Distributed Robust Synchronization Control of Multiple Heterogeneous Quadcopters with An Active Virtual Leader ⋆

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Abstract: This paper studies leader-following synchronization control of a group of multiple quadrotor unmanned aerial systems (UASs). A robust distributed scheme is developed to maintain the attitude motions of UASs with an active virtual leader. Complicated settings are considered in the design, where the topology is in a directed graph, and only one or some agents are connected to the leader. UASs can have different dynamic parameters. Also, some time-varying disturbances are added to the closed-loop system. A control protocol containing a robust term is proposed to each UAS to achieve asymptotic consensus. A rigorous mathematical proof and numerical example are presented to demonstrate the effectiveness of our scheme.

Keywords: Quadrotor, unmanned aerial systems (UASs), robust distributed scheme, synchronization control, leader-following, consensus and time-varying disturbance.

1. INTRODUCTION

Cyber physical systems is turned out as a new concept, leveraging the development of many emerging technologies on industrial autonomous systems. One of the main trends in this direction that has yielded considerable technological improvements is autonomous operation and cognition of collaborative set of multiple quadrotors in a networked environment (Um (2019); Montazeri et al. (2020)). Many interesting findings have been investigated under various settings, nevertheless, the major goal is to design proper controllers for the cooperative UASs operating in the real-world environments.

Quadrotors have a wide range of applications in a variety of hazardous contexts where human cannot operate easily. This includes areas such as nuclear decommissioning (Nemati and Montazeri (2018b); Burrell et al. (2018)), geospatial photography, volcano monitoring, and precision agriculture.

One of the most important considerations in the controller design is the presence of nonlinearities in the attitude dynamics. One approach to tackle this issue is to use a linear controller for the linearized dynamic model. However, this method can only be used in limited cases. To cover the full nonlinear operational range of the UAV, a nonlinear controller with feedback linearisation is required Zhou et al. (2010). Due to the presence of nonlinear dynamic uncertainties and unknown external disturbances on the UAS usually more sophisticated controllers are required.

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Robust and adaptive control are two typical control strategies for dealing with nonlinear functions with uncertain parameters. The goal of robust control is to dominate the unknown nonlinear term to control the closed-loop system. For example, in (Nemati and Montazeri (2018a)) a robust nonlinear sliding mode control technique is used to address the trajectory tracking problem of the attitude dynamic for a single UAV in the presence of disturbances and parametric uncertainties. Some other interesting results employing the robust control can be found in Huang and Chen (2004); Lewis et al. (2003).

On the other hand, by using adaptive control methods to deal with parametric uncertainties and the unknown external disturbances, the uncertain nonlinear terms are handled by estimating unknown parameters. Some studies in this respect can be found in Narendra and Annaswamy (1989); Astolfi et al. (2007) and with application in UAVs in Imran and Montazeri (2020); Imran et al. (2021).

Similar to the single setting, parametric uncertainties and unknown external disturbances are some of the main issues in designing control protocol under a collaborative setting. Both robust and adaptive methods are common approaches to deal with these issues. Some interesting results using robust control can be seen in Chen and Chen (2016); Zhu and Chen (2014) and using adaptive control in Lewis et al. (2013); Imran et al. (2019, 2022) for collaborative settings.

There are two typical control schemes for collaborative control Imran (2020). The first is centralized control. In this scheme, a central unit generates the control signals using the global information of the network. As a result, this technique is expensive in implementation. The second scheme is distributed/decentralized control. Under this situation, every agent generates its own controller without the global network of information. A local control protocol is generated by each agent based on the information received from some connected neighbours. Distributed controller is more realistic for practical settings and resource constrained agents. However, its design is more challenging, especially for systems with uncertainties. Moreover, for the system under a directed network, the corresponding Laplacian matrix is asymmetric, which significantly complicates the problem.

In this paper, a distributed robust scheme is developed for a leader-following consensus of multi-UAVs under a directed network. The system dynamic of each UAV is subject to different parameters as well as unknown time-varying external disturbances. Moreover, the global information of the states is not required for the control design, and only one or some agents have access to the leader. The synchronization controller of each agent is generated using the information of its own states and the relative angular position and velocity of the connected neighbours.

The remainder of the paper is organized as follows. The system dynamics of the UAVs is presented in section 2. This is followed by the distributed robust synchronization control of the attitude dynamic in Section 3. Then in Section 4, we present the numerical simulations to demonstrate the effectiveness of the proposed scheme in terms of achieving consensus in front of uncertainties. The paper is summarized with a recommendation for future work in the last section.

2. SYSTEM DYNAMICS OF MULTI-QUADROTOR UAVS

In this section, we present the attitude dynamic model of the multi-quadrotor UAVs. The attitude dynamic of the ith UAV (agent i) is expressed as

$$\ddot{\eta}_{2_i} = w_{1_i} f_i + w_{2_i}\tau_i + \delta_i,\ i = 1, \cdots, n, \tag{1}$$

where

$$\begin{aligned} w_{1_i} &= \mathrm{diag}\left[\frac{I_{y_i} - I_{z_i}}{I_{x_i}}\ \frac{I_{z_i} - I_{x_i}}{I_{y_i}}\ \frac{I_{x_i} - I_{y_i}}{I_{z_i}}\right] \in \mathbb{R}^{3\times 3} \\ w_{2_i} &= I_{M_i}^{-1} \in \mathbb{R}^{3\times 3} \\ f_i &= \left[\dot{\theta}_i\dot{\psi}_i\ \ \dot{\phi}_i\dot{\psi}_i\ \ \dot{\phi}_i\dot{\theta}_i\right]^\intercal \in \mathbb{R}^3. \end{aligned}$$

The vector $\eta_{2_i} = [\phi_i\ \theta_i\ \psi_i]^\intercal \in \mathbb{R}^3$ is an orientation vector composed of roll ϕ_i, pitch θ_i, and yaw ψ_i motions of agent i, and $\tau_i = [\tau_{\phi_i}\ \tau_{\theta_i}\ \tau_{\psi_i}]^\intercal \in \mathbb{R}^3$ is the torque vector acting on the body frame of the agent i in the rotational or attitude dynamics of the agent i and $I_{M_i} = \mathrm{diag}\,[I_{x_i}\ I_{y_i}\ I_{z_i}] \in \mathbb{R}^3$ is the inertia matrix of agent i, and $\delta_i = [\delta_{\phi_i}\ \delta_{\theta_i}\ \delta_{\psi_i}]^\intercal \in \mathbb{R}^3$ is the external disturbance of agent i. These disturbances satisfy the following boundaries

$$\|\delta_{\phi_i}\| \le d_{\phi_i},\ \|\delta_{\theta_i}\| \le d_{\theta_i},\ \|\delta_{\psi_i}\| \le d_{\psi_i} \tag{2}$$

where d_{ϕ_i}, d_{θ_i} and d_{ψ_i} are some constants.

Note that the disturbances δ_i in (2) are assumed to be unknown for the feedback control design. Moreover, the multi-UAV system studied in this paper is in the class of heterogeneous MASs. This indicates that each agent may have different inertia parameters and external disturbances.

The attitude dynamic (1) can be written in the compact from as follows

$$\begin{aligned} \ddot{\phi} &= w_1^1 f_1 + w_2^1 \tau_\phi + \delta_\phi \\ \ddot{\theta} &= w_1^2 f_2 + w_2^2 \tau_\theta + \delta_\theta \\ \ddot{\psi} &= w_1^3 f_3 + w_2^3 \tau_\psi + \delta_\psi \end{aligned} \tag{3}$$

where

$$\begin{aligned} \phi &= [\phi_1\ \cdots\ \phi_n]^\intercal \in \mathbb{R}^n \\ \theta &= [\theta_1\ \cdots\ \theta_n]^\intercal \in \mathbb{R}^n \\ \psi &= [\psi_1\ \cdots\ \psi_n]^\intercal \in \mathbb{R}^n \\ w_1^1 &= \mathrm{diag}\,[w_{1_1}(1,1)\ \cdots\ w_{1_n}(1,1)] \in \mathbb{R}^{n\times n} \\ w_1^2 &= \mathrm{diag}\,[w_{1_1}(2,2)\ \cdots\ w_{1_n}(2,2)] \in \mathbb{R}^{n\times n} \\ w_1^3 &= \mathrm{diag}\,[w_{1_1}(3,3)\ \cdots\ w_{1_n}(3,3)] \in \mathbb{R}^{n\times n} \\ w_2^1 &= \mathrm{diag}\,[w_{2_1}(1,1)\ \cdots\ w_{2_n}(1,1)] \in \mathbb{R}^{n\times n} \\ w_2^2 &= \mathrm{diag}\,[w_{2_1}(2,2)\ \cdots\ w_{2_n}(2,2)] \in \mathbb{R}^{n\times n} \\ w_2^3 &= \mathrm{diag}\,[w_{2_1}(3,3)\ \cdots\ w_{2_n}(3,3)] \in \mathbb{R}^{n\times n} \\ f_1 &= \left[\dot{\theta}_1\dot{\psi}_1\ \cdots\ \dot{\theta}_n\dot{\psi}_n\right]^\intercal \in \mathbb{R}^n \\ f_2 &= \left[\dot{\phi}_1\dot{\psi}_1\ \cdots\ \dot{\phi}_n\dot{\psi}_n\right]^\intercal \in \mathbb{R}^n \\ f_3 &= \left[\dot{\phi}_1\dot{\theta}_1\ \cdots\ \dot{\phi}_n\dot{\theta}_n\right]^\intercal \in \mathbb{R}^n \\ \tau_\phi &= [\tau_{\phi_1}\ \cdots\ \tau_{\phi_n}]^\intercal \in \mathbb{R}^n \\ \tau_\theta &= [\tau_{\theta_1}\ \cdots\ \tau_{\theta_n}]^\intercal \in \mathbb{R}^n \\ \tau_\psi &= [\tau_{\psi_1}\ \cdots\ \tau_{\psi_n}]^\intercal \in \mathbb{R}^n \\ \delta_\phi &= [\delta_{\phi_1}\ \cdots\ \delta_{\phi_n}]^\intercal \in \mathbb{R}^n \\ \delta_\theta &= [\delta_{\theta_1}\ \cdots\ \delta_{\theta_n}]^\intercal \in \mathbb{R}^n \\ \delta_\psi &= [\delta_{\psi_1}\ \cdots\ \delta_{\psi_n}]^\intercal \in \mathbb{R}^n \\ &. \end{aligned} \tag{4}$$

There exists an active virtual leader represented by the following dynamics

$$\begin{aligned} \ddot{\phi}_0 &= f_0^1(\phi_0, \dot{\phi}_0) \\ \ddot{\theta}_0 &= f_0^2(\theta_0, \dot{\theta}_0) \\ \ddot{\psi}_0 &= f_0^3(\psi_0, \dot{\psi}_0) \end{aligned} \tag{5}$$

where $\phi_0 \in \mathbb{R}$. $\theta_0 \in \mathbb{R}$ and $\psi_0 \in \mathbb{R}$ are the roll, pitch and yaw states of the leader, respectively. The function $f_0^1(\phi_0, \dot{\phi}_0) \in \mathbb{R}$, $f_0^2(\theta_0, \dot{\theta}_0) \in \mathbb{R}$ and $f_0^3(\psi_0, \dot{\psi}_0) \in \mathbb{R}$ are the time-varying nominal behavior of the leader. In other words, the dynamics (5) can be regarded as a reference or a command generator. These nominal behaviors satisfy the following boundaries

$$\|f_0^1(\phi_0, \dot{\phi}_0)\| \le f_m^1,\ \|f_0^2(\theta_0, \dot{\theta}_0)\| \le f_m^2,\ \|f_0^3(\psi_0, \dot{\psi}_0)\| \le f_m^3, \tag{6}$$

where f_m^1, f_m^2 and f_m^3 are some constants.

Let $\mathbf{0}_{n\times n} \in \mathbb{R}^{n\times n}$ and $\mathbf{0}_n \in \mathbb{R}^{n\times 1}$ are a matrix and vector with all elements being zero, respectively, and $\mathbf{I}_n \in \mathbb{R}^{n\times n}$ is the identity matrix. For convenience of the presentation, we can rewrite the global dynamic of agent i as follows

$$\dot{\xi}_\phi = \bar{A}\xi_\phi + g_\phi \tag{7}$$

$$\dot{\xi}_\theta = \bar{A}\xi_\theta + g_\theta \tag{8}$$

$$\dot{\xi}_\psi = \bar{A}\xi_\psi + g_\psi \tag{9}$$

where

$$\bar{A} = \begin{bmatrix} \mathbf{0}_{n\times n} & \mathbf{I}_n \\ \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} \end{bmatrix} \in \mathbb{R}^{2n\times 2n},\ \xi_\phi = \begin{bmatrix} \phi \\ \dot{\phi} \end{bmatrix} \in \mathbb{R}^{2n}$$
$$g_\phi = \begin{bmatrix} \mathbf{0}_n \\ w_1^1 f_1 + w_2^1 \tau_\phi + \delta_\phi \end{bmatrix} \in \mathbb{R}^{2n},\ \xi_\theta = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \in \mathbb{R}^{2n}$$
$$g_\theta = \begin{bmatrix} \mathbf{0}_n \\ w_1^2 f_2 + w_2^2 \tau_\theta + \delta_\theta \end{bmatrix} \in \mathbb{R}^{2n},\ \xi_\psi = \begin{bmatrix} \psi \\ \dot{\psi} \end{bmatrix} \in \mathbb{R}^{2n}$$
$$g_\psi = \begin{bmatrix} \mathbf{0}_n \\ w_1^3 f_3 + w_2^3 \tau_\psi + \delta_\psi \end{bmatrix} \in \mathbb{R}^{2n}.$$

By following a similar way, we can rewrite the dynamic of the leader as follows

$$\dot{\xi}_{\phi_0} = \bar{A}\xi_{\phi_0} + f_{\phi_0} \tag{10}$$
$$\dot{\xi}_{\theta_0} = \bar{A}\xi_{\theta_0} + f_{\theta_0} \tag{11}$$
$$\dot{\xi}_{\psi_0} = \bar{A}\xi_{\psi_0} + f_{\psi_0} \tag{12}$$

where

$$\xi_{\phi_0} = \begin{bmatrix} \phi_0 \\ \dot{\phi}_0 \end{bmatrix} \in \mathbb{R}^2,\ f_{\phi_0} = \begin{bmatrix} \mathbf{0}_n \\ f_0^1(\phi_0, \dot{\phi}_0) \end{bmatrix} \in \mathbb{R}^2$$
$$\xi_{\theta_0} = \begin{bmatrix} \theta_0 \\ \dot{\theta}_0 \end{bmatrix} \in \mathbb{R}^2,\ f_{\theta_0} = \begin{bmatrix} \mathbf{0}_n \\ f_0^2(\theta_0, \dot{\theta}_0) \end{bmatrix} \in \mathbb{R}^2$$
$$\xi_{\psi_0} = \begin{bmatrix} \psi_0 \\ \dot{\psi}_0 \end{bmatrix} \in \mathbb{R}^2,\ f_{\psi_0} = \begin{bmatrix} \mathbf{0}_n \\ f_0^3(\psi_0, \dot{\psi}_0) \end{bmatrix} \in \mathbb{R}^2.$$

In this section, the network topology is represented by a graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{1, 2, \cdots, n\}$ is a finite non-empty set of nodes and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the set of directed edges. The Adjacency matrix is represented by $\mathcal{A} = [a_{ij}]$, where $a_{ij} > 0$ if the edge $(j, i) \in \mathcal{E}$, $i \neq j$, $a_{ij} = 0$ if $i = j$. As a result, there is no self-loop. The Laplacian matrix is denoted by L. The relationship between the leader and follower is represented by $B = \mathrm{diag}(b), b = [b_1, \cdots, b_n]^\intercal$, where $b_i \geq 0$ the weight from agent i to the leader.

This paper investigates a general directed leader-following multi-UAVs setting under the following assumptions.

Assumption 1. The network topology of the followers at least contains a directed spanning tree. ∎

Assumption 2. At least one agent is connected to the leader ∎

Under the Assumption 1, the Laplacian matrix L at least has one zero eigenvalue, and the rest of eigenvalues have positive real part. It means that L is a positive semi-definite matrix. On another side, B is also a positive semi-definite matrix under Assumption 2, where at least one element of diagonal has a positive real part, and the rest eigenvalues are zero.

3. CONSENSUS CONTROL DESIGN

In this section, the proposed consensus controller for the attitude dynamic is presented. The presence of unknown time-varying disturbance in the attitude dynamics is an essential issue in designing an attitude consensus controller. A complicated setting is considered in this study, where every UAV may have different parameters. Moreover, there exists an active virtual leader, and only one or some of the UAVs are connected to the leader.

We define the local neighborhood synchronization error of agent i as follows

$$e_{1_{\phi_i}} = \sum_{j=1, i\neq j}^{n} a_{ij}(\phi_j - \phi_i) + b_i(\phi_0 - \phi_i)$$
$$e_{2_{\phi_i}} = \sum_{j=1, i\neq j}^{n} a_{ij}(\dot{\phi}_j - \dot{\phi}_i) + b_i(\dot{\phi}_0 - \dot{\phi}_i) \tag{13}$$
$$e_{1_{\theta_i}} = \sum_{j=1, i\neq j}^{n} a_{ij}(\theta_j - \theta_i) + b_i(\theta_0 - \theta_i)$$
$$e_{2_{\theta_i}} = \sum_{j=1, i\neq j}^{n} a_{ij}(\dot{\theta}_j - \dot{\theta}_i) + b_i(\dot{\theta}_0 - \dot{\theta}_i) \tag{14}$$
$$e_{1_{\psi_i}} = \sum_{j=1, i\neq j}^{n} a_{ij}(\psi_j - \phi_i) + b_i(\psi_0 - \psi_i)$$
$$e_{2_{\psi_i}} = \sum_{j=1, i\neq j}^{n} a_{ij}(\dot{\psi}_j - \dot{\psi}_i) + b_i(\dot{\psi}_0 - \dot{\psi}_i) \tag{15}$$

The main objective of the agent i is to follow the virtual leader as expressed by the following equation

$$\lim_{t\to\infty} \left[e_{1_{\phi_i}}(t)\ e_{2_{\phi_i}}(t)\ e_{1_{\theta_i}}(t)\ e_{2_{\theta_i}}(t)\ e_{1_{\psi_i}}(t)\ e_{2_{\psi_i}}(t) \right]^\intercal = \mathbf{0}_6 \tag{16}$$

where $\mathbf{0}_6 \in \mathbb{R}^6$ is a vector of zeros.

The global neighborhood synchronization error can be written as follows

$$e_{1_\phi} = -(L+B)(\phi - \mathbf{1}_n\phi_0)$$
$$e_{2_\phi} = -(L+B)(\dot{\phi} - \mathbf{1}_n\dot{\phi}_0) \tag{17}$$
$$e_{1_\theta} = -(L+B)(\theta - \mathbf{1}_n\phi_0)$$
$$e_{2_\theta} = -(L+B)(\dot{\theta} - \mathbf{1}_n\dot{\theta}_0) \tag{18}$$
$$e_{1_\psi} = -(L+B)(\psi - \mathbf{1}_n\psi_0)$$
$$e_{2_\psi} = -(L+B)(\dot{\psi} - \mathbf{1}_n\dot{\psi}_0), \tag{19}$$

where $\mathbf{1}_n$ is an $n \times 1$ vector with all elements chosen as 1.

From (3), (5), (17), (18) and (19), we can generate the dynamics error to be

$$\dot{e}_\phi = \begin{bmatrix} e_{2_\phi} \\ -(L+B)(w_1^1 f_1 + w_2^1\tau_\phi + \delta_\phi - \mathbf{1}_n f_0^1(\phi_0, \dot{\phi}_0)) \end{bmatrix} \tag{20}$$
$$\dot{e}_\theta = \begin{bmatrix} e_{2_\theta} \\ -(L+B)(w_1^2 f_2 + w_2^2\tau_\theta + \delta_\theta - \mathbf{1}_n f_0^2(\theta_0, \dot{\theta}_0)) \end{bmatrix} \tag{21}$$
$$\dot{e}_\psi = \begin{bmatrix} e_{2_\psi} \\ -(L+B)(w_1^3 f_3 + w_2^3\tau_\psi + \delta_\psi - \mathbf{1}_n f_0^3(\psi_0, \dot{\psi}_0)) \end{bmatrix}, \tag{22}$$

where

$$e_\phi = \begin{bmatrix} e_{2_\phi} \\ \dot{e}_{2_\phi} \end{bmatrix} \in \mathbb{R}^{2n},\ e_\theta = \begin{bmatrix} e_{2_\theta} \\ \dot{e}_{2_\theta} \end{bmatrix} \in \mathbb{R}^{2n},\ e_\psi = \begin{bmatrix} e_{2_\psi} \\ \dot{e}_{2_\psi} \end{bmatrix} \in \mathbb{R}^{2n}.$$

Before presenting our main results, we have two technical Lemmas used to prove the performance of the control protocol as follows

Lemma 3.1. *Lewis et al. (2013) If the network satisfies Assumption 1 and 2, then $L + B$ is a positive definite matrix.* ∎

Lemma 3.2. *Under the Assumption 1 and 2, there exists some positive constants γ_{ϕ_1}, γ_{ϕ_2}, γ_{θ_1}, γ_{θ_2}, γ_{ψ_1} and γ_{ψ_2} such that*

$$A_\phi = \begin{bmatrix} \boldsymbol{0}_{n\times n} & \boldsymbol{I}_n \\ -\gamma_{\phi_1}(L+B) & -\gamma_{\phi_2}(L+B) \end{bmatrix}$$

$$A_\theta = \begin{bmatrix} \boldsymbol{0}_{n\times n} & \boldsymbol{I}_n \\ -\gamma_{\theta_1}(L+B) & -\gamma_{\theta_2}(L+B) \end{bmatrix},$$

and

$$A_\psi = \begin{bmatrix} \boldsymbol{0}_{n\times n} & \boldsymbol{I}_n \\ -\gamma_{\psi_1}(L+B) & -\gamma_{\psi_2}(L+B) \end{bmatrix}.$$

∎

Proof: All eigenvalues of $L+B$ have positive real parts under the Assumption 1 and 2 as stated in Lemma 3.1. Let $P_\phi \in \mathbb{R}^{(n)\times(n)}$ be the positive definite matrix such that

$$P_\phi(L+B) + (L+B)^\intercal P_\phi = \mathbf{I}_n.$$

There exists a positive constant c_ϕ such that

$$P_\phi < 2c_\phi \mathbf{I}_n$$

and the Schur complement implies that

$$Q_{\phi_a} = \begin{bmatrix} \mathbf{I}_n & c_\phi\mathbf{I}_n - P_\phi \\ c_\phi\mathbf{I}_n - P_\phi & c_\phi^2\mathbf{I}_n \end{bmatrix} > 0.$$

Let

$$P_\phi = \begin{bmatrix} \gamma_{\phi_1}(L+B) & (L+B) \\ (L+B) & c_\phi(L+B) \end{bmatrix}$$

is the solution of Lyapunov function and a positive definite matrix for any $c_\phi\gamma_{\phi_1} > 1$ such that

$$P_\phi A_\phi + A_\phi{}^\intercal P_\phi = -Q_\phi, \tag{23}$$

where

$$Q_\phi = \gamma_{\phi_1} Q_{\phi_a} - Q_{\phi_b},\ Q_{\phi_b} = \begin{bmatrix} \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} \\ \mathbf{0}_{n\times n} & 2P_\phi \end{bmatrix}.$$

By selecting a sufficiently large $\gamma_{\phi_1} > 0$ and $\gamma_{\phi_2} = c_\phi\gamma_{\phi_1}$, thus Q_ϕ is a positive definite matrix.

By following a similar way, we can prove that both A_θ and A_ψ are Hurwitz by selecting a sufficiently large $\gamma_{\theta_1} > 0$, $\gamma_{\psi_1} > 0$, $\gamma_{\theta_2} = c_\theta\gamma_{\theta_1} > 1$ and $\gamma_{\psi_2} = c_\phi\gamma_{\phi_1} > 1$, thus

$$P_\theta A_\theta + A_\theta{}^\intercal P_\theta = -Q_\theta \tag{24}$$

$$P_\psi A_\psi + A_\psi{}^\intercal P_\psi = -Q_\psi \tag{25}$$

where

$$P_\theta = \begin{bmatrix} \gamma_{\theta_1}(L+B) & (L+B) \\ (L+B) & c_\theta(L+B) \end{bmatrix} > 0$$

$$P_\psi = \begin{bmatrix} \gamma_{\psi_1}(L+B) & (L+B) \\ (L+B) & c_\psi(L+B) \end{bmatrix} > 0$$

are the solution of the Lyapunov functions and

$$Q_\theta = \gamma_{\theta_1} Q_{\theta_a} - Q_{\theta_b},\ Q_\psi = \gamma_{\psi_1} Q_{\psi_a} - Q_{\psi_b}$$

$$Q_{\theta_a} = \begin{bmatrix} \mathbf{I}_n & c_\theta\mathbf{I}_n - P_\theta \\ c_\theta\mathbf{I}_n - P_\theta & c_\theta^2\mathbf{I}_n \end{bmatrix} > 0$$

$$Q_{\psi_a} = \begin{bmatrix} \mathbf{I}_n & c_\psi\mathbf{I}_n - P_\psi \\ c_\psi\mathbf{I}_n - P_\psi & c_\psi^2\mathbf{I}_n \end{bmatrix} > 0$$

$$Q_{\theta_b} = \begin{bmatrix} \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} \\ \mathbf{0}_{n\times n} & 2P_\theta \end{bmatrix},\ Q_{\psi_b} = \begin{bmatrix} \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} \\ \mathbf{0}_{n\times n} & 2P_\psi \end{bmatrix}.$$

∎

Next, we present the main result summarized in Theorem 3.1.

Theorem 3.1. Consider the system (1) under the Assumption 1 and 2. The consensus objective (16) is asymptotically achieved by selecting

$$\begin{aligned}\tau_{\phi_i} &= w_{2_i}(1,1)^{-1}\big(-w_{1_i}(1,1)f_i(1,1) + \gamma_{\phi_1}e_{1_{\phi_i}} \\ &\quad + \gamma_{\phi_2}e_{2_{\phi_i}} - k_{\phi_i}\mathrm{sign}(e_{1_{\phi_i}} + c_\phi e_{2_{\phi_i}})\big)\end{aligned} \tag{26}$$

$$\begin{aligned}\tau_{\theta_i} &= w_{2_i}(2,2)^{-1}\big(-w_{1_i}(2,2)f_i(2,1) + \gamma_{\theta_1}e_{1_{\theta_i}} \\ &\quad + \gamma_{\theta_2}e_{2_{\theta_i}} - k_{\theta_i}\mathrm{sign}(e_{1_{\theta_i}} + c_\theta e_{2_{\theta_i}})\big)\end{aligned} \tag{27}$$

$$\begin{aligned}\tau_{\psi_i} &= w_{2_i}(3,3)^{-1}\big(-w_{1_i}(3,3)f_i(3,1) + \gamma_{\psi_1}e_{1_{\psi_i}} \\ &\quad + \gamma_{\psi_2}e_{2_{\psi_i}} - k_{\psi_i}\mathrm{sign}(e_{1_{\psi_i}} + c_\psi e_{2_{\psi_i}})\big)\end{aligned} \tag{28}$$

where γ_{ϕ_1}, γ_{ϕ_2}, γ_{θ_1}, γ_{θ_2}, γ_{ψ_1} and γ_{ψ_2} are given in Lemma 3.2 such that A_ϕ, A_θ and A_ψ are Hurwitz; and the gains

$$k_{\phi_i} \geq d_{\phi_i},\ k_{\theta_i} \geq d_{\theta_i},\ k_{\psi_i} \geq d_{\psi_i}. \tag{29}$$

Proof: We can write the control inputs (26), (27) and (28) in the compact form as follows

$$\begin{aligned}\tau_\phi &= {w_2^1}^{-1}\big(-w_1^1 f_1 + \gamma_{\phi_1}e_{1_\phi} + \gamma_{\phi_2}e_{2_\phi} \\ &\quad - k_\phi\mathrm{sign}(e_{1_\phi} + c_\phi e_{2_\phi})\big)\end{aligned} \tag{30}$$

$$\begin{aligned}\tau_\theta &= {w_2^2}^{-1}\big(-w_1^2 f_2 + \gamma_{\theta_1}e_{1_\theta} + \gamma_{\theta_2}e_{2_\theta} \\ &\quad - k_\theta\mathrm{sign}(e_{1_\theta} + c_\theta e_{2_\theta})\big)\end{aligned} \tag{31}$$

$$\begin{aligned}\tau_\psi &= {w_2^3}^{-1}\big(-w_1^3 f_3 + \gamma_{\psi_1}e_{1_\psi} + \gamma_{\psi_2}e_{2_\psi} \\ &\quad - k_\psi\mathrm{sign}(e_{1_\psi} + c_\psi e_{2_\psi})\big).\end{aligned} \tag{32}$$

Therefore

$$\begin{aligned}\dot{e}_\phi &= A_\phi e_\phi + \begin{bmatrix} \mathbf{0}_n \\ (L+B)\mathbf{1}_n f_0^1 \end{bmatrix} \\ &\quad - \begin{bmatrix} \mathbf{0}_n \\ (L+B)\big(\delta_\phi - k_\phi\mathrm{sign}(e_{1_\phi} + c_\phi e_{2_\phi})\big) \end{bmatrix}\end{aligned} \tag{33}$$

$$\begin{aligned}\dot{e}_\theta &= A_\theta e_\theta + \begin{bmatrix} \mathbf{0}_n \\ (L+B)\mathbf{1}_n f_0^2 \end{bmatrix} \\ &\quad - \begin{bmatrix} \mathbf{0}_n \\ (L+B)\big(\delta_\theta - k_\theta\mathrm{sign}(e_{1_\theta} + c_\theta e_{2_\theta})\big) \end{bmatrix}\end{aligned} \tag{34}$$

$$\begin{aligned}\dot{e}_\psi &= A_\psi e_\psi + \begin{bmatrix} \mathbf{0}_n \\ (L+B)\mathbf{1}_n f_0^3 \end{bmatrix} \\ &\quad - \begin{bmatrix} \mathbf{0}_n \\ (L+B)\big(\delta_\psi - k_\psi\mathrm{sign}(e_{1_\psi} + c_\psi e_{2_\psi})\big) \end{bmatrix},\end{aligned} \tag{35}$$

where

$$k_\phi = \mathrm{diag}\,[k_{\phi_1}\ \cdots\ k_{\phi_n}] \in \mathbb{R}^{n\times n}$$
$$k_\theta = \mathrm{diag}\,[k_{\theta_1}\ \cdots\ k_{\theta_n}] \in \mathbb{R}^{n\times n}$$
$$k_\psi = \mathrm{diag}\,[k_{\psi_1}\ \cdots\ k_{\psi_n}] \in \mathbb{R}^{n\times n}.$$

We select the Lyapunov functions of (33), (34) and (35) to be

$$V_{e_\phi} = e_\phi^\intercal P_\phi e_\phi \tag{36}$$

$$V_{e_\theta} = e_\theta^\intercal P_\theta e_\theta \tag{37}$$

$$V_{e_\psi} = e_\psi^\intercal P_\psi e_\psi. \tag{38}$$

We define $\underline{\lambda}(.)$ and $\bar{\lambda}(.)$ to be the minimum and maximum of matrix (.), respectively. The time-derivative of Lyapunov functions (36), (37) and (38) can be calculated as follows

$$\begin{aligned}\dot{V}_{e_\phi} &\leq e_\phi^\intercal(P_\phi A_\phi + A_\phi^\intercal P_\phi)e_\phi + e_\phi^\intercal P_\phi r_\phi \\ &\quad + e_\phi^\intercal P_\phi(d_\phi - k_\phi\mathrm{sign}(e_{1_\phi} + c_\phi e_{2_\phi})) \\ &\leq -e_\phi^\intercal Q_\phi e_\phi + e_\phi^\intercal P_\phi r_\phi\end{aligned}$$

$$\leq -\underline{\lambda}(Q_\phi)\|e_\phi\|^2 + \bar{\lambda}(P_\phi)\|r_\phi\|\|e_\phi\| \tag{39}$$

$$\begin{aligned}\dot{V}_{e_\theta} &\leq e_\theta^\top(P_\theta A_\theta + A_\theta^\top P_\theta)e_\theta + e_\theta^\top P_\theta r_\theta \\ &\quad + e_\theta^\top P_\theta(d_\theta - k_\theta \mathrm{sign}(e_{1_\theta} + c_\theta e_{2_\theta})) \\ &\leq -e_\theta^\top Q_\theta e_\theta + e_\theta^\top P_\theta r_\theta \\ &\leq -\underline{\lambda}(Q_\theta)\|e_\theta\|^2 + \bar{\lambda}(P_\theta)\|r_\theta\|\|e_\theta\| \end{aligned} \tag{40}$$

$$\begin{aligned}\dot{V}_{e_\psi} &\leq e_\psi^\top(P_\psi A_\psi + A_\psi^\top P_\psi)e_\psi + e_\psi^\top P_\psi r_\psi \\ &\quad + e_\psi^\top P_\psi(d_\psi - k_\psi \mathrm{sign}(e_{1_\psi} + c_\psi e_{2_\psi})) \\ &\leq -e_\psi^\top Q_\psi e_\psi + e_\psi^\top P_\psi r_\psi \\ &\leq -\underline{\lambda}(Q_\psi)\|e_\psi\|^2 + \bar{\lambda}(P_\psi)\|r_\psi\|\|e_\psi\|, \end{aligned} \tag{41}$$

where

$$d_\phi = [d_{\phi_1}\ \cdots\ d_{\phi_n}]^\top \in \mathbb{R}^n,\ d_\theta = [d_{\theta_1}\ \cdots\ d_{\theta_n}]^\top \in \mathbb{R}^n$$

$$d_\psi = [d_{\psi_1}\ \cdots\ d_{\psi_n}]^\top \in \mathbb{R}^n,\ r_\phi = \begin{bmatrix} \mathbf{0}_n \\ (L+B)\mathbf{1}_n f_m^1 \end{bmatrix}$$

$$r_\theta = \begin{bmatrix} \mathbf{0}_n \\ (L+B)\mathbf{1}_n f_m^2 \end{bmatrix},\ r_\psi = \begin{bmatrix} \mathbf{0}_n \\ (L+B)\mathbf{1}_n f_m^3 \end{bmatrix}.$$

Then $\dot{V}_{e_\phi} \leq 0$, $\dot{V}_{e_\theta} \leq 0$, and $\dot{V}_{e_\psi} \leq 0$ if

$$\|e_\phi\| > \frac{\bar{\lambda}(P_\phi)\|r_\phi\|}{\underline{\lambda}(Q_\phi)} \tag{42}$$

$$\|e_\theta\| > \frac{\bar{\lambda}(P_\theta)\|r_\theta\|}{\underline{\lambda}(Q_\theta)} \tag{43}$$

$$\|e_\psi\| > \frac{\bar{\lambda}(P_\psi)\|r_\psi\|}{\underline{\lambda}(Q_\psi)}. \tag{44}$$

The Lyapunov functions (36), (37) and (38) satisfy the following conditions

$$\underline{\lambda}(P_\phi)\|e_\phi\|^2 \leq V_{e_\phi} \leq \bar{\lambda}(P_\phi)\|e_\phi\|^2 \tag{45}$$

$$\underline{\lambda}(P_\theta)\|e_\theta\|^2 \leq V_{e_\theta} \leq \bar{\lambda}(P_\theta)\|e_\theta\|^2 \tag{46}$$

$$\underline{\lambda}(P_\psi)\|e_\psi\|^2 \leq V_{e_\psi} \leq \bar{\lambda}(P_\psi)\|e_\psi\|^2. \tag{47}$$

Hence

$$V_{e_\phi} > \frac{\bar{\lambda}(P_\phi)^3\|r_\phi\|^2}{\underline{\lambda}(Q_\phi)^2} \tag{48}$$

$$V_{e_\theta} > \frac{\bar{\lambda}(P_\theta)^3\|r_\theta\|^2}{\underline{\lambda}(Q_\theta)^2} \tag{49}$$

$$V_{e_\psi} > \frac{\bar{\lambda}(P_\psi)^3\|r_\psi\|^2}{\underline{\lambda}(Q_\psi)^2}, \tag{50}$$

imply (42), (43) and (44). Therefore, the proof is completed. ∎

4. SIMULATION RESULTS

In this section, we numerically evaluate the performance of the proposed consensus protocol. The parameters of the multi-UAVs used in this simulation are presented in following Table.

Table 1. The parameters of a quadrotor UAVs.

Parameter Notation	Value (UAVs 1-3)	Value (UAVs 4-5)
I_x	$0.0069 kg.m^2$	$0.0082 kg.m^2$
I_y	$0.0069 kg.m^2$	$0.0082 kg.m^2$
I_z	$0.0129 kg.m^2$	$0.0190 kg.m^2$

The network topology of leader-following setting is represented by Figure 1. From the topology, we can calculate L

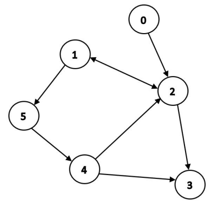

Fig. 1. The network topology of five agents and one leader

and B matrices of the network as represented by

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & 0 & -1 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix},\ B = \begin{bmatrix} 0&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \end{bmatrix}.$$

We maintain the attitude dynamics of heterogeneous UAVs using control protocols developed in Theorem 3.1. The following external disturbances are added to the closed-loop systems

$$\delta_\phi = [\sin(t)\ -\cos(t)\ -\sin(t)\ \cos(t)\ \sin(2t)]^\top$$

$$\delta_\theta = 0.1\,[-\sin(2t)\ -\cos(2t)\ \sin(t)\ \sin(t)\ \cos(t)]^\top$$

$$\delta_\psi = 0.01\,[\sin(t)\ \cos(3t)\ -\sin(2t)\ \cos(t)\ \cos(t)]^\top.$$

The gains of (26), (27) and (28) are selected as follows

$$\gamma_{\phi_1} = 70,\ c_\phi = 0.5,\ \gamma_{\theta_1} = 100$$

$$c_\theta = 0.3,\ \gamma_{\psi_1} = 50,\ c_\psi = 0.75$$

$$k_\phi = 1.2\mathbf{I}_5,\ k_\theta = 0.11\mathbf{I}_5,\ k_\psi = 0.012\mathbf{I}_5.$$

The initial conditions of the leader and followers are set to be

$$\phi_0(0) = 1.1,\ \theta_0(0) = 0.6,\ \psi_0(0) = 0$$

$$\phi(0) = [2\ 1.5\ 0\ -2\ -1.5]^\top$$

$$\theta(0) = [0.75\ 0.5\ -0.5\ -1\ 1]^\top$$

$$\psi(0) = [0.5\ 1\ 2\ 0.8\ -2]^\top.$$

The simulation results for synchronization control of the multi-UAVs are illustrated in Figure 2. From the results, we can verify that all states of every UAV can achieve the consensus objective (16), as concluded in Theorem 3.1. The synchronization error of each UAV converges to zero, as can be seen in Figure 3. The profiles of torque τ are presented in Figure 4.

5. CONCLUSION

We present a distributed leader-following consensus for attitude dynamics of multi-UAVs. The main contribution is to design a robust distributed consensus protocol for attitude dynamics of UAV i with the presence of unknown time-varying disturbances. A distributed controller is designed to maintain the synchronization control of heterogeneous UAVs with an active virtual leader. We verify the effectiveness of the consensus control in the rigorous mathematical proof. Also, a simulation of five UAVs is presented to demonstrate the performance of our scheme. It will be interesting to extend our scheme for 6-DOF with fully parametric uncertainties in future work.

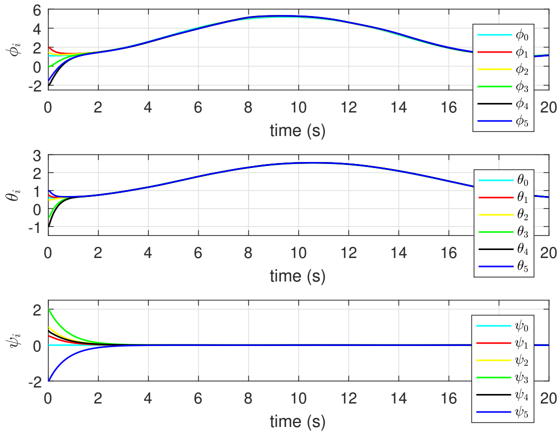

Fig. 2. The profile of ϕ_i, θ_i and ψ_i.

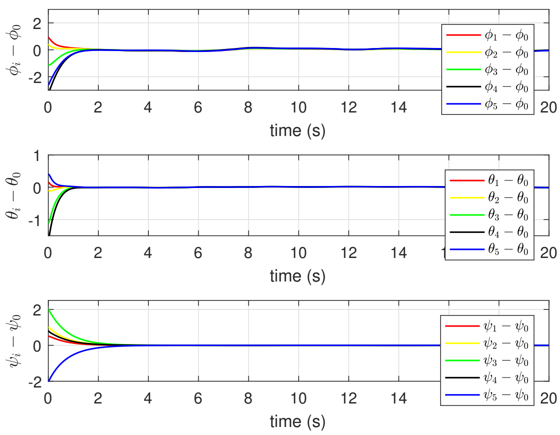

Fig. 3. The profile of consensus error.

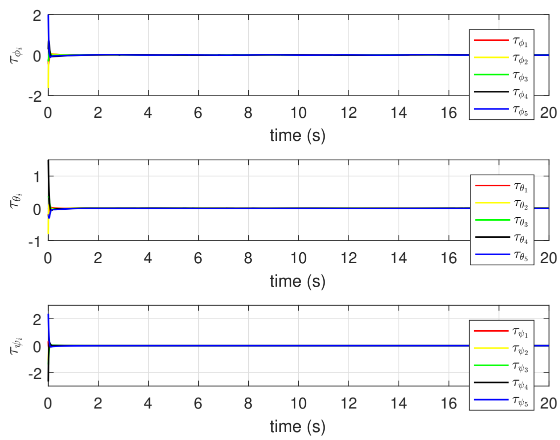

Fig. 4. The profile of τ.

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