Single-dimensional leg-level dynamic programming with booking-time dependent cancellation probabilities for revenue management

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Abstract: In this paper, an optimisation method is introduced that accounts for cancellations. We do so by estimating the opportunity cost of a booking between the time of booking and the expected time of cancellation. The formulation involves an estimate of the value of the state of the system at the time of cancellation (which is in the future), found through novel heuristics we introduce. The fare that is used to determine whether a product is available for sale, is adjusted by the risk the airline faces. We introduce an example which shows that there may be cases where it is optimal to reject a higher-priced product if the risk of cancellation is high, while accepting a lower-priced product. Simulations show increases in revenues against a traditional formulations that does not explicitly models cancellations. We show our method is robust against choice of heuristic, misjudgement of cancellation probability and forecasting errors.

Keywords: dynamic programming; single-dimensional state space; Poisson process; simulation; revenue management.

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1 Introduction

Cancellations in RM pose a risk to the company trying to maximise revenue. There are two different risks. The first risk, *is not being able to resell the unit of capacity (a hotel room, rental car, seat on a plane) before the product offering perishes (when time has passed, such as a night has passed, or a plane has taken off).* The second risk, is an implicit risk:*bookings on hand limit the number of units of capacity for sale. An increase in bookings on hand typically imply a positive price change, which in turn means a smaller customer base.* In the remainder of this paper, we will focus on the these problems in the context of the airline RM problem.

In the airline RM, cancellation rates vary greatly by point-of-sale (POS). The POS represents in what country a ticket is sold. Cancellation policies often depend on both the POS and origin and destination (OD) pair, but behaviour is typically driven by POS. Cancellation policies are therefore set accordingly by POS. Consequently, cancellations vary greatly by POS: naturally, POS's with low cancellation fees demonstrate a higher percentage of cancellations than POS with high fees. In practice, cancellation rates range from 20% to 60% in some cases.

The aforementioned risk of cancellation depends on time, as well as on the number of expected cancellations. Consider the time aspect. A booking and a corresponding cancellation one year before departure has a very low risk of type the first type of risk: there is sufficient time to resell this seat. Similarly, second type of risk is low since early on in the booking curve, capacity is likely still low and therefore plenty of availability. However, now consider a booking a year before departure and a cancellation an hour before departure. In this case, there is a total risk of the first type: there is no time left to resell this unit of capacity. The risk of type two depends how full the flight is: if it is at or near capacity, this one seat will have had an effect on availability and others would have been unable to purchase.

In practice, airlines combat this problem by overbooking. Overbooking is the process of selling more seats than physical capacity. Overbooking too many seats has financial consequences to the airline: in European territories, an airline is obliged to pay up to €600 and reaccommodate a passenger at the earliest possibility, even if this means rebooking on other airlines. Outside European territories, only the latter applies, but this still comes at a cost. Similarly, overbooking not enough results in empty seats which could have been sold. This shows the need for an accurate cancellation forecast and corresponding optimisation.

In our work, cancellation rates do not only depend on time of cancellation, but also on time booked. We have found substantial evidence that rates depending on class are not sufficient, but, rather, the time of booking is just as, if not more important. This is something that was shown in Hopman (2020).

This paper is structured as follows. We provide a literature review in Section 2. In Section 3, the formulation without cancellations is introduced. In Section 4 we introduce our dynamic program with cancellations. Section 5 covers results where we compare these two formulations. A discussion is given in Section 6.

2 Literature review

The end goal of RM is to determine the optimal booking policy. In this case, an optimal policy is the policy that maximises (network) revenue. The field of optimisation can be separated by assumptions on whether cancellations are modelled or not; by single resource or network optimisation; and, finally, by assumptions on demand: independent or customer choice. Cancellations can either be modelled directly (for example, by explicitly calculating cancellation probabilities and using these in an optimisation strategy) or indirectly (by removing cancellations from demand inputs, and assuming cancellations would not occur). Wang et al. (2015) provide an overview of challenges and progress in the RM field. They conclude nine emerging themes in RM and eight managerial shifts. One of these shifts is the change in demand forecasting from historical data to using big data techniques. It is interesting to note, however, that they do not mention cancellations at any stage. Next, we make the distinction between leg-level control, and network optimisation. Leg-level control are methods that optimise flights individually, network optimisation are methods that optimise an airline's network all at once. In a study conducted by Weatherford (2009), it was reported that 38% of the airlines that responded use a network RM system. Therefore, a significant number of airlines still use leg-level optimisation techniques and this type of optimisation is still relevant.

First, we will review leg-level optimisation followed by network optimisation.

2.1 Leg-level optimisation

The first works of optimisation in RM are credited to Beckmann (1958) and Littlewood (1972). Beckmann (1958) uses continuous demand and finds an optimal policy by calculating a series of integrals. Littlewood (1972) approaches the optimisation approach as a newsvendor problem. This approach is extended by Belobaba (1987). Belobaba calls his approach the expected marginal seat revenue (EMSR), and introduces a method to allow more than two classes. Under conditions, it can be shown that this method is optimal for two booking classes, but requires a heuristic for more than two. Both Littlewood and Belobaba assume aggregated demand over the booking curve; that is, there is no element of time. The first work that incorporated time was Lee and Hersh (1993). In this work, they describes a time-dependent model. Demand is assumed to be independent and follows a Poisson process, and no cancellations are assumed. This dynamic program was the basis of many publications to come, as well as in our earlier work (Hopman et al., 2017). We labelled this as *DPID* and will continue to do so in this paper. Lautenbacher and Stidham (1999), extends the work by Lee and Hersh (1993) by studying the underlying Markov chain. They show that the optimal policy, without the element of time, as put forth by Beckmann (1958) and Littlewood (1972), can be expressed as a similar policy as the work by Lee and Hersh (1993). In their work, where they assume there are no cancellations, no-shows nor overbooking, they show that the value function by Lee and Hersh (1993) is concave and as a result the optimal policy found by solving the dynamic program can be translated in an optimal booking policy in terms of a booking limit. A booking limit is the number of seats that are available for sale for a given product, which is the very output of Beckmann (1958) and Littlewood (1972). The inclusion of cancellations was first studied by Subramanian et al. (1999). In this work, they analyse a Markov decision process with cancellations, no-shows and overbooking. They allow multiple fare classes. Just like Lautenbacher and Stidham (1999), they exploit an equivalance with a problem from queuing theory to transform a multi-dimensional state space into a single-dimensional dynamic program, by assuming cancellation probabilities are independent of fare class. Gosavii et al. (2002) extend this work by lifting the assumption of having class-independent cancellation probabilities and compare the results of this method to the EMSR method of Belobaba (1987) and show revenue gains. Just as important is the robustness of this work. They show that underestimation of probability of cancellation results in greater loss than overestimation. This bias of underforecasting lower fare classes and overforecasting higher fare classes is in line with what we reported in Hopman et al. (2017). Boyd and Kallesen (2004) move away from the independent demand assumption by segmenting passengers between priceable (passengers that book by fare) and yieldable (passengers that book by product). Forecasting yieldable demand can be thought of as forecasting independent demand, for priceable demand a new forecasting technique needed to be developed. Belobaba and Hopperstad (2004) describe one approach to forecast priceable demand. They forecast demand for the lowest class, and then estimate sell up probabilities to higher classes. Simulation-based optimisation was studied by, for example, Van Ryzin and Vulcano (2008) or Vulcano et al. (2010). The dangers of buydown, which in the long term cause ‘spiral down’, are shown in a study by Cooper et al. (2006). They develop a mathematical model that defines when spiral down occurs. Work on efficient frontiers, such as that of Phillips (2012) and Fiig et al. (2012) show what the effects are of buydown. Fiig et al. (2010) study the optimisation of mixed fare structures, and create the notion of fare adjustment. Fare adjusment adjusts the fare value by the risk of customers purchasing a lower fare. A completely different approach is introduced by Frenk et al. (2016) Instead, they use the closing time of sale for each product as a decision variable. Through a survey Gönsch (2017) finds that most revenue managers are risk-adverse. He finds that most algorithms assume uncertainty of demand, but very few consider uncertainty of fares (most often, fares are assumed to be constant over time). Instead, Gönsch (2017) introduces several methods that concern risk-adverse RM and hypothesises that these are important in practice. Hopman (2020) discuss these problems from a true practical perspective.

2.2 Network optimisation

There have been attempts to make leg-level controls possible for network RM. One of these metrics is found using approximate dynamic programming. In this approach, the value function of the network RM problem is approximated by a sum of flight value functions. This approach is given in Hopman (2020). While all of these optimisation methods require some sort of approximation – after all, they suffer from the curse of dimensionality – the methods that are followed were originally *designed* for network RM.

Bertsimas and Popescu (2003) introduce the network RM problem. They first show the Bellman equation, assuming no cancellations, which state space consists of a vector with remaining capacities of every flight. The suboptimality of of bid prices in certain cases can be found in Talluri and Van Ryzin's (2006, p.90) book. However, in practice the consensus seems to be that a bid price strategy works well. well. In particular, it is agreed that frequent reoptimisation is necessary. Pimentel et al. (2018) show that infrequent reoptimisation can result in revenue losses of 6% in the hospitality industry. Shihab et al. (2019) choose a different technique to combat the curse of dimensionality by using Q-learning. Yet another approach is taken by Dai et al. (2019) They study the network RM problem with cancellations and no-shows and propose a deterministic, continuous-time and continuous-state model to solve it.

The RM problem is also tackled from another angle, by means of 'choice-based' RM. The dynamic program that follows is hurt by the curse of dimensionality, and subsequently there has been research to tackle this by approximating the solution by a (deterministic) linear program, see Liu and van Ryzin (2008), for example. There have been several other approaches. Zhang and Adelman (2009) use a different approach and approximate the dynamic program using a weighted basis function. Kunnumkal and Topaloglu (2010) propose another approximate dynamic programming approach. An approximate dynamic programming formulation for network RM under customer choice was introduced by Zhang and Adelman (2009), but lacks cancellations. Bront et al. (2009) propose an alternative and use column generation. Sierag et al. (2015) were the first ones to consider the RM problem including cancellations and customer choice. Sierag and van der Mei (2016) further analyses the single-leg RM problem under customer choice. The performance of choice-based RM is investigated by Carrier and Weatherford (2015). They use the passenger origin and destination simulator (PODS) (PODS, a large simulation model with real airline inputs) to show that optimising using MNL models outperforms standard forecasting, but is outperformed hybrid forecasting and fare adjustment. For a great overview of PODS, we refer the reader to Carrier (2003). For an extensive review of dependent demand RM, we refer to the work of Weatherford and Ratliff (2010). They provide an overview for both non-choice and choice-based methods, for both forecasting and optimisation.

The importance of explicitly modelling cancellations is studied by Petraru (2016). He provides heuristics to estimate cancellation policies and uses PODS to show how each of these methods perform. He shows revenue gains between 1% and 3% over methods that do not use these heuristics.

The context of overbooking and cancellations in the restuarant industry was reviewed by Tse and Poon (2017). He considers cancellations, overbooking and walk-ins in the restaurant industry and uses a binomial distribution to model cancellations. For more background of cancellations in the aviation industry, please refer to Hopman (2020).

Another framework for overbooking and cancellations in airlines is put forth by Sulistio et al. (2008) They introduce different methods, that are based on probability (a fixed overbooking percentage), risk aversiveness (overbooking as a function of risk) and service level (having the expected number of denied boarding passengers less than some level).

3 Traditional formulation (no cancellations)

We begin by repeating the traditional dynamic programming formulation, introduced by Talluri and Van Ryzin (2006). Consider:

- λ_j is the arrival rate of class j, $j = 1, ..., J$
- f_j the fare of product/class j, $f_1 \geq f_2 \geq ... \geq f_J$
- x the remaining capacity
- t the time unit, $t = 1, ..., T$
- $R(t)$ a random variable, with $R(t) = p_j$ if a demand for class j arrives, and 0 otherwise.

Suppose we have discretised time in such a way that in each time slice, we can have at most one arrival. Also note that $P(R(t) = p_j) = \lambda_j(t)$. When presented with an arrival, we need to decide whether to receive the current revenue, given by the random variable R_t, and move to the next time unit with one unit of capacity less, or reject this arrival request but have the same number of capacity in the next time unit. Therefore, introduce an indicator variable $u \in (0, 1)$, which is what we want to maximise over:

$$R(t)u + V_{t+1}(x - 1)$$

Now define $V_t(x)$ as the value function that represents the expected revenue-to-go given t units of time left and x units of capacity.

$$V_t(x) = E\left(\max_{u \in (0,1)} R(t)u + V_{t+1}(x - u)\right) \tag{1}$$

We denote equation (1) as *DPID* to indicate this is the traditional dynamic programming formulation, without cancellations.

Equation (1) implies that $u = 1$, that is, accept a given request, if and only if:

$$\begin{aligned} &R(t) * 1 + V_{t+1}(x - 1) \geq R(t) * 0 + V_{t+1}(x) \\ &\rightarrow R(t) + V_{t+1}(x - 1) \geq V_{t+1}(x) \\ &\rightarrow R(t) \geq V_{t+1}(x) - V_{t+1}(x - 1) \end{aligned} \tag{2}$$

Having identified this, introduce:

$$\Delta V_t(x) = V_t(x) - V_t(x - 1). \tag{3}$$

$\Delta V_t(x)$ can be thought of a point-estimate for the gradient in the x direction. Now, taking the expected value of $R(t)$, we obtain the optimal policy:

$$\text{Accept a given product } j \text{ if and only if: } f_j > \Delta V_{t+1}(x) \tag{4}$$

4 New formulation

Define f_j as the fare of product j. The refund percentage is given by ρ_j. Therefore, the value returned to the customer if cancelled is equal to $\rho_j f_j$. Let $c_j(t, \tau)$ be the probability that product j booked at time t is cancelled at time τ. For more information how this is calculated, please refer to Hopman (2020). Let $\zeta_j = \sum_{\tau=t+1}^{T} c_j(t, \tau)$, such that $1 - \zeta_j$ represents the probability that product j is not cancelled.

To define our new formulation, we consider two new random variables. $R(t)$, is as before, represents the direct reward for accepting product j. The second random variable, $C(t)$, represents the future (negative) revenue of the system of a cancellation. Next, we introduce $O(t, x(t))$ which is the opportunity cost of one seat up to time t. This opportunity cost is of course dependent on the state of the system x at time t.

$$R(t) = \begin{cases} f_j + V_t(x-1) & \text{if a request arrives for product } j \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Since a request for product j arrives using a Poisson process with $\lambda_j(t)$, we have $P(R(t) = f_j + V_t(x-1)) = \lambda_j(t)$.

$$C(t) = \begin{cases} -\rho_j f_j & \text{if a request for cancellation for product } j \text{ occurs} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Equation (6) says that the future reward depends on whether a cancellation request arrives. If a product cancels, the airline has to refund $\rho_j f_j$ to the customer. If it does not cancel, there is no (negative) revenue. A cancellation request having booked at time t occurs at time τ with $c_j(t, \tau)$, so we have $P(C(t) = -\rho f_j) = c_j(t, \tau)$.

Next, introduce:

$$O(\tau, x(\tau)) = \begin{cases} V_\tau(\tilde{s}) - V_\tau(\tilde{s}-1) & \text{if a product } j \text{ is cancelled} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

Similarly, we only suffer an opportunity cost in the future if a booking will cancel (if it does not, it is only evaluated against the opportunity cost $V_{t+1}(x) - V_{t+1}(x-1)$), so we have $P(O(\tau, x(\tau)) = V_\tau(\tilde{s}) - V_\tau(\tilde{s}-1)) = c_j(t, \tau)$. It is not cancelled, we do not incur any future opportunity cost. Of course, a-priori we do not know this, and we also do not know what state, $\tilde{s}$, we are in at a future time τ.

Equation (7) says that if a product is cancelled at time τ, with probability $c_j(t, \tau)$, the airline lost the opportunity of that one seat up to that point τ. We use the $\tilde{s}$ to indicate that this itself is a random variable. This term, $V_\tau(\tilde{s})$ term itself introduces complexity. After all, since a-priori we do not know when this request when this request will come. Similarly, we do not know what state (how many seats) the system will be in. In the next section, we will discuss simple heuristics to estimate $V_\tau(\tilde{s})$.

The expected reward from accepting a booking is then equal to:

$$E[R(t)] + E[C(t)] - E[O(t)] \tag{8}$$

We make this distinction so that it is clear to the reader what the revenue is that the airline expects: it receives a direct revenue given by $R(t)$, has to offer a refund given by $C(t)$ if cancelled, and loses $O(t)$ revenue in between. Therefore, the new Bellman equation follows that of equation (1), and is equal to:

$$V_t(x) = E\left(\max_{u\in(0,1)} u(R(t)+C(t)-O(t)) + (1-u)V_{t+1}(x)\right) \quad (DPC) \tag{9}$$

Similar to the naming convention of equation (1), we name equation (9) as *DPC*, our dynamic programming formulation including cancellations.

The optimal policy can be derived similarly. Maximising the term, taking $u = 1$, or, similarly, accepting a product j, happens if and only if:

$$\begin{aligned}
&1 * (R(t)+C(t)+O(t)) + (1-1)V_{t+1}(x)\\
&> 0 * (R(t)+C(t)+O(t)) + (1-0)V_{t+1}(x)\\
&\rightarrow E[R(t)+E[C(t)]+E[O(t)] > V_{t+1}(x)
\end{aligned} \tag{10}$$

We will derive the optimal policy below.

$E[R(t)]+E[C(t)]+E[O(t)] > V_{t+1}(x)$, which implies

$$\begin{aligned}
&= f_j + V_t(x) + \sum_{\tau=t+1}^{T} c_j(t,\tau)(-\rho_j f_j) + \left(1 - \sum_{\tau=t+1}^{T} c_j(t,\tau)\right) * 0\\
&- \sum_{\tau=t+1}^{T} c_j(t,\tau)(V_\tau(\tilde{s}) - V_\tau(\tilde{s}-1)) + \left(1 - \sum_{\tau=t+1}^{T} c_j(t,\tau)\right) * 0 > V_{t+1}(x)
\end{aligned}$$

Removing zero-valued terms, we obtain:

$$\begin{aligned}
&= f_j + V_t(x) + \sum_{\tau=t+1}^{T} c_j(t,\tau)(-\rho_j f_j)\\
&- \sum_{\tau=t+1}^{T} c_j(t,\tau)(V_\tau(\tilde{s}) - V_\tau(\tilde{s}-1)) > V_{t+1}(x)
\end{aligned}$$

Since $-\rho_j f_j$ does not depend on τ, and we have defined

$$\zeta_j = \sum_{\tau=t+1}^{T} c_j(t,\tau), \text{ we have}$$

$$= f_j + V_t(x-1) - \zeta_j\rho_j f_j - \sum_{\tau=t+1}^{T} c_j(t,\tau)(V_\tau(\tilde{s}) - V_\tau(\tilde{s}-1)) > V_{t+1}(x)$$

Collecting terms, we obtain:

$$= f_j(1-\zeta_j\rho_j) + V_t(x-1) - \sum_{\tau=t+1}^{T} c_j(t,\tau)(V_\tau(\tilde{s}) - V_\tau(\tilde{s}-1)) > V_{t+1}(x).$$

Finally, rearranging terms, we have:

$$= f_j(1-\zeta_j\rho_j) - \sum_{\tau=t+1}^{T} c_j(t,\tau)(V_\tau(\tilde{s}) - V_\tau(\tilde{s}-1)) > V_{t+1}(x) - V_{t+1}(x-1)$$

Note that the right hand side is as before, the opportunity cost of capacity in the next time unit.

Let us define $f_j^* = f_j(1 - \zeta_j \rho_j)$. Next, define $\Delta V_t(x) = V_t(x) - V_t(x-1)$. Substituting these terms, we obtain the following policy, given in equation (11):

Accept a given product j if and only if:

$$f_j^* > \Delta V_{t+1}(x) + \sum_{\tau=t+1}^{T} c_j(t,\tau)(V_\tau(\tilde{s}) - V_\tau(\tilde{s}-1)) \tag{11}$$

The term on the left, f_j^*, can be seen as the expected value of the fare of a request. It is given by the fare adjusted by its cancellation probability ζ_j, as well as its refund percentage ρ_j.

The first term on the right, $\Delta V_{t+1}(x)$, is the opportunity cost of a unit of capacity in the next time stage. Next, consider the $V_\tau(\tilde{s}) - V_{t+1}(x-1)$ term inside the summation. This difference shows an opportunity cost between time unit t and time of cancellation τ. This opportunity cost is weighted by the probability of time of cancellation τ. Equivalently, we can write equation (11), as follows:

Accept a given product j if and only if:

$$f_j^* - \sum_{\tau=t+1}^{T} c_j(t,\tau)(V_\tau(\tilde{s}) - V_\tau(\tilde{s}-1)) > \Delta V_{t+1}(x) \tag{12}$$

This formulation and corresponding optimal policy of equations (1) and (4), respectively, are a special case of equations (9) and (11). After all, in absence of cancellations, we have $\zeta_j = 0$ and $\rho_j = 0$ for all j, so we have $f_j^* = f_j(1 - \zeta_j \rho_j) = f_j$. Similarly, no cancellations imply $c_j(t,\tau) = 0$ for all t, τ, j, so the summation of equation (11) disappears and reduced to $f_j > \Delta V_{t+1}(x)$, which was shown in equation (4).

The problem is solved by substituting the optimal policy of equation (11) into equation (9):

$$\begin{aligned} V_t(x) &= E[R(t) + C(t) + O(t)] \\ &= \sum_{j=1}^{n} \lambda_j(t) \left(f_j^* - \sum_{\tau=t+1}^{T} c_j(t,\tau)(V_\tau(\tilde{s}) - V_\tau(\tilde{s}-1)) \right)^{+} \end{aligned} \tag{13}$$

Where the $^+$-notation in equation (13) indicates that we take the maximum of this term and zero – therefore, the term is replaced by zero if negative (and, equivalently, if the left hand side of equation (12) is not greater than the right hand side).

Equation (13) shows the power of this heuristic: the state-space is still one-dimensional, x. In practice, it is very important to make this a feasible approach. In Hopman (2020), we discuss practical limitations of the RM system. Specifically, it is discussed that reoptimising the most basic formulation, equation (1), is impossible to do daily in practice. Therefore, the work of Sierag et al. (2015), for example, which uses more than a single dimension in the state space, cannot easily be used in practice and it is critical to keep the state space one-dimensional.

4.1 Estimating $V_\tau(\tilde{s})$

As mentioned in Section 4, we need a way to estimate the value of $V_\tau(\tilde{s})$. When deciding to accept a request through the optimal policy as defined in equation (11), it is unknown what state the system, $\tilde{s}$, is at time τ, for all τ. Since we do not know the state of the system, we do not know its corresponding value function. In this section, we propose a heuristic to estimate this value.

1. Solve the optimal policy of equation (1), ignoring any cancellations in the optimisation process.
2. Generate n different arrival processes different cancellation processes.
3. Simulate the acceptance of products in these processes using equation (4), cancellations arrive according to the simulated cancellation process.
4. For every time unit t, $t = 1, ..., T$ and simulation number k, $k = 1, ..., n$ record the state of the system x and corresponding value function. Denote these as x_t^k and $V_t^k(x_t^k)$.

The heuristics are as follows. Note that in our notation, $V_t(x)$ represents the actual value function, as derived using equation (1). On the other hand, V_t^k represents the recorded value of the value function of simulation k at time t. Next, let us define:

$$1_{it} = \begin{cases} 1 & \text{if the system is in state } i \text{ at time } t,\ i = 0, 1, ..., C;\ t = 1, ..., T \\ 0 & \text{otherwise} \end{cases}$$

Next, define:

$$\chi_i(t) = \frac{1}{n}\sum_{i=1}^{n} 1_{it}$$

$\chi_i(t)$ denotes the proportion of being in state i at time t. Let $\widehat{\chi(t)}$ represent the most likely state for a time value t that is, $\widehat{\chi(t)} = \max(\chi_1(t), \chi_2(t), ...)$. Using this notation, we have defined the following heuristics:

$$V_t^{H_1}(x) = \frac{1}{n}\sum_{k=1}^{n} V_t^k(x_t^k) \qquad \forall t \tag{14}$$

$$V_t^{H_2}(x) = med(V_t^k(x_t^k)) \qquad \forall t \tag{15}$$

$$V_t^{H_3}(x) = \max(V_t^k(x_t^k)) - \min(V_t^k(x_t^k)) \qquad \forall t \tag{16}$$

$$V_t^{H_4}(x) = V_t(\left\lfloor \frac{C}{2} \right\rfloor) \qquad \forall t \tag{17}$$

$$V_t^{H_5}(x) = \sum_{i=1}^{C} \chi_i(t) V_t(x_i) \qquad \forall t \tag{18}$$

$$V_t^{H_6}(x) = V_t(\bar{\chi(t)}) \qquad \forall t \tag{19}$$

$$V_t^{H_7}(x) = \frac{1}{6}\sum_{i=1}^{6} V_t^{H_i}(x) \qquad \forall t \tag{20}$$

$$V_t^{H_8}(x) = med(V_t^{H_1}(x), ..., V_t^{H_6}(x)) \qquad \forall t \tag{21}$$

$$V_t^{H_9}(x) = V_t(1) \qquad \forall t \tag{22}$$

$V_t^{H_1}$ calculates the average over all n simulations, for a given time unit. $V_t^{H_2}$ is done in similar fashion, but takes the median. The $V_t^{H_3}$ heuristic takes the difference of largest and smallest value. $V_t^{H_4}$ does not use any simulated values, but instead takes the value function evaluated at half capacity. In heuristic $V_t^{H_5}$, we first obtain the proportions of being in a given state at time t, and using these as weights to obtain an estimate. These same proportions are used in heuristic $V_t^{H_6}$, but rather than weighing, we only take the most common value (statistical mode) as an estimate for the state we find ourselves in. Finally, heuristics $V_t^{H_7}$ and $V_t^{H_8}$ take the average and median, respectively, over the first six heuristics.

Note that in these heuristics, we have defined two approaches: estimating the value function directly, or estimating the state first, then plugging in the result into a value function.

5 Results

This section is organised as follows. First, we discuss the setup of our simulation in Subsection 5.1. Next, we discuss the performance of the different heuristics H in Subsection 5.2. Having identified what heuristic to use in our calculations, we present the results in Subsection 5.3. We will review the robustness of the model against the different scenarios in this same section. Next, three examples are given in Subsection 5.4, where we compare the DPC and $DPID$ methods.

5.1 Simulation setup

In this section, we will discuss the setup of our simulation. Revenues will be compared between five scenarios:

1. *base*: This is the baseline scenario. Cancellations are calculated using the approach described in Hopman (2020). Arrival processes are simulated using ‘perfect’ demand estimates; that is, the same $Poisson(\lambda)$ process is used to forecast demand and to simulate.
2. *CxlEarlier*: In this scenario, we optimise according to the *base* scenario, but in the arrival processes cancellations occur earlier than expected.
3. *CxlLater*: This is the opposite scenario: in this case, we simulate cancellations that occur later than planned.
4. *FcOver*: When constructing the optimal policy, we purposely overforecast by 20%.
5. *FcUnder*: Similarly, in this scenario we purposely underforecast by 20%.

The *base* case contains a like-for-like comparison between the $DPID$ and DPC method. Next, we have also developed four different scenarios that measure the sensitivity of the $DPID$ and DPC methods with respect to incorrect estimates of cancellation times and levels of demand. The *CxlEarlier* and *CxlLater* scenarios were constructed to identify robustness of $c_j(t, \tau)$ in the optimisation. We proposed this as further research in Hopman (2020). The *FcOver* and *FcUnder* scenarios are used to study how the model performs when forecasts are incorrect. Combinations of these are not studied, so we can isolate the effects of misjudgements in either cancellation probabilities or forecasts.

The unconstrained demand factors, the ratio of unconstrained demand forecast and aircraft capacity, are given in Table 1.

Table 1 Demand factors for different scenarios

Type	Base	CxlEarlier	CxlLater	FcOver	FcUnder
Scenario	1.09	1.09	1.09	1.31	0.87

Table 1 shows that the base case has sufficient (mean) demand to fill the plane at 109%. These demand factors are kept for the *CxlEarlier* and *CxlLater* scenarios. For the *FcOver* and *FcUnder* scenarios, we have scaled the base demand factor by 1.2 and 0.8, respectively. This results in scenarios with demand factors of 1.31 and 0.87. For these scenarios, we assume cancellation occur according to the base case.

The (unconstrained) demand distribution is given in Table 2. Unconstraining is done through the framework outlined in Price et al. (2019). The fares are also shown.

Table 2 Distribution of demand by time and fares for different classes. DCPs (data collection points) range from 1 (earliest) to 9 (just before departure)

Class	*DCP1*	*DCP2*	*DCP3*	*DCP4*	*DCP5*	*DCP6*	*DCP7*	*DCP8*	*DCP9*	*Total*	*Fare*
1	0.24	0.10	0.05	0.05	0.05	0.10	0.05	0.19	0.19	0.19	1000
2	0.25	0.12	0.09	0.06	0.06	0.09	0.12	0.16	0.03	0.29	750
3	0.21	0.12	0.12	0.09	0.09	0.09	0.09	0.09	0.09	0.51	500

From Table 2, it becomes evident that the majority of the demand is expected to come from class 3. Most of this demand comes in earlier DCPs (1, 2 and 3). Demand from class 2 is also expected earlier one, but closer to departure, in DCPs 7 and 8, 28% of its demand is forecast. Class 1 has a similar demand curve, but the proportion of demand is expected to arrive even closer to departure in DCPs 8 and 9. The last column displays the fares: these are 1,000, 750 and 500. The data used for our simulations is based on real data, but we have aggregated demand up to fare family level. This not only helps us obtain better demand estimates, but it will also enable us to study the effects better. To stay consistent with literature, we use the terminology of 'class'.

To speed up simulations, we have chosen a capacity of 50 seats. n = 500 simulations were performed for each scenario and for different heuristics H, H_1 through H_9. Demand is assumed to follow a Poisson process. Time is discretised in such a way that the probability of having two booking requests in the same time unit, is chosen at ϵ = 0.001. As the demand grows, we need more time units to satisfy this condition. The control mechanism, checking the (adjusted) fare against the bid price, is achieved by first finding the closest time unit, and looking up the bid price for this unit of time. Finding the closest time unit is achieved by first converting both the time units of both the reference (simulation demand) and target (bid price) vectors to absolute time, finding the closest match, and converting the index of that closest match back to the time unit of the target vector. In case of a duplicate match, the earlier unit of time is used.

5.2 Robustness on H

Before we look into the model's results, we will discuss the robustness of H. Recall that we use H to estimate future states, which depend on an a-priori unknown state of the system x_t. We refer to equations (14) through (22) in Subsection 4.1 to gain an understanding on how these were constructed. After running 500 simulations, the estimates are as shown in Figure 1.

As time draws closer, the gradient estimate, as shown in equation (3), declines. This makes sense, since the DPID formulation, from equation (1), is non-increasing in both t and x. For a proof, we refer the reader to Talluri and Van Ryzin (2006).

Figure 1 Comparison of different ways H_1 through H_9 to estimate V_t (see online version for colours)

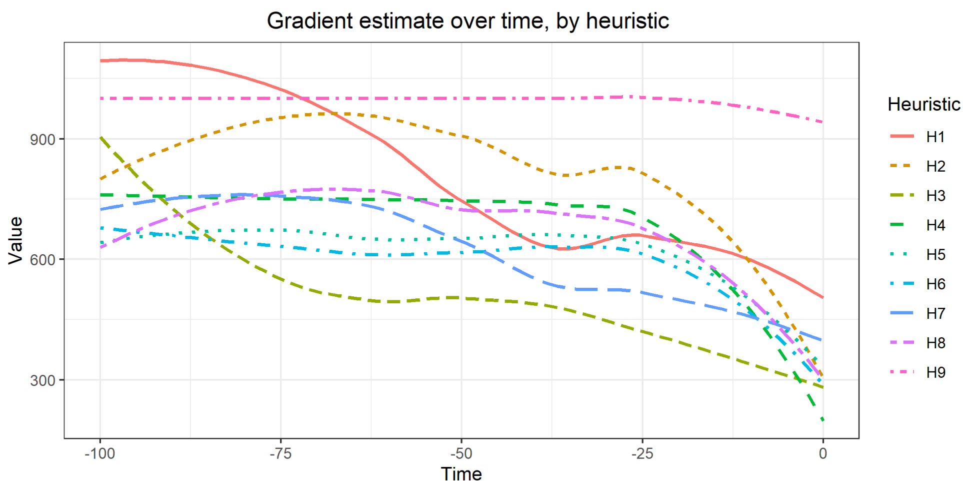


Table 3 Relative revenue performance of H by class, scaled to H_1

Class	*H1*	*H2*	*H3*	*H4*	*H5*	*H6*	*H7*	*H8*	*H9*
1	1.000	0.989	0.989	1.000	0.987	0.995	0.989	0.978	0.992
2	1.000	0.980	1.000	0.980	0.993	1.000	0.993	1.000	1.000
3	1.000	1.003	0.999	1.002	1.002	0.999	1.002	1.004	1.003

Consider heuristic H_1. This line represents the gradient over time, averaged over all simulations. In absence of cancellations, this means on average the lowest price fare class, 3, start to become available only close to departure. The most aggressive heuristic, H_9, is equal to the highest fare, 1,000, until close to departure when it starts to decrease. This again, is as expected: this heuristic measures last-seat availability. In effect, we impose the risk of having a final seat for sale for every booking we decide to accept or not. We will not go into detail how to interpret other heuristics as these are impossible to intuitively gain an understanding how these are constructed.

The most import part of this heuristic, is, of course the revenue it generates. We compare the revenues by fare class in Table 3. This is the result of 500 different arrival processes, which are being evaluated against every value function through simulation.

Table 3 shows the relative revenues scaled to the results obtained by H_1. We note very few differences between the different heuristics. None of the heuristics increases class 1 sales. At best, this is matched by H_4, but this heuristic performs worst out of all heuristics at selling class 2. Heuristic H_8 is able to sell more customers to class 3, but does not sell class 1 well: 2.2% less than H_1. In Table 4, we show the total relative revenues.

Table 4 Relative revenue performance of H, total, scaled to H_1

H1	*H2*	*H3*	*H4*	*H5*	*H6*	*H7*	*H8*	*H9*
1.000	0.997	0.997	0.999	0.997	0.998	0.998	0.997	1.000

Table 4 analyses the (total) mean revenue generated using heuristics H_1 through H_9. Note that H_1 performs best. Interestingly, the most aggressive heuristic, H_9 performs just as well. H_5 performs worst. Looking at Table 3, we observe that this is the result of selling less class 1 and 2. However, we observe that the average revenues performance are very close: using the worst heuristic we obtain revenues that are 0.3% lower than the best performing heuristic. We, therefore, conclude that our method is very robust against choice of heuristic. Considering that H_1 performs best, we have chosen to use this heuristic for the results we will show in the next section.

5.3 Model results

In this section, we review the performance of our formulation. We have divided this section in two parts: the first part, Subsection 5.3.1, covers the performance in terms of revenues and accepted passengers. The second part, Subsection 5.3.2, looks into the underlying processes and investigates customer behaviour and compares this to the traditional DP formulation, $DPID$.

5.3.1 Revenue

Having identified our choice of H, we now present our results. We will review the accepted passengers by method and scenario, and then investigate the differences in the number of passengers accepted by class. Table 5 shows the revenues.

Table 5 Revenue performance for different scenarios, comparing $DPID$ with DPC

Scenario	*Base*	*CxlEarlier*	*CxlLater*	*FcOver*	*FcUnder*
DPID	29,075	31,450	29,250	25,000	30,975
DPC	29,875	30,975	31,950	29,725	30,725
Performance	1.03	0.98	1.09	1.19	0.99

Table 5 show the performance between $DPID$ and DPC methods. For the base case, our algorithm outperforms the standard formulation by 3%. Looking at the performance when we misjudge cancellation probabilities, our model performs worse if cancellations happen earlier than we expect, and performs better if cancellations happen later than expected. We note that the performance is not linear: we lose 2% if they occur earlier, but gain 9% if they happen later. When looking at the forecasting scenarios, we show a significant improvement over the $DPID$ method when we overforecast, with a slight revenue loss when we underforecast.

When comparing the robustness of revenues between scenarios, we note a better performance for the *CxlEarlier* method, a 8.2% difference for $DPID$ and 3.7% for DPC. For *CxlLater*, the $DPID$ method is more robust: 0.6% compared to 6.8%. When overforecasting, DPC is more robust: 0.5% against 16% difference for $DPID$. Finally, when underforecasting, the robustness of revenues of DPC are again more favourable: 2.9% in comparison to 6.5% of $DPID$. In summary, we show minimal revenue losses in two out of five scenarios, and a much more robust revenues.

In Table 6 we look into the distribution of accepted passengers. Comparing our method with the traditional formulation, on average we accept slightly less class 1 passengers. This is consistent across the different scenarios, except the *CxlLater* scenario. Looking at class 2, we see a sharp decrease in number of accepted passengers. This seems to indicate that the value proposition, its fare and the risk of cancellation and taking up a unit of capacity, is not worth it.

To illustrate why this may happen, consider again Table 2. Here, we see a substantial amount of demand in DCP 5 through DCP 9 (46%). This roughly represents the last 25 days of the booking curve. Now consider Figure 2.

Figure 2 Adjusted fare by time of booking, by class (see online version for colours)

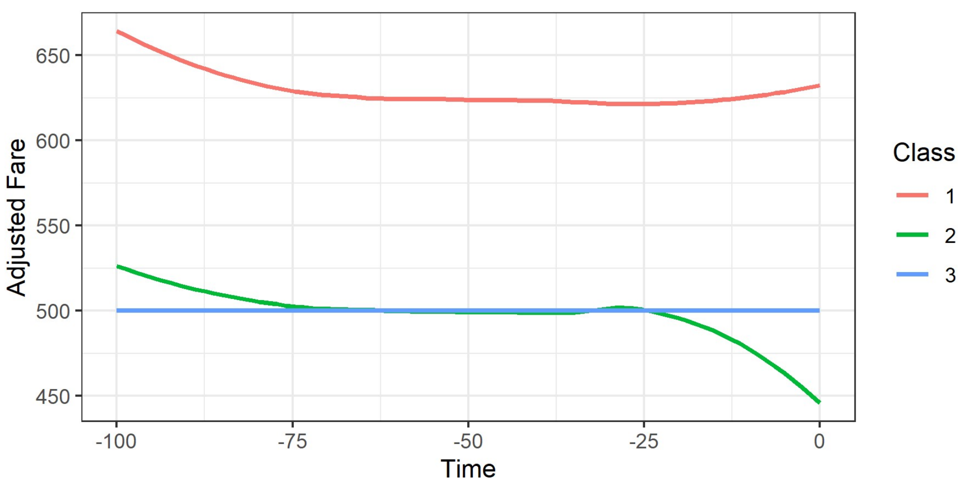


Figure 2 shows the adjusted fare by time of arrival. Here, we see that the adjusted fare for class 2 decreases below the value of class 3 (recall that class 3 is unable to cancel). Therefore, it is a better decision to accept class 3 at this stage. This behaviour explains why the algorithm accepts more of class 2, and much more of class 3. We will investigate this behaviour in more detail in the next section where we provide an example, Subsection 5.4.

Looking at the robustness between scenarios, we note that class 1 is the most robust and class 3 is the least robust, both in absolute and relative numbers. Class 2 show very consistent performance.

Lastly, we note very consistent total number of passengers in the DPC case: these range from 44.4 in the lowest case to 47.6 in the highest case: an absolute difference of 3.2 passengers. For the $DPID$ method, this ranges from 32.7 to 38.9, a difference of 6.2 customers.

Table 6 Passenger count, by method and scenario

Class	*1*		*2*		*3*		*Total*	
scenario	*DPC*	*DPID*	*DPC*	*DPID*	*DPC*	*DPID*	*DPC*	*DPID*
Base	10.30	11.20	10.10	21.10	24.00	4.10	44.40	36.40
FcOver	8.90	9.00	10.30	16.60	26.20	7.10	45.40	32.70
FcUnder	11.70	12.10	10.50	24.10	22.30	1.60	44.50	37.80
CxlEarlier	11.50	13.10	10.10	21.80	23.80	4.00	45.40	38.90
CxlLater	11.20	10.90	10.20	21.00	26.20	5.20	47.60	37.10

To understand the effects of a wrong cancellation prediction, consider Figure 3.

Figure 3 Example of wrong $c_j(t, \tau)$ curves (see online version for colours)

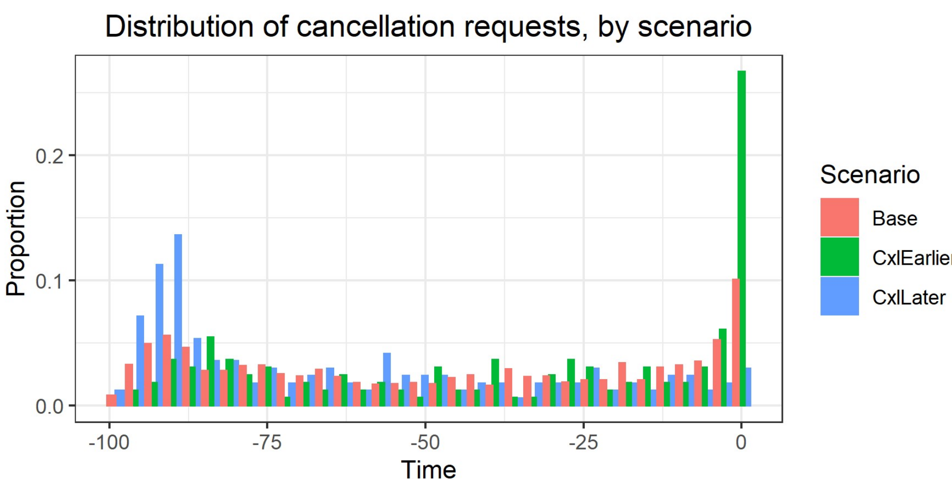


Figure 3 shows the distribution of cancellation rates for the *base*, *CxlEarlier* and *CxlLater* case for a booking made t = 100 days in advance. Note that we recognise the U shape we have seen in Hopman (2020), with slightly more mass toward 0, indicating that this booking request is more likely to cancel late than early. For the *CxlLater* case, we put more mass on early cancellations in the arrival processes. Similarly, for the *CxlEarlier* case, we put more mass close to departure. Consider the implications of this: if we expect passengers to cancel late, in small values of t, we penalise bookings (since we have higher probabilities $c_j(t, \tau)$) harsher than we should have: we would have rejected someone. Similarly, if we expect passengers to cancel early, we do not penalise them as harsh. Moreover, we would have less booked passengers than we expected at early time units t, which will cause availability to open up more classes and we get a chance to accept more demand. We suspect this is why the revenue gains are not symmetric.

5.3.2 Customer behaviour

In this section, we will review customer behaviour. First, let us review the average number of cancellations. These are shown in Table 7.

Table 7 Average number of cancellations, by method and scenario

Scenario	*Base*	*CxlEarlier*	*CxlLater*	*FcOver*	*FcUnder*
DPID	18.44	17.06	21.42	16.02	20.98
DPC	16.82	15.38	19.46	16.00	19.34
Difference	–1.62	–1.68	–1.96	–0.02	–1.64

Table 7 shows that for every scenario, we report a lower number of cancellations. Keep in mind that the accepted number of passengers for DPC is higher than the $DPID$ on average; refer to Table 6. Here, we see that the total number of passengers is about eight larger for the DPC method. Despite having a larger number of passengers, we record a lower number of cancellations. This is true, except for the overforecasting, which shows a minimal gain.

Keep in mind that cancellation rates are percentages of bookings that are forecasted to cancel. When forecasting bookings and cancellations independently, one may end up with more cancellations than bookings. Working with bookings and a cancellation rate mitigates this problem.

The number of cancellations is robust for the *CxlEarlier* and *FcOver* cases, compared to the *Base* case. However, in practice, an overall lower number of cancellations for the *Base* is important since this will make overbooking easier and less risky.

Looking into the underlying process into more detail, Table 8 shows the average number of requests that were cancelled, accepted and rejected. Note that the average number of accepted requests can exceed the capacity (in this case, 50), as long as there is capacity at the time of the request.

Table 8 Average number of accepted, rejected and cancelled requests, by method and scenario

Type	*Cancelled*		*Accepted*		*Rejected*	
scenario	*DPC*	*DPID*	*DPC*	*DPID*	*DPC*	*DPID*
Base	16.82	18.44	64.16	57.90	46.96	52.16
FcOver	16.00	16.02	62.74	51.92	26.30	35.80
FcUnder	19.34	20.98	66.24	62.00	66.76	70.22
CxlEarlier	15.38	17.06	60.36	54.36	50.02	55.80
CxlLater	19.46	21.42	68.22	62.42	49.44	53.34

Table 8 shows the mean number of accepted and rejected requests. We have included the cancelled requests as well for sake of completion, this was earlier shown in Table 7.

Note that the number of accepted requests is higher. This is the result of accepting more passengers (which we saw in Table 6) but, adjusted for those additional passengers, we seem to accept the right customers are the number of cancellations is lower. This leads to the observed cancellation rates, which we show in Table 9.

Table 9 Mean observed cancellation rates, different scenarios

Scenario	*DPC*	*DPID*
Base	0.26	0.32
FcOver	0.26	0.31
FcUnder	0.29	0.34
CxlEarlier	0.25	0.31
CxlLater	0.29	0.34

The observed cancellation rate in Table 9 is defined as the ratio of the number of cancellations and the number of accepted requests. Note that we observe less observed cancellation rates in all scenarios: this seems to indicate that the passengers we accept, are less likely to cancel. In reality, cancellations create uncertainty and angst in analysts that judge how much to overbook. For this reason this is another positive result in practice.

Table 10 shows the coefficient of variation of the observed cancellation rates, which is the ratio of the standard deviation and mean: $c_v = \frac{\sigma}{\mu}$.

Table 10 Coefficient of variation of cancellation rates, different scenarios

Type	*Cancelled*		*Accepted*		*Rejected*	
scenario	*DPC*	*DPID*	*DPC*	*DPID*	*DPC*	*DPID*
Base	0.30	0.27	0.07	0.08	0.22	0.15
FcUnder	0.27	0.27	0.07	0.08	0.16	0.14
FcOver	0.23	0.19	0.05	0.08	0.35	0.20
CxlEarlier	0.23	0.26	0.05	0.07	0.21	0.16
CxlLater	0.25	0.23	0.07	0.08	0.22	0.17

Note: Smaller values represent better values.

Table 10 show the variability in the number of cancellations report. Looking at the cancellation requests, we observe a relatively higher variability for the *Base*, *FcUnder*, *FcOver* and *CxlLater* cases. Interestingly, *CxlEarlier* is the only scenario where the variability is lower, as compared to the $DPID$ method. The number of accepted requests are more stable for all scenarios. The largest difference is in the number of rejected requests: for the *Base* case, the coefficient of variation is 46% higher. In other scenarios, such as *FcUnder*, this coefficient is variation is closer to the $DPID$ case. We look into the number of rejected requests in Table 11.

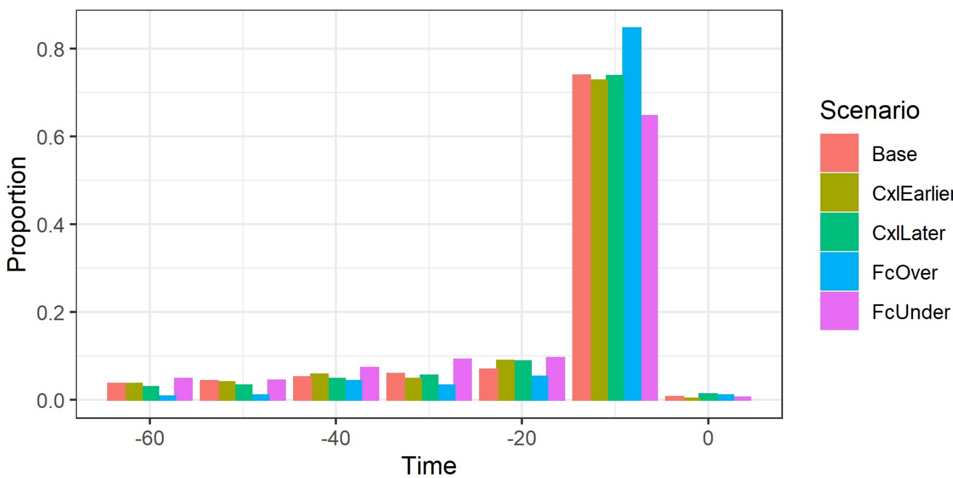
In Table 11, we look at the average number of rejected requests, conditioning on the fact they were accepted in the $DPID$ formulation. Particularly interesting are the rejected requests for class 1. For the base case, we reject, on average, three customers willing to buy class 1. However, looking back at Table 6, the final passenger count for DPC in the base case only had 0.9 less of class 1 booked. This seems to indicate that out of the three bookings that $DPID$ accepted, on average, 2 of those bookings took up a valuable unit of capacity that was cancelled at some stage. We also confirm that the DPC model rejects a lot more of class 2 demand, that the $DPID$ formulation did accept.

Table 11 Mean number of rejected requests in DPC that were accepted in $DPID$, different scenarios

Class	*Base*	*CxlEarlier*	*CxlLater*	*FcOver*	*FcUnder*
1	3.00	3.30	2.80	2.40	2.80
2	15.50	15.50	16.70	9.90	19.20
3	4.50	3.60	5.10	5.10	3.00

Figure 4 shows the distribution of the times arriving requests were rejected by scenario.

Figure 4 Distribution of time of rejection, by scenario (see online version for colours)



In Figure 4, we have bucketed time in multiples of ten days, and calculated the proportion of rejections in that time bucket. Comparing the different scenarios, we first note very similar patterns across different scenarios. For all scenarios, the majority of rejections arrive between 10 and 0 days before departure. Intuitively, this makes sense: occupying a unit of capacity closer to departure is riskier to the airline that someone occupying a seat early in the booking curve.

5.4 Example

In this section, we will provide an examples of a simulation, comparing the performance of $DPID$ and DPC. The revenue for DPC in this example is 31,250, compared to the revenue of $DPID$ of 2,600. This represents a revenue improvement of 20%.

Figure 5 Example 1 – comparison of availability of classes over time, DPC vs. $DPID$ (see online version for colours)

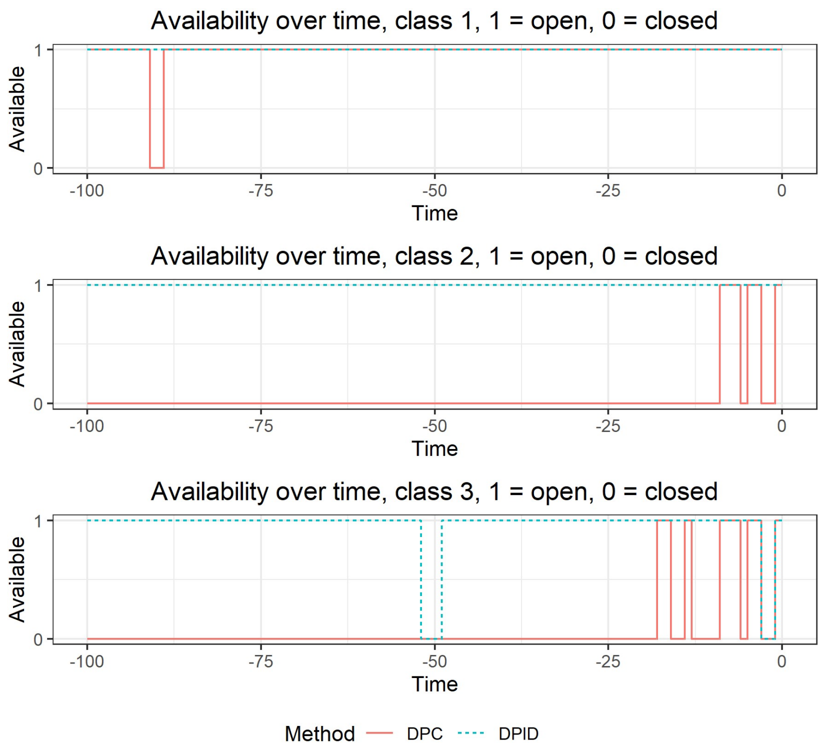


Figure 6 Relative fare adjustment by time of booking (see online version for colours)

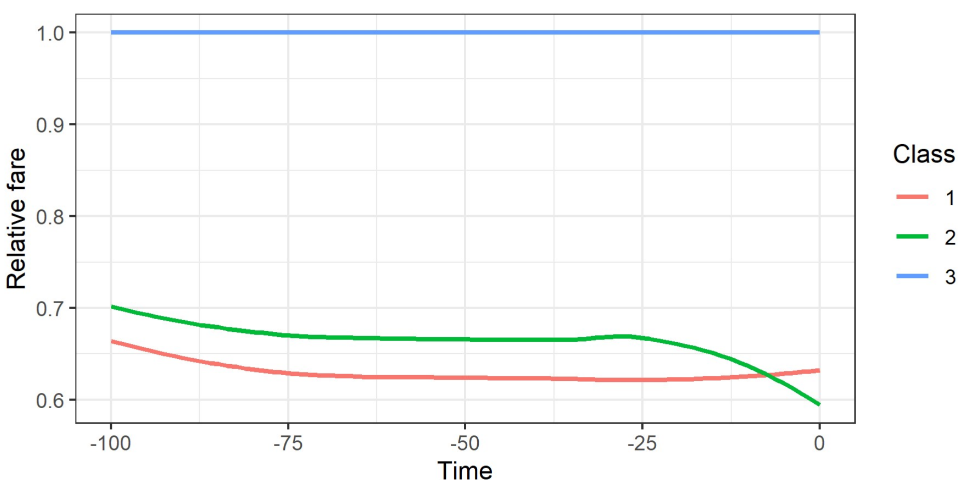
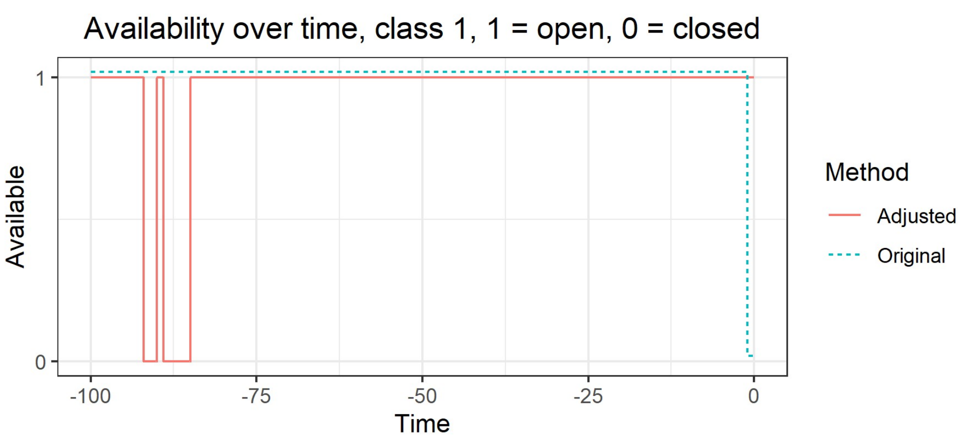


Figure 5 shows the availability over time of this example for class 1, 2 and 3 in the top, middle and bottom graph. A value of 1 represents this class is available for sale, while 0 means this class was closed. We introduce a slight jitter to avoid overlapping lines in the figures. The solid line represents the availability for DPC, while the dotted line is the availability for $DPID$. A first interesting observation in the top graph is that class 1 is closed for 2 days, at t = –91 and t = –90. From the middle and bottom graphs, we see that these classes are also closed for the DPC. It is important to add that these classes were not closed, because capacity was exhausted. On the contrary, the formulation expects sufficient of demand later on, and this combined with the adjusted fare of class 1, that incorporates the estimated effect of taking up a unit of capacity, means it is optimal not to accept any class. While we only show one example in this section, we have seen this other simulations too, even for extended periods of time. Another example of the availability for class 1 in a different simulation is shown in Figure 7.

Table 12 Number of rejected requests in DPC that were accepted in $DPID$, by class

	Class 1	*Class 2*	*Class 3*
Rejections	0	12	3

Figure 7 Example 2 – comparison of availability of class 1 over time, DPC vs. $DPID$ (see online version for colours)



Looking at the availability of class 2, we note that this class is closed from the beginning of the booking curve for the DPC method, while it is available throughout of $DPID$. This is a phenomenon of the $DPID$ that was reported earlier in Hopman et al. (2017). Now consider the time close to departure. Here, the algorithm opens class 2 for brief moments of time. Compare this availability with class 3 now: this class is open for a few days roughly two weeks before departure, while class 2 is closed. This is the result of the adjustment of fare for class 2, which we earlier highlighted with Figure 2. Figure 6 shows this in a different way by calculating the relative fare adjustment.

From Figure 6, we observe that the relative fare adjustment drops closer to departure, and is strictly declining from roughty t = –25. The relative fare evaluated against the bid price drops from 68% to only 60% of its actual fare. This causes class 2 to be closed, while class 3 [that is not adjusted, since $c_j(t, \tau) = 0$ for all t] is open and is sold. This example shows non-nested availability in the last stages of the booking curve. This results in an increased number of rejected requests, as compared to the $DPID$ model, which we show in Table 12.

Table 12 shows the number of rejected requests that were accepted for the $DPID$ model. Note that in this simulation, we reject arriving requests from class 2, a direct result of the aforementioned fare adjusted value.

6 Discussion

In this section, we provide a discussion of the results and provide opportunities for further research.

In our earlier work (Hopman, 2020) we found that the time of cancellation is dependent on the time it was booked. Intuitively, this makes sense. However, in the literature, it is assumed a product cancels or not. It is seen as a Bernoulli process, which disregards time. In our earlier work, we provide a framework on how to estimate cancellation probabilities depending on t. However, while this modelling is important, it should not be the objective itself: after all, the revenue that is generated using this model is what really matters.

In our formulation, we evaluate a product against the current opportunity cost, and the future opportunity cost of a unit of capacity weighted by the time-dependent. A-priori, we do not know the future state we are in. For this reason, we proposed different heuristics H. Estimating this term can be difficult, and there is no intuitive way to determine what heuristic of estimation is optimal. We have to rely on simulations to determine what method works best. These heuristics are based on the state of the system, at time t and simulation number i, $x_i(t)$. When we aggregate value function trajectories, such as H_1, it is impossible to calculate the gradient at that point. After all, the value function is defined at x_i = 0, 1, ..., 50, that is, the value function only exists at (whole) points of x_i. To approximate the gradient, we compared the value function estimate to the list of the true value function, find the associated x_i, and use this as a gradient. Suppose we find an estimate at some time t, with heuristic H_1. Let this estimate be $h(t)$. To approximate the gradient, we find:

$$\arg\min_i \big(abs(h(t) - V_t(x_i))\big) \tag{23}$$

Let i_t^* be the i that minimises equation (23). Then, we approximate by:

$$\frac{\partial H(t)}{\partial x} = \Delta V_t(x_{i^*}) \tag{24}$$

Note that this is an approximation that may impact the values of H. However, as we have shown in Subsection 5.2, the choice of this parameter is very robust against revenues. We therefore conclude that this approximation in finding the gradient is reasonable.

We have expressed f_j, adjusted for cancellation risk by f_j^*. We prefer to keep the adjustment term on the left-hand side of the equation, such as was done in equation (12). This is preferred, since this enables airlines with the $DPID$ formulation to easily implement our model, by using a fare adjustment. This feature, adjusting the fare that is used in the optimisation, is present in all RM systems.

Looking at the results, we observe that our DPC model outperforms the $DPID$ for most scenarios. We also stress that while the revenues increase, load factors increase too. This is important since in practice, lots of 'professionals' still use load factors as a benchmark to see how a flight is performing. Even these days, analysts and managers alike are hard to convince that lower load factors may result in higher revenues. Fortunately, we accomplish both increases in revenues and load factors.

Looking at the different scenarios, misjudging time of cancellations, and over- or underforecasting, we found the DPC method to have relatively robust revenues and booking class distributions. This will, in the long run, ensure more stable forecasts. And, in turn, ensure more reliable value function estimates and corresponding bid prices.

It is not immediately clear how this method works in network RM. The currently most common method, which assumes that the network dynamic program is a sum of flight-level dynamic program, shown in this paper, can easily be adapted to incorporate our work. Another way for further research is the robustness against multiple scenarios, different demand curves, different fare structures and more fare classes. Furthermore, it will be interesting to study structural properties of our dynamic programming formulation. Lastly, in Table 10, we showed the coefficient of variation. One finding we brought up in Subsection 5.3.2 is the relatively high coefficient of variation of rejected requests. It was not immediately clear why the number of rejected requests has a higher variance for the DPC model, and this is something that may be studied.

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