, p. 4). [4] If confirmed, this would be a powerful insight, which parallels similar ones obtained in industrial dynamics for firm size and growth rates (Ijiri and Simon 1977; Sutton 1997).

In this paper, however, we show that log-normality is not generally the case for Italian HCE distributions. We employ the "Survey of Household Income and Wealth" (SHIW) provided by the Bank of Italy and we study HCE distributions with a parametric approach both in the aggregate and conditioned to the age of the household head (as reported in the survey) for a sequence of 8 waves from 1989 to 2004.[5] Unlike in Battistin et al. (2007), who only check goodness of fit (GoF) employing graphical tools and Kolmogorov–Smirnov (KS) tests, we run a wider-range battery of GoF tests that overcome the well-known power limitations of the KS test.

Our main result is that in Italy, for all the waves under study and for the majority of age classes, the logs of HCE distributions are not normal but can be satisfactorily approximated by asymmetric exponential-power densities. This family of distributions features five parameters and allows one to flexibly model asymmetries in both the third and the fourth moment. Indeed, our statistical tests often reject the hypothesis that logs of HCE display zero-skewness and normal kurtosis. On the contrary, one invariably detects significant positive/negative skewness and asymmetry in the tail behavior. More specifically, the large majority of logged HCE distributions exhibit thick lower tails together with upper tails thinner than Gaussian ones. This evidence is quite robust to a series of further checks involving, e.g., estimation with robust statistics.

[3] Of course, one can derive log-normality of consumption directly from the hypothesis that individual consumption is approximately equal to permanent income (Friedman 1957). Notice, however, that permanent income is not observable in practice.

[4] Notice that log-normality of individual consumption stems from log-normality of permanent income only if a long list of restrictions do indeed hold. Let aside the very hypothesis that individuals act as utility maximizers, one also needs that the random-shock process obeys some form of central-limit theorem and marginal utility is linear in log consumption.

[5] Italian income distributions for SHIW data have been extensively studied in Clementi and Gallegati (2005b). They find that income is log-normal in the body and power-law in the upper tail. More on the relationships between income and consumption distributions is in Sect. 4.

The basic message is that, at least for Italy, it seems impossible to come up with a statistical description of consumption data that can compress the existing heterogeneity in HCE distributions, so as to avoid a higher number of degrees of freedom in parametric characterizations. In other words, the existing, statistically-detectable, departures from log-normality prevent us from providing a simple two-parameter density that fits both aggregate and disaggregate HCE distributions. This in turn casts some doubts on the possibility to explain observable HCE distributions by means of simple, invariant models based on permanent income and Gibrat's law hypotheses.

The rest of the paper is organized as follows. In Sect. 2 we describe the database that we employ in the analysis. Section 3 reports on our empirical results and related robustness checks. Section 4 presents a speculative discussion on the implications of our findings. Finally, Sect. 5 concludes.

2 Data

Our empirical analysis is based on the "Survey of Household Income and Wealth" (SHIW) provided by the Bank of Italy. The SHIW is one of the main sources of information on household income and consumption in Italy. Indeed, the quality of the SHIW is nowadays very similar to that of surveys in other comparable countries like France, Germany and the U.K.[6]

The SHIW was firstly carried out in the 1960s with the goal of gathering data on incomes and savings of Italian households. Over the years, the survey has been widening its scopes. Household are now asked to provide, in addition to income and wealth information, also details on their consumption behavior and even their preferred payment methods. Since then, the SHIW was conducted yearly until 1987 (except for 1985) and every 2 years thereafter (the survey for 1997 was shifted to 1998). In 1989 a panel section consisting of units already interviewed in the previous survey was introduced in order to allow for time comparison.

The present analysis focuses on the period 1989–2004. We therefore have 8 waves. The sample used in the most recent surveys comprises about 8,000 households (24,000 individuals), distributed across about 300 Italian municipalities. The sample is representative of the Italian population and is based on a rotating panel targeted at 4,000 units.

Available information includes data on household demographics (e.g., age of household head, number of household components, geographical area, etc.), disposable income, consumption expenditures, savings, and wealth. In this paper, we employ yearly data on aggregate HCEs.[7] We study both unconditional and age-conditioned distributions, where age conventionally refers to the household head (on the problems related to assigning household-level data to its members, see for example, Attanasio 1999, Sect. 2.2). Consumption figures have been cleaned from outliers and converted,

[6] SHIW data are regularly published in the Bank's supplements to the Statistical Bulletin and made publicly available online at the URL http://www.bancaditalia.it/statistiche/indcamp/bilfait. We refer the reader to Brandolini (1999) for a detailed overview on data quality and main changes in the SHIW sample design.

[7] Data for disaggregated expenditure categories (e.g., nondurables, food, durables, etc.) are also available. See Sect. 5.

when necessary, to Euros (i.e., for the period 1989–2000). Furthermore, HCEs have been weighted by using appropriate sample weights provided by the Bank of Italy. Finally, we have deflated all consumption expenditure figures so as to obtain *real* HCE distributions. To do so, we have employed the Consumer Price Index deflator (yearly series based on year 2000) provided within the OECD Economic Outlook.[8]

More formally, our data structure consists of the aggregate distribution of yearly household (real) expenditure for consumption $\{C_{h,t}\}$, where $h = 1, \ldots, H_t$ stands for households and $t \in T = \{1989, 1991, 1993, 1995, 1998, 2000, 2002, 2004\}$ are survey waves. Since in each wave there were many cases of unrealistic (e.g., zero or negative) consumption figures, we decided to drop such observations and to keep only strictly positive ones. We also dropped households for which consumption expenditures were larger than yearly income (as reported in the SHIW). Therefore, we ended up with a changing (but still very large) number of households in each wave (H_t). HCE distribution is complemented with information on the age of household head in wave t ($A_{h,t}$). We employ this variable to condition HCE distributions. More specifically, in line with Battistin et al. (2007), we consider the following age breakdown: $A = \{A_1; \ldots; A_8\} = \{\leq 30; 31\text{–}35; 36\text{–}40; 41\text{–}45; 46\text{–}50; 51\text{–}55; 56\text{–}60; \geq 61\}$, which generates sufficiently homogeneous subsamples as far as the number of observations is concerned.[9] In each wave, we then build the distributions $\{C_{h,t} | A_{h,t} \in A_k\}$, with $k = 1, \ldots, 8$. As usual, we will mainly employ natural logs of real consumption expenditure figures, defined as $c_{h,t} = \log(C_{h,t})$. Age-conditioned distributions will thus read $\{c_{h,t} | A_{h,t} \in A_k\}$, for $k = 1, \ldots, 8$ and $t \in T$.

3 Towards a characterization of household consumption expenditure distributions

In this Section, we shall explore the statistical properties of Italian HCE distributions and their evolution over time.[10] We are interested in answering four related questions: (i) Did HCE distributions exhibit structural changes over time? (ii) Can aggregate and age-conditioned HCE distributions be well-approximated by log-normal densities? (iii) If not, which are the causes of departures from log-normality? (iv) If HCE distributions are not log-normal, can one find alternative, better statistical descriptions of HCE distributions across age classes and time?

3.1 Time-evolution of HCE distributions

Let us begin with a descriptive analysis of HCE distributions and their evolution over time. Table 1 reports descriptive statistics for (real) aggregate $\{C_{h,t}\}$ distributions.

8 Available at http://www.sourceoecd.org.

9 The only exception is the class ($\geq$61), which contains about 2,000 observation in each year. We decided not to further disaggregate this class in order to keep the analysis as close as possible to Battistin et al. (2007).

10 Additional details on the statistical analyses presented in this Section are available from the Authors upon request. All exercises were performed using MATLAB®, version 7.4.0.287 (R2007a).

Table 1 Moments of aggregate HCE distributions versus waves

Stats	Waves							
	1989	1991	1993	1995	1998	2000	2002	2004
Mean	194.044	182.233	181.363	180.051	175.263	182.445	182.685	187.132
SD	250.789	251.174	226.596	190.695	190.301	200.845	220.376	213.916
Skewness	3.357	3.732	3.294	2.655	3.100	2.830	3.215	3.253
Kurtosis	18.503	22.892	17.486	12.655	17.777	14.304	17.709	17.546
N Obs	7424.000	7208.000	6241.000	6274.000	5606.000	6292.000	6376.000	6277.000

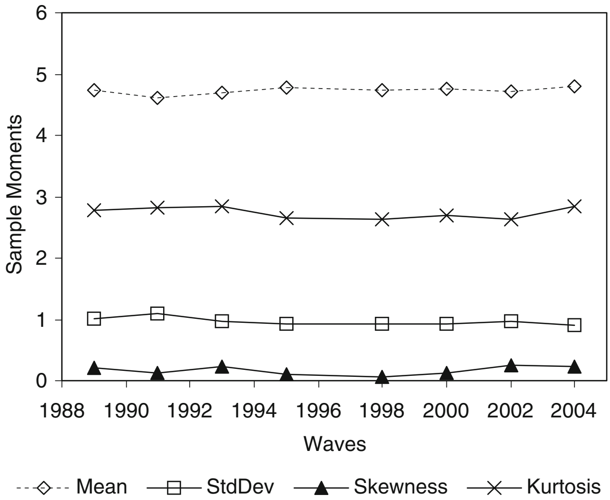

Fig. 1 Moments of aggregate logs of HCE distributions versus waves

Simple inspection shows that HCE sample moments are quite stable over time. Such evidence is confirmed by Fig. 1, where the first four sample moments of logged HCE distributions $\{c_{h,t}\}$ are plotted against time. This means that, notwithstanding many households did probably move back and forth across quantiles, HCE distributions did not dramatically change their structural properties. This is a strong result, also in light of the introduction of the Euro in 2002.

Surprisingly enough, HCE distributions appear to be quite stable over time also when one conditions to age classes. Figure 2 reports plots of sample moments versus waves for two age classes (left: 41–45; right: $\geq$61). As it can be easily seen, also within age classes HCE distributions have remained quite stable over the years. Furthermore, one does not detect any evident trends in the first moments of the HCE distributions when, in each wave, they are plotted against age classes (see Fig. 3 for wave 2004).

Notice also that if HCE distributions were lognormal, their logs would have been normally distributed, with zero skewness and kurtosis equal to 3. On the contrary, Table 2 shows that for logged aggregate HCE distributions some positive skewness always emerges, while kurtosis levels fluctuate slightly below the normal threshold.

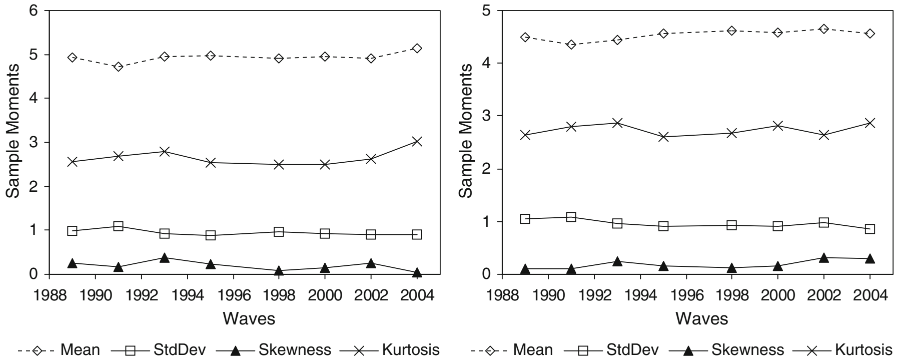

Fig. 2 Moments of logs of age-conditioned HCE distributions versus waves. *Left* Class 4 (41–45). *Right* Class 8 (≥61)

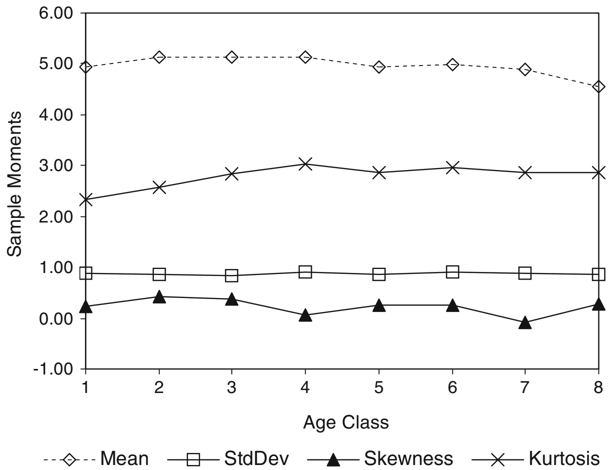

Fig. 3 Moments of age-conditioned logs of HCE distributions versus age classes in wave 2004

This holds true in general also for age-conditioned logged distributions, see again Table 2. It is interesting to note that while logged HCE distributions appear to be right-skewed for almost all age classes and years, kurtosis is in the majority of cases below 3 (more on that below). Of course, to decide whether these departures from the normal benchmark are significant or not, one needs a more formal battery of statistical tests. This is what we shall do in the next section.

3.2 Are HCE distributions log-normal?

To check whether HCE distributions are log-normal (or equivalently if their logs are normal), we have used a battery of three normality tests: Lilliefors (Lilliefors 1967), Jarque-Bera (Bera and Jarque 1980, 1981) and Quadratic Anderson–Darling (Anderson and Darling 1954). These tests are known to perform better than comparable ones

Table 2 Skewness (top panel) and kurtosis (bottom panel) of logged HCE distributions (on aggregate and within age classes) versus waves

Age classes	Waves							
	1989	1991	1993	1995	1998	2000	2002	2004
Skewness								
Aggregate	0.201	0.128	0.231	0.103	0.059	0.124	0.248	0.221
≤30	0.452	0.186	0.358	0.203	0.048	0.016	0.130	0.228
31–35	0.440	0.320	0.398	0.083	0.011	0.079	0.196	0.427
36–40	0.329	0.289	0.403	0.030	0.164	0.060	0.289	0.372
41–45	0.254	0.161	0.388	0.224	0.089	0.156	0.249	0.051
46–50	0.258	0.107	0.132	0.095	0.080	0.176	0.408	0.251
51–55	0.278	0.136	0.265	0.038	−0.189	0.156	0.083	0.263
56–60	0.207	0.100	0.270	0.054	0.031	0.016	0.127	−0.074
≥61	0.110	0.113	0.237	0.159	0.122	0.163	0.313	0.289
Kurtosis								
Aggregate	2.778	2.824	2.836	2.652	2.629	2.699	2.638	2.848
≤30	3.066	2.583	2.698	2.318	2.519	2.170	2.254	2.322
31–35	2.959	2.898	2.735	2.602	2.553	2.611	2.531	2.577
36–40	2.753	2.866	2.909	3.440	2.828	2.748	2.597	2.839
41–45	2.558	2.687	2.798	2.530	2.492	2.501	2.628	3.022
46–50	2.631	2.628	2.648	2.649	2.650	2.575	2.617	2.854
51–55	2.791	2.915	2.711	2.486	2.596	2.529	2.687	2.950
56–60	2.750	3.011	2.764	2.712	2.622	2.975	2.797	2.872
≥61	2.632	2.794	2.862	2.609	2.670	2.812	2.632	2.873

(e.g., KS test) in terms of power (see D'Agostino and Stephens 1986; Thode 2002, for details). More specifically, the Lilliefors test adapts the KS test to the case where parameters are unknown. In this sense, it can benchmark results obtained in Battistin et al. (2007), who, as already mentioned, only employ the less-performing KS test. Finally, the Jarque-Bera test is known to perform better in presence of outliers, which is a commonly-detected problem for consumption data (more on this in Sect. 3.3).

Table 3 reports GoF results for logs of aggregate and age-conditioned HCE distributions. Aggregate distributions are never log-normal, while in the age-conditioned case, only for 9 distributions (out of 64) the three tests are simultaneously unable to reject (at 5%) the null hypothesis of log-normality (in boldface). Log-normality seems to be slightly more pervasive in age classes 36–40 and 56–60, and in the years 1991, 1995 and 1998.

The above GoF evidence casts some doubts on whether consumption distributions can be well-approximated—in Italy—by log-normal densities. This seems to be true, for all waves under study, both at the aggregate level and after one conditions to age classes.

Table 3 Goodness-of-fit tests for the null hypothesis of normal logs of HCE distributions

		Waves							
Age Class	Test	1989	1991	1993	1995	1998	2000	2002	2004
Aggregate	Lilliefors	0.025 (0.000)	0.023 (0.000)	0.025 (0.000)	0.019 (0.000)	0.017 (0.002)	0.019 (0.000)	0.036 (0.000)	0.023 (0.000)
	JB	65.371 (0.000)	28.831 (0.000)	62.612 (0.000)	42.774 (0.000)	35.355 (0.000)	39.652 (0.000)	100.242 (0.000)	57.291 (0.000)
	AD2	8.181 (0.000)	6.751 (0.000)	7.756 (0.000)	4.208 (0.000)	2.545 (0.000)	4.689 (0.000)	14.509 (0.000)	5.722 (0.000)
≤ 30	Lilliefors	0.0518 (0.0000)	**0.0415 (0.0920)**	0.0474 (0.1060)	0.0644 (0.0140)	0.0373 (0.0287)	0.0622 (0.0400)	0.0760 (0.0030)	0.0552 (0.0080)
	JB	20.5164 (0.0010)	**5.0113 (0.0690)**	7.5121 (0.0160)	6.9433 (0.0310)	2.0264 (0.0514)	6.5595 (0.0290)	6.2203 (0.0450)	6.0806 (0.0490)
	AD2	0.0000 (0.0000)	**0.0920 (0.0573)**	0.1060 (0.0220)	0.0140 (0.0003)	0.7087 (0.0688)	0.0400 (0.0063)	0.0030 (0.0033)	0.1080 (0.0129)
31-35	Lilliefors	0.0417 (0.0110)	0.0390 (0.0350)	0.0555 (0.0010)	0.0306 (0.0352)	0.0285 (0.0671)	0.0405 (0.0164)	0.0561 (0.0060)	0.0615 (0.0080)
	JB	20.4006 (0.0000)	10.3319 (0.0080)	12.3648 (0.0070)	7.2821 (0.0168)	7.0633 (0.0183)	2.6383 (0.2315)	5.6840 (0.0570)	10.9717 (0.0150)
	AD2	0.0110 (0.0000)	0.0350 (0.0005)	0.0010 (0.0000)	0.4350 (0.0975)	0.6781 (0.0840)	0.1624 (0.0019)	0.0060 (0.0026)	0.0080 (0.0004)
36-40	Lilliefors	0.0454 (0.0010)	0.0566 (0.0000)	0.0351 (0.1357)	**0.0305 (0.1112)**	**0.0280 (0.4525)**	**0.0254 (0.5482)**	0.0454 (0.0120)	0.0642 (0.0000)
	JB	14.2769 (0.0030)	8.8226 (0.0140)	14.6828 (0.0030)	**4.0984 (0.1148)**	**2.8605 (0.2334)**	**1.7654 (0.3968)**	11.3930 (0.0090)	11.2770 (0.0090)
	AD2	0.0010 (0.0000)	0.0000 (0.0000)	0.1357 (0.0005)	**0.3112 (0.1687)**	**0.4525 (0.4312)**	**0.5482 (0.5391)**	0.0120 (0.0003)	0.0000 (0.0001)
41-45	Lilliefors	0.0402 (0.0030)	0.0347 (0.0240)	0.0544 (0.0000)	0.0415 (0.0210)	0.0318 (0.1780)	0.0374 (0.0400)	0.0514 (0.0030)	**0.0249 (0.5942)**
	JB	15.8387 (0.0010)	6.7070 (0.0330)	16.9326 (0.0020)	10.0249 (0.0120)	6.6776 (0.0360)	8.6106 (0.0170)	9.2720 (0.0100)	**0.2449 (0.8786)**
	AD2	0.0030 (0.0000)	0.0240 (0.0020)	0.0532 (0.0000)	0.0210 (0.0006)	0.1780 (0.0365)	0.0400 (0.0014)	0.0030 (0.0006)	**0.5942 (0.6104)**
46-50	Lilliefors	0.0360 (0.0170)	0.0322 (0.0550)	0.0251 (0.0295)	0.0274 (0.0243)	**0.0275 (0.2702)**	0.0394 (0.0130)	0.0616 (0.0000)	0.0389 (0.0370)
	JB	13.5895 (0.0050)	5.6765 (0.0560)	5.1119 (0.0710)	4.4812 (0.1050)	**4.1129 (0.1185)**	8.5301 (0.0210)	21.6100 (0.0010)	6.6417 (0.0460)
	AD2	0.0170 (0.0008)	0.0550 (0.0116)	0.4395 (0.2849)	0.2743 (0.0784)	**0.2702 (0.1643)**	0.0130 (0.0031)	0.0000 (0.0000)	0.0370 (0.0074)
51-55	Lilliefors	0.0424 (0.0000)	**0.0234 (0.3407)**	0.0374 (0.0160)	0.0258 (0.3412)	0.0370 (0.0230)	0.0362 (0.0240)	0.0366 (0.0250)	0.0512 (0.0000)
	JB	11.9020 (0.0050)	**2.7986 (0.2322)**	10.7778 (0.0090)	7.5082 (0.0320)	8.6477 (0.0240)	9.4538 (0.0130)	3.6450 (0.1551)	7.6226 (0.0280)
	AD2	0.0000 (0.0001)	**0.3407 (0.1141)**	0.0160 (0.0008)	0.3412 (0.0293)	0.0230 (0.0005)	0.0240 (0.0047)	0.0250 (0.0177)	0.5061 (0.0001)
56-60	Lilliefors	0.0469 (0.0010)	0.0326 (0.0570)	0.0429 (0.0060)	**0.0240 (0.4385)**	0.1255 (0.0429)	0.0363 (0.0350)	**0.0249 (0.4611)**	0.0267 (0.0281)
	JB	7.5154 (0.0310)	1.2201 (0.5210)	9.1354 (0.0150)	**2.7416 (0.2278)**	3.6515 (0.1605)	0.0458 (0.9760)	**2.7667 (0.2425)**	5.0774 (0.1187)
	AD2	0.0010 (0.0006)	0.0570 (0.0162)	0.0060 (0.0002)	**0.4385 (0.5161)**	0.4629 (0.0208)	0.0350 (0.0445)	**0.4611 (0.2329)**	0.3881 (0.0878)
≥ 61	Lilliefors	0.0167 (0.1210)	0.0296 (0.0000)	0.0255 (0.0020)	0.0276 (0.0000)	0.0230 (0.0060)	0.0176 (0.0540)	0.0469 (0.0000)	0.0295 (0.0000)
	JB	17.3600 (0.0000)	9.8501 (0.0110)	24.1645 (0.0010)	26.1708 (0.0000)	14.3017 (0.0010)	14.9425 (0.0030)	58.9140 (0.0000)	41.7881 (0.0000)
	AD2	0.1210 (0.0032)	0.2410 (0.0000)	0.0020 (0.0000)	0.0650 (0.0000)	0.0060 (0.0022)	0.0540 (0.0032)	0.3256 (0.0000)	0.0782 (0.0000)

Monte-Carlo (one-tailed) p-values in round brackets. *JB* Jarque-Bera test, *AD*2 Anderson–Darling test. Boldface entries: The null hypothesis is never rejected (at 5%)

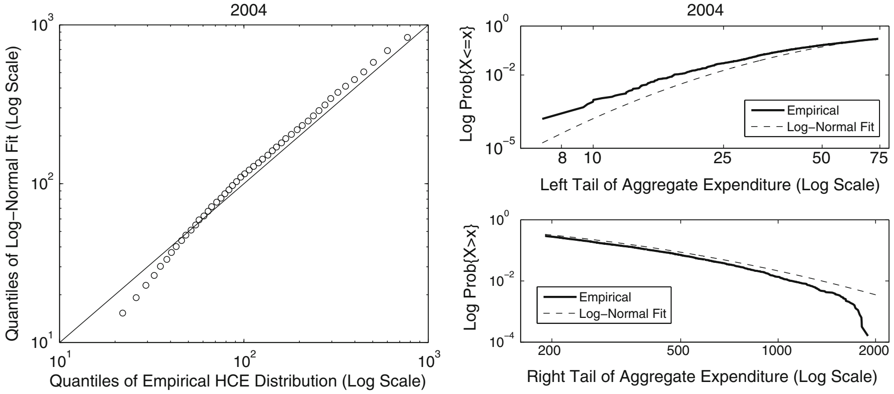

Fig. 4 Aggregate 2004 HCE distribution. *Left* QQ-plot of the quantiles of the empirical HCE distribution versus the quantiles of lognormal fit (log scales on both axes). *Right* Cumulative distribution function $F(x) = Prob\{X \leq x\}$ for the smallest 30% of observations (top) and complementary cumulative distribution function $1 - F(x) = Prob\{X > x\}$ for the largest 30% of observations (bottom). Log scales on both axes

Our statistical findings are also detectable through standard graphical analyses. As an example, the left panel of Fig. 4 presents for wave 2004a QQ-plot of the aggregate HCE distribution versus its log-normal fit (cf. for details Embrechts et al. 1997; Adler et al. 1998; Reiss and Thomas 2001). Even a simple visual inspection suggests that the empirical HCE is not close to a log-normal. Departures from log-normality emerge not only in the tails, but to some extent also in the central part of the distribution. What is more, the lower (respectively, upper) tail seems thicker (respectively, thinner) than its counterpart in the lognormal fit. This is confirmed by a magnification of the lower and upper tail behavior of the cumulative distribution function (CDF), see the right panels of Fig. 4. Notice how the CDF of aggregate 2004 HCE data is larger (respectively, smaller) than the associated log-normal fit for the smallest (respectively, largest) 30% of the distribution.

A more robust way to assess the relative thickness of the distribution tails is to take a non-parametric perspective and apply the Hill estimator (Hill 1975). Table 4 reports estimates for Hill's tail parameter α together with optimal tail sizes k^* and asymptotic standard deviations.[11] It is easy to see that right tails appear to be thinner than left ones, as the associated point estimator for α turns out to be larger. Notice, however, that standard deviations of α-estimates are in general relatively large. This is because the procedure to optimally choose the cutoff parameter k^* often selects a relatively small tail size (on this point, cf. Embrechts et al. 1997, p. 341). In fact, the simple stability statistic suggested by Loretan and Phillips (1994) to test for the difference

[11] In order to select the most appropriate value for the cutoff (tail-size) parameter (k^*), we have employed here the procedure discussed in Drees and Kaufmann (1998). See Lux (1996, 2000, 2001) for details. Standard deviations have been computed by exploiting asymptotic normality, see e.g. Hall (1982); Embrechts et al. (1997).

between upper and lower Hill's tail estimates[12] provides mixed indications: for 49% of (aggregate and age-conditioned) distributions, there appears to be no statistically-significant difference between the upper and lower tail estimates, whereas in all other cases the left tail is statistically thicker than the right one (at 5%).[13]

All that hints to the necessity of studying in more detail the possible asymmetries emerging between the lower and upper HCE tails, e.g. by extending our parametric analysis and going beyond a simple two-parameter statistical model. We shall go back to this issue in Sect. 3.4.

3.3 Robustness checks

As discussed in Battistin et al. (2007), consumption and income data generally suffer from under reporting (especially in the tails) and outliers, and Italian data are not an exception (Brandolini 1999). In order to minimize the effect of gross errors and outliers, we have studied distribution moments and normality GoF tests with two alternative strategies.

First, we have employed robust statistics to estimate the moments of HEC distributions (Huber 1981). More specifically, following Battistin et al. (2007), we have used median (MED) and mean absolute deviation (MAD) as robust estimators for location and scale parameters. Furthermore we have estimated the third moment with quartile skewness (Groeneveld and Meeden 1984) and kurtosis using Moors's octile-based robust estimator (Moors 1988). Results confirm, overall, our previous findings. Robust moments for the logs of HEC are stable over time (and within each wave, across age classes). Aggregate and conditioned (logs of) HEC distributions display a significant excess skewness, while robust kurtosis values are statistically different (according to standard bootstrap tests) from their expected value in normal samples (i.e. 1.233). Again, upper tails appear to be in general relatively light. Furthermore, we have computed normality tests on logs of consumption distributions standardized using robust statistics. More formally, for any given logged consumption distribution $\{c\}$, we have computed Lilliefors, Jarque-Bera and Quadratic Anderson–Darling tests on:

$$\tilde{c} = \frac{c - MED(c)}{MAD(c)}. \tag{1}$$

Table 5 reports p-values for the three tests in the aggregate and age-conditioned cases. Overall, results confirm the evidence obtained without robust standardization: aggregate distributions are never log-normal, whereas now for 11 conditioned HCE

[12] That is $(\hat{\gamma}_r - \hat{\gamma}_l)/[\sigma^2(\hat{\gamma}_l) + \sigma^2(\hat{\gamma}_r)]^{1/2}$, where $\hat{\gamma}_r = \hat{\alpha}_r^{-1}$ and $\hat{\gamma}_l = \hat{\alpha}_l^{-1}$ are the reciprocal of right and left Hill tail estimates, and $\sigma^2(\cdot)$ is their sample variance.

[13] The results of the Hill-estimator analysis must be taken with a little caution (see the discussion in Pictet et al. 1998). Indeed, it is known that asymptotic results are valid only under a number of regularity conditions that are often impossible to test in finite samples. This may introduce a relevant bias in the analysis. Furthermore, and most important here, the Hill statistic is a maximum-likelihood estimator only if some stringent parametric assumptions on the underlying HCE distributions are made, which may not necessarily apply to the case studied in this paper.

Table 4 Estimates for Hill's α tail statistic

(a) Left Tail

Age Class	Statistic	1989	1991	1993	1995	1998	2000	2002	2004
		Waves							
Aggregate	$\hat{\alpha}$	2.545	2.248	3.089	2.813	2.154	2.804	2.667	2.927
	k^*	180	165	133	345	388	226	138	181
	$\sigma(\hat{\alpha})$	0.190	0.175	0.268	0.151	0.109	0.187	0.227	0.218
≤ 30	$\hat{\alpha}$	1.762	2.285	2.415	2.194	2.143	2.507	2.099	2.765
	k^*	201	111	78	127	145	84	71	69
	$\sigma(\hat{\alpha})$	0.124	0.217	0.273	0.195	0.178	0.274	0.249	0.333
31-35	$\hat{\alpha}$	1.829	1.709	2.227	1.934	1.682	1.803	1.737	1.973
	k^*	54	156	79	137	177	148	161	235
	$\sigma(\hat{\alpha})$	0.249	0.137	0.251	0.165	0.126	0.148	0.137	0.129
36-40	$\hat{\alpha}$	1.822	1.759	2.185	2.067	2.015	2.021	2.086	2.287
	k^*	90	131	118	168	104	120	130	268
	$\sigma(\hat{\alpha})$	0.192	0.154	0.201	0.159	0.198	0.185	0.183	0.140
41-45	$\hat{\alpha}$	2.275	2.113	1.919	2.143	2.535	2.171	1.964	2.430
	k^*	61	78	97	129	78	116	125	309
	$\sigma(\hat{\alpha})$	0.291	0.239	0.195	0.189	0.287	0.202	0.176	0.138
46-50	$\hat{\alpha}$	1.845	2.132	2.166	2.125	2.147	2.363	1.976	2.526
	k^*	50	59	102	110	114	85	99	180
	$\sigma(\hat{\alpha})$	0.261	0.278	0.215	0.203	0.201	0.256	0.199	0.188
51-55	$\hat{\alpha}$	1.925	1.496	1.981	2.175	2.561	1.882	1.655	2.335
	k^*	62	126	94	110	77	163	167	304
	$\sigma(\hat{\alpha})$	0.244	0.133	0.204	0.207	0.292	0.147	0.128	0.134
56-60	$\hat{\alpha}$	2.021	2.140	1.922	2.472	2.304	1.812	2.266	2.332
	k^*	68	72	133	63	143	188	83	530
	$\sigma(\hat{\alpha})$	0.245	0.252	0.167	0.311	0.193	0.132	0.249	0.101
≥ 61	$\hat{\alpha}$	2.172	1.858	2.083	1.850	2.307	2.504	1.835	2.805
	k^*	61	107	118	111	85	48	120	331
	$\sigma(\hat{\alpha})$	0.278	0.180	0.192	0.176	0.250	0.361	0.168	0.154

(b) Right Tail

Age Class	Statistic	1989	1991	1993	1995	1998	2000	2002	2004
		Waves							
Aggregate	$\hat{\alpha}$	3.296	3.913	3.478	3.099	2.524	3.546	3.583	3.151
	k^*	540	495	399	1 035	1 164	678	414	543
	$\sigma(\hat{\alpha})$	0.142	0.176	0.174	0.096	0.074	0.136	0.176	0.135
≤ 30	$\hat{\alpha}$	4.029	2.635	3.434	3.084	3.578	3.037	2.696	2.800
	k^*	106	68	68	93	61	82	115	194
	$\sigma(\hat{\alpha})$	0.391	0.320	0.416	0.320	0.458	0.335	0.251	0.201
31-35	$\hat{\alpha}$	2.387	2.122	2.027	2.277	2.934	2.329	2.455	2.480
	k^*	88	73	158	179	81	88	116	351
	$\sigma(\hat{\alpha})$	0.254	0.161	0.217	0.170	0.326	0.248	0.228	0.132
36-40	$\hat{\alpha}$	2.467	2.848	5.725	3.562	2.462	3.581	3.082	3.144
	k^*	67	54	41	58	122	63	74	130
	$\sigma(\hat{\alpha})$	0.301	0.388	0.894	0.468	0.223	0.451	0.358	0.276
41-45	$\hat{\alpha}$	2.326	3.491	2.511	2.924	3.995	2.702	3.116	3.263
	k^*	100	69	134	102	77	139	97	141
	$\sigma(\hat{\alpha})$	0.233	0.420	0.217	0.290	0.455	0.229	0.316	0.275
46-50	$\hat{\alpha}$	2.755	2.743	2.253	2.412	2.365	2.707	2.468	2.876
	k^*	69	107	146	150	181	194	144	230
	$\sigma(\hat{\alpha})$	0.332	0.265	0.186	0.197	0.176	0.194	0.206	0.190
51-55	$\hat{\alpha}$	2.868	2.410	3.002	3.043	2.595	2.579	2.705	3.218
	k^*	87	115	81	100	165	144	119	152
	$\sigma(\hat{\alpha})$	0.308	0.225	0.334	0.304	0.202	0.215	0.248	0.261
56-60	$\hat{\alpha}$	2.410	2.427	2.653	2.971	2.316	2.709	2.482	3.025
	k^*	209	93	103	94	118	111	119	150
	$\sigma(\hat{\alpha})$	0.167	0.252	0.261	0.306	0.213	0.257	0.228	0.247
≥ 61	$\hat{\alpha}$	2.081	2.517	3.395	2.227	2.811	3.292	2.568	3.238
	k^*	79	71	61	156	78	54	191	171
	$\sigma(\hat{\alpha})$	0.234	0.299	0.435	0.178	0.318	0.448	0.186	0.248

Left and right tail estimates for aggregate and age-conditioned HCE distributions versus waves are reported together with the optimal tail size k^* (selected with the Drees and Kaufmann (1998) procedure) and asymptotic standard deviations $\sigma(\hat{\alpha})$ of α tail statistic estimates

Table 5 Robustly-standardized distributions

		Waves							
Age Class	Test	1989	1991	1993	1995	1998	2000	2002	2004
Aggregate	Lilliefors	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.000
	JB	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	AD2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
≤ 30	Lilliefors	0.000	**0.114**	0.112	0.008	0.071	0.027	0.003	0.101
	JB	0.000	**0.074**	0.023	0.027	0.032	0.039	0.042	0.055
	AD2	0.000	**0.057**	0.022	0.000	0.119	0.006	0.003	0.013
31-35	Lilliefors	0.008	0.030	0.003	0.438	0.068	0.065	0.006	0.010
	JB	0.001	0.009	0.006	0.164	0.016	0.029	0.047	0.011
	AD2	0.000	0.001	0.000	0.097	0.041	0.074	0.003	0.000
36-40	Lilliefors	0.001	0.000	0.118	**0.333**	**0.459**	**0.566**	0.008	0.000
	JB	0.006	0.014	0.000	**0.108**	**0.213**	**0.405**	0.011	0.010
	AD2	0.000	0.000	0.001	**0.169**	**0.431**	**0.539**	0.000	0.000
41-45	Lilliefors	0.000	0.021	0.001	0.020	0.210	0.048	0.001	**0.602**
	JB	0.002	0.035	0.002	0.012	0.032	0.024	0.017	**0.875**
	AD2	0.000	0.002	0.000	0.001	0.037	0.001	0.001	**0.610**
46-50	Lilliefors	0.018	0.077	0.045	**0.058**	**0.268**	0.018	0.000	0.026
	JB	0.003	0.051	0.082	**0.101**	**0.116**	0.017	0.001	0.029
	AD2	0.001	0.012	0.028	**0.079**	**0.164**	0.003	0.000	0.007
51-55	Lilliefors	0.001	**0.358**	0.020	0.361	0.023	0.019	0.019	0.000
	JB	0.010	**0.234**	0.010	0.026	0.018	0.011	0.149	0.029
	AD2	0.000	**0.114**	0.001	0.029	0.001	0.005	0.018	0.000
56-60	Lilliefors	0.000	0.064	0.003	**0.449**	**0.465**	0.036	**0.474**	0.029
	JB	0.027	0.140	0.012	**0.240**	**0.138**	0.081	**0.233**	0.057
	AD2	0.001	0.016	0.000	**0.516**	**0.521**	0.044	**0.233**	0.022
≥ 61	Lilliefors	0.143	0.000	0.001	0.000	0.010	0.043	0.000	0.000
	JB	0.003	0.010	0.000	0.000	0.003	0.001	0.000	0.000
	AD2	0.003	0.000	0.000	0.000	0.002	0.003	0.000	0.000

Monte-Carlo (one-tailed) p-values for goodness-of-fit tests. Null hypothesis: normal logs of HCE distributions. *JB* Jarque-Bera test, *AD2* Anderson–Darling test. Boldface entries: The null hypothesis is never rejected (at 5%)

distributions log-normality cannot be rejected (2 more than before). All this implies that existing statistically-significant departures from log-normality are not due to outliers.

Second, we used both our original data and robustly-standardized samples to compute normality tests on sub-samples of logged HEC distributions obtained by truncating the upper or lower tail. More precisely, we defined left- and right-truncated distributions by cutting either the lower or the upper $x\%$ of the distribution. We then ran standard truncated-GoF normality tests (Chernobay et al. 2005) by allowing $x \in \{5, 10, \ldots, 25, 30\}$. In all our exercises (not shown), we ended up with p-values which were *even lower* than those obtained for the full samples, irrespective of whether the $x\%$ of the lower or the upper tail was removed.

3.4 Fitting asymmetric exponential-power densities to HCE distributions

The foregoing analysis shows that HCE distributions can hardly be described by means of log-normal distributions. In other words, the same family of two-parameter density is not able to describe the existing, statistically-detectable, heterogeneity in HCE distributions. This is true both at the aggregate level and at the age-conditioned level, across years. The underlying cause of this distributional heterogeneity seems to be the presence/absence of: (i) positive/negative skewness; (ii) leptokurtic/platykurtic behavior of the distribution as a whole. However, it may well be that any given HCE distribution displays tails that look different between each other. This can happen if e.g. the upper (respectively, lower) tail is thinner (respectively, thicker) than a normal one. Furthermore, HCE distributions might exhibit a variability that is larger on the right (left) of their median or modal value than it is on its left (right).

To possibly accommodate all these departures from a well-behaved log-normal statistical model, we propose here to fit *the logs of* HCE distributions with a higher-parameterized, more flexible, distribution family known as the *asymmetric exponential power* (AEP). The density of the AEP family[14] reads:

$$g(x; a_l, a_r, b_l, b_r, m) = \begin{cases} K^{-1} e^{-\frac{1}{b_l}\left|\frac{x-m}{a_l}\right|^{b_l}}, & x < m \\ K^{-1} e^{-\frac{1}{b_r}\left|\frac{x-m}{a_r}\right|^{b_r}}, & x \geq m \end{cases}, \tag{2}$$

where $K = a_l b_l^{1/b_l} \Gamma(1 + 1/b_l) + a_r b_r^{1/b_r} \Gamma(1 + 1/b_r)$, and Γ is the Gamma function. The AEP features five parameters. The parameter m controls for location. The two a's parameters control for scale to the left (a_l) and to the right (a_r) of m. Larger values for a's imply—*coeteris paribus*—a larger variability. Finally, the two b's parameters govern the left (b_l) and right (b_r) tail behavior of the distribution. To illustrate this point, let us start with the case of a symmetric EP, i.e. when $a_l = a_r = a$ and $b_l = b_r = b$. It is easy to check that if $b = 2$, the EP boils down to the normal distribution. In that case, the correspondent HCE distribution would be log-normal. If $b < 2$, the EP displays tails thicker than a normal one, but still not heavy. In fact, for $b < 2$, the EP configures itself as a medium-tailed distribution, for which all moments exist. In the case $b = 1$ we recover the Laplace distribution. Finally, for $b > 2$ the EP features tails thinner than a normal one but still exponential.

It is easy to see that when one allows for different left-right a- and b-parameters, the AEP can encompass a wealth of different shapes. In Fig. 5 we plot the log-density[15] of the AEP in both the symmetric and asymmetric case for different parameter values. Notice how the AEP can easily pick up across-distribution heterogeneity in skewness and kurtosis, but also within-distribution heterogeneity concerning variance and tail-thickness behaviors.

In what follows, we fit AEP densities to both aggregate and age-conditioned *logged* HCE distributions. Parameters are estimated via maximum likelihood.[16]

To check whether logs of HCE distributions can be satisfactorily described by AEP densities, we firstly estimate AEP parameters via ML and then we employ a battery

[14] The AEP density turns out to be a very good statistical model for many economic variables, like firm growth-rates and market-price returns. See Bottazzi and Secchi (2006b) and references cited therein for details.

[15] Thus, in the normal case one would end up with a parabolic log-density shape.

[16] See Agrò (1995) and Bottazzi and Secchi (2006b) for technical details. Estimation has been carried out with the package SUBBOTOOLS, available online at http://www.cafed.eu.

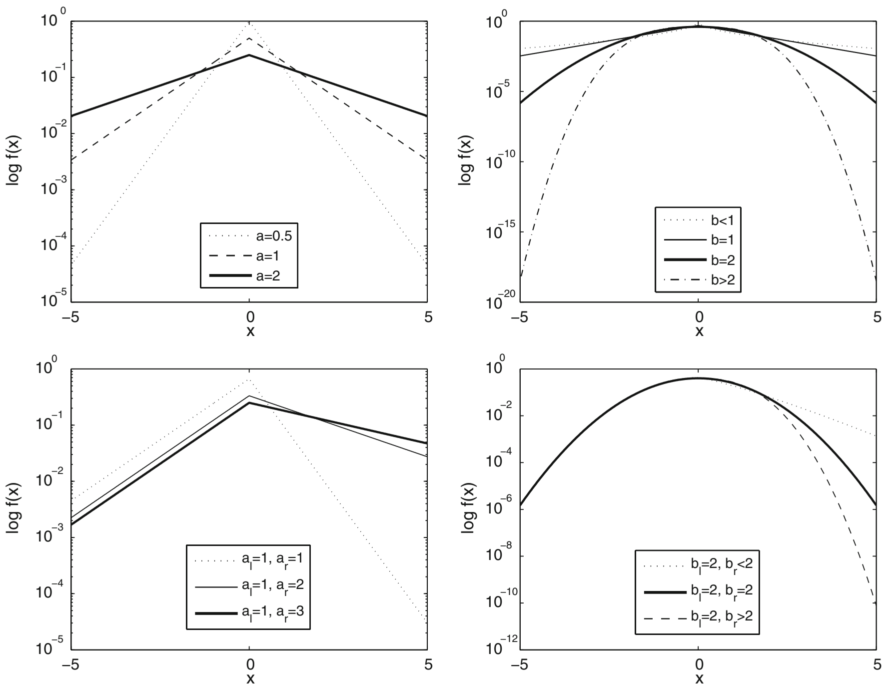

Fig. 5 The asymmetric exponential-power density (logs) for different parameter values. *Top-Left* Symmetric case for increasing shape-parameter (a). Other parameter values: $m = 0$, $b = 1$. *Top-Right* Symmetric case for increasing tail-parameter (b). Other parameter values: $m = 0$, $a = 1$. *Bottom-Left* Asymmetric case for increasing right shape-parameter (a_r). Other parameter values: $m = 0$, $b_l = b_r = 1$. *Bottom-Right* Asymmetric case for increasing tail-parameter (b_r). Other parameter values: $m = 0$, $a_l = a_r = 1$

of GoF tests based on empirical distribution function (EDF) statistics. More specifically, we run three widely-used EDF-based GoF tests: Kuiper (KUI), Cramér-Von Mises (CVM) and Quadratic Anderson–Darling (AD2), with small-sample modifications usually considered in the literature (the AD2 test statistic employed here is analogous to that used above to test for normality: for more formal definitions, see D'Agostino and Stephens 1986, Chapter 4, Table 4.2). We also compute a KS test to benchmark our results to those in Battistin et al. (2007). Notice, however, that the former three tests are known to perform better—in terms of e.g. power—than the KS in all practical situations (Thode 2002). All p-values for the test statistics are computed by running Monte-Carlo simulations (1,000 replications) under the null hypothesis that the empirical sample comes from an AEP with unknown parameters; see Capasso et al. (2009) for a discussion.

Table 6 reports test statistics and Monte-Carlo p-values for the four GoF tests. Notice how AEP fits perform dramatically better than normal ones. p-Values are almost always larger than 0.05, meaning that (if one takes 5% as the relevant significance level) in almost all the cases the logs of HCE distributions can be statistically described by AEP densities (and not by normal distributions). There are only three exceptions to this rule (in boldface). Indeed, for the age class 41–45, all four tests are rejected at 5% for the 1989 and 1995 waves. Borderline cases (still at 5%) are represented by the aggregate distribution and the 51–55 age class in 1989. However, if one lowers the significance level at 1% in almost every situation all four tests pass.

The main reason why AEP distributions are able to better explain, from a statistical perspective, the logs of HCE distributions appears to be evident from Table 7, where ML estimates of AEP parameters are shown. Let us focus on $\hat{a}$'s and $\hat{b}$'s parameter point estimates (the estimate for the location parameter m is not relevant, as it closely tracks the mode of the distribution). It is easy to see that in about 79% of cases (i.e., 57/72) the right tail of the distribution appears to be thinner than the left tail ($\hat{b}_r > \hat{b}_l$). Furthermore, about 89% of all distributions display a right tail which is thinner than a normal one ($\hat{b}_r > 2$). Conversely, in 47% of times $\hat{b}_l < 2$, meaning that the left tail is thicker than a normal one. Right thin tails are also associated to higher $\hat{a}$'s parameters. In fact, about 90% of distributions obey the condition $\hat{a}_r > \hat{a}_l$.

To assess more robustly whether the left tail parameter is statistically smaller than the right tail parameter (i.e., whether the lower tail is thicker than the upper tail), one should firstly study the distributions of $\hat{b}_r$ and $\hat{b}_l$ estimators; then accordingly compute confidence intervals; and finally perform hypothesis-testing exercises. Unfortunately, the analytical distributions of the estimators are not available and can only be proxied using simulation. As this typically requires a huge computational effort (cf. for example Fagiolo et al. 2008), here we take a more direct (but less precise) approach and we employ the results in Bottazzi and Secchi (2006b) to compute the asymptotic standard errors (SEs) of all our parameter estimates (see Table 7, in round brackets). Notice that SEs are not always negligible, especially for tail parameters. More precisely, let us follow Agrò (1995) and take as a reference Cramér-Rao confidence intervals (CRCI) of the form $[\hat{b}_* - 2SE(\hat{b}_*), \hat{b}_* + 2SE(\hat{b}_*)]$, where $* \in \{r, l\}$. Simple back-of-the-envelope calculations show that only in 13 out of 72 cases (18%) either $\hat{b}_r$ belongs to the CRCI of $\hat{b}_l$ or viceversa. This signals that, for such distributions, more rigorous small-sample hypothesis tests could possibly accept the hypothesis that the tail parameters are equal. Notice, however, that only 8 out of these 13 cases refer to distributions for which $\hat{b}_l < \hat{b}_r$. Hence, in the 86% (i.e., (57-8)/57) of cases where point estimates satisfy $\hat{b}_l < \hat{b}_r$, this inequality appears to be also statistically robust.[17] Furthermore, as far as aggregate logged HCE distributions are concerned, the inclusion of either point estimate in the opposite CRCI arises only in one single case (i.e., in 1989 $\hat{b}_r$ belongs to the CRCI of $\hat{b}_l$). This may indicate that the impossibility to reject tail-estimate equality could indeed depend on the relatively smaller sample size of age-conditioned HCE distributions: a further look at the distributional properties of the estimators in finite samples seems therefore required.[18]

[17] A more stringent criterion is to check for the occurrences of overlaps between the two CRCIs. This happens for 17 distributions only, 12 of which satisfy $\hat{b}_l < \hat{b}_r$.

[18] A possible, but very computationally-expensive, way to address this issue (and one of the first points in our agenda) would be to set up log-likelihood (LL) ratio tests where one compares the LL obtained under the null hypothesis of symmetric EP fit versus the LL obtained by fitting the AEP. By simulating the distribution of the LL ratio test, one could check whether the relative gain in fitting an AEP to the data is statistically significant or not. In the positive case, this procedure can allow one to more-robustly conclude that $\hat{b}_r \neq \hat{b}_l$ (for an application to data with smaller sample sizes, cf. Fagiolo et al. 2008).

Table 6 Goodness-of-fit tests for the null hypothesis of AEP-distributed logs of HCEs

Age Class	Test	Waves 1989	1991	1993	1995	1998	2000	2002	2004
Aggregate	KS	1.146 (0.061)	0.722 (0.094)	0.713 (0.104)	0.666 (0.157)	0.533 (0.457)	1.016 (0.064)	0.489 (0.599)	0.640 (0.198)
	KUI	1.904 (0.084)	1.146 (0.137)	1.212 (0.075)	1.257 (0.052)	1.064 (0.228)	1.634 (0.082)	0.950 (0.443)	1.249 (0.053)
	CVM	0.250 (0.003)	0.074 (0.054)	0.080 (0.037)	0.062 (0.101)	0.055 (0.139)	0.157 (0.052)	0.039 (0.325)	0.071 (0.063)
	AD2	1.586 (0.074)	0.438 (0.039)	0.792 (0.062)	0.428 (0.054)	0.385 (0.072)	0.920 (0.071)	0.243 (0.348)	0.570 (0.098)
≤ 30	KS	0.408 (0.868)	0.520 (0.501)	0.401 (0.889)	0.857 (0.024)	0.844 (0.551)	0.531 (0.461)	0.688 (0.129)	0.667 (0.226)
	KUI	0.789 (0.786)	0.971 (0.399)	0.798 (0.777)	1.236 (0.061)	1.271 (0.422)	0.988 (0.356)	1.186 (0.097)	1.151 (0.159)
	CVM	0.018 (0.861)	0.041 (0.280)	0.030 (0.509)	0.135 (0.004)	0.132 (0.688)	0.026 (0.608)	0.053 (0.154)	0.085 (0.131)
	AD2	0.204 (0.504)	0.278 (0.242)	0.191 (0.560)	0.646 (0.004)	0.682 (0.571)	0.206 (0.498)	0.280 (0.240)	0.316 (0.226)
31-35	KS	0.544 (0.416)	0.499 (0.577)	0.495 (0.584)	0.603 (0.272)	0.742 (0.345)	0.768 (0.059)	0.749 (0.958)	0.411 (0.860)
	KUI	0.984 (0.366)	0.887 (0.598)	0.980 (0.373)	1.018 (0.302)	1.396 (0.218)	1.383 (0.079)	1.410 (0.819)	0.788 (0.791)
	CVM	0.035 (0.391)	0.025 (0.640)	0.037 (0.350)	0.046 (0.217)	0.101 (0.314)	0.075 (0.053)	0.088 (0.795)	0.018 (0.864)
	AD2	0.237 (0.371)	0.225 (0.416)	0.240 (0.356)	0.257 (0.296)	0.445 (0.198)	0.436 (0.059)	0.461 (0.684)	0.154 (0.771)
36-40	KS	0.621 (0.235)	0.665 (0.158)	0.457 (0.702)	0.481 (0.626)	0.663 (0.163)	0.515 (0.518)	0.686 (0.132)	0.875 (0.050)
	KUI	1.139 (0.143)	1.270 (0.050)	0.893 (0.586)	0.941 (0.468)	1.072 (0.215)	0.793 (0.784)	1.215 (0.070)	1.455 (0.067)
	CVM	0.074 (0.054)	0.056 (0.135)	0.026 (0.612)	0.022 (0.753)	0.039 (0.323)	0.031 (0.486)	0.063 (0.098)	0.068 (0.080)
	AD2	0.511 (0.015)	0.326 (0.143)	0.248 (0.328)	0.168 (0.687)	0.208 (0.483)	0.208 (0.488)	0.367 (0.090)	0.443 (0.048)
41-45	KS	**0.855 (0.024)**	0.520 (0.504)	0.502 (0.568)	**0.853 (0.025)**	0.328 (0.982)	1.416 (0.994)	0.694 (0.126)	0.423 (0.828)
	KUI	**1.562 (0.004)**	0.858 (0.643)	0.880 (0.609)	**1.337 (0.030)**	0.650 (0.973)	2.150 (1.001)	1.012 (0.313)	0.741 (0.880)
	CVM	**0.129 (0.004)**	0.028 (0.559)	0.028 (0.566)	**0.082 (0.035)**	0.015 (0.940)	0.308 (1.021)	0.033 (0.428)	0.023 (0.702)
	AD2	**0.785 (0.002)**	0.158 (0.747)	0.256 (0.302)	**0.506 (0.016)**	0.147 (0.809)	1.668 (0.774)	0.208 (0.486)	0.216 (0.451)
46-50	KS	0.624 (0.227)	0.470 (0.667)	0.835 (0.700)	0.849 (0.781)	0.428 (0.815)	0.687 (0.132)	0.833 (0.178)	0.810 (0.241)
	KUI	1.140 (0.143)	0.822 (0.732)	1.263 (0.719)	1.272 (0.623)	0.825 (0.720)	1.115 (0.163)	1.211 (0.145)	1.242 (0.205)
	CVM	0.065 (0.089)	0.024 (0.691)	0.109 (0.667)	0.179 (0.827)	0.021 (0.795)	0.085 (0.025)	0.091 (0.056)	0.125 (0.056)
	AD2	0.391 (0.066)	0.168 (0.690)	0.600 (0.679)	0.683 (0.730)	0.197 (0.537)	0.454 (0.034)	0.618 (0.087)	0.670 (0.029)
51-55	KS	**0.829 (0.033)**	0.460 (0.691)	0.533 (0.458)	0.438 (0.774)	0.650 (0.180)	0.476 (0.649)	0.498 (0.579)	0.671 (0.153)
	KUI	**1.268 (0.047)**	0.816 (0.745)	0.921 (0.513)	0.855 (0.649)	1.159 (0.125)	0.899 (0.571)	0.942 (0.467)	1.093 (0.186)
	CVM	**0.073 (0.048)**	0.021 (0.792)	0.038 (0.337)	0.028 (0.559)	0.064 (0.095)	0.035 (0.394)	0.029 (0.525)	0.046 (0.215)
	AD2	**0.489 (0.022)**	0.230 (0.397)	0.308 (0.175)	0.245 (0.337)	0.392 (0.064)	0.230 (0.397)	0.238 (0.366)	0.346 (0.116)
56-60	KS	0.893 (0.057)	0.796 (0.042)	0.478 (0.637)	0.401 (0.889)	0.372 (0.944)	0.870 (0.022)	0.381 (0.925)	0.365 (0.950)
	KUI	1.316 (0.038)	1.411 (0.015)	0.858 (0.644)	0.781 (0.808)	0.714 (0.912)	1.211 (0.075)	0.755 (0.852)	0.729 (0.894)
	CVM	0.094 (0.068)	0.076 (0.051)	0.028 (0.549)	0.023 (0.723)	0.022 (0.732)	0.052 (0.154)	0.017 (0.906)	0.019 (0.852)
	AD2	0.515 (0.074)	0.390 (0.066)	0.250 (0.321)	0.140 (0.854)	0.172 (0.667)	0.275 (0.247)	0.133 (0.891)	0.150 (0.797)
≥ 61	KS	0.741 (0.078)	0.517 (0.514)	0.464 (0.683)	0.448 (0.730)	0.715 (0.103)	0.345 (0.972)	0.636 (0.202)	0.786 (0.068)
	KUI	1.401 (0.056)	0.990 (0.353)	0.823 (0.729)	0.884 (0.602)	1.363 (0.054)	0.675 (0.956)	1.144 (0.139)	1.202 (0.083)
	CVM	0.082 (0.034)	0.042 (0.265)	0.028 (0.559)	0.024 (0.692)	0.079 (0.042)	0.017 (0.898)	0.043 (0.260)	0.058 (0.118)
	AD2	0.542 (0.098)	0.287 (0.217)	0.207 (0.491)	0.173 (0.662)	0.405 (0.058)	0.138 (0.869)	0.249 (0.326)	0.361 (0.095)

KS Kolmogorow-Smirnov test, *KUI* Kuiper test, *CVM* Cramér-Von Mises test, *AD2* Quadratic Anderson–Darling test. Monte-Carlo (one-tailed) p-values in round brackets. Boldface entries: The null hypothesis is always rejected (at 5%)

Table 7 Maximum-likelihood estimates for AEP parameters

Age Class	Par Est (Std Err)	Waves 1989	1991	1993	1995	1998	2000	2002	2004
Aggregate	b_l	2.1100 (0.0577)	1.4800 (0.0353)	1.9600 (0.0583)	2.0600 (0.0690)	2.8000 (0.0920)	2.0000 (0.0637)	1.7200 (0.0570)	2.2300 (0.0357)
	b_r	2.1600 (0.0685)	2.8400 (0.0685)	2.2300 (0.0710)	2.7600 (0.1030)	2.2200 (0.1135)	2.6000 (0.0925)	3.0000 (0.0815)	2.4500 (0.0420)
	a_l	0.8790 (0.0361)	0.6840 (0.0220)	0.7650 (0.0338)	0.7540 (0.0389)	1.0700 (0.0545)	0.7640 (0.0369)	0.5590 (0.0269)	0.8140 (0.0362)
	a_r	1.2000 (0.0368)	1.6000 (0.0311)	1.2100 (0.0366)	1.2100 (0.0443)	0.9340 (0.0497)	1.2100 (0.0417)	1.5300 (0.0352)	1.0200 (0.0350)
	m	4.4800 (0.0535)	3.9300 (0.0378)	4.3500 (0.0515)	4.4400 (0.0590)	4.8300 (0.0735)	4.4400 (0.0565)	4.0100 (0.0437)	4.6400 (0.0520)
≤ 30	b_l	1.8264 (0.0997)	2.6838 (0.1979)	3.4784 (0.1553)	2.8409 (0.2123)	2.3229 (0.2218)	2.0270 (0.1323)	1.6824 (0.1078)	1.4698 (0.1098)
	b_r	1.9099 (0.0638)	2.5343 (0.0955)	2.1067 (0.1008)	4.7809 (0.1409)	5.5946 (0.1323)	5.9859 (0.0900)	4.7128 (0.0992)	4.6543 (0.0849)
	a_l	0.6375 (0.0333)	1.0720 (0.0634)	0.9439 (0.0554)	0.3336 (0.0736)	0.4333 (0.0789)	0.5746 (0.0509)	0.4023 (0.0427)	0.3939 (0.0473)
	a_r	1.1933 (0.0353)	1.3895 (0.0556)	1.1355 (0.0553)	1.9250 (0.0735)	1.9339 (0.0726)	1.8933 (0.0494)	1.8632 (0.0493)	1.8796 (0.0441)
	m	4.0828 (0.0516)	4.3805 (0.0873)	4.6277 (0.0813)	3.6510 (0.1034)	3.8154 (0.1070)	3.9950 (0.0742)	3.8786 (0.0676)	3.9322 (0.0648)
31-35	b_l	2.5369 (0.0953)	1.4394 (0.0940)	1.1079 (0.0439)	1.4773 (0.0834)	1.6434 (0.0812)	1.7550 (0.0961)	2.0736 (0.0460)	2.8661 (0.0355)
	b_r	1.8686 (0.0862)	2.5804 (0.0956)	2.8514 (0.0925)	3.7219 (0.1100)	3.3895 (0.1267)	2.8100 (0.0983)	3.0101 (0.0897)	2.8389 (0.0574)
	a_l	0.8025 (0.1088)	0.5577 (0.0319)	0.3579 (0.0118)	0.5240 (0.0318)	0.5804 (0.0301)	0.7265 (0.0416)	0.6399 (0.0164)	0.4790 (0.0137)
	a_r	1.1243 (0.1069)	1.6217 (0.0458)	1.6403 (0.0267)	1.4988 (0.0469)	1.3814 (0.0466)	1.2432 (0.0494)	1.3074 (0.0307)	1.3745 (0.0222)
	m	4.4728 (0.1507)	3.8696 (0.0553)	3.9560 (0.0214)	4.2383 (0.0550)	4.5930 (0.0529)	4.6114 (0.0668)	4.6097 (0.0310)	4.4728 (0.0248)
36-40	b_l	2.1609 (0.0934)	0.9380 (0.0744)	3.2251 (0.2264)	1.0805 (0.1123)	2.9043 (0.0629)	1.8151 (0.1512)	2.2951 (0.0646)	2.3470 (0.0767)
	b_r	2.0893 (0.0671)	3.0883 (0.1044)	1.6099 (0.0917)	2.8363 (0.0894)	1.6710 (0.0658)	2.5732 (0.1126)	2.5396 (0.0932)	1.8137 (0.0571)
	a_l	0.7643 (0.1124)	0.3794 (0.0176)	0.9303 (0.0620)	0.4707 (0.0342)	1.0649 (0.0292)	0.7106 (0.0563)	0.7197 (0.0197)	0.7209 (0.0296)
	a_r	1.1940 (0.0939)	1.7964 (0.0344)	0.8543 (0.0457)	1.3487 (0.0428)	0.7860 (0.0147)	1.0961 (0.0595)	1.2988 (0.0338)	0.9504 (0.0298)
	m	4.5031 (0.1436)	3.5896 (0.0329)	4.9635 (0.0777)	4.3117 (0.0561)	5.0316 (0.0329)	4.7428 (0.0842)	4.5627 (0.0363)	4.9490 (0.0437)
41-45	b_l	2.5612 (0.0457)	1.4617 (0.1194)	1.6333 (0.0533)	1.8705 (0.1627)	3.8072 (0.2257)	0.6505 (0.1808)	1.2828 (0.1421)	1.6956 (0.1202)
	b_r	2.5086 (0.1945)	3.1394 (0.1894)	2.3128 (0.1037)	3.0148 (0.1545)	2.0921 (0.1281)	5.3062 (0.2161)	3.3669 (0.1500)	2.2922 (0.0891)
	a_l	0.8341 (0.0120)	0.6264 (0.0380)	0.5325 (0.0167)	0.5753 (0.0501)	1.3160 (0.0913)	0.2537 (0.0677)	0.3950 (0.0556)	0.7260 (0.0419)
	a_r	1.2709 (0.0294)	1.7432 (0.0576)	1.3212 (0.0308)	1.3335 (0.0622)	0.8500 (0.0699)	2.1415 (0.0817)	1.6184 (0.0631)	1.0751 (0.0407)
	m	4.6130 (0.0194)	3.9111 (0.0657)	4.3436 (0.0316)	4.4315 (0.0789)	5.2057 (0.1061)	3.6109 (0.1029)	4.0068 (0.0850)	4.8672 (0.0580)
46-50	b_l	2.8096 (0.0979)	1.4084 (0.0953)	1.3657 (0.1841)	1.7577 (0.2901)	1.9047 (0.1388)	2.0853 (0.1283)	2.1231 (0.1797)	1.7792 (0.1110)
	b_r	2.2515 (0.0884)	3.4740 (0.0845)	3.4019 (0.1092)	3.0167 (0.1903)	2.8639 (0.1529)	2.7611 (0.1080)	2.7404 (0.1872)	3.1136 (0.0877)
	a_l	0.9484 (0.1181)	0.5992 (0.0337)	0.6125 (0.0631)	0.6877 (0.1000)	0.7266 (0.0531)	0.7180 (0.0473)	0.7567 (0.0746)	0.6186 (0.0425)
	a_r	1.1956 (0.1098)	1.7433 (0.0407)	1.7715 (0.0489)	1.4712 (0.0810)	1.2562 (0.0638)	1.2382 (0.0335)	1.1886 (0.0798)	1.3936 (0.0404)
	m	4.7793 (0.1502)	4.0615 (0.0461)	4.1525 (0.0809)	4.0397 (0.1237)	4.3729 (0.0832)	4.4334 (0.0563)	4.4656 (0.1095)	4.4622 (0.0590)
51-55	b_l	2.1609 (0.1170)	1.7359 (0.1561)	2.0888 (0.1292)	2.1000 (0.0277)	3.2167 (0.1677)	2.0871 (0.1778)	1.3787 (0.0671)	1.4694 (0.0858)
	b_r	1.9891 (0.1022)	2.3749 (0.1922)	2.2832 (0.1465)	3.4695 (0.1786)	1.5723 (0.1536)	3.1234 (0.1685)	3.5366 (0.1177)	2.0241 (0.0614)
	a_l	0.8357 (0.0876)	0.7851 (0.0628)	0.7684 (0.0495)	0.7527 (0.0094)	1.4808 (0.0595)	0.6823 (0.0604)	0.5149 (0.0252)	0.6065 (0.0323)
	a_r	1.1406 (0.1198)	1.3103 (0.0785)	1.2501 (0.0588)	1.3319 (0.0220)	0.5256 (0.0674)	1.3621 (0.0712)	1.5937 (0.0403)	1.1296 (0.0290)
	m	4.6808 (0.1345)	4.4028 (0.1014)	4.4891 (0.0785)	4.5970 (0.0071)	5.4916 (0.0899)	4.3501 (0.0916)	3.8658 (0.0457)	4.5630 (0.0444)
56-60	b_l	1.8000 (0.1111)	1.0879 (0.1203)	1.2046 (0.2269)	2.0186 (0.0848)	2.2544 (0.2138)	1.2993 (0.0815)	2.1298 (0.1564)	2.6124 (0.0548)
	b_r	2.3144 (0.1564)	2.9803 (0.1892)	2.8425 (0.1547)	2.6568 (0.1199)	2.6424 (0.1293)	3.1377 (0.1295)	2.3666 (0.1396)	1.6653 (0.0549)
	a_l	0.7367 (0.0486)	0.5377 (0.0477)	0.4703 (0.0746)	0.7786 (0.0213)	0.9005 (0.0810)	0.5924 (0.0262)	0.8440 (0.0630)	1.1678 (0.0150)
	a_r	1.2507 (0.0718)	1.6693 (0.0644)	1.5856 (0.0786)	1.1368 (0.0365)	1.1019 (0.0712)	1.4766 (0.0414)	1.1392 (0.0665)	0.6601 (0.0218)
	m	4.3758 (0.0793)	3.8507 (0.0580)	3.7869 (0.1087)	4.5036 (0.0385)	4.6072 (0.1075)	4.1829 (0.0463)	4.3743 (0.0937)	5.2561 (0.0255)
≥ 61	b_l	2.2891 (0.0160)	1.2890 (0.0649)	2.1234 (0.1909)	2.6800 (0.1079)	2.5814 (0.1423)	2.5327 (0.0761)	1.5162 (0.1196)	2.6018 (0.0886)
	b_r	2.7515 (0.0812)	3.1477 (0.0844)	2.2968 (0.1042)	2.8594 (0.1203)	2.7732 (0.1599)	2.9858 (0.0754)	3.0719 (0.1011)	2.9253 (0.0521)
	a_l	1.1169 (0.1226)	0.6045 (0.1199)	0.8207 (0.0582)	0.8811 (0.0435)	0.9528 (0.0649)	0.9494 (0.0274)	0.4568 (0.0479)	0.8324 (0.0286)
	a_r	1.1290 (0.1042)	1.7444 (0.1210)	1.1313 (0.0518)	1.0749 (0.0515)	1.0168 (0.0539)	0.9387 (0.0347)	1.6372 (0.0379)	0.9385 (0.0251)
	m	4.4717 (0.1508)	3.5217 (0.1642)	4.2002 (0.0816)	4.4132 (0.0704)	4.5531 (0.0718)	4.5833 (0.0466)	3.7797 (0.0623)	4.4634 (0.0390)

Logs of aggregate and age-conditioned HCE distributions versus waves. Standard errors in parentheses

Nevertheless, if one focuses on goodness of fit and point-estimate results, the overall evidence seems to confirm our conjecture about the existence of a double-faced heterogeneity in HCE distributions. On the one hand, HCE distributions, even if they belong to the same family, are characterized, as expected, by very different parameter values. On the other hand, each HCE distribution displays very different structural properties as far as its left and right tails are concerned: the right tail is typically thinner than the left tail. This implies that a two-parameter, log-normal distribution is not enough to statistically model HCE distributions. As a consequence, one needs to employ higher-parameterized distributions that, as happens with the AEP, are able to account for these two levels of heterogeneity.

To further elucidate this point, Fig. 6 plots AEP fits to aggregate HCE distributions in the period considered (similar patterns emerge also for age-conditioned HCE distributions). Notice how AEP densities are able to satisfactorily characterize aggregate data. This is true especially as far as asymmetry in tail thickness of logged density is concerned, a feature that could never be accommodated by a two-parameter density like the normal one. Figure 7 shows instead some examples of AEP fits to age-conditioned distributions (in wave 1991). Again, despite the smaller sample size of age-conditioned HCE distributions, all plots seem quite satisfactory at least from a visual perspective.

4 Discussion

The foregoing findings convey two methodological messages. First, moments estimators (whether they are robust statistics or not) are not always able to shed light on within-distribution heterogeneity. Whereas HCE kurtosis levels hinted to the presence of tails thinner than normal as a whole, EP fits have shown that this could be probably due to the presence of a thick left tail coexisting with a thin right tail. Therefore, our study not only suggests that moments higher than the second one do indeed matter, but it also stresses the importance of investigating the existence of within-distribution asymmetries in higher moments. Second, and relatedly, the use of too generic statistical models (like the log-normal) might hinder the exploration of more subtle statistical properties, as the co-existence of tail asymmetries.

In fact, the AEP turned out to be an extremely flexible, but still parsimonious, family of densities capable of accounting for the extreme heterogeneity found in the data. This is important because without the AEP one should have employed two different distributions in order to account for different left and right tail behaviors. Having a single distribution that does the job is not only more elegant, but also more parsimonious from a statistical viewpoint.

Furthermore, our exercises show that alternative families of densities like the Lévy-stable (Nolan 2006) and the generalized hyperbolic (Barndorff-Nielsen 1977), which can only account for tails thicker than normal ones, are not able to statistically describe our data, in the sense that GoF tests are always rejected. This implies, once again, that a key feature of the logs of HCE distributions is within- and cross-distribution heterogeneity in tail behavior.

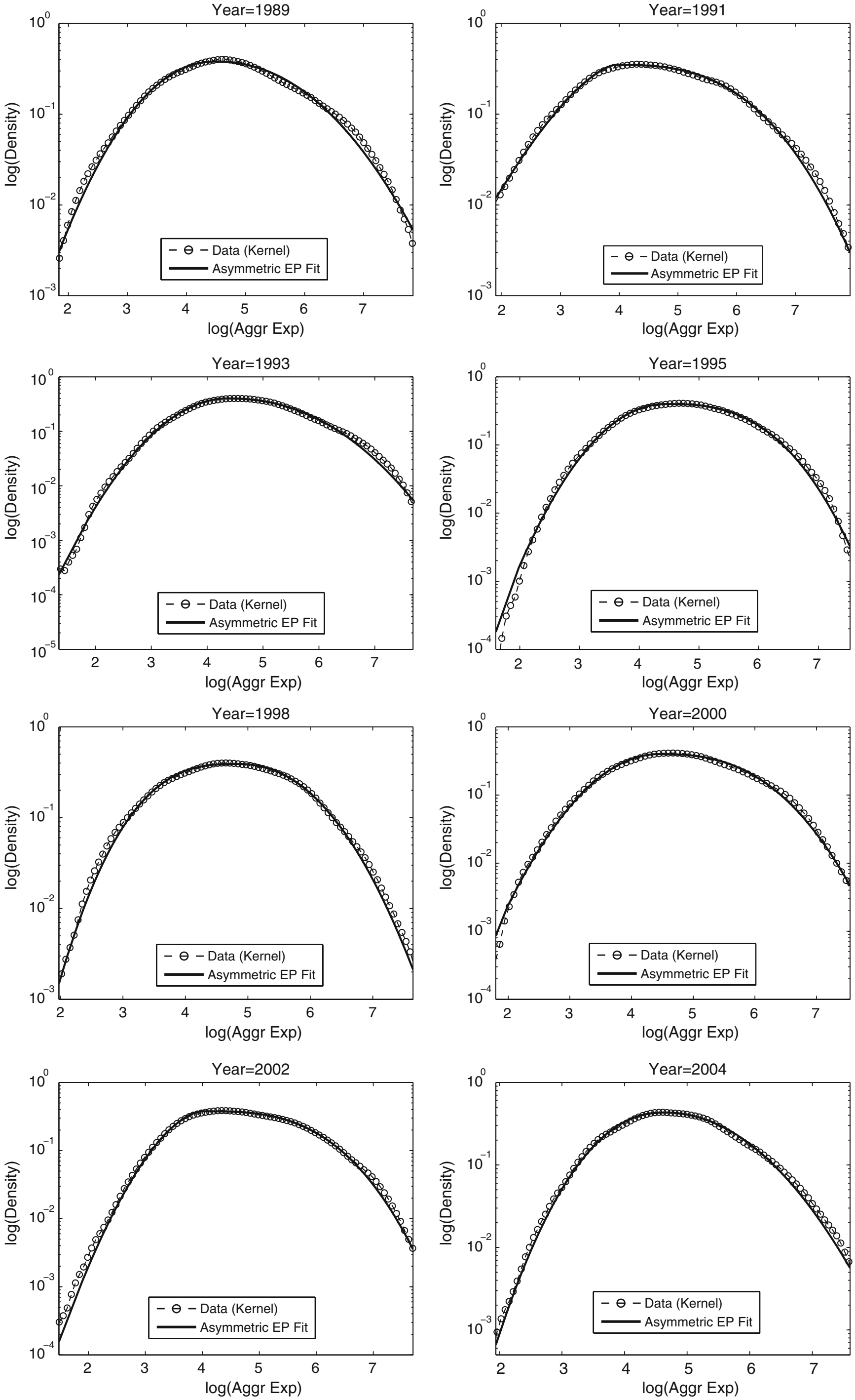

Fig. 6 AEP fits for aggregate HCE distributions

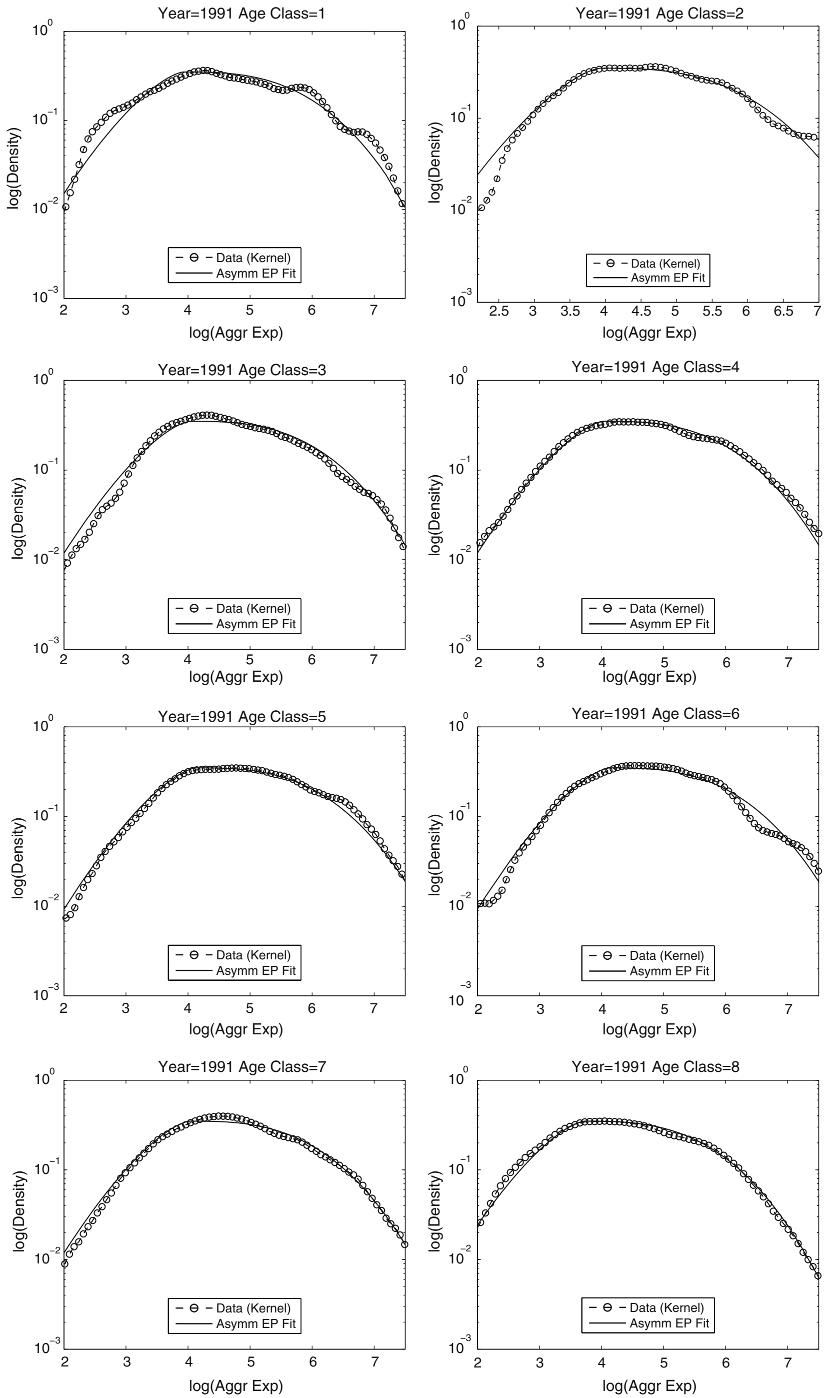

Fig. 7 Some examples of AEP fits for age-conditioned HCE distributions (wave 1991). Age classes: 1:≤30; 2:31–35; 3:36–40; 4:41–45; 5:46–50; 6:51–55; 7:56–60; 8:≥61

No matter whether a symmetric or asymmetric exponential-power fits the data, the existence of tails different from Gaussian ones for the logs of HCE distributions implies that the results in Battistin et al. (2007) do not apply for Italy. Furthermore, even the weaker statement that consumption is more log-normal than income seems to be rejected by our data. Indeed, it is well-established (Clementi and Gallegati 2005b) that Italian household income is log-normal in the body and power-law in the tail. Our results about thick-left and thin-right tails in HCE distributions suggest that, if any, it is income that is more log-normal than consumption. In other words, income and consumption distributions seem to belong to two different density families. Log of income displays a thick upper tail but features a standard-normal lower tail. On the contrary, logged consumption displays a thinner-than-normal upper tail and features a thicker-than-normal lower tail.

What does the foregoing statistical description add to our understanding of consumption? To begin with, a parametric approach can shed light on the nature of the existing heterogeneity in cross-section HCE distributions. The fact that the logs of HCE distributions can be robustly characterized—over the years—by AEP densities with thick lower tails and thin upper tails, enable us to better understand how consumption is distributed across households and to go beyond standard representative-hypothesis assumptions where only the first (and sometimes the second) moment matters. Since heterogeneity has been shown to be of crucial importance for aggregation (Forni and Lippi 1997), a deeper knowledge of HCE distributional properties might hopefully help to better grasp the statistical properties of consumption dynamics at the macro level.

A second important reason why knowledge of cross-section HCE distributional properties may be important is that they can be considered as stylized facts that theoretical models should be able to replicate and explain. Of course, their "unconditional-object" status prevents us from univocally finding *the* generating process: as discussed in Brock (1999), there can be many alternative generating processes that could generate a given unconditional object such as a HCE distribution as their long-run equilibrium.

Nevertheless, knowledge about distributional properties of unconditional objects may help us in restricting the scope of the analysis. In other words, knowing that logs of HCE are not log-normal but AEP-distributed gives us some information on which generating processes one may exclude. For example, rejection of the null hypothesis that consumption is log-normally distributed irrespectively of disaggregation casts some serious doubt on whether some sort of Gibrat's law is at work. If one indeed assumes—like in Battistin et al. (2007)—that a simple multiplicative process applies to permanent income and marginal utility, the resulting limit HCE distributions would be log-normal. If this were the stylized fact to be replicated, one would have come very close to Champernowne's indication about the strategy to follow in modeling economic phenomena (in his case, income dynamics):

> The forces determining the distribution of incomes in any community are so varied and complex, and interact and fluctuate so continuously, that any theoretical model must either be unrealistically simplified or hopelessly complicated. We shall choose the former alternative but then give indications that the introduction

of some of the more obvious complications of the real world does not seem to disturb the general trend of our conclusions. (Champernowne 1953, p. 319)

On the contrary, it appears that our results suggest to leave the "unrealistically simplified" domain: if stylized facts to be replicated are much more complicated than what a Gibrat's law would have implied, also their theoretical explanation in terms of Gibrat's dynamics could hardly hold.

However, the fact that our data reject simple multiplicative models *à la* Gibrat does not necessarily mean that the alternative explanation is "hopelessly complicated". The idea is that a fruitful strategy to replicate and explain what we have found in Italian data is to build simple-enough stochastic equilibrium models like those developed in industrial dynamics to explain the emergence of power-laws and Laplace distributions for firm size and growth rates (Bottazzi and Secchi 2006a; Fu et al. 2005). Along similar lines, one might attempt to single out the basic forces driving the generating process of cross-section distributions (e.g., imitation, innovation, fulfilment of basic needs, optimal choice under budget constraints, etc.) and find necessary conditions for the emergence of the observed stylized facts.[19]

5 Conclusions

In this paper we have studied the statistical properties of Italian household consumption expenditure (HCE) distributions. We have found that, contrary to Battistin et al. (2007), HCE distributions, both in the aggregate and within homogeneous age classes, are not log-normally distributed. Goodness-of-fit tests have allowed us to conclude that HCE distributions can be well approximated by asymmetric exponential-power densities. We have shown that departures from log-normality are due to a pervasive heterogeneity that is present at two different levels. First, given the same parametric model, consumption distributions differ, as expected, in their parameters. Second, within each given distribution, the lower tail behaves differently from the upper tail. More specifically, our results indicate that, in the majority of cases, logged HCE distributions display a thick lower tail coexisting with a thin upper tail. These results hold vis-à-vis a series of further checks involving e.g. robust-statistic estimation.

The fact that the AEP performs better than a normal density in statistically describing HCE distributions suggests not only that higher moments matter, but also that within-distribution asymmetries in tail behavior can be important. It must be also noticed that the AEP density has been extensively used to statistically model many economic variables and indicators related to growth rates or returns expressed as differences between log levels (e.g., growth rates of firm size, returns of log prices, etc.; see Bottazzi and Secchi 2006b). To our best knowledge, this is the first time that the AEP density is shown to provide a good approximation for the logs of the levels of a given variable (i.e., consumption expenditures).

Several extensions to the present study can be conceived. First, it would be interesting to apply the same methodology employed above to U.K. and U.S. data to investigate

[19] This is quite in tune with the evolutionary agenda on the economics of micro- and macro-consumption patterns, see for example Aversi et al. (1999). Complementary ideas are in Witt (2001, 2007).

whether one could detect in other countries the same departures found in Italian HCE distributions. This could enable us to compare more thoroughly our findings with those in Battistin et al. (2007).

Second, one might explore the distributional properties of HCE data disaggregated over consumption categories (durables, non-durables, etc.) and study whether the tail behavior observed in the aggregate can be traced back to some particular consumption category or it is the mere effect of aggregation. Furthermore, since HCE distributions for consumption categories are likely to be correlated, it would be interesting to characterize the distributional properties of the joint G-dimensional HCE distribution (where G is the number of observable consumption categories).

Finally, another issue worth to be addressed concerns the statistical characterization of consumption budget-share (CBS) distributions (where for any given household the consumption budget-share for good g is simply a number in the unit interval defined as the share of expenditure for good g over total household consumption expenditure). Indeed, the fact that HCE distributions—disaggregated over consumption categories—are not statistically independent makes very hard to make predictions about the shape of CBS distributions, even if we knew how HCE marginals are distributed.

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References

Adler R, Feldman R, Taqqu M (1998) A practical guide to heavy tailed data. Birkhäuser, Boston

Agrò G (1995) Maximum likelihood estimation for the exponential power function parameters. Commun Stat 24:523–536

Anderson T, Darling D (1954) A test for goodness-of-fit. J Am Stat Assoc 49:765–769

Attanasio O (1999) Consumption demand. In: Taylor J, Woodford M (eds) Handbook of macroeconomics. Elsevier Science, Amsterdam

Aversi R, Dosi G, Fagiolo G, Meacci M, Olivetti C (1999) Demand dynamics with socially evolving preferences. J Evol Econ 8:353–408

Axtell R (2001) Zipf distributions of U.S. firm sizes. Science 293:1818–1820

Barndorff-Nielsen O (1977) Exponentially decreasing distributions for the logarithm of particle size. Proc Roy Soc A 353:401–419

Battistin E, Blundell R, Lewbel A (2007) Why is consumption more log normal than income? Gibrat's law revisited. IFS Working Papers WP08/07, The Institute for Fiscal Studies, London, UK

Bera A, Jarque C (1980) Efficient tests for normality, homoscedasticity and serial independence of regression residuals. Econ Lett 6:255–259

Bera A, Jarque C (1981) Efficient tests for normality, homoscedasticity and serial independence of regression residuals: Monte Carlo evidence. Econ Lett 7:313–318

Bottazzi G, Secchi A (2006a) Explaining the distribution of firm growth rates. Rand J Econ 37:235–256

Bottazzi G, Secchi A (2006b) Maximum likelihood estimation of the symmetric and asymmetric exponential power distribution. Working Paper 2006/19 Laboratory of Economics and Management (LEM), Sant'Anna School of Advanced Studies, Pisa, Italy

Brandolini A (1999) The distribution of personal income in post-war Italy: source description, data quality, and the time pattern of income inequality. Temi di Discussione (Economics Working Papers), No. 350, Bank of Italy, Rome

Brock WA (1999) Scaling in economics: a reader's guide. Ind Corp Change 8(3):409–446

Capasso M, Alessi L, Barigozzi M, Fagiolo G (2009) On approximating the distributions of goodness-of-fit test statistics based on the empirical distribution function: the case of unknown parameters. Adv Complex Syst (forthcoming)

Champernowne D (1953) A model of income distribution. Econ J 63:318–351
Chatterjee A, Yarlagadda S, Chakrabarti B (eds) (2005) Econophysics of wealth distributions. Springer-Verlag Italia, Milan
Chernobay A, Rachev S, Fabozzi F (2005) Composites goodness-of-fit tests for left-truncated loss samples. Tech. Rep., University of California at Santa Barbara
Clementi F, Gallegati M (2005a) Pareto's law of income distribution: evidence for Germany, the United Kingdom, and the United States. In: Chatterjee A, Yarlagadda S, Chakrabarti B (eds) Econophysics of wealth distributions. Springer-Verlag Italia, Milan
Clementi F, Gallegati M (2005b) Power law tails in the Italian personal income distribution. Physica A 350:427–438
D'Agostino R, Stephens M (1986) Goodness of fit techniques. Marcel Dekker, New York
Drees H, Kaufmann E (1998) Selecting the optimal sample fraction in univariate extreme value estimation. Stoch Proc Appl 75:149–172
Embrechts P, Kluppelberg CP, Mikosh T (1997) Modelling extremal events. Springer, Berlin
Fagiolo G, Napoletano M, Roventini A (2008) Are output growth-rate distributions fat-tailed? Some evidence from OECD countries. J Appl Econ 23:639–669
Forni M, Lippi M (1997) Aggregation and the microfoundations of dynamic macroeconomics. Clarendon Press, Oxford
Friedman M (1957) A theory of the consumption function. Princeton University Press, Princeton
Fu D, Pammolli F, Buldyrev S, Riccaboni M, Matia K, Yamasaki K, Stanley H (2005) The growth of business firms: theoretical framework and empirical evidence. Proc Natl Acad Sci USA 102:18801–18806
Gibrat R (1931) Les Inégalités Economiques. Librairie du Recueil Sirey, Paris
Groeneveld R, Meeden G (1984) Measuring skewness and kurtosis. Statistician 33:391–399
Hall P (1982) On some simple estimates of an exponent of regular variation. J Roy Stat Soc B 44:37–42
Hall R (1978) Stochastic implications of the life cycle-permamnent income hypothesis: theory and evidence. J Polit Econ 86:971–987
Hill B (1975) A simple general approach to inference about the tail of a distribution. Ann Stat 3:1163–1173
Hohnisch M, Pittnauer S, Chakrabarty M (2002) Empirical regularities in distributions of individual consumption expenditure. Int J Mod Phys C 13:541–549
Huber P (1981) Robust statistics. Wiley, New York
Ijiri Y, Simon H (1977) Skew distributions and the sizes of business firms. North-Holland, New York
Kalecki M (1945) On the Gibrat distribution. Econometrica 13:161–170
Lee Y, Amaral LAN, Canning D, Meyer M, Stanley HE (1998) Universal features in the growth dynamics of complex organizations. Phys Rev Lett 81:3275–3278
Lilliefors H (1967) On the Kolmogorov-Smirnov tests for normality with mean and variance unknown. J Am Stat Assoc 62:399–402
Loretan M, Phillips P (1994) Testing the covariance stationarity of heavy-tailed time series. J Emp Fin 1:211–248
Lux T (1996) The stable Paretian hypothesis and the frequency of large returns: an examination of major german stocks. Appl Fin Econ 6:463–475
Lux T (2000) On moment condition failure in German stock returns: an application of recent advances in extreme value statistics. Emp Econ 25:641–652
Lux T (2001) The limiting extremal behaviour of speculative returns: an analysis of intra-daily data from the Frankfurt Stock Exchange. Appl Fin Econ 11:299–315
Moors J (1988) A quantile alternative to kurtosis. Statistician 37:25–32
Nolan J (2006) Stable distributions. In: Models for heavy tailed data. Birkhäuser, Boston
Pictet O, Dacorogna M, Müller U (1998) Hill, bootstrap and jacknife estimators for heavy tails. In: Adler R, Feldman R, Taqqu M (eds) A practical guide to heavy tailed data. Birkhäuser, Boston
Reiss R.-D, Thomas M (2001) Statistical analysis of extreme values. Birkhäuser Verlag, Basel
Souma W (2001) Universal structure of the personal income distribution. Fractals 9:463–470
Subbotin M (1923) On the law of frequency of errors. Matematicheskii Sbornik 31:296–301
Sutton J (1997) Gibrat's legacy. J Econ Lit 35:40–59
Thode HCJ (2002) Testing for normality. Marcel Dekker, New York
Witt U (2001) Special issue: learning to consume—a theory of wants and the growth of demand. J Evol Econ 11:23–36
Witt U (2007) Evolutionary economics and psychology. Papers on Econonics and evolution 2006–13, Max Planck Institute of Economics, Evolutionary Economics Group, Jena, Germany