

Push-over Analysis of Optimized Steel Frames

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Abstract-The traditional optimization methods are effective when dealing with small-scale problems. However, for large-scale problems, these methods fail to obtain optimal solutions, and after a long operation, several solutions are obtained. New methods, known as metaheuristics, have provided new implementations to be used in many applications. They have enabled the resolution of many complex industrial and technical problems. They have the merits of avoiding local optima and finding optimal solutions, due to their ease of understanding, flexibility, adaptation simplicity, and ability to get out of local optima traps. This article aims to model a 2D metal frame gantry with two spans and two levels already optimized by ROBOT Millennium software in order to show the effect of structural optimization in the pre-design phase and of obtaining its non-linear behavior by the pushover method. Three optimal dimensional configurations of this gantry were taken into account and the best was chosen, one which satisfied an adequate behavior in the non-linear domain while respecting the CM66 and Eurocode3 regulations.

Keywords-optimization; vulnerability; push-over method; non-linear behavior

I. INTRODUCTION

There is an increasing tendency to analyze structures using probabilistic information on loads, geometry, material properties, and boundary conditions. Design notes were prepared manually and as new and complex requirements and demands arose, much more urgent actions were taken to meet these requirements. Progress has been made in the construction technology sector, especially in the search for productivity gains, which has led to two main thrusts [1]:

- Industrialization, therefore prefabrication of building elements, with the verification of the validity of an object previously designed and checking its operation afterwards.

- The creation of the computer, i.e. an efficient and fast tool to automate the calculation and the verification of construction elements.

Optimization algorithms are written to minimize a function (to maximize a function, it will be simple to minimize its opposite). Optimization is a method that uses mathematics and computer science, seeking to model, analyze, and solve analytically or numerically the problems of determining which solution(s) satisfy a quantitative objective while respecting possible restraints. The quality of the results and predictions depends on the relevance of the model and the efficiency of the used algorithm. Traditional and heuristic optimization algorithms have been widely used to find the global optimum in a problem or cost function. The reason may be their reliable approaches to solve difficult optimization problems. Many metaheuristic algorithms have been proposed during the recent years, such as the Ant Lion optimizer [2], the Artificial Algae algorithm [3], the Binary Bat algorithm [4], the Black Hole algorithm [5], the Binary Cat swarm optimization [6], the Firefly algorithm [7], the Fish Swarm algorithm [8], and the Grey Wolf optimizer [9]. Some examples of the first generations of these algorithms are the genetic algorithm [10], genetic programming [11], evolutionary programming [5], Tabu search [12], and simulated annealing [13]. These are metaheuristic algorithms that are also classified into three sections [14, 15], which are: evolutionary algorithms, physics-based algorithms, and intelligent algorithms.

The traditional approach to the optimization of steel structures is essentially based on minimizing the weight of the structure while taking into account the reliability and/or vulnerability which is calculated using mathematical and numerical methods such as the Finite Element Method (FEM) to obtain a detailed structural response.

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The aim of this work is to show the effect of structural optimization in the pre-design phase, using ROBOT Millennium software, on the final response of the non-linear behavior of a steel portal frame through a comparative study of three different configurations. We will model a two-span, two-level portal frame in a simple steel frame and then proceed to determine its shear forces and bending moments using the SAP2000 software while respecting the CM66 and EUROCODE 3 codes. Finally, we try to determine the vulnerability of this gantry frame by the non-linear static pushover method.

II. MATERIALS AND METHODS

A. Optimization Problems and Criteria

Optimization problems can be classified as single-objective or multi-objective, depending on the number of objective functions [16-17]. In Civil Engineering, building optimization studies often use up to 2 objective functions [18], with the exception of 3-objective functional optimization in different techniques where the objective function [19] can be linear or non-linear, convex or non-convex, unimodal or multimodal, differentiable or not, continuous or discontinuous, expensive or unaffordable in terms of computing. This results in heuristic and meta-heuristic optimization methods, simulation-based optimization, and surrogate-based optimization [20-21].

Most real-world optimization problems are described with several, often conflicting, objectives or criteria that need to be optimized simultaneously [22]. For the steel frame building design optimization problem, the variables are the components of the structure, since the variation of their characteristics directly influences the global production cost criterion chosen as the optimization objective in this research [23]. A steel frame is the result of the assembly of different components. This assembly must be designed to ensure that the resulting structure is suitable for the intended use. The early design phase, prior to any detailed calculation, concretely implies a process of analysis and verifications that can be adopted at the "maximum" according to the class of the cross-sections (Eurocode3). A comparative study will be carried out on a two-dimensional steel portal frame using the profiles available on the Algerian market, namely IPE, IPN, HEA, and HEB in order to have optimal dimensioning in the plastic domain.

B. Optimization Module in ROBOT Millennium

In order to facilitate the work, the use of ROBOT Millennium is proposed, which has a vast set of tools simplifying the study of structures. In ROBOT Millennium, the notion of objects and the creation of the model of the structure are carried out with typical construction objects: beams, columns, bracing, floors, and walls. During the study stage, the elements of the structure take on specific attributes of their own (including regulatory attributes), thus, at the model definition stage, all the regulatory parameters of the structure are defined, which allows the regulatory analysis to be carried out immediately after the static calculations. The same applies to the nodes. The notion of nodes has lost its traditional meaning since they are automatically defined during the creation of the various objects. The available optimization criteria are: weight, maximum and minimum section height, minimum flange

thickness, and minimum web thickness. The option Calculations for the entire profile family is available. The entire accessible profile database will be scanned to find the optimal profile. The sizing module is used to find the optimal profile in terms of stress. For example, we see that ROBOT recommends an IPE 240 that is identical to our initial choice (a first estimate). However, the verification of the IPE 240 in deflection shows that the profile is insufficient. Indeed, the deflection is the dimensioning element in certain structures, so it would be logical to make dimensioning in deflection. Unfortunately, automatic dimensioning in deflection is impossible. To justify this observation, we must explain how sizing works.

III. CASE STUDY

The main objective of this project was to design a building structure in CM66. However, the achievement of this objective was done in several steps. First, it was necessary to:

- Determine the load applied to the structure.
- Determine the different types of possible structures.
- Carry out the design and finally choose the most advantageous design in relation to the client's needs.

A. Description of the Structure

The project is located in Oran city, that is located in an average seismic activity zone (Zone II-a) according to the Algerian earthquake code RPA99, in zone B for snow density, and zone I for wind force according to the RNV2013. Depending on the plan view, the dimensions of the structure are shown in Table I.

TABLE I. DIMENSIONS OF THE STRUCTURE

Total length	Total width	Height from the ground level	Height of the ground floor	Height of the floors	Total height of the building
30.64m	13.4m	2.80m	3.80m	3.80m	57m

The light roof is made of TN 40 industrialized ribbed sheet metal, transoms, purlins, and bracing. The purlins and trusses are made of IPE profiles. The columns are made of HEA profiles, the distance between two purlins is 1.3m, and the external masonry walls have 6m of high. The cladding is made of the same type of sheet metal as the TN 40 roof. The cladding rails are fixed to posts or possibly to posts. The loads to the building are transmitted to the ground through the insulated reinforced concrete footings. The value of the permissible soil stress is 2.5 bars. The characteristics of reinforced concrete in the foundations are: Strength of the concrete $f_{c28} = 25$ MPa, elasticity modulus of concrete $E_c = 32164$ MPa, safety factor of concrete $\gamma_c = 1.5$, safety factor of steel $\gamma_s = 1.15$, and yield steel stress $F_c = 400$ MPa. The steel elements are made with steel grade S235, $f_y = 2350$ daN/cm². The modulus of elasticity of the steel is $E = 2100000$ daN/cm². The connections are made by means of ordinary and high strength welds and bolts.

A calculation with ROBOT software:

- Defines a structure made up of portal frames (columns + truss) in accordance with the project data. The wind actions are Calculate
- Creates the load combinations
- Analyzes the structure and dimensions of the different elements of the structure (purlins, cladding rails, truss elements, columns, and posts).
- Calculates the connections (truss elements, truss-post, and post foot).

Subsequently, the gantry was analyzed using Etabs software to apply the pushover method and see the vulnerability of the gantry. The type of structure is a planar portal frame with a total mass of 594.79kg. Figure 1 shows the studding gantry in CM with the optimized metallic profiles. However, three configurations were selected after the first step of the present study, which is the optimization of the structure with different profile sections available in the Algerian market, especially HEA, HEB, IPE, and IPN profiles.

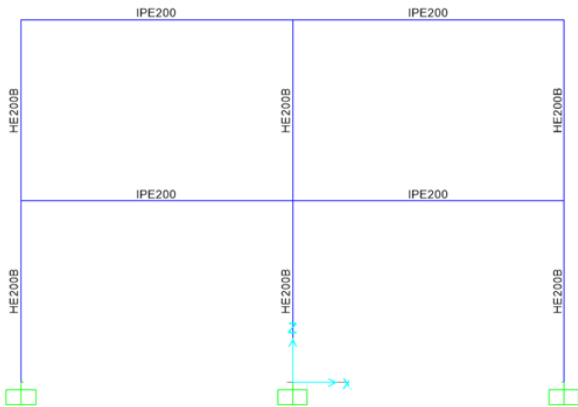


Fig. 1. The metal frame of the case study.

B. Deformation of the Gantry

Each variant was subjected to nonlinear analysis according to the pushover method. Figure 2 shows the deformation form and the plastic hinges of the gantry with the optimized metallic profiles for all three configurations.

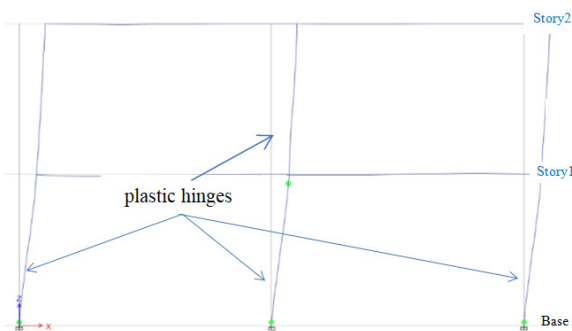


Fig. 2. Shape deformation with plastic hinges.

IV. RESULTS AND DISCUSSION

In this section, we present and compare the results. We illustrate and compare the results of two frames of the same number of stories according to the period, base shear, maximum displacement, and stiffness.

A. First Variant (Columns : HEB180, Beams: HEA240)

Table II represents the distribution of plastic hinges in the metallic gantry according to the first configuration. From Table II, we can see that 80% of the plastic hinges are in the first segment (A-IO), 20% in the second segment (IO-LS), 0% in the 3rd segment (LS-CP), and 0% in the last segment, which corresponds to failure or collapse. Figure 3 shows the evolution of the capacity curve and the performance parameters of the studied structure according to the first configuration. The capacity curve (Figure 3(a)) is defined by the elastic state corresponding to an elastic shear force of $V_y = 58.37kN$ for an elastic displacement of $d_y = 30.5mm$ and initial stiffness $K_0 = 1913.91KNm$ which represents the product of the elastic shear force and the elastic displacement. The ultimate limit state corresponding to the shear force is $V_u = 92.70kN$. From Figure 3(b) and Table II, it can be said that the performance point, which represents the intersection of the response spectrum curve with the capacity curve, is at the point that connects the shear force of 342.30kN with the displacement of 18.7mm.

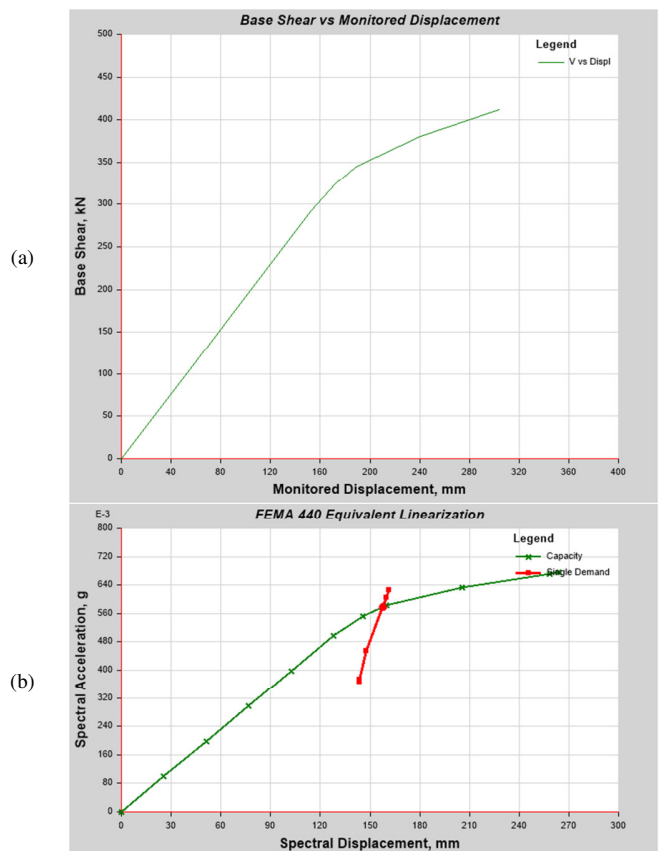


Fig. 3. Response of the nonlinear analysis of the first variant. (a) Capacity curve, (b) performance parameters.

TABLE II. BASE SHEAR, MAX DISPLACEMENT, AND STIFFNESS OF THE FIRST VARIANT OF THE TWO-STOREY FRAME

Step	Monitored displacement (mm)	Base force (kN)	A-B	B-C	C-D	D-E	>E	A-IO	IO-LS	LS-CP	>CP	Total
0	0.1	0	20	0	0	0	0	20	0	0	0	20
1	30.5	58.37	20	0	0	0	0	20	0	0	0	20
2	60.9	116.74	20	0	0	0	0	20	0	0	0	20
3	91.3	175.12	20	0	0	0	0	20	0	0	0	20
4	121.7	233.49	20	0	0	0	0	20	0	0	0	20
5	152.1	291.87	19	1	0	0	0	20	0	0	0	20
6	172.9	324.75	18	2	0	0	0	19	1	0	0	20
7	189	344.92	17	3	0	0	0	19	1	0	0	20
8	240.7	380.51	16	4	0	0	0	17	3	0	0	20
9	298.1	409.22	15	5	0	0	0	16	4	0	0	20
10	304.1	411.71	15	5	0	0	0	16	4	0	0	20

TABLE III. BASE SHEAR, MAX DISPLACEMENT, AND STIFFNESS OF THE SECOND VARIANT OF THE TWO-STOREY FRAME

Step	Monitored displacement (mm)	Base force (kN)	A-B	B-C	C-D	D-E	>E	A-IO	IO-LS	LS-CP	>CP	Total
0	0.3	0	20	0	0	0	0	20	0	0	0	20
1	30.7	32.50	20	0	0	0	0	20	0	0	0	20
2	61.1	65.00	20	0	0	0	0	20	0	0	0	20
3	64.6	68.71	19	1	0	0	0	20	0	0	0	20
4	113.6	107.05	17	3	0	0	0	20	0	0	0	20
5	144	128.05	17	3	0	0	0	20	0	0	0	20
6	183.5	151.65	16	4	0	0	0	20	0	0	0	20
7	213.9	168.96	16	4	0	0	0	18	2	0	0	20
8	244.3	186.27	16	4	0	0	0	18	2	0	0	20
9	274.7	203.58	16	4	0	0	0	17	3	0	0	20
10	304.3	219.88	15	5	0	0	0	16	4	0	0	20

TABLE IV. BASE SHEAR, MAX DISPLACEMENT, AND STIFFNESS OF THE THIRD VARIANT OF THE TWO-STOREY FRAME

Step	Monitored displacement (mm)	Base force (kN)	A-B	B-C	C-D	D-E	>E	A-IO	IO-LS	LS-CP	>CP	Total
0	0.4	0	16	4	0	0	0	20	0	0	0	20
1	30.8	13.74	16	4	0	0	0	20	0	0	0	20
2	51.9	23.31	15	5	0	0	0	20	0	0	0	20
3	82.3	33.85	15	5	0	0	0	20	0	0	0	20
4	112.7	44.39	15	5	0	0	0	20	0	0	0	20
5	143.1	54.93	15	5	0	0	0	20	0	0	0	20
6	182.6	66.94	14	6	0	0	0	19	1	0	0	20
7	213	75.80	14	6	0	0	0	17	3	0	0	20
8	254	86.37	13	7	0	0	0	17	3	0	0	20
9	284.4	93.85	13	7	0	0	0	16	4	0	0	20
10	304.4	98.76	12	8	0	0	0	16	4	0	0	20

B. Second Variant (Columns : HEB180, Beams: IPE 200)

Table III represents the distribution of the plastic hinges in the metallic gantry. We can see that 80% of the plastic hinges are in the first segment (A-IO), 20% in the second segment (IO-LS), 0% in the 3rd segment (LS-CP), and 0% in the last segment, which corresponds to failure or collapse. Figure 4 shows the evolution of the capacity curve and the performance parameters of the studied structure according to the second configuration. The capacity curve (Figure 4(a)) is defined by the elastic state corresponding to an elastic shear force of $V_y = 32.50\text{kN}$ for an elastic displacement of $d_y = 30.7\text{mm}$ and initial stiffness $K_0 = 1058.63\text{kNm}$ which represents the product of the elastic shear force and the elastic displacement. The ultimate limit state corresponding to the shear force is

$V_u = 219.88\text{kN}$. From Table III and Figure 4(b), it can be said that the performance point, which represents the intersection of the response spectrum curve with the capacity curve, is at the point that connects the shear force of 0.00kN with the displacement of 0.0m . So, the performance point is not found.

C. Third Variant (Columns : HEB140, Beams: IPE180)

Table IV represents the distribution of plastic hinges in the metallic gantry. From Table IV, we can see that 80% of the plastic hinges are in the first segment (A-IO), 20% in the second segment (IO-LS), 0% in the 3rd segment (LS-CP), and 0% in the last segment, which corresponds to failure or collapse. Figure 5 shows the evolution of the capacity curve and the performance parameters of the studied structure according to the third configuration.

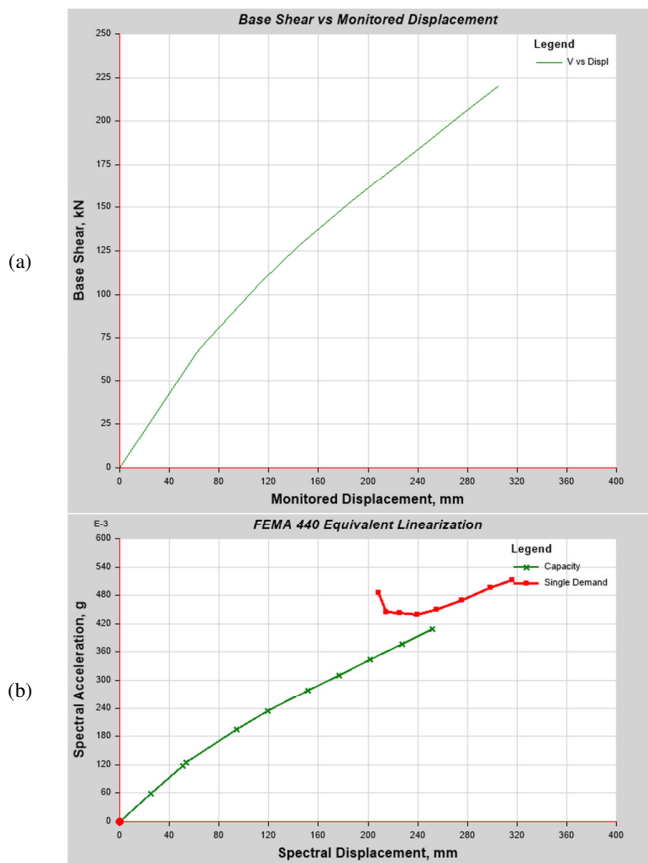


Fig. 4. Response of the nonlinear analysis of the second variant. (a) Capacity curve, (b) performance parameters.

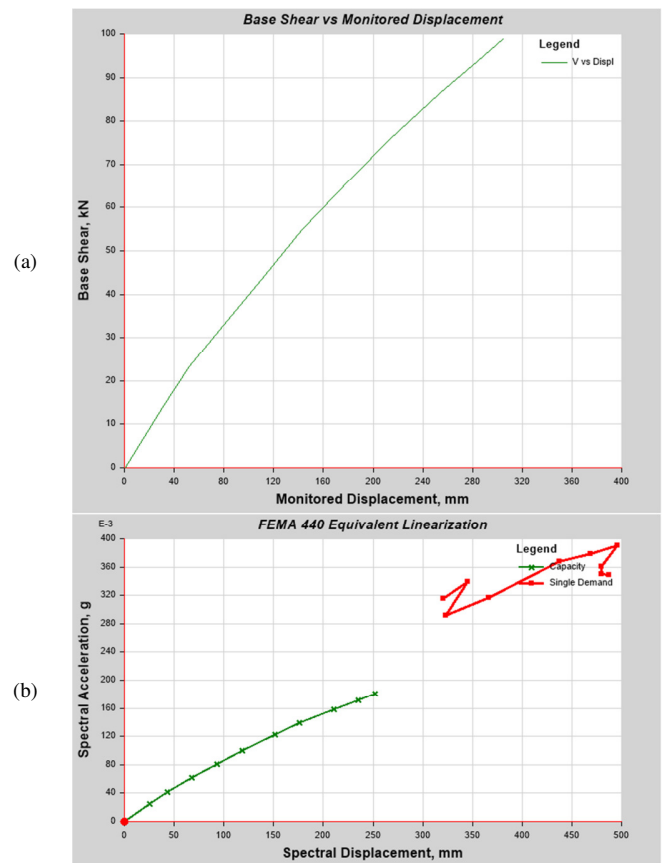


Fig. 5. Response of the nonlinear analysis of the third variant. (a) Capacity curve, (b) performance parameters.

The capacity curve (Figure 5(a)) is defined by the elastic state corresponding to the elastic shear force of $V_y = 13.74\text{kN}$ for an elastic displacement of $d_y = 30.8\text{mm}$ and initial stiffness $K_0 = 446.10\text{kNm}$ which represents the product of the elastic shear force and the elastic displacement. The ultimate limit state corresponding to the shear force is $V_u = 98.77\text{kN}$. From Table IV and Figure 5(b), it can be said that the performance point, which represents the intersection of the response spectrum curve with the capacity curve, is at the point that connects the shear force of 0.00 kN for the displacement of 0.0m . So, the performance point is not found.

D. Comparison between the Results of the Treated Variants

After analyzing each model separately, the results obtained are compared according to the capacity curve and the following criteria: elastic shear force, displacement, initial stiffness, and ultimate shear force as detailed in Table V. Regarding to the elastic shear force, it can be easily noticed that variant 1 presents the highest value compared to the other two, mainly due to the geometrical characteristics of the profiles used in this variant. In the same context, variant 1, gives the lowest elastic displacement. It also presents the highest initial stiffness and ultimate shear force.

It can be said that the columns have good resistance to the applied loads and the under dimensioning of these elements can lead to low resistance.

TABLE V. COMPARISON BETWEEN THE TREATED VARIANTS

Variant	1	2	3
Elastic shear force V_y (kN)	58.37	32.50	13.74
Elastic displacement d_y (m)	0.0305	0.0307	0.0308
Initial stiffness K_0 (kN/m)	1913.91	1058.63	446.10
Ultimate shear force V_u (kN)	92.70	219.88	98.77

On the other hand, it can be said that variant 1 has the highest weight among the considered variants. This has a considerable influence on the cost of the structure. However, among the three different optimized configurations, the first variant with (HEB180 for columns and HEA240 for beams) shows the best-optimized configuration in ROBOT compared to the other two optimized configurations.

V. CONCLUSIONS

The analysis showed that the selected models and the interpretation of the results, allow us to conclude that the choice of the profiles is appropriate, either for the columns and/or for the crossbeams, which ensure an adequate resistance, and add in the weight of the structure. The latter is one of the major criteria among the main optimization parameters. It should be noted that many questions are still open in the field of behavior of steel frames, in particular questions related to the dependence of the response on the non-linear domain.

Finding optimal solutions to engineering problems is always a difficult task. Due to the many conditions and limitations, structural engineers usually have to proceed and repeat through trial and error, in search of better design solutions in terms of improved structural performance and cost reduction.

In this paper, nonlinear analysis of a 2D steel frame was conducted and the results were compared for three different optimized configurations. The results obtained showed that in order to properly design a metallic gantry frame that meets the regulatory requirements with a light structure and minimum cost, it is necessary to introduce several parameters in the form of an algorithm that allows us to arrive at the most appropriate profiles in a simple way. For this reason, we can say that the use of genetic algorithms in this case is adequate. They are beneficial for tackling problems that are considered difficult or that require a lot of computation time with a classical algorithmic approach. It is also easy to show that when using genetic algorithms, one can improve the solution speed and make it possible to solve something in adequate time that used to be very time consuming, given the complexity of the data of physical problems.

Future work will focus on three-dimensional models to properly analyze this type of steel structure.

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