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# HYDRAULIC COMPUTATIONS FOR STEPPED CONCRETE OVERLAYS OF EMBANKMENT DAMS

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**Abstract.** One method for rehabilitating and modernizing embankment dams is the addition of a concrete overlay that protects the embankment and allows floods to pass safely over the dam. Roller compacted concrete is commonly used, which makes it practical to use a stepped construction that also enhances energy dissipation. Analysis of flow conditions over such structures requires computation of aerated flow and its effects on flow depth, training wall design, and energy dissipation. This paper describes SpillwayPro, an energy-based water surface profile calculation tool for smooth spillway chutes (Wahl et al. 2019), recently improved to also analyze stepped chutes. The tool is applicable to a wide range of chute slopes, including flatter slopes typical of embankment dam overlays and steeper slopes encountered on concrete gravity dams. An energy-based analysis allows SpillwayPro to be applied to situations differing from the idealized configurations covered by available empirical approaches, such as non-constant slopes, varying step heights, and converging chutes. SpillwayPro integrates water surface profile calculations, aerated flow effects, and cavitation analysis, which is potentially important for steeper slopes and large unit discharges. Simultaneous calculation of smooth and stepped-chute flow profiles enables rapid assessment of the energy dissipation benefits of steps, as well as a comparison of the aerated flow and cavitation issues for smooth vs. stepped chute alternatives.

## 1 INTRODUCTION

Roller compacted concrete (RCC) overlays are a common rehabilitation option for small to medium-height (up to about 70 ft) embankment dams with insufficient spillway capacity and high hazard potential, meaning their failure would probably cause loss of human life. Such dams have often received this hazard potential classification due to downstream urban development after their original construction. A stepped RCC overlay is a relatively inexpensive option for protecting such dams against overtopping erosion and breach, providing additional spillway capacity and dissipating energy associated with spillway flows.

While stepped construction is economical and provides energy dissipation benefits, it also presents some challenges from a hydraulic analysis perspective. Steps force early initiation of aerated flow which causes bulking of flow depths, and both steps and aeration have a strong influence on friction factors and energy dissipation. In addition, steps create the potential for cavitation that might damage the chute, especially in areas upstream from the inception of aerated flow.

The foundation for designing any spillway chute—or for that matter any open channel waterway—is the calculation of a water surface profile. The HEC-RAS program developed by the U.S. Army Corps of Engineers is commonly applied for this purpose to many hydraulic engineering problems, but HEC-RAS is not equipped for analysis of steep, stepped, or highly aerated chutes, and provides no analysis of cavitation potential. Another, older program with similar limitations is WSPRO, developed by the U.S. Geological Survey and distributed by the Federal Highway Administration (FHWA) as HY−7.

The technology in SpillwayPro was developed gradually over several decades at the Bureau of Reclamation. Early FORTRAN codes were developed to compute standard-step water surface profiles for self-aerated flows in smooth open-channel chutes (Falvey 1980). Later versions of these programs added support for cavitation analysis and the design of aerators and spillway profiles with designed pressure characteristics that limit cavitation damage potential (Falvey 1990). SpillwayPro puts the engineering calculations in these earlier programs into Visual Basic for Applications subroutines executed in a Microsoft Excel workbook (Wahl et al. 2019), with the spreadsheet interface used to organize input and output data. Most recently, SpillwayPro has been improved (Wahl and Falvey 2022a) with modern aerated flow calculations for smooth chutes (Wilhelms and Gulliver 2005a,b) and capabilities for analyzing stepped chutes. SpillwayPro offers more rapid analysis of important aspects of spillway hydraulics than can be achieved with physical or computational fluid dynamics (CFD) models. SpillwayPro is available for free download by the public (Wahl and Falvey 2022b).

This paper will focus on SpillwayPro's features related to stepped chutes and particularly chutes with slopes typical of the downstream face of overlayed embankment dams. Wahl and Falvey (2022a) provides a more complete description that includes steeper stepped chutes and the new aerated flow analysis methods for smooth chutes.

## 2 SPILLWAYPRO WATER SURFACE PROFILES: BASIC CONCEPTS

SpillwayPro uses the standard step method (Chow 1959) to compute water surface profiles using the energy equation. In supercritical channels the calculations proceed in a downstream direction, starting from either a critical section or a section that is already supercritical. The spillway profile is defined by a series of station and elevation values and associated cross section dimensions. The stations need not be located exactly at the position of step tips, but they should define the profile of the so-called pseudobottom of the flow defined by a smooth curve through the step tips. Although this paper will focus on stepped chutes, the program always performs parallel calculation of water surface profiles for the stepped profile and a smooth chute following the pseudobottom profile. This enables direct comparison of the effects of incorporating steps into a spillway chute design.

SpillwayPro's implementation of the energy equation accounts for streamline curvature that tends to increase or decrease piezometric pressures against a spillway face. However, curvature effects can only be addressed as long as the curvature is not so extreme that the assumption of gradually varied flow is violated. The curvature associated with ogee crest spillway shapes often exceeds this limit, causing rapidly varied flow to occur immediately downstream from the critical section of a spillway. SpillwayPro calculations should start at a station downstream from the start of gradually varied flow, and SpillwayPro will assist the user to determine an appropriate initial flow depth at this station that is consistent with the energy head available to drive flow over the crest. Details of this process are provided in Wahl et al. (2019) and Falvey (1990). Ogee crest shapes are not common in RCC dam overlay applications, but a similar concept applies; the flow immediately over the brink of the crest involves streamline curvature that usually exceeds gradually varied flow limits, so computations should begin a short distance down the slope, but far enough upstream that flow is not yet aerated.

## 3 FLOW DEVELOPMENT IN STEPPED CHUTES

Flows of interest in most stepped chute spillways are generally in the skimming flow regime, where flow depths are large enough that steps operate as submerged roughness elements with the main flow skimming over the tips of the steps. In contrast, at low flow rates each step functions as a local critical depth control with a hydraulic jump and brief transition back to subcritical flow on the step tread. SpillwayPro is only intended to analyze flow in the skimming regime, as defined by $h/d_c < 1/(1.2 - 0.325 \tan \theta)$ where $h$ is the step height, $d_c$ is critical depth, and θ is the slope angle. Figure 1 shows the important flow zones for stepped chutes experiencing skimming flow. Hunt & Kadavy (2018b) provide empirical relations for calculating many of the parameters shown in this figure. SpillwayPro uses empirical relations to locate the beginning of aerated flow and to estimate air concentrations, but the energy equation and other analytical equations are used for most of the other depicted parameters.

SpillwayPro is able to analyze chutes with slopes steeper than 2H:1V (common for spillways of concrete gravity dams) and those with a slope of 2:1 or flatter which are typical of embankment dam overlays. Since many research studies have adopted the 2H:1V slope as a dividing line between flatter and steeper slopes, different equations are used for each regime. This paper will focus on the relations used for the flatter slopes, which are applicable to most embankment overlay projects. Some rockfill dams have slopes as steep as 1.5H:1V. These can be analyzed in SpillwayPro using the equations for steeper slopes. For more details see Wahl and Falvey (2022a).

### 3.1 Aeration Inception Point

SpillwayPro determines the aeration inception point for smooth and stepped chutes using the dominant methods in the literature for each situation. For smooth chutes the inception point is determined by computing the growth of boundary layer thickness from the start of the chute and comparing to the computed flow depth. When the boundary layer thickness exceeds 80 percent of the flow depth, inception of aeration is presumed to occur. Although the mechanism for aeration inception in stepped chutes is similar (boundary layer interacting with the free surface), in stepped chute flows the distance to the aeration inception point is usually computed from empirical relations based on the roughness Froude number, $\mathsf{F}^* = q/\sqrt{g \sin \theta (h \cos \theta)^3}$, where $g$ is the acceleration due to gravity, $q$ is the discharge per unit width, and $h \cdot \cos(\theta)$ represents the step height measured normal to the chute slope. The sin(θ) term is often included but may be an artifact of the historical development of the roughness Froude number from other dimensionless parameters (Wahl 2022).

For 2:1 and flatter slopes, SpillwayPro calculates the distance to the inception point, $L_i$, using relations developed from tests of overtopping flow on embankment overlays at 4:1, 3:1, and 2:1 slopes (Hunt et al. 2014). A slight adjustment of the relation occurs at $\mathsf{F}^*$=28:

$$L_i = 5.19(\mathsf{F}^*)^{0.89}(h \cos \theta) \text{ for } 0.1 < \mathsf{F}^* \leq 28 \quad (1)$$

$$L_i = 7.48(\mathsf{F}^*)^{0.78}(h\cos\theta) \text{ for } 28 < \mathsf{F}^* < 10^5 \quad (2)$$

These equations apply to steps with square-edged vertical faces. SpillwayPro includes an option to adjust the inception point for cases where a 45° beveled face is used, which simplifies construction and may provide a more durable face. Hunt et al. (2022) showed that the inception point moves upstream compared to a vertical step face of the same height and also for comparable values of F* and the effective roughness height.

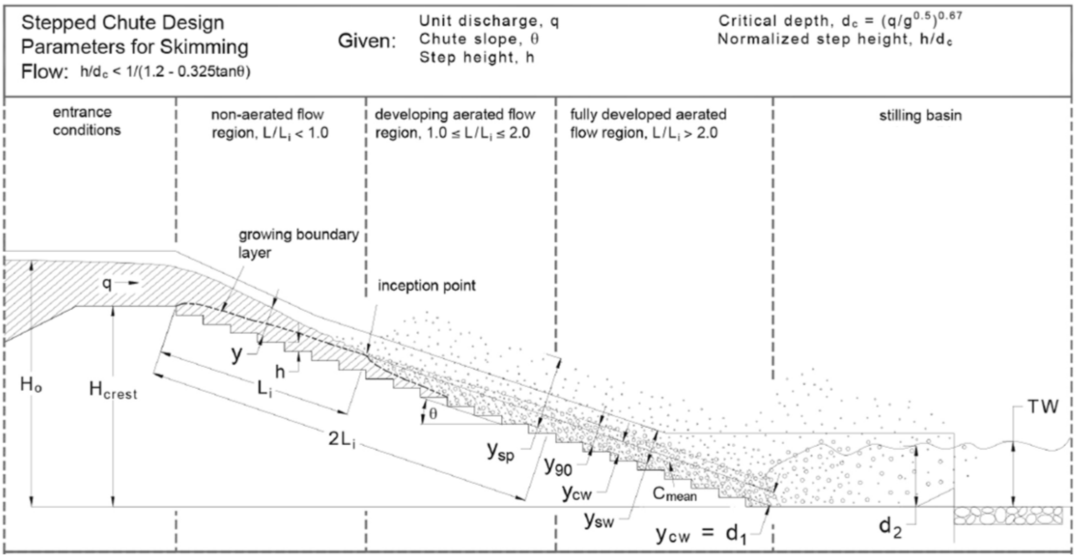


**Figure 1.** Dominant flow zones for stepped chute overlays of embankment dams (adapted from Hunt & Kadavy 2018b).

## 3.2 Fully Developed Aerated Flow

SpillwayPro does not attempt to model the distribution of air within the water column, but instead estimates only the mean air concentration of the entire flow. The basis for computing air concentration in the developing zone is the air concentration in the fully developed region, which is expressed as $\overline{C}_{u,90}$. The 90 subscript indicates that this mean air concentration represents an average computed up to the level at which the local air concentration is 90 percent. Above this level the flow is mostly air with a spray of water. Tests conducted at 2:1, 3:1, and 4:1 slopes (Hunt et al. 2014) provide Equation (3) for slopes of 2:1 or flatter.

$$\overline{C}_{u,90} = 0.0645 + 0.216(h/d_c) + 0.453\sin\theta \quad (3)$$

If the embankment slope changes, the fully developed air concentration values are updated for the new slope. If the slope flattens, the air concentration at the present station may exceed the newly computed value of $\overline{C}_{u,90}$. If so, deaeration begins, and the air concentration will tend back toward the new $\overline{C}_{u,90}$ value. The deaeration rate is computed using a rate function developed by Kramer (2004) until the concentration drops back to the new $\overline{C}_{u,90}$ value.

## 3.3 Transitional Aerated Flow

For 2:1 and flatter slopes, the variation of air concentration between the inception point and the fully developed zone is a straightforward linear relation starting from 11% lower than the fully developed uniform value at a dimensionless flow distance $L/L_i$ = 1.0 and increasing to the uniform value at $L/L_i = 2.0$ (Hunt et al. 2014). With the air concentration calculated, the bulked depth of the flow mixture is determined, and a safety factor multiple is applied to compute the elevation of training walls at each station.

### 3.4 Friction Factor

SpillwayPro directly calculates the water surface profile of the equivalent clear-water flow using the standard step method (energy equation) and then uses the empirical air concentration estimates to determine the mixture flow depth. To enable calculation of the water surface profile, values of the Darcy friction factor, $f$, are needed. Many investigations of stepped spillway flows have not determined friction factors but have instead used empirical relations to directly calculate the water surface profile and residual energy of the flow. Studies that have quantified $f$ directly have related it to chute slope, relative step height, and relative step tip spacing, but there is significant scatter in these relations. Most studies have utilized testing conducted in the fully developed aerated zone, so effects of aeration are implicit, although scale effects may be significant in lab-scale studies.

For chutes on typical embankment slopes (2:1 or flatter), SpillwayPro estimates the value of $f$ based on energy loss data from studies of flow upstream and downstream from the inception point (Hunt & Kadavy 2010a, 2010b; Meireles & Matos 2009). On a 4:1 slope, Hunt & Kadavy (2010a) found that the relative energy loss upstream from the inception point can be estimated as $\Delta H/H_o = 0.3(L/L_{i*})$, where $\Delta H$ is the energy loss, $H_o$ is the available head from the upstream reservoir to the point of interest in the chute, $L$ is the flow distance along the slope from the start of the steps, and $L_{i*}$ is the empirically predicted inception length from the same reference point. Similar losses were observed on a 2:1 slope by Meireles & Matos (2009). Downstream from the inception point (Hunt & Kadavy 2010b), the relative energy loss was $\Delta H/H_o = 1 - (L/L_{i*} + 0.51)^{-0.87}$. The friction factor can then be estimated as $f = 8(y_{cw}/y_c)^3(\mathrm{d}H/\mathrm{d}L)$ where $y_{cw}$ is the clear-water depth, $y_c$ is critical depth, and $\mathrm{d}H/\mathrm{d}L$ is the derivative of the energy head function determined from the relative energy loss relations. Values of $y_{cw}/y_c$ can be estimated from empirical relations (Hunt et al. 2014). A resulting typical relation between $f$ and $L/L_{i*}$ is shown in Figure 2. The friction factor is initially low as the boundary layer induced by the steps begins to develop and affects just a fraction of the flow, then approaches an equilibrium value downstream from the inception point. The discontinuity in $f$ at the inception point is relatively minor but may represent a real increase in turbulent mixing at the inception point.

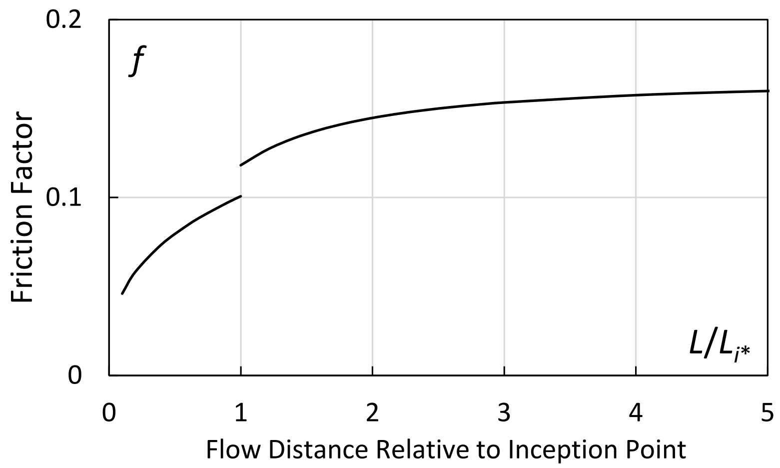


**Figure 2.** An example of typical variation of the stepped chute friction factor upstream and downstream from the inception point.

It should be noted that the discussion above applies to the friction factor of the chute floor only. SpillwayPro presumes hydraulically smooth training walls at each edge of the chute and determines a wetted perimeter-weighted average friction factor for the complete cross section. A future revision could offer an option to neglect this adjustment, which would be appropriate when the stepped overlay spans the full width of the dam and wraps around the abutments.

### 3.5 Cavitation Check

Although RCC overlays have not typically been applied to high dams in which cavitation would be a common issue, large unit discharges in stepped chutes do have potential to create damaging cavitation upstream from the aeration inception point. Downstream from the inception point, there is little risk of cavitation damage since air concentrations build quickly and some quantity of air is rapidly present near the boundary.

SpillwayPro provides the cavitation index of the flow at each station and a corresponding incipient cavitation index value, $\sigma_c$, using the relation $\sigma_c = 4f$ (Frizell et al. 2013). This relation is based on data from several investigations of different types of roughness elements, including water tunnel tests of a stepped chute floor conducted in a reduced atmospheric pressure condition that allowed incipient cavitation to be observed. When applying this relation, SpillwayPro conservatively uses the bottom friction factor estimated for fully developed flow (i.e., the limiting value illustrated in Fig. 2). Downstream from the inception point, SpillwayPro estimates the bottom air concentration using a relation developed for slopes steeper than 2:1 (Boes & Hager 2003). There has been little research on the development of bottom air concentration in flatter stepped chutes, but this equation developed for steeper slopes is well behaved for flatter slopes and performs as one would intuitively expect, with bottom air concentration diminishing as slope flattens toward zero. However, without research specific to this slope range, the accuracy of the rate of reduction vs. slope is an open question.

### 3.6 Energy Dissipation and Stilling Basin Estimation

SpillwayPro calculates the specific energy at each station of the stepped-chute profile, and results can be compared to the smooth-chute profile to see energy dissipation benefits. An energy (Coriolis) coefficient $\alpha$ is included to account for nonuniform velocity profiles, using equations provided by Hunt et al. (2014) for the range of typical embankment-overlay slopes. Values range from 1.01 to 1.17, depending on the relative step height and chute slope. Energy dissipation provided by steps reduces the required stilling basin length and the tailwater depth needed to create a hydraulic jump. SpillwayPro uses the clear-water depth at the last station of the profile to calculate the conjugate depth and estimated basin length for the common USBR Type I, II, III (Peterka 1958), and Low-Froude Number (George 1978) stilling basins. (Note that the Low-Froude Number basin referred to here is an updated and more compact design than the Type IV basin described in Peterka [1958] for a similar range of Froude numbers). Results are displayed in spreadsheet cells at the top of the smooth- and stepped-chute hydraulic results pages, and the cell colors are highlighted in green when the incoming flow Froude number is within the suggested range for each stilling basin, or red when the flow condition is outside of the recommended range. There is an inherent assumption that the basin length below stepped chutes can be estimated from the equivalent clear water depth and Froude number. Recent experiments on Type I basins (Stojnic et al. 2021) have shown that the normalized hydraulic jump length is significantly extended when flow is delivered to the basin from a stepped chute. However, testing of Type III and Type IV basins (Frizell et al. 2012; Hunt & Kadavy 2018a) has shown adequate performance below stepped chutes using the smooth-chute guidance applied using the clear-water depth.

### 3.7 Converging Walls, Changing Slopes, Varying Step Heights

SpillwayPro's energy-based calculation method gives it some capability to analyze situations that differ from idealized conditions of constant chute width, slope, and step height.

Convergence of a chute is a common situation encountered on embankment dam overlay projects. SpillwayPro's calculations should be accurate if the structure plan is arched so that the gradient of the spillway slope is aligned with the training walls. However, if the abutments and training walls cross the gradient of the slope, SpillwayPro will not accurately simulate the concentrated flow that develops in the groin areas. For these situations, studies specific to these types of convergent chutes should be consulted (Hunt et al. 2012, Zindovic et al. 2016).

Changes of slope or step height can be readily accommodated, although SpillwayPro takes a relatively simple approach to using such changes. If the slope flattens or step height is reduced in the downstream direction, the inception length will increase and SpillwayPro will simply extend the inception point, unless it has already been reached. If the slope steepens or step height increases, the inception length will be reduced. SpillwayPro makes no adjustment for the fact that initially smaller or larger steps may have caused a different growth rate of the boundary layer; the inception point is determined as though the entire chute was comprised of the new steps. Recent research by Hunt & Kadavy (2021) may offer a better means of estimating the combined effects of a changed step height.

In addition to affecting the inception length, slope and step height changes will affect the fully developed air concentration, transitional air concentration, friction factors, energy coefficients, and other parameters discussed previously, partly because some of these calculations depend on the inception length. SpillwayPro assumes that once the inception point is reached, a subsequent change in slope or step height will not change it; calculations that depend on the inception length will use the actual inception point, not the one that would have occurred if the entire chute had been constructed with the new slope or step height.

SpillwayPro is also able to handle the situation of a smooth chute transitioning to steps, perhaps at a location of steepening slope. In this case, the program will separately determine the inception points for smooth-chute flow with a boundary layer growing from the start of the chute vs. stepped-chute inception distance beginning from the first step. Aeration will be initiated at earliest of these two conditions.

## 4 VALIDATION

Wahl and Falvey (2022a) describe the comparison of SpillwayPro results to three validation scenarios: prototype field measurements of aerated flow in the smooth chute at Aviemore Dam (New Zealand) by Cain & Wood (1981); empirical equations of Boes & Hager (2003b) for the steep chute of an example gravity dam spillway; and empirical equations of Hunt et al. (2014) for a hypothetical dam overlay application on a 3:1 slope. Results for all of these cases were reasonable, within a few percent of measured or empirically predicted air concentrations and specific energy values. The empirical equations used for comparison come from laboratory studies performed at sizes sufficient to minimize scale effects. At this time there is no known source of good prototype-scale field data for an embankment overlay application. Validation against more complex scenarios such as a mid-chute slope change (Mirza et al. 2017) or step-height change (Hunt & Kadavy 2021) would be valuable but has not yet been performed.

## 5 CONCLUSIONS

SpillwayPro is an energy-based water surface profile calculation tool that has been recently improved to enable analysis of stepped chutes. The use of a standard-step solution method allows it to be applied to situations that may differ from the idealized cases for which empirical flow profile equations are available, including non-constant slopes, varying step heights, and converging chutes. SpillwayPro integrates water surface profile calculations, aerated flow effects, and cavitation analysis. Simultaneous calculation of smooth and stepped-chute flow profiles enables rapid assessment of the energy dissipation benefits of steps, as well as a comparison of the aerated flow and cavitation issues for smooth vs. stepped chute alternatives.

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