

A Population Dynamics Approach to Viral Marketing

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A Population Dynamics Approach to Viral Marketing

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Abstract. The symbiosis of Social Media and viral campaigns has recently become ubiquitous. In many recent phenomena (e.g., the Cambridge Analytica scandal), rumours in viral marketing programs are present without being even noticed by consumers. Yet, the study of population dynamics and its complex patterns of interaction remains largely elusive. Here, we propose an agent-based Marketing referral model to study the impact on firms' dissemination and profitability of biased behavior in a population of opportunistic individuals. We show that those agents only interested in collecting rewards without any brand recognition are responsible for most of Marketing campaign success and dissemination, for a large range of different cost structures, network characteristics, and number of invites. This effect is further amplified whenever the difference between the cost of using the service and the reward collected after bringing a new customer is higher.

Keywords: Rumour and Viral Marketing, Referral Programs, Social Networks

1 Introduction

Nowadays, firms have multiple tools to boost sales. However, none of them is as powerful as the price [13]. The influence that prices establish in sales and companies revenues are by far, the most pre-eminent success factor in the corporate world. Price sensitivity, fully described by Ramirez et. al [20], is the consumption marginal impact on a price variation, where a decrease in product price has, all else equal, a positive impact in consumption levels [17,26]. Nonetheless, companies are becoming threatened by a growing proliferation of new products and competitors. Their capacity to differentiate the essential features of their products is key to define clients' connectedness to the brand. To improve such a relationship, new Marketing methods are being tested to accelerate the customer journey towards the acquisition of a product and brand dissemination. One of those is a Marketing Referral program. Such a program is defined as leading consumers to self-advertise a given service or product in their network, in exchange for a reward. As described in [27] this method, also known as word-of-mouth referral (WOM referral), happens when a consumer indicates a relative/friend as a potential buyer/consumer. In exchange for his loyalty, the client receives a reward to use in a future acquisition of the service/product. Some authors [27,29], describe such marketing operation as a

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”win-win” strategy, where the firm gets a new client and reward the older one for keeping his loyalty to the brand.

As seen, brand loyalty has a central role in such programs. Youjae Yi and Hoseong Jeon show that companies have loyal consumers when those acquire periodically the same type of products or services [31]. Having clients connected to the brand empowers companies to differentiate themselves from competition avoiding price wars. Goldsmith showed the negative relationship between price sensitivity and brand loyalty [12], where consumers elasticity to price variations is lower and more receptive to pay higher prices relative to other product with lower connection [10,12,31]. Whenever such price sensitivity is higher and brands are not well-known, companies tend to use promotions or discounts, both to gain new clients and increase the link with those who already known the brand. Despite being occasionally seen as harmful, making use of an addictive constant flow of discount [10], it is widespread across all industries [11]. Even a Marketing Referral program can be seen as a discount program, due to the fact of giving a reward to those who bring new customer to the company. The main difference with the present strategy is the customized and targeted way the discount work, where only those who search for reward will profit from the program. Therefore, we can argue that somehow promotes an individual opportunistic behavior from the consumer side. Despite the usage of such Marketing methods, it is undeniable the lack of studies on the emergent collective dynamics associated with the proliferation of a brand in the presence of opportunistic agents. Therefore, here we resort to complex systems tools developed across disciplines — from theoretical ecology to economics — to study how this type of rewards influence the number of users, and how the opportunistic agents change the company’s profit structure, leading to a successful or unsuccessful Marketing strategy to boost brand recognition and consumption.

2 Marketing Referral Programs and Growth Hacking Marketing

From the emergence and recent developments in the technological world, new firms and ways of approaching the market have flourished. In Dalaman works [14], he mentioned that this dramatic evolution in the enterprise ecosystem enlarges the need to test other Marketing methods to keep the client base and gain market share. Online Marketing was undoubtedly one of the most important changes in the industry. Empowered with cybernetic tools, companies are now able to communicate with current and potential customers in a customized and personalized fashion [9, 14]. Such an environment was also responsible for creating the conditions for the emergence of Growth Hacking Marketing. Companies like Uber, Airbnb, Facebook, Dropbox, Hotmail, LinkedIn, Instagram, and Twitter are some of the most well-known firms of this new Marketing Era. As described by Seal Ellis [9] and Andrew Chen [6], Growth Hacking is fully connected to the increasing demand to acquire new clients, but is mostly dedicated for keeping them as loyal customers for persistent and future consumption. The Growth Hacking concept [8, 9] can be decomposed into four elements of new business’ scalability: reward engine, economical engine, growth engine, and market engine. Through the reward component, the firm gives incentives to current clients to invite friends and relatives. The economical engine traces the costs of acquiring and maintaining a cus-

tomers. Only if those costs are lower than the benefits of having an additional client, the company business model is scalable. Growth Engine relates to the capability of promoting initiatives to increase market share, consisting in the definition of a market segment that can easily understand what the product is, but also, the appropriate client profile for such product [8].

Marketing referral programs are an extension of the growth hacking concept. Such programs are based on the paradigm of keeping clients connected to the brand through a constant flow of interactions based on a reward system [5, 8]. This reward system embodies a policy of bringing a new consumer for a foreknown benefit [5]. Christophe Van Den Bulte [29] summarizes why companies should and are building a growing customer base, based on such new Marketing techniques. Firstly, the company does not need any form of previous knowledge about clients' friendship network. By promoting this invitation process, clients freely give unknown information about themselves and their peers. Secondly, Marketing referral programs are relatively cheap to implement, given the fact of low initial fixed investment. Additionally, such Marketing strategy is a natural process to management and trace. Finally, the new clients, that were attracted by their peers have much higher segmentation degree, relative to other known Marketing strategies [28–30].

Accordingly to Nielsen's report [22], only 42% of the inquired people, throughout the globe, reported trust in online advertisement. Nonetheless, when asked about the word-of-mouth (WOM) referral impact in their consumption decisions, close to 83% of the sample confirm some peer influencing, from their relatives and friends [15, 22]. In order to properly function such invitation-based process, reward magnitude and clients network are relevant to its success. In the first place, the propensity to invite someone is, at the end of the day, the function of the reward they are about to capture. Such tendency will explain how many people each client desires to invite. In Jiwan Jeong research [15], regardless of the reward, it is quite unlikely to have someone willing to invite more than 20 people. By doing so, and relative to other strategies cost structure, Marketing referral programs are being seen as quite advantageous to companies [30].

However, Kuester et al. [7, 15, 30] mentioned the fact that such campaigns are, indeed, easy to spread and apply. This is especially prominent in an Era of social media networks, where sharing with friends is a daily common activity [30]), but may also responsible to enlarge the opportunistic population only reward interested, with no brand connection or recognition [7]. Opportunistic players have being studied since the nineties, by Alan Dick and Kunan Basu [7]. Sometimes cited as spurious loyalty, such consumers thrive in search of promotions, discounts or any other type of price reduction. Despite not being seen as brand promoters, their action may have a positive impact on company external recognition. Kuester proved that even without believing in firm' value and its products, Opportunistic players spread invitations throughout their networks in exchange for a reward [30]. Moreover, those so-called counter-attitudinal referrals, it is showed not only their positive impact both on profitability and expansion of a business [7, 30], but also a positive feedback loop towards brand loyalty and brand recognition. Perhaps, contrarily to what most of us would expect, the studies conducted so far, indicate that counter-attitudinal referrals effect has a counter-intuitive negative

relationship with reward: as the reward volume is lower, the impact on brand recognition and initiations increase [7, 30].

What it is the impact of a population with Opportunistic behavior in a Marketing referral program? Given the lack of academic literature on the combination of complex systems, viral Marketing and population dynamics, it is our intention to measure the impact of an opportunistic population has in a firm that follows a Marketing referral program, in terms of brand dissemination and profitability. By doing so, it can be found network structure or an optimal reward policy to maximize the positive impact of such programs in the sales and reputation of the firm.

The document is structured in the following manner: Section 2 presents the agent-based model description, whose entities will be part of a Marketing referral program. Section 3 discusses the main findings and address its implications to the company and to the customer journey. Finally, the last section debates the proposed model limitations, as well as the next steps in this research stream and final remarks.

3 Methods: An Agent-Based Marketing Referral Model

The framework here presented was inspired by computer simulated agent-based models in well-mixed and structured populations [4]. This class of models have the ability of agent definition and interaction and through sampling method, arrive to a close relation with what happens in real-life situations.

Rewarding under a referral system. We consider the situation where a firm starts a Marketing Referral Program to grow its consumer base and clients. As noted by Truvov et al. [28], "typically initiated by a small group of founders who send out invitations (...) to the members of their own personal networks". This process relies on rewarding those who bring non-users and turn them into customers. As standard practice in several real-life program examples [21, 23], we assume a fixed reward R to every individual that invite someone into the company. In some fashion, it resembles a Susceptible-Infected-Recovered model (SIR model) in a well-mixed Population [2, 16, 19], where a non-user is susceptible to be converted, an infected one it is when the agent is an active user and finally the state of recover whenever there is an exit of the program. Important to note, that whenever we refer to the term "invites", we are only considered those who are effective, meaning, invites that convert officially a non-user into a client (susceptible to infected). Since there is a constraint on the number of people an individual knows to convert into the customer, there is a maximum number of invites (I) a person can make, where $\sum_{i=1}^N Invites \leq \bar{I}$. Such constraint can also be seen as an agent's degree in the network, meaning, an agent can only invite other individuals that he or she is connected to. Therefore, in a complex network environment, the maximum amount of invites a node can make is equal to its degree ($Invites_i = \kappa$).

Actions and Profiles. Within such a model, individual actions are grouped into two distinct profiles: Loyal and Opportunistic. Whenever an individual is loyal to a given brand (or product), her decision to consume (C) is set to be independent of any Marketing Referral Program. Thus, she is going to consume with a probability of ρ and

invite a friend to consume with $1 - \rho$. On the other hand, Opportunistic players will only consume if the amount collected through companies' reward system is higher or equal to the cost of consuming the product. Therefore, while it is lower than the stated value, its action is set to invite others (non-users), until it reaches the amount needed (when $\sum_{i=1}^N R > C$). If an individual with the stated profile finds himself in the situation of not having nobody to invite (to accumulate the necessary rewards to consume), he will exit the referral program. Moreover, product or brand loyalty is not constant over time; there is a likelihood of changing towards other and similar products. The probability of exiting the client base, ε , defines whether an individual continues to consume (or invite), or decides to exit the program.

Population Dynamics. Based on computer simulation, evolution proceeds in the discrete space (steps). As stated earlier, each runs starts with a random distribution of Loyal and Opportunistic agents, chosen using a uniform probability distribution (UPD). Each simulation is executed for a large number g of generations. We do not consider new entries without being invited by someone. Only the first N individuals were able to enter the simulation without any invitation.

4 Results and Discussion

To address our baseline challenge, which is to shed light on the influence of rewards on the number of users, and how opportunistic agents change the company's profit structure, we perform one set of 2×10^4 runs. Within those simulations, we measure what would change when the incentives to invite are higher and whether the number of invitations per person was impacting the dissemination of the service. For simplicity, each run starts with just one individual of each profile (loyal and opportunistic). In the next two subsections, we begin by assuming a complete graph or network, where every agent can interact with all others in the population. This assumption is then dropped by considering a structured community. We study the same model, where this time, agents are constrained to interact solely with their acquaintances (their 1st neighbours). We adopt an interaction network defined by a scale-free network, which built through the algorithm of growth and preferential attachment proposed by Barabasi and Albert [3]. By doing so, we can identify the role played by a structured population with a heterogeneous degree distribution in the overall dynamics.

4.1 Social Propagation and Population Dynamics

Let us assume that each time an user invites someone into the service, receives a credit or reward of 5 monetary units (u.m). In Figure 1, we can see the difference in the number of active users across time when the cost of using the app increases, meaning, for a fixed reward, we measure the impact of different cost structures. Additionally, It is also displayed (Figure 1) the importance of a maximum number of invites each users can use throughout his experience in the service/app.

Firstly, we can show the impact of a higher service cost. As it increases the difficulty on obtaining a free-ride usage of the service, by the opportunistic population, the

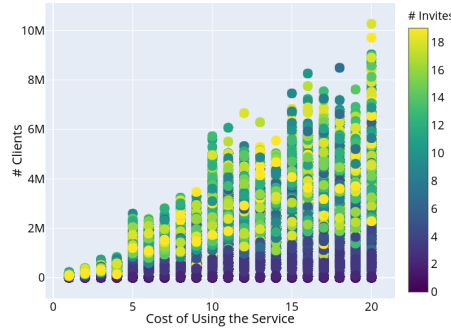


Fig. 1: Number of Service Users and the Cost of consuming each of its Unit (Color Bar as the number of allowed invites)

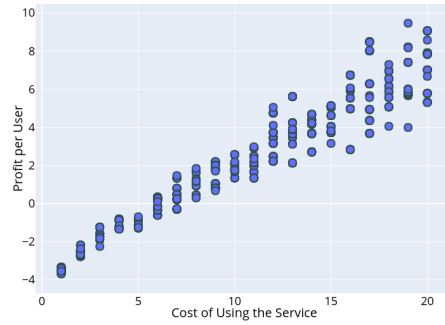


Fig. 2: Profit per user as function of service cost structure (whereas the reward is set to 5 monetary units)

number of active users increase. If it is quite straightforward that, all else equal, as a firm charges more his clients for his products or services, firm's profits are higher (as reported in Figure 2); yet, it says virtually nothing relative to active users. Those are greater in number just by leveraging the actions the opportunistic behavior promotes: if it is not possible to acquire the service with the current collected rewards, then the agent needs to invite more people into the company, earning afterwards its reward. Since the loyal population is independent on the ratio between the cost of the service and the reward given per invite, this part of the population is, therefore, not responsible for this phenomena. However, if the reward is large enough to avoid sending an invite, an opportunistic player will delay its decision of disseminate the service while its credits are not yet finished.

Secondly, we consider interesting to mention the impact of imposing a limit in the number of invites each active user can send to a new user. As captured in Figure 1, if the firm imposes limitations on the number of invites, this will lead to a poor performance in the final number of active users. Still, this effect is amplified whenever the difference between the reward and the cost of using the service is highest. One possible explanation may come from acknowledging that opportunistic players will only consume whenever they have enough accumulated reward to pay the service. If the ratio cost/reward is higher, consuming can only be achieved by a relatively higher number of invitations. By limiting it, opportunistic agents would eventually die out and stop using/consuming the service (as well as inviting others). Therefore, it is possible to conclude that the number of allowed invitation is important, gaining additional relevance as the cost of using the service gets higher.

Thirdly, it is also possible to measure, within the current environment and holding the assumption of a fixed reward of 5 monetary units (u.m.), the break-even point of the firm. At Figure 2, independently of the number of allowed invites per user, any cost structure below 7 ($ratio = \frac{7}{5} = 1.4$) makes the profit per user below zero. This means that this marketing campaign would not generate any value to the firm. However, for ratios above that threshold would make the present marketing referral campaign

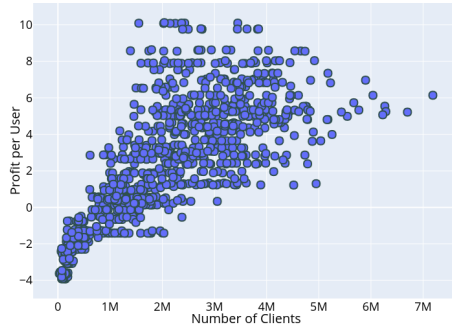


Fig. 3: Profit per user as function of service active users (whereas the reward is set to 5 monetary units and the cost ranges from 1 to 20)

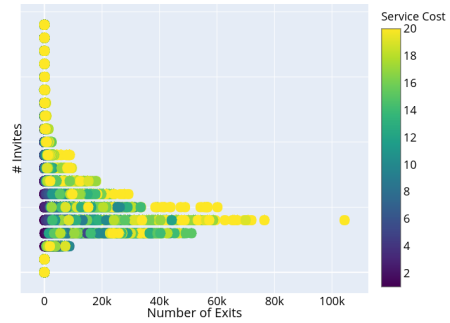


Fig. 4: Number of Users that exited the firm and the Cost of consuming each of its Unit (Color Bar is associated with the cost of using the service)

responsible not only to bring more users into the firm, but also enhance its the profits (as reported in Figure 2).

4.2 The Influence of Service Cost Structure

Having said that a higher cost structure would promote the action of inviting outsiders into the service (from opportunistic agents), it also enhance the number of exits, whenever there are nobody to invite and no credits to use. Additionally, we need also to discuss how it scales over time: *Does this marketing campaign improves as its grows? Does it have increasing returns to scale?*

In Figure 3 we can argue that these type of Marketing Referral campaigns and programs are, for most of tested settings, profitable as they grow in active users. So, we may conclude the high likelihood of being in the presence of increasing returns to scale, meaning, as more people are entering in the service, the profit per user increases (regardless of the needed reward payment for those who have brought these new users). Indeed, even in a simplistic approach like the one here presented, any rational agent would be more prone to stop using a service that it is quite expensive. On one side, as the cost increases, all else equal, opportunistic players need more invites to support the same level of consumption. If it is not possible, they will opt out of the service. On the other side, the population that it is already connected to the brand (loyal agents) is more reluctant to stop consuming a service, being therefore almost an independent and exogenous consumption to the model. Most of the discussed can be found on Figure 4. At a first glance, we can confirm the previously explained phenomena of a lower level of invitation brings fewer people into the service, leading unquestionably to lower levels of exits (fewer people to leave). However, as the number of invites grows and the firm acquires more active users, the number of people opting out the service also increases, with a more prominent value whenever the cost of consumption is higher.

Nonetheless, it is quite remarkable the fact that as the number of invites grows *ad infinitum*, the reasons to stop consuming the service and opt out the firm are completely

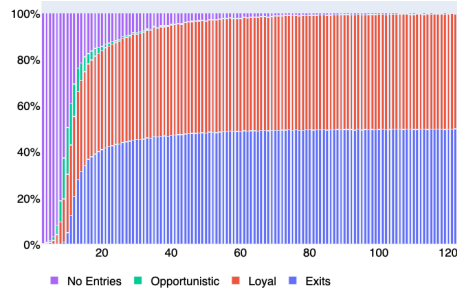


Fig. 5: Rate of Spreading over Time (**Susceptible**: No Entries; **Infected**: Opportunistic, Loyal; **Recovered**: Exits)

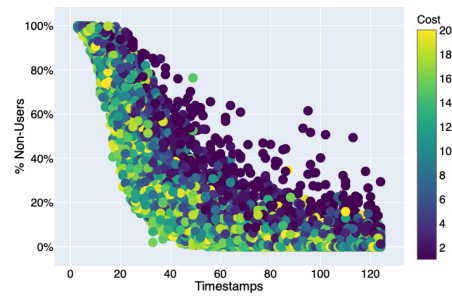


Fig. 6: Rate of Non-Users over time (Color Bar is associated with the cost of using the service)

exogenous. If an agent i is loyal to the brand, the agent is, by definition, independent of the number of allowed invites and to the cost of using the service. On the contrary, if the agent has an opportunistic view and act based upon the collected credits, he is dependent on both cost and invitations permitted. Yet, if this player is capable of inviting an infinitude number of non-users, he would be able to pay the service regardless of the prices asked, just by collecting the rewards of bring new people in.

4.3 Network Impact on Propagation and Profitability

Until this point, we assumed a complete graph where every individual can interact to any other. Nonetheless, real social networks are often sparse and show a strong degree heterogeneity [1, 3] (few hubs with multiple connections and most nodes with fewer links). In this section, we test what would change by adopting a Barabasi-Albert scale-free network with 10^3 nodes and an average degree z of 2, 6, 10 and 20^5 .

On one hand, the first striking evidence is the close relation to previous section results. As displayed in Figure 5, as time passes by this Marketing referral program propagates into the entire network, with just a single active user or client (infected node) in the beginning. After only 50 timestamps, most nodes are either active users (infected) or former users (recovered). This may lead us to infer that despite network characteristics such marketing strategy is a winner at spreading.

Additionally, with such model, the Opportunistic population tends to die out quickly as they run out of people (nodes) to invite. Such effect is even larger when node degree is lower. Yet, disappearance of Opportunistic individuals is a feature that it is still quite elusively studied. As presented earlier, every firm is fighting for gaining loyal customers. If by using such Marketing technique, and relying on opportunist players to reach out loyal agents that are not yet connected to the brand, firms increase loyal population. And, as time goes by, they get rid of those only interested in rewards, we may argue it is a win-win strategy.

⁵A Network with a graph of n nodes that is grown by attaching new nodes each with $m = z/2$ links that are preferentially attached to existing ones with high degree [3]

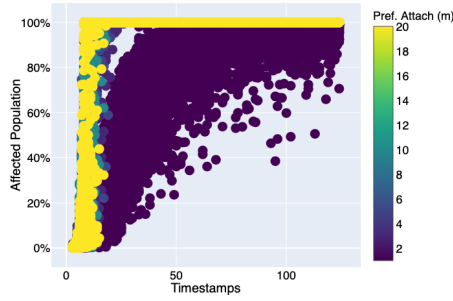


Fig. 7: Affected Population (active or exits) in % over Time (Color bar represents Network Preferential Attachment)

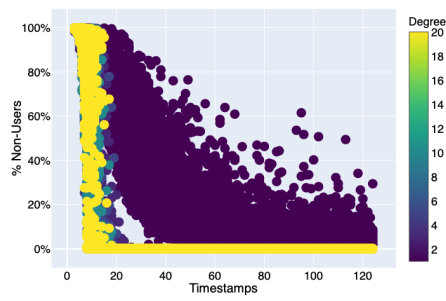


Fig. 8: Non-active or susceptible population in % of total population over time (Color bar as first node degree)

On the other hand, we do confirm the positive impact on increasing the cost of using the service. In Figure 6 we show that the percentage of non-users, (i.e., the susceptible population) diminish at a quicker pace as cost values are increasing (for all the scale-free networks tested, and assuming a constant reward of 5 u.m.). As long as the Opportunistic population has collected enough rewards, it does not disseminates further invitations. Thus, the referral program suffers a delay with a lower rate of spreading. From these insights, further questions were raised, namely the first user degree impact on propagation, as well as, network preferential attachment, meaning how important the initial node is in the network and how many connections each node has.

In order to tackle the stated questions, we conducted additional analysis on the impact of inner network characteristics, plotted in Figure 7 and Figure 8. In the first one, different average degrees z were tested to gain sensibility on how a more connected network could impact on the dissemination of the Marketing program. As seen in Figure 7, when z is larger the amount of non-affected population quickly diminishes. As expected, our results suggest that sparser graphs delivers, all else equal, a lower rate of program dissemination. Once more, despite population profile, if an individual knows less people, the amount of invites we can share are, by definition, lower in number.

Perhaps more important than the structure of the network, is the initial conditions of the first user. On the last chart (Figure 8), it is possible to acknowledge the relevance of the degree of the first user. As we start with a node with a high degree (i.e., a hub or an *influencer*), we see an faster dissemination rate with a consistent fall of the susceptible population in a few rounds or timestamps. This key aspect give strength, in our view, to those who promote their brands with well-known and connected people, in order to boost brand recognition and adoption.

5 Conclusions and Final Remarks

In the present work, we propose a new approach to strength the connection between agent-based models embodied in social networks and the Marketing field. Despite the nature and structure of population network, we show a close relation between population dynamics and Marketing Referral Programs performance. Opportunistic players'

behavior helps, under most conditions, to propagate the brand into a population that it is still not actively using the service.

Within such environment, we help to demonstrate that as the gap between the collected reward and the cost of using the service strengthens, Marketing referral programs gain more from this population. Initial node characteristics may boost the stated, with hubs (higher degree) and well-connected networks having, naturally, the best spreading rates. Yet, even in a sparse graph with a poorly connected node as the "first-infected", Marketing Referral Program tend to prosper consistently over time. Nonetheless, further studies must be pursued for a better understanding of the Marketing Corporate World. Here we propose some insights of what we think it is relevant to pursue in future work.

Different Network Structures. We have shown the importance of node characteristics, such as the degree, or network features (preferential attachment and network sparsity) in such Marketing campaigns. Despite so, multiple other networks were not yet tested. Further studies must be pursued to measure the impact of different network structures and its time-varying changes. In a context of a social media network, it is important to add the co-evolution over time of the network, whose nodes and links can be replaced, added or deleted.

Competitors best-response. In the present model, we are assuming no competition in the market, meaning, there is no more firms offering the same type of services. Yet, example like digital personal transport services (Uber, Bolt, Kapten, etc.) or portable electric scooter (Lime, Hive, Bird, among others) show that mimic the service is quite easy and spreading the brand can be done using similar Marketing strategy. Though, some conclusions done previously can be put in question where a simple increase in price not only boost the opportunistic behavior of spreading invites to non-users, but also promotes more exits into similar firms and/or services. Therefore, jeopardizing in some sort of fashion the effect wanted of boosting active users.

Reputation and Conversion. Another factor not taking into account was the public acknowledgement of being an opportunistic player. As deeply studied by several authors [18, 24, 25], reputation has a central role in human-human interactions. Despite being, at an individual level, profitable and, most probably, the best action to be made (in this setting), we should not assume perfect rationality. Additionally, we do not consider the hypothesis of the correlation between who is making the invitation and the new-user, meaning, empirically we may argue if loyal users tend to attract, all else equal, more loyal new users relative to the opportunistic players. Thirdly, by imposing a fixed percentage of the population as loyal/opportunist we are not allowing players to inter-change their behaviour throughout the simulation which can naturally happen.

Effectiveness Cross-Comparison. As discussed earlier, these type of Marketing programs are self-sustainable and can be, under specific and controlled settings, profitable to the company. However, the present study lack of real-life confirmation and a greater cross-comparison with several other Marketing techniques to test if it is, under no reasonable doubt, consistently better. In the Era of Social Media and influencers, we are

seeing more often than not brands using well-known and connected people to self-advertise their products and services. Including such examples on the presented model would empower the analysis, specifically to confirm whose nodes in the network are best to help a Marketing referral Program to succeed (higher degree, centrality or any other network measure).

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